Lie Algebras from Octonion Algebraic Orientation Covariant Adjacent Algebras

Author: Richard D. Lockyer

Email: rick@octospace.com

Abstract

Attempts to apply Octonion Algebra to the Standard Model have involved creating a matrix algebra representing Octonion Algebra operating on itself, an adjacent algebra. Each used only one of sixteen Octonion division algebra orientations, and favored left application. This manuscript first demonstrates how proper use of Octonion Algebra for mathematical physics *must* give consideration to all sixteen in an algebraic orientation covariant fashion if one is to avoid pathological outcomes when the same analysis is repeated using another orientation.

Properly constructed 8x8 adjacent algebra matrices generate an algebraic orientation covariant left/right application inclusive group also closed for all Octonion algebraic orientation changes. Orientation closed pairs of conjugacy classes lead to sixteen class equivalence partitions. Eight bracket/orientation closed Lie algebras are created from them, which demonstrate the orientation of each 8x8 adjacent algebra matrix position is fixed. Seven Lie algebras have four ideals in 4x4 submatrices that pair up to occupied rows/columns directly related to the seven Quaternion subalgebra indexes or their basic quad index set. From these, fourteen unique skew-Hermitian Dirac gamma matrix bases for su(4) are created, and within each six skew-Hermitian Pauli matrix su(2) bases. 28 unique su(2) bases span the 8x8 matrix format with a bracket/orientation closed Lie algebra. Skew-Hermitian bases for adjacent algebra su(n: 2-8) Lie algebras can be placed in any nxn submatrix position using linear combinations of su(2) Lie algebras. Two distinct representations of su(2) ⊕ su(3) are presented that combine into the direct sum su(2) ⊕ su(3) ⊕ su(3).

***

The need for Octonion algebraic orientation covariant representations

It is fairly well understood that there are 480 different "multiplication tables" for Octonion Algebra. These come about from 30 different ways to partition the seven non-scalar Octonion basis elements into the seven Quaternion subalgebras, and most importantly 16 out of $2^7 = 128$ ways to orient these subalgebras such that they form a division algebra. Most mathematicians/physicists desiring to use Octonion Algebra naively pick one of the 480 ways and ignore the others. I fully agree any one of the 30 Quaternion partitions may be freely and singularly chosen, but one way is orders of magnitude more illuminating, which is clearly shown below. However, allow me to first disabuse anyone of the notion using only one of the 16 division algebra orientations is a free choice with a simple example.

For now, take it on faith the results below in each of the 16 orientations are correct as presented. A full definition of these and a map for changing final results to what they would be if a different Octonion orientation were used will follow shortly.

Define two Octonion algebraic elements **A** and **B** as

$\mathbf{A} = \sin(\theta)\, e_2 + \cos(\theta)\, e_3$
$\mathbf{B} = \cos(\theta)\, e_6 + \sin(\theta)\, e_7$

Their product is

$\mathbf{A}*\mathbf{B} = \sin(\theta)\cos(\theta)\ \{e_2*e_6 + e_3*e_7\} + \sin^2(\theta)\ e_2*e_7 + \cos^2(\theta)\ e_3*e_6$

Using the Octonion Algebra outlined below, its 16 different division algebra orientation choices yield

$e_2*e_6 = \pm e_4 \quad e_3*e_7 = \pm e_4 \quad e_2*e_7 = \pm e_5 \quad e_3*e_6 = \pm e_5$

Doing the product **A*B** in each of the 16 orientations, we can generally represent these results as

$a\ e_4 + b\ e_5$ where a can be $\{\pm \sin(2\theta), 0\}$ and b can be $\{\pm 1, \pm \cos(2\theta)\}$

Which is the correct result? We must never lose, yet do lose parametric functionality with some choices of algebraic orientation but not others. We cannot exclusively choose one orientation over another without some unjustifiable by-hand, single anointment selection. These disparate outcomes must be taken as *seriously pathological*, thus needing a remedy.

The answer to our question must be that none of representations for a and b are correct results. The signages on the four product terms are governed by four different Quaternion subalgebra algebraic orientations. The correct answer for **A*B** mandates the use of algebraic orientation indicators with ± signage set by a free choice of one of the 16 proper Octonion Algebra orientations taken as the reference orientation, and results generated by exclusively using it. My choice has always been what I have called **R0** (defined below). Augmenting with appropriate algebraic orientation indicators s_3, s_7, s_{11}, and s_{15} that also will be generally defined subsequently, the proper result representation is

$\mathbf{A}*\mathbf{B} = \{\sin(\theta)\cos(\theta)\ s_3 + \sin(\theta)\cos(\theta)\ s_7\}e_4 + \{-\sin^2(\theta)\ s_{11} + \cos^2(\theta)\ s_{15}\}e_5$

To properly use Octonion Algebra, we must augment the fundamental algebraic element addition rule to state two coefficients scaling the same basis element cannot be combined in any fashion unless they have the same algebraic orientation indicator. Beyond avoiding pathological situations like above, this has the advantage of allowing us to do full calculations using only one reference orientation Octonion Algebra. This will naturally partition results into sets scaling both the same basis element and same algebraic orientation indicator where *all set members* will either change sign or not when the Octonion algebraic orientation is changed up. Moreover, this result can be simply mapped to what it would have been had we used any of the other fifteen proper Octonion orientations without a total redo of the process in another orientation.

I cannot overstress the following bottom line:

When utilizing Octonion Algebra in mathematical physics, the free choice between definable algebraic orientation options is only assured to create consistent results between selections if orientation options are intrinsically managed in what should be called an algebraic orientation covariant fashion, such as with the algebraic orientation indicator set defined below. The use of these algebraic orientation indicators or some other isomorphic set is not optional if pathological outcomes are to be avoided.

When we attempt to do physics with Octonion Algebra, we are looking for mathematical descriptions of observables that must not change descriptive sign should we move between free choices of applied algebraic orientation. The Octonion intrinsic basis element e_0 is identical to +1 so its product with any other intrinsic basis element as well as any product $e_x * e_x$ is fixed form for all orientation choices, so we will always have one algebraic orientation indicator enumerated here as $s_0 = +1$, which tells us all product terms thus scaled will not change sign for any Octonion algebraic orientation change, defining what can be called an algebraic orientation invariant, and therefore a candidate for an observable.

We might also have theoretical hints that something hidden from direct experimental detection is lurking. These could be found in sets of not algebraic orientation invariant expressions where each term in the set is scaled by the same Octonion basis element and same algebraic orientation indicator. These sets could be forced to algebraic orientation invariant status by simply assigning a zero value to the sum of all set members. I have called these *homogeneous equations of algebraic orientation constraint*. Thus, any Octonion algebraic expression cast in an algebraic orientation covariant fashion may be forced to fully algebraic invariant status. Of course, the homogeneous equations must be theoretically consistent, either a priori or through enlightenment.

The case for applying Octonion Algebra to The Standard Model

Some researchers claim since the Standard Model was not initially created with Octonion Algebra, and numerous ways to duplicate features of its group structures do not require Octonion Algebra, this algebra is non-essential to understanding the model. The Octonion methods tend to use Clifford algebras, which lean on fully anticommutative sets easily and too numerously created from the anticommutative Octonion non-scalar basis elements. The selection of a suitable set amongst many suitable alternatives becomes arbitrary, giving less credibility to the use of Octonion algebra.

On the other hand, numerous researchers (notably my first introduction three decades ago by Geoffry Dixon: reference [11], as well as Nichol Furey, Mia Hughes references [13] through [21]) as well as others, have made connections between the model and a matrix algebra created by Octonion Algebra acting on itself sometimes called an adjoint algebra, sometimes a multiplication algebra. The descriptor adjoint is overworked in mathematics, and multiplication algebra seems redundant. I prefer to call these *adjacent algebras* instead, implying a direct relationship to something different. The rub with all of these approaches is they are not done in an algebraic orientation covariant fashion. I will be reworking this approach below in its proper algebraic orientation covariant form.

It might be reassuring to further justify the use of Octonion Algebra to at least explain the group structure of The Standard Model if not derive it or expand it. One limitation of the model is its inability to cover gravitation. The biggest obstacle to this is the assumption that gravitation is described by intrinsic curvature of 4D space-time suggested by Einstein. Without a known to me historical accounting, I surmise Einstein properly felt it was quite impossible to fit within a four-dimensional representation the two classical central forces: charge-electric field and mass-gravitational field using a scalar-vector potential function theory, since the nature of the two forces are quite disparate but their calculus would not be. There is nothing of an electro-magnetic induction nature involving mass. He thus had two options, increase the fundamental dimension of the math past four, or look elsewhere for gravitation. He chose the latter, perhaps as a path of least resistance with some mathematical physicists of the time barking loudly over putting time on an equal footing as the 3D space of our perception.

I will submit a better approach is to start off integrating electrodynamics and gravitation, not quantum mechanics and general relativity, by increasing the dimensionality of the coordinate system and finding an appropriate associated algebra, staying with potential functions that cleanly cover electrodynamics *and* gravitation, at least at the macro scale. We take our first clue how this might be done from classical electrodynamics, where we have two disparate types of 3D vectors, axial (magnetic field) and polar (electric field). Their disparate nature should lead us to assume we must at least double up on the spatial coordinates to individually cover both, and we should explore the vector products between and within each to determine the proper algebraic structure required to cover both simultaneously.

We find axial-axial vector products will result in axial vectors, and they are by nature oriented. We should easily see that the magnetic field lives in some Quaternion Algebra created by the curl product rule on two polar types. The vector product of two polar vectors is an axial vector, and the vector product of polar and axial is polar. The obvious conclusion here is the electric field cannot live in a Quaternion Algebra. It can however algebraically live in three of the four Octonion *basic quad* dimensions of the Quaternion subalgebra the magnetic field lives in, where basic quad := the four of eight intrinsic Octonion basis elements not within a given Quaternion subalgebra set.

We can set aside a free choice of one of the seven intrinsic Octonion non-scalar basis elements to be paired with the scalar basis element, defining it as non-spatial complex, albeit with an anti-commutating imaginary. The remaining six basis elements double cover our 3D perception space, with a free choice of one of four Quaternion subalgebras not including the non-spatial non-scalar basis selected for the home of the magnetic field, or more generally its three non-scalar bases the basis set for axial vectors. Once selected, the basic quad is known, and the non-Quaternion 3D spatial dimension basis for polar type vector assigned to the electric field is set.

The 3D xyz basis pairings for the 3D perception axial and polar bases are set by the three Quaternion subalgebras the chosen non-spatial non-scalar basis element resides in. Each will additionally have one axial and one polar basis element for the same free choice of x, y or z physical direction.

We would then have a scalar potential function, and triplets of polar and axial vector potential functions, with differential calculus properly defined for the Octonion framework. The electric field would be found in typical fashion as the sum of scalar differentiation on polar vector potentials and polar differentiation on the scalar potential. The magnetic field in typical fashion would be the rotational vector curl of polar differentiation on polar vector potentials.

Gravitation would of course live in the same Quaternion subalgebra the magnetic field lives in. The gravitational field would be the sum of scalar differentiation on the axial potential functions and axial differentiation on the scalar potential, thus creating a central force algebraically distinct from the charge central force as it must be.

The calculus would produce additional rotational curls from mixing axial/polar differentiations on polar/axial potential functions, as well as incorporating the curls involving the non-spatial non-scalar basis. I have generally categorized these as *glue*.

All of this and more is covered in reference [6], where 4D electrodynamics is covered in harmony with algebraically distinct gravitation without the need to bring in any additional field and stress-energy-momentum tensor structures. These are instead intrinsically and beautifully covered by the structure of Octonion algebra.

Since Octonion Algebra has the dimensionality and algebraic structure to integrate electrodynamics and gravitation within, it would be foolish not to lean on it to augment The Standard Model in order to cover both.

Right and Left proper Octonion orientations and algebraic orientation indicator definitions

I have done a reasonable effort of describing algebraic variance indicators within the referenced document section below. Rather than sending you off on a search of my position by reading all, let me (somewhat) briefly describe the program.

A superior representation of what has been called Cayley-Dickson algebras, basically hypercomplex algebras of dimension 2^n for integer n, is what I have called the family $\mathscr{H}_m$ where m = 2^n. They are fully discussed in reference [9]. These representations recognize the range of basis index enumerations fully span a binary number of n bits, and that the standard structure constant basis element product rule $e_a * e_b = s_{abc}\, e_c$ may be written without loss of generality most simply as $e_a * e_b = \pm e_{a\wedge b}$ where ^ is the binary bit-wise Boolean logic exclusive-or operator (xor: typical computer software operator ^). This basis element product definition is independent of the choice of indexes a and b, but the ± sign choice is dependent on a and b. If a=b, a=0 or b=0, the sign has a singular choice i.e., a non-oriented product, and if $a \neq b \neq 0$ the sign choice is set by the orientation choice for the Quaternion subalgebra formed with basis set $\{e_0, e_a, e_b, e_c = e_{a\wedge b}\}$, for *any* hypercomplex algebra of dimension 2^n, $n \geq 2$.

We also find $a \wedge b \wedge c = a \wedge b \wedge a \wedge b = 0$ for any non-zero and different a, b and c of any Quaternion (sub)algebra, which tells us we can find all Quaternion subalgebras simply by determining the full complement of three unlike and non-zero n-bit binary numbers in the range 0 to $2^n - 1$ that xor to zero. The orientation choice for one Quaternion subalgebra does not confront the orientation choice of any other, so the full complement of orientation choices for any 2^n dimensional hypercomplex algebra comes down to a free choice of orientations for each of its Quaternion subalgebras. This xor to zero rule is recognizable as one of the 30 ways to partition the seven non-scalar basis elements into sets of three, each representing Quaternion subalgebras of the Octonions. I see no legitimate reason to use any of the other 29, this one is orders of magnitude more illuminating, and will be used throughout this manuscript.

This counters the obtuse Cayley-Dickson doubling product structure which is only capable of producing one doubled dimension orientation from a single source orientation. Beyond this, each hypercomplex algebra dimension actually stands on its own, with connection to lower dimension hypercomplex algebras as subordinate subalgebras and not relying on them for its existence. Perhaps Cayley-Dickson doubling should be preserved only for historical purposes. $\mathscr{H}_m$ is a superior parsimonious representation for 2^n dimension hypercomplex algebras.

We find Octonion Algebra lives within the family $\mathscr{H}_8$ which has seven freely oriented Quaternion subalgebras, giving us $2^7 = 128$ choices. However, only 16 of these form division algebras, and thus proper Octonion. The other 112 orientations each exhibit 24 primitive zero divisors, and such oriented 8-dimensional hypercomplex algebras are all seen to be one of seven Quaternion subalgebra orientations off of a proper Octonion division algebra. I like the name Broctonion Algebras (BRoken OCTONION) for these, very much. This is covered in reference [3].

A lot of words so far to simply conclude the algebraic orientation indicators we seek are singularly connected to individual Quaternion subalgebras.

Let us now freely pick one of 16 proper Octonion algebraic orientations as our default reference Octonion Algebra algebraic orientation. I call this Right Octonion choice **R0**. Enumerate the seven Quaternion subalgebras with the xor to 0 choice, and then define **R0** algebraic orientations by indicating structure constant index order for the seven Quaternion subalgebras where each = +1, such that the result is a division algebra. For all Octonion orientations we will implement, the following optimal n valued enumerations for each Quaternion subalgebra are also shown.

$(n{=}1){:}s_{642} = (n{=}2){:}s_{541} = (n{=}3){:}s_{743} = (n{=}4){:}s_{123} = (n{=}5){:}s_{572} = (n{=}6){:}s_{761} = (n{=}7){:}s_{653} = +1$

These are equivalently represented by ordered permutation product rules sequentially as

$(e_6, e_4, e_2), (e_5, e_4, e_1), (e_7, e_4, e_3), (e_1, e_2, e_3), (e_5, e_7, e_2), (e_7, e_6, e_1), (e_6, e_5, e_3)$

They are called permutation rules because the three even permutations of a given ordered set yield the product rules defined as first * second = + the third, whereas the odd permutations of the given order indicate first * second = – the third, covering all six product combinations.

It is interesting to note for any n value above selecting a particular triplet {a, b, c}, the enumerated triplets this index n appears in are triplet indexes a, b and c. For example pick index 2 selecting $(n{=}2){:}s_{541}$, e_2 is found in the structure constants indexed n = 5, 4 and 1 as $(n{=}5){:}s_{572}$, $(n{=}4){:}s_{123}$, and $(n{=}1){:}s_{642}$ all include 2. We will also see below that this enumeration will permit us to see structure in sets with different indicators utilizing the xor of their indexes, whereas any other Quaternion subalgebra enumeration of seven would obscure but not remove the intrinsic structure.

Octonion Algebra splits the 16 proper Octonion orientations into two chiral partitions of 8 members each, which I call *Right Octonions **R0** through **R**7*, and *Left Octonions **L0** through **L**7*. The maps **Rn** ↔ **Ln** change the orientations of all seven Quaternion subalgebras. The maps **R0** → **Rn** and **L0** → **Ln** require **R0** (**L0**) orientation negation of only the four Quaternion subalgebras that exclude basis element e_n. These two and their compositions are the *only* Quaternion orientation changes that maintain division algebra status. With specification of **R0** above, all 16 Octonion orientations are now defined.

Ordered permutation product rules are used to characterize what is a Right Octonion form vs. a Left Octonion form. Gather the three permutation product rules that include any one of the non-scalar basis elements. Cyclically shift (no rule change) each to place the common basis element centrally. For *any* **Rn** orientation and *any* common basis element, the three basis elements on the right side of the three product rules will be another Quaternion set, and those on the left side will not be. Conversely, all **Ln** have the Quaternion set on the left side and not the right. *This is an important clean, consistent structural difference between Right and Left Octonion algebraic orientations*.

When we build Octonion algebraic elements with multiple products, each final result product term has an ordered multi-product history. Each of the oriented products within this product history will change sign under the maps **Rn** ↔ **Ln**. If the total number of oriented products is odd, maps **Rn** ↔ **Ln** will negate the sign of the final product term, and if the total number is even, maps **Rn** ↔ **Ln** will not change the sign. To cover this, we must include in our algebraic orientation indicators a parity bit representing an odd or even number of oriented products in the product history of each final result product term.

Two different non-scalar basis elements uniquely select a particular Quaternion subalgebra. Their product resultant basis element and sign are fully described by the orientation of this Quaternion subalgebra. We can cover both parity and relevant n indexed Quaternion subalgebra enumeration as above by defining our algebraic orientation indicators as s_x where index $x := 2n + \text{parity}$ (parity 0 even, 1 odd), ranging 0 to 15. Index $x = 0$ refers to the sole non-oriented, algebraic invariant result. Essentially, we have $s_0 = +1$, so omitting it simplifies things. From this we can augment the **R0** intrinsic basis element product table as the following with all indicator indexes odd since the table represents a single product, oriented or not.

R0	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_0	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	e_1	$-e_0$	$s_9\, e_3$	$-s_9\, e_2$	$-s_5\, e_5$	$s_5\, e_4$	$-s_{13}\, e_7$	$s_{13}\, e_6$
e_2	e_2	$-s_9\, e_3$	$-e_0$	$s_9\, e_1$	$-s_3\, e_6$	$s_{11}\, e_7$	$s_3\, e_4$	$-s_{11}\, e_5$
e_3	e_3	$s_9\, e_2$	$-s_9\, e_1$	$-e_0$	$-s_7\, e_7$	$-s_{15}\, e_6$	$s_{15}\, e_5$	$s_7\, e_4$
e_4	e_4	$s_5\, e_5$	$s_3\, e_6$	$s_7\, e_7$	$-e_0$	$-s_5\, e_1$	$-s_3\, e_2$	$-s_7\, e_3$
e_5	e_5	$-s_5\, e_4$	$-s_{11}\, e_7$	$s_{15}\, e_6$	$s_5\, e_1$	$-e_0$	$-s_{15}\, e_3$	$s_{11}\, e_2$
e_6	e_6	$s_{13}\, e_7$	$-s_3\, e_4$	$-s_{15}\, e_5$	$s_3\, e_2$	$s_{15}\, e_3$	$-e_0$	$-s_{13}\, e_1$
e_7	e_7	$-s_{13}\, e_6$	$s_{11}\, e_5$	$-s_7\, e_4$	$s_7\, e_3$	$-s_{11}\, e_2$	$s_{13}\, e_1$	$-e_0$

This table gives us the *algebraic orientation covariant* := (form invariance across all orientations) product of two naked (intrinsic) basis elements. This covariance means we should be able to devise a sign change - no change map for each s_x that will take this **R0** orientation result to the equivalent result had we used any other Octonion orientation. This map will be presented in the next section.

Smap: Mapping results from one Octonion orientation to equivalent results using another orientation

When forming meaningful Octonion algebraic expressions, we must of course pick a single orientation and exclusively use its product rules. When this process also includes algebraic orientation indicators on oriented product terms, we have everything needed to map any result created with the original Octonion orientation to what we would have achieved if a different Octonion orientation was used without repeating the process from the beginning using the alternate Octonion orientation.

The new result will have the same algebraic orientation indicator on any given product term, but its ± scale sign may or may not change sign. How the scale sign changes must be precisely how the associated algebraic orientation indicator changes. Staying with our default reference orientation **R0**, all algebraic orientation changes are shown relative to it in the following table, so column labeled **R0** is all +1. Also, s_0 is an algebraic invariant +1 as indicated in the first row.

Smap	**R0**	**R1**	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**	**L0**	**L1**	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**
s_0	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
s_2	+1	–1	+1	–1	+1	–1	+1	–1	+1	–1	+1	–1	+1	–1	+1	–1
s_4	+1	+1	–1	–1	+1	+1	–1	–1	+1	+1	–1	–1	+1	+1	–1	–1
s_6	+1	–1	–1	+1	+1	–1	–1	+1	+1	–1	–1	+1	+1	–1	–1	+1
s_8	+1	+1	+1	+1	–1	–1	–1	–1	+1	+1	+1	+1	–1	–1	–1	–1
s_{10}	+1	–1	+1	–1	–1	+1	–1	+1	+1	–1	+1	–1	–1	+1	–1	+1
s_{12}	+1	+1	–1	–1	–1	–1	+1	+1	+1	+1	–1	–1	–1	–1	+1	+1
s_{14}	+1	–1	–1	+1	–1	+1	+1	–1	+1	–1	–1	+1	–1	+1	+1	–1
s_1	+1	+1	+1	+1	+1	+1	+1	+1	–1	–1	–1	–1	–1	–1	–1	–1
s_3	+1	–1	+1	–1	+1	–1	+1	–1	–1	+1	–1	+1	–1	+1	–1	+1
s_5	+1	+1	–1	–1	+1	+1	–1	–1	–1	–1	+1	+1	–1	–1	+1	+1
s_7	+1	–1	–1	+1	+1	–1	–1	+1	–1	+1	+1	–1	–1	+1	+1	–1
s_9	+1	+1	+1	+1	–1	–1	–1	–1	–1	–1	–1	–1	+1	+1	+1	+1
s_{11}	+1	–1	+1	–1	–1	+1	–1	+1	–1	+1	–1	+1	+1	–1	+1	–1
s_{13}	+1	+1	–1	–1	–1	–1	+1	+1	–1	–1	+1	+1	+1	+1	–1	–1
s_{15}	+1	–1	–1	+1	–1	+1	+1	–1	–1	+1	+1	–1	+1	–1	–1	+1

The genesis of this table comes from the above-mentioned Quaternion subalgebra orientation maps that preserve division algebra status. The algebraic orientation indicator index is twice the Quaternion index n + parity, so the first 8 rows are invariant followed by indicators for Quaternion subalgebras 1 through 7 with even parity, meaning the map **R0** → **Rn** requires the negation of all Quaternion subalgebras excluding basis element e_n. This is precisely where all –1 table entries are in the first 8 rows and columns. Even parity states there is no orientation change moving from **Rn** → **Ln** and the maps **L0** → **Ln** duplicate maps **R0** → **Rn** so the top right 8x8 section duplicates the top left. The maps **R0** → **Rn** are agnostic to the parity bit, so the odd parity bottom 8 rows and left 8 columns are again the same 8x8 section as the top two. Odd parity requires the map **Rn** → **Ln** to additionally negate all Quaternion orientations, so our bottom right 8x8 section is the negation of the identical other three.

Notice the table matrix of ±1 values is the Octonion ever-popular Hadamard matrix, now 16x16 systematically doubling the systematic basis element product sign 8x8 Hadamard as indicated by the negation of only the bottom right quadrant, which is the definition of Hadamard systematic. This systematic form starts with the upper left 2x2 cell systematically doubled to 4x4 with bottom right 2x2 negated, to 8x8 doubled in kind, to our 16x16 systematic format.

This table appears in Hadamard systematic format *only* because we used the xor to zero Quaternion index partition process. *This is reason enough to make the claim it is superior to the other 29*. This Hadamard structure is present in any Quaternion enumeration, but the table gets row/column scrambled in the other 29. I say ever-popular because Hadamard structure should be considered *the spine of Octonion Algebra structure*, it will additionally reappear below in the guise of diagonal matrices which we will define now.

Define eight 8x8 Hadamard diagonal matrices **H**[n] where the diagonal members are sequentially row n of the top left 8x8 section of the Smap table.

A significant feature of Hadamard tables is a row composition formed by products of all same column values for two selected rows is always a table row, i.e., this composition is closed for the table. This is equivalent to the matrix product of two **H**[] matrices will always result in an **H**[] matrix. The identity is the all +1 row, and each row is its own inverse under the composition. Hadamard table rows thus form a group under the composition operation, or equivalently the **H**[] matrices form a group with group operation matrix multiplication.

When in systematic form, the composition is group element by element index identically isomorphic to the xor group X(4), where X(n) is a group where the members are integers 0 through $2^n - 1$, and the group operation is xor. Enumerating the rows top to bottom as 0-15, we can easily see the composition of row a and row b is row a^b. Understanding the xor operation is bit-wise, it is easy to see the composition is identical to the effect of the product of two algebraic orientation indicators, so we have the definition for the product of two algebraic orientation indicators is

$$s_a\, s_b = s_{a\wedge b}$$

By virtue of the fact the table is closed for row composition, any number of products in any final product term product history will always be oriented by a single algebraic orientation indicator within the set s_0 to s_{15}. This is analogous to the fact orientation options for any $\mathscr{H}_m$ is fully described by the orientation of its full complement of Quaternion subalgebras. Thus, the very same algebraic orientation covariance of a single product extends to any possible Octonion algebraic expression with any depth of product history.

The above table allows us to now define the map *Smap*. Assume one has an algebraic orientation covariant object properly oriented sign-wise for its creation Octonion orientation. Moving to another orientation will negate all algebraic orientation indicators where the columns for the two orientations within the associated row have different signs. Since our default orientation is **R0** and the table was constructed to have its column all +1, the Smap from **R0** to any other Octonion orientation negates the row label algebraic orientation indicators that have –1 in the “to orientation” column.

The focus of this paper is mapping Octonion Algebra to its adjacent algebra utilizing 8x8 matrices that can be brought to bear on group theory, which requires the associative property whereas Octonions are generally non-associative. We will no longer deal below with Octonion algebraic expressions Smap importantly addresses. At this point then, it would be instructive to drill down on the Smap concept as it applies to algebraic orientation covariant matrices.

An important characterization of the 8x8 matrices we will generate below is how many positions are not zero, and where they are within the matrix. Define generally the non-zero matrix position count by concatenating “nz” + “count” + “form”, i.e., *nz8forms* are matrices with 8 non-zero positions. Two matrices with 8 non-zero positions are nz8form equivalent if their non-zero members are in the same matrix positions but not necessarily equal. We can also talk generally about two matrices being *nzform equivalent* if they have the same number of non-zero components and all are at the same matrix locations independent of content. An important partition of a set of matrices would then be by equivalent nzforms.

Since Smap is strictly a map on algebraic orientation indicator signs, it can only map a given matrix to another nzform equivalent matrix that has the same algebraic orientation indicators on all non-zero matrix positions. Any algebraic invariant matrix will be unmodified, and a matrix will map to its negation only if all positions are algebraic variants.

A set of algebraic orientation covariant objects is called *closed for Smap* if the results of all 16 Smaps operating on any set object produces the same object or another in the set. Doing all 16 Smaps on a single object will produce all objects in the Smap closed set it belongs to, since repeating this process on any member of this Smap closed set will duplicate the same Smap closed set. If this single element generated Smap closed set is not a subset of the original set, the original set is not closed for Smap. If it is a subset, it can be snipped from the original set, and what remains is then independently dispositioned for Smap closure.

Members of a group(set) also closed for Smap are assured to partition into multiple Smap closed subsets if the group(set) size is greater than 16, and in many cases, this will happen for sizes less than 16. Any single group(set) member will be a member of only one Smap closed partition of the full group(set) in much the same way as conjugacy classes. Group(set) closure for Smap is determined by generating all Smap partitions. If no new elements are created in the process, the group(set) is closed for Smap.

We will be looking for sets of adjacent algebra matrices that form proper groups closed for Smap. Just as we can generate a group from a set of matrices that do not yet form a group by adding new members that are the result of the group operation on two existing members until no more new ones are created, we can take a group not closed for Smap and form new members until the group closes for Smap. Non-group members are created, so closing the group iteratively with Smap closure will lead to proper groups also closed for Smap. Ideally, we would like to have a paradigm where the generating set for a group naturally forms one that is also closed for Smap.

When we do the full set of Smaps on a given matrix formed with **R0** to generate the Smap closed set it belongs to, we may find we get two separate and distinct subsets, one subset generated by **R0** → **Rn** Smaps and the other subset generated by **R0** → **Ln** Smaps, where both subsets are individually closed for **R0** → **Rn** Smaps. The union of the two subsets is closed for Smap, but **R0** → **Ln** Smaps will map one subset into the other.

Define then an *Smap partition* of an Smap closed set as two subsets created where the first is only **Rn** Smaps on the generator matrix, and the other only **Ln** Smaps on the same generator matrix.

Smap partition := [[Right Smap generated subset], [Left Smap generated subset]]

If we always specify groups(sets) that pair {element, its negation}, elements with fully algebraic invariant content or fully variant content will always show up in both subsets of their Smap closed partition. Elements that are a mix of invariant and non-invariant can show up as disjoint related pairs in the **R0** → **Rn** and **R0** → **Ln** subsets, or the **R0** → **Ln** subset may be empty indicating the mapped to element is not in the group(set), in which case Smap closure is only partially for **R0** → **Rn**. When the two subsets are the same, the group(set) is achiral.

When the Smap subsets are disjoint, they are indicating sets of elements with a different yet related fundamental chiral structure. Left and right representations of the same essence or the like. The particular generator matrix is of no concern, different choices might only swap which partition subset is shown first. This is not important due to the natural symmetry of each individually closed for **R0** → **Rn** Smaps and **R0** → **Ln** Smaps exchanging the two.

If we find a set that is closed for **R0** → **Rn** Smaps, yet empty for all **R0** → **Ln** Smaps, we can form a companion set by performing the Smap **R0 → L0** on all set members to form a new set. The union of the two companions will be closed for Smap having oriented member Smap partitions with disjoint content.

We will find an example of this below when we build the Lie algebra su(8). As built, it is closed for **R0 → Rn** Smaps, but for oriented members is empty for the **R0 → Ln** Smaps, i.e., it is chiral. We build its opposite chirality companion with the Smap **R0 → L0** on all members and state the Smap partition form

Adjacent Lie algebra su(8) := [$\{su(8)_{right}\}$, $\{su(8)_{left}\}$]

Maximal commutating and anticommutating sets

Matrix set or group members can be partitioned into extremely important subsets of mutual commutation and mutual anticommutation. Building these out, we find the group/set structure provides us with a number of important partitions called *maximal commutating sets*, and *maximal anticommutating* sets. The maximal commutating sets give a heads up on maximal sets of matrices that can be added, followed by exponentiation of the sum equal to the commutative product of the exponentiations of each sum member. The maximal anticommutative sets gives the full complement of maximal 1-forms for building Clifford Algebras.

They come in sets of specific sizes, each size with some number of instances. They are called maximal because no other group/set member outside the maximal set will commute(anticommute) with all maximal set members. Thus, no maximal set is a subset of a larger maximal set. Set reduction by element elimination does not produce a maximal set, therefore no collision happens with any smaller existing maximal set, they are somewhat orthogonal.

An important use of set reduction is forming Clifford Algebras from maximal anti-commutating sets using members for 1-forms. Maximal sets we will find below have a good chance of producing the scaled identity with the product of all, thus not forming a universal Clifford Algebra. This is remedied by removing one from the set, which then becomes the pseudo-scalar (n-1)-form.

The maximal anticommutative sets also quantify the fact that for our target adjacent algebras there are too many available to simply dive in to a Clifford Algebra paradigm for answers. Just considering the “positive” members neglecting their negations, the initial order 256 adjacent algebra groups for left and right action defined below have 2688 size three maximal anticommutating sets, 64512 size five maximal anticommutating sets, and 36864 size seven anticommutating sets. The optimal order 1024 group we will also build below has 15456 size three maximal anticommutating sets, 12096 size five maximal anticommutating sets, and 576 size seven maximal anticommutating sets.

The Clifford algebra attached by some to the Standard Model is $Cl_{0,6}$. The 7-forms for all size seven maximal anticommutating sets here each scale the identity matrix, hence do not form universal Clifford algebras unless one member is dropped, which becomes the 6-form for a $Cl_{0,6}$ Clifford algebra. All considered, there are too many different choices to build $Cl_{0,6}$ Clifford algebras, with little selection help from this somewhat narrow view of the structure of Octonion Algebra, leaning only the intrinsic anticommutation of non-scalar basis elements.

Below, we will only minimally but most importantly use Clifford Algebra to optimally enumerate group members such that some gradation is structured in, and meaningful structures such as conjugacy classes, normal subgroups and quotient group cosets can be found in consecutive or otherwise predictable group element indexes.

Algebraic orientation covariant left and right action Octonion adjacent algebra matrix representation

With an understanding now of Octonion algebraic orientation, we can (yes, *finally*) get back to the matter at hand.

Our ultimate goal is using the highly directive structure of Octonion Algebra to derive matrix-based Lie algebras for special unitary Lie groups. We must provide a way for our non-associative algebra to give us necessarily associative matrices. We can do this by having the algebra "act on itself". The classical method (see reference [11]) to generate these adjacent algebra matrices is to start with a vector built from sequentially indexed Octonion intrinsic basis elements as

$\mathbf{V} = [e_0, e_1, e_2, e_3, e_4, e_5, e_6, e_7]$

Next, create new vectors formed by "the algebra acting on itself" by multiplying **V** from the left or from the right by each intrinsic Octonion basis element in turn, defining the "left action" and "right action" respectively.

Left action $\mathbf{A_{L_x}} := e_x * \mathbf{V}$ $\quad\quad$ $x = 0, 1, 2, \ldots 7$
Right action $\mathbf{A_{R_x}} := \mathbf{V} * e_x$ $\quad\quad$ $x = 0, 1, 2, \ldots 7$

We must consider both right and left side application since Octonion Algebra is not strictly commutative.

We can duplicate left and right actions of the algebra acting on itself with real valued matrices acting naturally on the unmodified **V** as a column vector as follows, defining "adjacent algebra matrices" $g_L[x]$ and $g_R[x]$

$\mathbf{A_{L_x}}[a] = g_L[x]_{ab}\, V_b$
$\mathbf{A_{R_x}}[a] = g_R[x]_{ab}\, V_b$

The rub with this well-travelled path is the adjacent algebra matrices were simply signed permutation matrices where each row and column have a single non-zero entry either +1 or –1 accounting only for the basis element product sign and product result basis, created from a singular by-hand initial choice of Octonion Algebra orientation.

Clearly, if some other identically partitioned Quaternion subalgebra Octonion orientation was used, the non-zero positions would remain the same but some of the signs would not. Hopefully the first section above has convinced you picking a singular Octonion algebraic orientation to the exclusion of all others is not appropriate. We must include appropriate algebraic orientation indicators to represent the adjacent algebra matrices in an *algebraic orientation covariant* fashion. Setting the signage using our reference Octonion orientation **R0**, we have the following basic set of eight pairs of adjacent algebra matrices using the left and right actions labeled as such:

$$g_L[0] = \begin{bmatrix} +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \end{bmatrix}$$

$$g_R[0] = \begin{bmatrix} +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \end{bmatrix}$$

$g_L[1] =$

$$\begin{bmatrix} 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_9 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{13} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{13} & 0 \end{bmatrix}$$

$g_R[1] =$

$$\begin{bmatrix} 0 & +1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_9 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{13} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{13} & 0 \end{bmatrix}$$

$g_L[2] =$

$$\begin{bmatrix} 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_9 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{11} \\ 0 & 0 & 0 & 0 & +1s_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{11} & 0 & 0 \end{bmatrix}$$

$g_R[2] =$

$$\begin{bmatrix} 0 & 0 & +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_9 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{11} \\ 0 & 0 & 0 & 0 & -1s_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{11} & 0 & 0 \end{bmatrix}$$

$g_L[3] =$

$$\begin{bmatrix} 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_9 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_7 & 0 & 0 & 0 \end{bmatrix}$$

$g_R[3] =$

$$\begin{bmatrix} 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_9 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_7 & 0 & 0 & 0 \end{bmatrix}$$

$g_L[4] =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_7 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_7 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$g_R[4] =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_7 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_7 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$g_L[5] =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & +1s_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$g_R[5] =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \\ 0 & -1s_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$g_L[6] =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{13} \\ 0 & 0 & 0 & 0 & -1s_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 & 0 \\ 0 & 0 & +1s_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{15} & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{13} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$g_R[6] =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{13} \\ 0 & 0 & 0 & 0 & +1s_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 & 0 \\ 0 & 0 & -1s_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_{13} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$g_L[7] =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$g_R[7] =$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The nice specific form of these matrices is an outcome of enumerating each of the seven Quaternion subalgebras with the index xor to zero rule, yet another justification for the xor to zero rule superiority. The eight non-zero matrix elements in each of these reside within four different contiguous 2x2 submatrices amongst contiguous 2x2 all zero entry submatrices. The $g_L[x]$ and $g_R[x]$ for even x are all contiguous 2x2 submatrix diagonal members and the $g_L[x]$ and $g_R[x]$ for odd x are the contiguous 2x2 submatrix transverse members.

Each of the eight $g_x[0]$ through $g_x[7]$ have eight non-zero entries, one on each row and each column. There are no non-zero positional intersections between them, and the union (sum) of all 8 has no zero positions. Together, they span all positions of an 8x8 matrix. We should consider these distinct nzforms might lead us to basis sets for 8x8 matrices.

We also find

Determinant $\|g_x[n]\| = +1$	algebraic invariant even though $g_x[1$ through $7]$ are not
$g_x[0]\ g_x[0] = \mathbf{I}\ \ \mathbf{I} = \mathbf{I}$	$\mathbf{I}$ is the 8x8 identity matrix
$g_x[a]\ g_x[a] = -\mathbf{I}$	a = 1 to 7
$g_x[0]\ g_x[a] = g_x[a]\ g_x[0]$	any a
$g_x[a]\ g_x[b] = -\ g_x[b]\ g_x[a]$	anti-commutative for $a \neq b$, any 1 to 7
$g_x[a]\ g_x[b]$ is nzform compatible with $g_x[a\hat{}b]$	within nzform compatibility with $e_a * e_b = s_{ab(a\hat{}b)}\ e_{(a\hat{}b)}$

This implies that the matrix products $g_x[a]\ g_x[b]$ within nzform compatibility is a mostly faithful representation of the basis element products $e_a * e_b$, and as represented there is a one-to-one correlation (map) between $g_x[n]$ and e_n.

The anticommutative products of different non-scalar Octonion basis elements makes it abundantly clear the difference between left action and right action is the negation of all oriented matrix locations, as indicated.

Rather than trying to fit all of this into some tensoring structure of division algebras which might appear at this point somewhat ad hoc and by hand, we have a straightforward chance within group theory for a more holistic representation since we have adjacent algebra representation matrices with determinant +1, so group closure is possible. We will put our full faith and confidence in group theory. While not unexpectedly, matrices $g_x[0]$ through $g_x[7]$ do not form groups, but each will generate a group through completion. This is covered in the next section.

Moreover, understand that all adjacent algebra matrices are designed to act naturally on the set of Octonion basis elements. They are linear operators mapping the vector space of Octonion algebraic elements to themselves, i.e., endomorphisms of Octonion Algebra algebraic elements.

Left and Right action Octonion Adjacent Algebra Groups

The sets $g_L[0]$ through $g_L[7]$ and $g_R[0]$ through $g_R[7]$ separately generate order 256 groups I will call $\mathbf{g^L_{256}}$ and $\mathbf{g^R_{256}}$ respectively, or $\mathbf{g_{256}}$ when common features are discussed but with the restriction these features apply individually to $\mathbf{g^L_{256}}$ *or* $\mathbf{g^R_{256}}$. Both groups partition into 128 matrix and matrix negation pairs. To delineate between, we will call "positive elements" one of the pair and "negative elements" the other.

We find neither $\mathbf{g^L_{256}}$ or $\mathbf{g^R_{256}}$ are closed for Smap $\mathbf{R0 \rightarrow Rn}$ and $\mathbf{R0 \rightarrow Ln}$ partitions. Off the top, we can consider them unsuitable for our purposes. However, the structure of both allows us to pull out meaningful alternatives to our fundamental adjacent algebra $g_L[]$ and $g_R[]$ matrices we will use later.

Both $\mathbf{g^L_{256}}$ and $\mathbf{g^R_{256}}$ share the same set of 32 diagonal matrix elements which form normal subgroups. Since the product of any diagonal matrix and some particular nzform matrix remains the same nzform, we can simply arrange the group elements into eight quotient group cosets of like nzform where the kernel is the set of diagonal elements, and one nzform representative used to generate all of its coset members through group operation on the kernel. This eight-coset structure is intrinsic to all adjacent algebra groups we will consider so is important enough to give a name to. Define it as $\text{coset}_8[x]$.

$\text{coset}_8[0]$:= normal subgroup kernel: all diagonal matrix group elements
$\text{coset}_8[1]$:= product of kernel and group element nzform compatible with $g_L[1]$ for $\mathbf{g^L_{256}}$ and $g_R[1]$ for $\mathbf{g^R_{256}}$
$\text{coset}_8[2]$:= product of kernel and group element nzform compatible with $g_L[2]$ for $\mathbf{g^L_{256}}$ and $g_R[2]$ for $\mathbf{g^R_{256}}$
$\text{coset}_8[3]$:= product of kernel and group element nzform compatible with $g_L[3]$ for $\mathbf{g^L_{256}}$ and $g_R[3]$ for $\mathbf{g^R_{256}}$
$\text{coset}_8[4]$:= product of kernel and group element nzform compatible with $g_L[4]$ for $\mathbf{g^L_{256}}$ and $g_R[4]$ for $\mathbf{g^R_{256}}$
$\text{coset}_8[5]$:= product of kernel and group element nzform compatible with $g_L[5]$ for $\mathbf{g^L_{256}}$ and $g_R[5]$ for $\mathbf{g^R_{256}}$
$\text{coset}_8[6]$:= product of kernel and group element nzform compatible with $g_L[6]$ for $\mathbf{g^L_{256}}$ and $g_R[6]$ for $\mathbf{g^R_{256}}$
$\text{coset}_8[7]$:= product of kernel and group element nzform compatible with $g_L[7]$ for $\mathbf{g^L_{256}}$ and $g_R[7]$ for $\mathbf{g^R_{256}}$

An appropriate name for the "group element nzform compatible with…" is the *coset builder*, and is applicable to all adjacent algebra groups we will construct.

Next define our $\text{coset}_8[x]$ builders generally as cbl[x] for $\mathbf{g^L_{256}}$ and cbr[x] for $\mathbf{g^R_{256}}$, where cbl[0] = cbr[0] = the identity matrix. We could certainly use cbl[x] = $g_L[x]$ for $\mathbf{g^L_{256}}$ and cbr[x] = $g_R[x]$ for $\mathbf{g^R_{256}}$, but a more optimal enumeration would be to use eight members of a related $Cl_{0,3}$ Clifford algebra where the three 1-forms are taken from $g_L[x]$ for $\mathbf{g^L_{256}}$ and $g_R[x]$ for $\mathbf{g^R_{256}}$. Group closure assures us any Clifford algebra so constructed will consist of group members. Also, we will find cbl[x] is the same nzform as $g_L[x]$ and cbr[x] is the same nzform as $g_R[x]$, so identical groups are constructed replacing $g_L[x]$ with a Clifford algebra cbl[x] and replacing $g_R[x]$ with a Clifford algebra cbr[x] for $\text{coset}_8[x]$ builders.

Defining the normal subgroup of diagonal matrices as $\{\mathbf{D}\}$ we will then declare our optimal group $\text{coset}_8[n]$ element enumeration to be the matrix products

$\mathbf{g^L_{256}}$ $\text{coset}_8[n]$:= cbl[n] $\{\mathbf{D}\}$ for n: 0 to 7

$\mathbf{g^R_{256}}$ coset$_8$[n] := cbr[n] {**D**}   for n: 0 to 7

There are multiple ways to generate cbl[n] and cbr[n] from g_L[] and g_R[], and what follows is just one of these ways and *not* a requirement. All products within either g_L[1-7] or g_R[1-7] are anticommutative, but we must avoid the triplet indexes of our seven Quaternion subalgebras to insure the 3-form does not scale the identity, so we would have 28 different options for 1-forms equivalently for both left and right action.

Looking at the nzforms for the unions $g_L[0] \cup g_L[1]$ and $g_R[0] \cup g_R[1]$, we find both are block diagonal 2x2 non-zero submatrices. The natural association for the 3-forms would thus be some $\mathbf{g_{256}}$ group member nzform equivalent to $g_L[1]$ and $g_R[1]$. Since the matrix product $g_x[a]\, g_x[b]$ is nzform compatible with $g_x[a\text{\^{}}b]$, we can achieve our desired $g_L[1]$ and $g_R[1]$ equivalent nzform 3-forms from 1-form assignments $g_x[a]$, $g_x[b]$ and $g_x[c]$ provided $a \neq b \neq c \neq 0$ and a^b^c = 1.

We can achieve this as well as matching the odd-even gradation with odd-even coset$_8$[n] n index selecting our 1-forms most naturally for x = r(ight) or l(eft) actions

cbx[3] = $g_x[3]$	1-form	
cbx[5] = $g_x[5]$	1-form	
cbx[7] = $g_x[7]$	1-form	
cbx[2] = $g_x[5]\, g_x[7]$	2-form	nzform equivalent to $g_x[2]$
cbx[4] = $g_x[3]\, g_x[7]$	2-form	nzform equivalent to $g_x[4]$
cbx[6] = $g_x[5]\, g_x[3]$	2-form	nzform equivalent to $g_x[6]$
cbx[1] = $g_x[3]\, (g_x[5]\, g_x[7])$	3-form	nzform equivalent to $g_x[1]$

This gives succinct and optimal definitions for left and right action adjacent algebra matrix groups:

$\mathbf{g^L_{256}}$:= {cbl[]} {**D**}
$\mathbf{g^R_{256}}$:= {cbr[]} {**D**}

Itemizing these non-identity Clifford algebra coset$_8$[] builders {cbl} and {cbr} we will also use below:

cbl[1] =

$$\begin{bmatrix} 0 & +1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ +1s_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \end{bmatrix}$$

cbr[1] =

$$\begin{bmatrix} 0 & +1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ +1s_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \end{bmatrix}$$

cbl[2] =

$$\begin{bmatrix} 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_2 & 0 & 0 & 0 & 0 \\ +1s_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & +1s_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \end{bmatrix}$$

cbr[2] =

$$\begin{bmatrix} 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_2 & 0 & 0 & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & +1s_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \end{bmatrix}$$

cbl[3] =

$$\begin{bmatrix} 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_9 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_7 & 0 & 0 & 0 \end{bmatrix}$$

cbr[3] =

$$\begin{bmatrix} 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_9 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_7 & 0 & 0 & 0 \end{bmatrix}$$

cbl[4] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & -1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ +1s_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

cbr[4] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & +1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1s_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

cbl[5] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & +1s_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

cbr[5] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \\ 0 & -1s_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

cbl[6] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_2 \\ 0 & 0 & 0 & 0 & -1s_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & +1s_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ +1s_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

cbr[6] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_2 \\ 0 & 0 & 0 & 0 & -1s_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & +1s_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1s_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

cbl[7] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

cbr[7] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Before abandoning the left action and right action groups, something interesting.

The following 2x2 matrices form a basis set for all 2x2 matrices

t[0] = $\begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix}$ t[1] $\begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix}$ t[2] $\begin{bmatrix} 0 & +1 \\ +1 & 0 \end{bmatrix}$ t[3] $\begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix}$

We can build bases for larger matrices by forming matrix direct products with these. The dimension of the new matrix will double with each direct product from the set, so we could create 8x8 matrices with the direct product of any three. We will create 64 different 8x8 matrices by letting the t[] indexes for the three used freely range 0 to 3. These matrices have simple algebraic invariant ±1 members, so cannot be directly compared to our algebraic orientation covariant $\mathbf{g}_{256}$ matrices.

As build however, $\mathbf{g}_{256}$ matrix signage is correct for the particular orientation used, and we could drop all algebraic orientation indicators and have something to compare. Doing so, we find all 64 members of the direct product set match members in the not oriented group $\mathbf{g}^{\mathbf{L}}_{256}$ but do not match members in the not oriented group $\mathbf{g}^{\mathbf{R}}_{256}$.

This begs the question, is the left action representation more fundamental? It seems to be the one preferred by others. The answer is no. The deficiency is the outcome of our two groups not being closed for Smap. This will be remedied below.

Group $\mathbb{A}$: The algebraic orientation covariant adjacent algebra matrix group closed for Smap

We could form this group by simply taking the union of representations $\mathbf{g}^{\mathbf{L}}_{256}$ and $\mathbf{g}^{\mathbf{R}}_{256}$ then completing the group, but we would find this group to be much larger than necessary. Instead, we will produce the group $\mathbb{A}$ by taking as group generators the union of our Clifford algebra left and right coset builders cbl[x] and cbr[x] defined above, then performing the group completion. It was non-essential to use the Clifford Algebra coset_8[] builders for $\mathbf{g}^{\mathbf{L}}_{256}$ and $\mathbf{g}^{\mathbf{R}}_{256}$, the very same groups are constructed either way. We will find it imperative now to use Clifford algebra cbl[x] and cbr[x] and not g_L[x] and g_R[x] in order to build a suitable minimal order group with meaningful results.

Doing so, we find $\mathbb{A}$ builds out to an order 1024 group where every "positive" element is accompanied by its negation. Every element is an 8x8 nz8form matrix. Again, we have a set of diagonal matrices that form a normal subgroup, now with order 128 instead of 32 as above. These diagonal matrices are different than above in that all are algebraic orientation invariants.

As above, we can build out the full group with eight coset builders scaling our full diagonal set normal subgroup. We are free to choose either the cbl[] set or the cbr[] set for our coset builders, since each will build out the very same group, albeit with different element index sequencing.

We will be ahead of the game understanding the structure of this group if we optimally set the sequence of members in the group. First up is the normal subgroup of diagonal members. The group center **Z** is {**I**, –**I**} and we find the first subgroup product partition for the diagonals is {**H**} {**Z**}, with **H**[n] was defined above as our eight non-oriented diagonal Hadamard matrices. This forms a supremely important order 16 normal subgroup of $\mathbb{A}$ which we will park in the first 16 group positions. The diagonal matrix normal subgroup is completed with the additional product with the set **CP** defined as

CP = {**I**, cbl[1]cbr[1], cbl[2]cbr[2], cbl[3]cbr[3], cbl[4]cbr[4], cbl[5]cbr[5], cbl[6]cbr[6], cbl[7]cbr[7]}

The full set of 128 diagonal matrices **D** := {**CP**} ({**H**} {**Z**}) is optimally enumerated where ({**H**} {**Z**}) implies the sequence **H**[0] {**Z**} followed by **H**[1] {**Z**} … This ordered set is multiplied by **CP**[0] followed by **CP**[1]… setting the final optimal sequence of diagonal members.

The full group 𝔸 is given by eight coset_8[x] = cbl[x] **D**, x: 0 to 7.

Each product cbl[x]cbr[x] in **CP** is commutative. From this it is easy to see how we can use either cbl or cbr as coset_8[] builders, since cbl[x] and cbr[x] square to –**I** and the group includes negations of all members. The choice of using cbl[] or cbr[] will simply impact the coset_8[] member sequence, not its content.

𝔸 has group closure and is closed for Smap which was our objective. It has the following characteristics.

1. All members have an algebraic orientation invariant determinant of +1
2. The 512 different elements can be found as positive elements at 𝔸[x] for even x, and their negations at 𝔸[x+1]
3. The first m = 2^n for n: 1 to 9 members form a normal subgroup of 𝔸
4. Cosets of quotient group 𝔸 / {**Z**}, being an element and its negation, trivially have identical symmetric or antisymmetric status, as well as have identical commutative and anticommutative properties with other group members.
5. The product 𝔸[a] 𝔸[b] and element 𝔸[a^b] are in the same quotient group 𝔸 / {**H**}{**Z**} coset.
6. All quotient groups using kernels from (3.) are abelian, and are isomorphic to the xor group X(10–n)
7. The characteristic polynomial for every member is algebraic orientation invariant, thus so are their eigenvalues
8. Members produce six different sets of eigenvalues, i = standard commutative square root of –1
    a. +1 multiplicity 8
    b. –1 multiplicity 8
    c. +1 multiplicity 4, –1 multiplicity 4
    d. +1 multiplicity 2, –1 multiplicity 6
    e. +1 multiplicity 6, –1 multiplicity 2
    f. +i multiplicity 4, –i multiplicity 4
    g. +1 multiplicity 2, –1 multiplicity 2, +i multiplicity 2, –i multiplicity 2
9. Non-diagonal members are either symmetric, antisymmetric or a mix of both
10. Members square to one member of normal subgroup {**H**} {**Z**} found at element index 0 through 15
    a. All symmetric members square to +**I** (group element index 0)
    b. All antisymmetric members square to –**I** (group element index 1)
    c. Mixed symmetry members square to one of group element index 2 through 15, the non-identity Hadamard matrices and their negations
11. All non-diagonal members have 0 values on the diagonals

In consideration of items 10 consistent with Clifford Algebra standard $Cl_{p,q}$ designating p for squares to +**I**, q for squares to –**I**, add to definitions *p-type* and *q-type* the name *n-type* for members satisfying 10-c. Each of these n-types form a four-member power group.

There are too many matrices to put all up in an appendix, it would definitely be a can't see the forest for the trees type of thing. This group is only an intermediate step in the ultimate goal of producing Lie algebras. I will however refer to the group by member element indexes below. These integers are better for comparative purposes than matrices would be anyhow.

Group 𝔸 conjugacy class Smap considerations and element squared class equivalences

One of the most important partitions of any group is its set of conjugacy classes. The conjugacy classes for group 𝔸 non-diagonal oriented members are closed for **R0** → **Rn** Smaps, but **R0** → **Ln** Smaps fully map all members of one conjugacy class to the member set of another. Conjugacy class Smap closure requires the union of two conjugacy classes. The interplay between conjugacy classes and group 𝔸 Smap partitions grouped by coset_8[] is shown in Appendix 1.

If we look at the square of group 𝔸 members, it can be shown that this is within sign a conjugacy class Smap pair equivalence. The conjugacy class pairs are as follows in Smap partition format

Pair[0] = [[cc[8][7]], [cc[8][9]]]	square to +**H**[0]
Pair[1] = [[cc[8][8]], [cc[8][10]]]	square to –**H**[0]
Pair[2] = [[cc[8][11]], [cc[8][13]]]	square to –**H**[0]
Pair[3] = [[cc[8][12]], [cc[8][14]]]	square to +**H**[0]
Pair[4] = [[cc[8][15]], [cc[8][17]]]	square to –**H**[0]
Pair[5] = [[cc[8][16]], [cc[8][18]]]	square to +**H**[0]
Pair[6] = [[cc[8][19]], [cc[8][21]]]	square to –**H**[0]
Pair[7] = [[cc[8][20]], [cc[8][22]]]	square to +**H**[0]
Pair[8] = [[cc[8][23]], [cc[8][25]]]	square to –**H**[0]
Pair[9] = [[cc[8][24]], [cc[8][26]]]	square to +**H**[0]
Pair[10] = [[cc[8][27]], [cc[8][29]]]	square to –**H**[0]
Pair[11] = [[cc[8][28]], [cc[8][30]]]	square to +**H**[0]
Pair[12] = [[cc[8][31]], [cc[8][33]]]	square to –**H**[0]
Pair[13] = [[cc[8][32]], [cc[8][34]]]	square to +**H**[0]

Pair[14] = [[cc[16][0]], [cc[16][1]]]	square to ±**H**[4]
Pair[15] = [[cc[16][2]], [cc[16][3]]]	square to ±**H**[2]
Pair[16] = [[cc[16][4]], [cc[16][5]]]	square to ±**H**[6]
Pair[17] = [[cc[16][6]], [cc[16][7]]]	square to ±**H**[4]
Pair[18] = [[cc[16][8]], [cc[16][10]]]	square to ±**H**[1]
Pair[19] = [[cc[16][9]], [cc[16][11]]]	square to ±**H**[5]
Pair[20] = [[cc[16][12]], [cc[16][13]]]	square to ±**H**[4]
Pair[21] = [[cc[16][14]], [cc[16][17]]]	square to ±**H**[3]
Pair[22] = [[cc[16][15]], [cc[16][16]]]	square to ±**H**[7]
Pair[23] = [[cc[16][18]], [cc[16][21]]]	square to ±**H**[2]
Pair[24] = [[cc[16][19]], [cc[16][22]]]	square to ±**H**[1]
Pair[25] = [[cc[16][20]], [cc[16][23]]]	square to ±**H**[3]
Pair[26] = [[cc[16][24]], [cc[16][27]]]	square to ±**H**[2]
Pair[27] = [[cc[16][25]], [cc[16][29]]]	square to ±**H**[5]
Pair[28] = [[cc[16][26]], [cc[16][28]]]	square to ±**H**[7]
Pair[29] = [[cc[16][30]], [cc[16][35]]]	square to ±**H**[6]
Pair[30] = [[cc[16][31]], [cc[16][33]]]	square to ±**H**[1]
Pair[31] = [[cc[16][32]], [cc[16][34]]]	square to ±**H**[7]
Pair[32] = [[cc[16][36]], [cc[16][41]]]	square to ±**H**[6]
Pair[33] = [[cc[16][37]], [cc[16][40]]]	square to ±**H**[5]
Pair[34] = [[cc[16][38]], [cc[16][39]]]	square to ±**H**[3]

The size 8 conjugacy class Smap pairs all square to a common **H**[0] sign, while the size 16 conjugacy class Smap partitions square with half +**H**[x ≠ 0] and half – **H**[x ≠ 0]. The natural sign separation in the size 8 tells us to do the same sign partitioning on each n-type square to ±**H**[x ≠ 0] set. This need not be a "by hand" move. All we need to do is look at the maximal commutating subsets built from the full set of Pair[x] elements.

When we do so, we find all members of Pair[x] where there is a single square to sign (p-types and q-types) are fully commutative sets. However, when we look at the n-type Pair[x], each the union of two size 16 conjugacy classes, we find two separate size 16 maximal commutating sets, eight members from each of the two paired conjugacy classes. Each member of one maximal commutating set will square to +**H**[y] and the other maximal set members will all square to –**H**[y]. We also find both maximal commutating sets remain closed for Smap.

We now have multiple instances of 16 types of "square to" one element of normal subgroup {**H**} {**Z**} found at $\mathbb{A}$[0 through 15]. Each square to partition is Smap closed, and is a fully commutative set.

The "square to" class equivalence is telling us a worthy set of partitions for the non-diagonal oriented set of members in our Smap closed $\mathbb{A}$ group is taking the union of all members of common square to $\mathbb{A}$[x], x: 0-15. This gives us the full complement of size 8 conjugacy that square to $\mathbb{A}$[0] (p-type) and square to $\mathbb{A}$[1] (q-type) partitions, and the full complement of size 16 conjugacy classes (n-type) that square to $\mathbb{A}$[2 through 15].

For brevity, define **Sq**[x] := the partition of oriented $\mathbb{A}$ members that square to $\mathbb{A}$[x], x: 0 through 15.

Sq[0] has 112 members, 16 from each coset_8[1 through 7]. Its members within each coset_8[x] sub-partition are fully commutative. For each x, coset_8[x] sub-partitions of **Sq**[0] are the union of the eight symmetric members of cbl[x] {**H**} {**Z**} and eight symmetric members of cbr[x] {**H**} {**Z**}. They are the full complement of p-types in group $\mathbb{A}$.

Sq[1] has 112 members, 16 from each coset_8[1 through 7]. Its members within each coset_8[x] sub-partition are fully commutative. For each x, coset_8[x] sub-partitions of **Sq**[1] are the union of the eight antisymmetric members of cbl[x] {**H**} {**Z**} and eight antisymmetric members of cbr[x] {**H**} {**Z**}. They are the full complement of q-types in group $\mathbb{A}$.

Sq[$2 \le x \le 15$] each have 48 members, 16 n-types from three separate coset_8[y] where the three y indexes are the non-scalar basis indexes we enumerated for Quaternion subalgebra index n = integer(x/2). Together, these span the full complement of n-types in group $\mathbb{A}$.

Mapping **Sq**[x] to Lie bracket algebras

We can map each **Sq**[x] set into Lie algebras by feeding all pairs of **Sq**[x] members into the standard matrix Lie algebra representation *bracket* defined as [A, B] = C = matrix products AB – BA, and registering all unique results.

It is important to be precise on what I will mean by the phrase *closed for bracket*. We will sometimes get matrix C, and other times matrix 2C for a different choice of matrices A and B. For the purpose of determining Lie Algebra element itemization and closure for bracket we will take C and 2C to be one result with a different bracket structure constant.

The sets we will build below will be made by expansion of some generating set by appending unique bracket outcomes until the set closes, much like group closure from a generating set. If the closure algorithm produces A, B and {A+B}, they will be considered three separate members. This will be important since we may find both the linearly independent subset and remainder dependent subset have individual significance.

While strictly speaking, an algebra does not distinguish between a basis element and its negation, it will be helpful to sequence Lie algebra set enumerations with C and –C in consecutive indexes as {**0**, C_1, –C_1, C_2, –

C_2, … } where **0** is the null matrix always present since [A, A] = **0** for all A, and negations that are always present in a set closed for bracket since [A, B] = –[B, A]. Including null and negations makes the closure test algorithm straight-forward.

Doing so for each **Sq**[x], the outcome of bracket on all pairs of members will sometimes be an nz8form and other times an nz4form. These sets of unique bracket outcomes will form a Lie algebra closed for bracket and Smap after the first seed pass on any **Sq**[x].

The Lie algebra seeded from the full complement of p-type symmetric matrices **Sq**[0] has no symmetric members. All nz8form bracket results are q-types, in fact the full complement of q-types in group 𝔸.

The Lie algebra seeded from the full complement of q-type antisymmetric matrices **Sq**[1] also has no symmetric members, and all **Sq**[1] members are bracket outcomes included in the Lie algebra. All bracket result nz8forms seeded from **Sq**[1] are again the full complement of q-types in group 𝔸.

The nz4form bracket outcomes for **Sq**[0] and **Sq**[1] are also identical sets, so we find **Sq**[0] and **Sq**[1] seeded Lie algebras are identical.

The Lie algebras seeded from exclusively n-type matrices **Sq**[2 through 15] have no n-type bracket outcomes. They have nz4forms and q-type nz8forms. As with Lie algebras seeded from **Sq**[0] and **Sq**[1], Lie algebras seeded by **Sq**[even x>0] are identical to those seeded by **Sq**[1 + even x>0].

We can then identify eight different Lie algebras as **LA**[x: 0 through 7], where **LA**[x] is sourced from group 𝔸 **Sq**[2x] members that square to our positive Hadamard matrices **H**[x].

The seven Lie algebras **LA**[x: 1-7] have 73 members including **0** and 36 element-negation pairs. The 72 non-null members of **LA**[x] are 48 nz8forms in 24 element-negation pairs sourced from the antisymmetric members of three different sets cbl[m] {**H**} {**Z**} and cbr[m] {**H**} {**Z**}. The three m are the triplet indexes for Quaternion subalgebra x. The remaining 24 members coming in 12 element-negation pairs are nz4forms, which partition into four sets of **LA**[x: 1-7] ideals each with three element-negation pairs.

The nz4forms of **LA**[x: 1-7] and **LA**[y: 1-7, ≠ x] have no intersection, so combined, the ideals of Lie Algebras **LA**[x: 1-7] specify 168 separate nz4forms. Additionally we have ideal positive member brackets for each ideal Id[n] given by $[Id_{na}, Id_{nb}] = 2Id_{nc}$, $[Id_{nb}, Id_{nc}] = 2Id_{na}$, $[Id_{nc}, Id_{na}] = 2Id_{nb}$.

Lie algebra **LA**[0] has 281 members. The 280 non-null members consist of 112 nz8forms and 168 nz4forms. The 112 nz8forms are seven sets of 16 antisymmetric members from cbl[1-7] {**H**} {**Z**} and cbr[1-7] {**H**} {**Z**}, the full complement of q-types. The 168 **LA**[0] nz4forms are the union of all independent nz4forms (ideals) for Lie algebras **LA**[1-7]. It collectively has no ideals. We see then that Lie algebras **LA**[1-7] are each Lie subalgebras of Lie algebra **LA**[0].

Each nz8form in every Lie algebra **LA**[x: 1-7] is the unique sum of two nz4forms of the same Lie algebra, and each of the nz4forms are linearly independent. We can form a linearly independent set by simply discarding all nz8forms. What remains is the null matrix **0** and the four ideal sets of each **LA**[x: 1-7]. Name these preliminary Lie algebras consisting now of just the ideals from **LA**[x: 1-7] as **IDLA**[x: 1-7].

The 25 entries in each preliminary **IDLA**[x: 1-7] form a linearly independent Lie algebra closed for bracket and closed for Smap, precisely what we will continually demand. As an aside, if we were to take just the nz8forms of any **LA**[x: 1-7], we would find they do not close for bracket, generating nz4forms when the bracket is with one antisymmetric member of cbl[x] {**H**} {**Z**} and one antisymmetric member of cbr[x] {**H**} {**Z**}.

Each of these nz4forms exist in a 4x4 submatrix of the adjacent algebra 8x8 matrix format. We can reduce the matrix dimension by discarding all full zero rows and columns. Doing so, the four ideals split into two different sets of four 4x4 submatrices of four kept and four discarded rows/columns where what is kept for one is what is discarded by the other. Moreover, observe the following itemization of the kept rows/columns for the two types of reduced matrices for each **LA**[x: 1 – 7]:

IDLA[1] separately keeps rows/columns {0, 2, 4, 6} and rows/columns {1, 3, 5, 7}
IDLA[2] separately keeps rows/columns {0, 1, 4, 5} and rows/columns {2, 3, 6, 7}
IDLA[3] separately keeps rows/columns {0, 3, 4, 7} and rows/columns {1, 2, 5, 6}
IDLA[4] separately keeps rows/columns {0, 1, 2, 3} and rows/columns {4, 5, 6, 7}
IDLA[5] separately keeps rows/columns {0, 2, 5, 7} and rows/columns {1, 3, 4, 6}
IDLA[6] separately keeps rows/columns {0, 1, 6, 7} and rows/columns {2, 3, 4, 5}
IDLA[7] separately keeps rows/columns {0, 3, 5, 6} and rows/columns {1, 2, 4, 7}

Recognize what we remarkably have here:

IDLA[n] separately keeps rows/columns Quaternion[n] indexes and Quaternion[n] basic quad indexes.

We can now split each **IDLA**[n] into two separate partitions as follows.

IDLA[1][0] := members whose reduction keeps rows/columns {0, 2, 4, 6}
IDLA[1][1] := members whose reduction keeps rows/columns {1, 3, 5, 7}
IDLA[2][0] := members whose reduction keeps rows/columns {0, 1, 4, 5}
IDLA[2][1] := members whose reduction keeps rows/columns {2, 3, 6, 7}
IDLA[3][0] := members whose reduction keeps rows/columns {0, 3, 4, 7}
IDLA[3][1] := members whose reduction keeps rows/columns {1, 2, 5, 6}
IDLA[4][0] := members whose reduction keeps rows/columns {0, 1, 2, 3}
IDLA[4][1] := members whose reduction keeps rows/columns {4, 5, 6, 7}
IDLA[5][0] := members whose reduction keeps rows/columns {0, 2, 5, 7}
IDLA[5][1] := members whose reduction keeps rows/columns {1, 3, 4, 6}
IDLA[6][0] := members whose reduction keeps rows/columns {0, 1, 6, 7}
IDLA[6][1] := members whose reduction keeps rows/columns {2, 3, 4, 5}
IDLA[7][0] := members whose reduction keeps rows/columns {0, 3, 5, 6}
IDLA[7][1] := members whose reduction keeps rows/columns {1, 2, 4, 7}

These **IDLA**[n][x] do not span their 4x4 reduced matrix positions. What is missing are the diagonal elements which for now are all zero. We want to fully span each 4x4 submatrix, so we must append appropriately positioned elements of some form within new 8x8 members such that our dimensional reduction places them in diagonal positions of each reduced 4x4 matrix.

All current non-null **IDLA**[x][n] members are real and antisymmetric, so would fit into a skew-Hermitian unitary matrix scheme. With an eye on generating SU(n) Lie groups, whose Lie algebras are necessarily skew-Hermitian, we should look to purely imaginary elements using the standard complex number imaginary basis "i" on the diagonals and require them to have zero trace.

We just went through a dimensional reduction from 8x8 to 4x4. The next reduction to look for would be from our 4x4 to 2x2 matrices of a Lie Algebra su(2) nature. To this end, append to each **IDLA**[x][n] 8x8 nz2forms using {+i, –i} and separately its negation, where the different positions of each +imaginary and –imaginary within the 8x8 matrix format reduce to all six combinations of two out of four target 4x4 diagonal positions.

Generating algebraic orientation covariant su(2) Pauli and su(4) Dirac matrices

The obvious next step is to close for bracket each now 4x4 spanning **IDLA**[n][x]. Doing so, we find the closure to be proper Lie algebras consisting of the null matrix **0** and 33 pairs of {element-negation} appending new additional nz4forms along with off 4x4 diagonal nz2forms. We have 15 element-negation pairs of nz4forms, and 18 element-negation pairs of nz2forms.

Once again, we can remove all linear dependencies within these member sets by removing all nz4forms, each the sum of two nz2form members of the same **IDLA**[n][x]. Unlike above where the linearly dependent nz8forms did not close for bracket, our linearly dependent nz4form members separately close for bracket and Smap, as do the nz2form members taken in isolation.

The finished **IDLA**[n][x] nz4forms have the appearance of 15 element-negation pairs of augmented Dirac gamma matrices with algebraic orientations proper for their station in the 8x8 matrix adjacent Octonion Algebra setting. Call these fourteen sets Dirac[n: 1-7][x: 0, 1]. Dirac[n][0] sets occupy Quaternion[n] indexed rows and columns in the 8x8 adjacent algebra forms, while Dirac[n][1] sets occupy Quaternion[n] basic quad indexed rows and columns in the 8x8 adjacent algebra forms.

Each Dirac[n][x] member is an nz4form skew-Hermitian unitary matrix with a 4x4 reduced matrix algebraic invariant determinant of +1. The 15 reduced to 4x4 matrix positive members are 3 diagonal forms and four flavors of each of the 3 Dirac gamma matrix nzforms. The positive members of Dirac[n][x] form bases for 14 separate algebraic orientation covariant su(4) symmetry Octonion measure spaces. Their positive matrices are itemized in Appendix 2.

As itemized and indexed in Appendix 2, each of the fourteen Dirac[n][x] Lie algebras have identical bracket tables. As shown, the three rows of four non-diagonal matrices of positive member Dirac[n][1] indicate with the first two columns the positive members of two **LA**[n] ideals, and the last two columns are two maximal commutating sets. The lack of row/column intersection between Dirac[n][0] and Dirac[n][1] indicates matrix products of elements between the two are always the null matrix **0**.

This orthogonality and the fact Dirac[n][0] and Dirac[n][1] have identical bracket tables allows us to add each Dirac[n][0][x] and Dirac[n][1][x] member matrix to form an nz8form direct sum Dirac set once again with bracket table identical to all other Dirac[n][x]. Each matrix in these direct sums are nz8form skew-Hermitian unitary matrices with an algebra orientation invariant determinant of +1.

We find the 18 nz2form element-negation pairs of each **IDLA**[n][x] have the appearance of complexified Pauli matrices $i\sigma_1$, $i\sigma_2$, and $i\sigma_3$, with assigned algebraic orientations proper for their station in the 8x8 matrix adjacent Octonion Algebra setting. Call these Pauli[n: 1-7][x: 0,1].

The members of each Pauli[n][x] set are skew-Hermitian unitary matrices with a 2x2 reduced matrix algebraic invariant determinant of +1, where the 18 positive members form bases for six separate algebraic orientation covariant su(2) Lie Algebras.

Within a given Pauli[n][x], each of its six separate su(2) basis triplets show up in two other Pauli[m][x], and each of the three occurrences are identical. We therefore have only 28 unique triplets of skew-Hermitian algebraic orientation covariant su(2) Lie Algebra bases. These unique Pauli[n][x] 2x2 submatrices of the 8x8 adjacent algebra form occupy all 28 row/column pairs of the full 8x8 matrix, thus spanning the full 8x8 adjacent algebra matrix format. These 28 are itemized in Appendix 3. Along with the null matrix and their negations, the set of 169 elements are closed for both bracket and Smap.

If we look closely at the algebraic orientations of each position within the 8x8 adjacent algebra matrices for all Dirac[n][x] and Pauli[n][x], they are singularly defined. This is the most important takeaway for the somewhat laborious path we just followed to justify this conclusion.

The sum of all coset$_8$[n] builders cbl[n] or cbr[n] also span the oriented non-diagonal 8x8 adjacent matrix positions, and cbr[] and cbl[] equivalent positional algebraic orientation indicators match those of all nz2form members in Appendix 3.

This is important. The fixed algebraic orientations for our endomorphism linear operator 8x8 adjacent algebra matrix positions are generated by the 1-form and 2-form Clifford algebra choice equivalently defining cbl[x] and cbr[x] algebraic orientations. The choice between 28 different Clifford algebras is a free choice that determines which Octonion basis elements represent 3D axial vectors (2-forms) and 3D polar vectors (1-forms). The impact of each of these 28 choices are itemized in Appendix 4.

If we take any of the 28 matrices in Appendix 4, and keep just the rows and columns of its 1-form choice along with row/column 0, the resultant 4x4 submatrix off-diagonals are algebraic invariants, true to the fact polar vectors are not oriented. Doing the same for the three 2-forms and the 3-form row/column set, all 4x4 submatrix off-diagonal locations are oriented since both 2-forms and the 3-form are by nature oriented.

Since the su(2) nz2forms span the 8x8 matrix, we can state any possible adjacent algebra generated orientation covariant su(n) Lie algebra skew-Hermitian basis set must be built from Appendix 3 members or linear combinations thereof.

Using the techniques below to generate the su(4) Lie algebra occupying the [0, 1-form index] set of row/columns for our current selection we get the following positive basis set for this su(4) set of Dirac matrices.

$$\text{Dirac}[1] = \begin{array}{cc} & \begin{array}{cccc} 0&3&5&7 \end{array} \\ \begin{array}{c} 0\\3\\5\\7 \end{array} & \left[\begin{array}{cccc} +i&0&0&0\\ 0&-i&0&0\\ 0&0&+i&0\\ 0&0&0&-i \end{array}\right] \end{array} \quad \text{Dirac}[3] = \begin{array}{cc} & \begin{array}{cccc} 0&3&5&7 \end{array} \\ \begin{array}{c} 0\\3\\5\\7 \end{array} & \left[\begin{array}{cccc} +i&0&0&0\\ 0&+i&0&0\\ 0&0&-i&0\\ 0&0&0&-i \end{array}\right] \end{array} \quad \text{Dirac}[5] = \begin{array}{cc} & \begin{array}{cccc} 0&3&5&7 \end{array} \\ \begin{array}{c} 0\\3\\5\\7 \end{array} & \left[\begin{array}{cccc} +i&0&0&0\\ 0&-i&0&0\\ 0&0&-i&0\\ 0&0&0&+i \end{array}\right] \end{array}$$

$$\text{Dirac}[7] = \begin{array}{cc} & \begin{array}{cccc} 0&3&5&7 \end{array} \\ \begin{array}{c} 0\\3\\5\\7 \end{array} & \left[\begin{array}{cccc} 0&+i&0&0\\ +i&0&0&0\\ 0&0&0&+i\\ 0&0&+i&0 \end{array}\right] \end{array} \quad \text{Dirac}[9] = \begin{array}{cc} & \begin{array}{cccc} 0&3&5&7 \end{array} \\ \begin{array}{c} 0\\3\\5\\7 \end{array} & \left[\begin{array}{cccc} 0&+i&0&0\\ +i&0&0&0\\ 0&0&0&-i\\ 0&0&-i&0 \end{array}\right] \end{array} \quad \text{Dirac}[11] = \begin{array}{cc} & \begin{array}{cccc} 0&3&5&7 \end{array} \\ \begin{array}{c} 0\\3\\5\\7 \end{array} & \left[\begin{array}{cccc} 0&+1&0&0\\ -1&0&0&0\\ 0&0&0&+1\\ 0&0&-1&0 \end{array}\right] \end{array} \quad \text{Dirac}[13] = \begin{array}{cc} & \begin{array}{cccc} 0&3&5&7 \end{array} \\ \begin{array}{c} 0\\3\\5\\7 \end{array} & \left[\begin{array}{cccc} 0&+1&0&0\\ -1&0&0&0\\ 0&0&0&-1\\ 0&0&+1&0 \end{array}\right] \end{array}$$

$$\text{Dirac}[15] = \begin{array}{cc} & \begin{array}{cccc} 0&3&5&7 \end{array} \\ \begin{array}{c} 0\\3\\5\\7 \end{array} & \left[\begin{array}{cccc} 0&0&0&+i\\ 0&0&+i&0\\ 0&+i&0&0\\ +i&0&0&0 \end{array}\right] \end{array} \quad \text{Dirac}[17] = \begin{array}{cc} & \begin{array}{cccc} 0&3&5&7 \end{array} \\ \begin{array}{c} 0\\3\\5\\7 \end{array} & \left[\begin{array}{cccc} 0&0&0&+i\\ 0&0&-i&0\\ 0&-i&0&0\\ +i&0&0&0 \end{array}\right] \end{array} \quad \text{Dirac}[19] = \begin{array}{cc} & \begin{array}{cccc} 0&3&5&7 \end{array} \\ \begin{array}{c} 0\\3\\5\\7 \end{array} & \left[\begin{array}{cccc} 0&0&0&+1\\ 0&0&+1&0\\ 0&-1&0&0\\ -1&0&0&0 \end{array}\right] \end{array} \quad \text{Dirac}[21] = \begin{array}{cc} & \begin{array}{cccc} 0&3&5&7 \end{array} \\ \begin{array}{c} 0\\3\\5\\7 \end{array} & \left[\begin{array}{cccc} 0&0&0&+1\\ 0&0&-1&0\\ 0&+1&0&0\\ -1&0&0&0 \end{array}\right] \end{array}$$

Dirac[23] = $$\begin{array}{cc} & \begin{matrix}0 & 3 & 5 & 7\end{matrix} \\ \begin{matrix}0\\3\\5\\7\end{matrix} & \begin{bmatrix}0 & 0 & +i & 0\\0 & 0 & 0 & +i\\+i & 0 & 0 & 0\\0 & +i & 0 & 0\end{bmatrix}\end{array}$$

Dirac[25] = $$\begin{array}{cc} & \begin{matrix}0 & 3 & 5 & 7\end{matrix} \\ \begin{matrix}0\\3\\5\\7\end{matrix} & \begin{bmatrix}0 & 0 & +i & 0\\0 & 0 & 0 & -i\\+i & 0 & 0 & 0\\0 & -i & 0 & 0\end{bmatrix}\end{array}$$

Dirac[27] = $$\begin{array}{cc} & \begin{matrix}0 & 3 & 5 & 7\end{matrix} \\ \begin{matrix}0\\3\\5\\7\end{matrix} & \begin{bmatrix}0 & 0 & +1 & 0\\0 & 0 & 0 & +1\\-1 & 0 & 0 & 0\\0 & -1 & 0 & 0\end{bmatrix}\end{array}$$

Dirac[29] = $$\begin{array}{cc} & \begin{matrix}0 & 3 & 5 & 7\end{matrix} \\ \begin{matrix}0\\3\\5\\7\end{matrix} & \begin{bmatrix}0 & 0 & +1 & 0\\0 & 0 & 0 & -1\\-1 & 0 & 0 & 0\\0 & +1 & 0 & 0\end{bmatrix}\end{array}$$

These show the maximal su(4) Lie algebra members occupying row/columns [0, 1-forms] are algebraic invariants. Likewise, any su(2) Lie algebra members resident within this 4x4 submatrix will be algebraic invariants.

Now that we have justified the fact that all linear operator adjacent algebra matrices have fixed orientations, we can cut to the chase scene so to speak by selecting the Appendix 4 symmetric orientation matrix appropriate for our choice of 3D polar/axial Octonion bases, call it S_{ij}. Next set up the 28 su(2) orientations from this choice by assigning the algebraic orientation indicator indicated in S_{ij} to the two off diagonal su(2) members relevant to row/column set [i, j]. From these, any su(n) submatrix or su(8) full 8x8 matrix Lie algebra can be created appropriately for our chosen mapping for Octonion Algebra into 4D perception space.

Generating all fundamental algebraic orientation covariant su(n) Lie algebra matrix sets

Above, we generated Dirac su(4) Lie algebras built exclusively from nz4form matrices that closed for bracket to the desired $n^2 - 1$ dimension of 15 positive basis matrices. This will not be possible for all adjacent algebra su(n), only additionally for su(8). Lie algebra su(8) will be seen to be buildable exclusively from nz8forms, closing for bracket with 63 positive basis matrices. However, any su(n: 2 through 8) can be built with off-diagonal basis members defined by su(2) members. Define these as *fundamental* su(n) Lie algebras. The justification for calling them fundamental is 2x2 su(2) matrices are the smallest dimension where forming the adjoint matrix can exchange non-zero members in its transpose operation. Additionally, larger dimension skew-Hermitian matrices are built with linear combinations of them.

Pick *any* like row/column set nxn submatrix of our 8x8 adjacent matrix form, determine all $n! / (2! (n-2)!)$ pairs of row/column indexes in its definition to build all $n! / (n-2)!$ off-diagonal real and imaginary positive members from the Pauli matrices they determine. If we take just this off-diagonal set as the generating set, bracket closure will additionally pick up the Pauli diagonals, leading to $3 \{n! / (2! (n-2)!)\}$ positive non-null matrices in the bracket closure, which exceeds the required $n^2 - 1$ count.

Take the definition of orthogonality between matrices A and B to mean their Hilbert-Schmidt inner product is zero. The overage comes from the fact that while all positive off-diagonals are Hilbert-Schmidt orthogonal, the positive diagonals are not. We can find $n - 1$ linear combinations of them which together with the off-diagonals are all mutually orthogonal, giving $n! / (n-2)!$ off-diagonal positive members plus $n - 1$ positive diagonal members for the proper total of $n^2 - 1$ members for the su(n) Lie algebra basis set.

The following typical diagonal sets can be used when the su(n) off-diagonal members are su(2) members, taking the first $n - 1$ diagonals in this list. Each are linear combinations of the full set of relevant su(2) diagonals. They span the required zero trace diagonal bases, and the Hilbert-Schmidt inner product of any two along with the off-diagonals is zero. The unnormalized diagonals are

Diagonal[2] := [+i, –i, 0, 0, 0, 0, 0, 0]
Diagonal[3] := [+i, +i, –2i, 0, 0, 0, 0, 0]
Diagonal[4] := [+i, +i, +i, –3i, 0, 0, 0, 0]

Diagonal[5] := [+i, +i, +i, +i, –4i, 0, 0, 0]
Diagonal[6] := [+i, +i, +i, +i, +i, –5i, 0, 0]
Diagonal[7] := [+i, +i, +i, +i, +i, +i, –6i, 0]
Diagonal[8] := [+i, +i, +i, +i, +i, +i, +i, –7i]

The diagonal set for the su(n) Lie algebra basis for Lie group SU(n) are the n – 1 Diagonal[2] through Diagonal[n].

They are shown in sequential matrix row/column positions 0 through 7 which can be mapped to any relocation row/column sets in ascending order, e.g., for row/column set [1, 3, 5], the diagonals are

[0, +i, 0, –i, 0, 0, 0, 0]
[0, +i, 0, +i, 0, –2i, 0, 0]

Appending these n – 1 diagonals to the full complement of off-diagonals produces a mutually orthogonal set of $n^2 - 1$ 8x8 matrices, the correct size for an su(n) basis. However, this set is not closed for bracket, since closure will pick up all of the 2x2 Pauli diagonals we replaced with the diagonal sets just defined.

The full set of fundamental linearly independent su(n) Lie algebras given as sets of our 28 su(2) Pauli Lie algebras is given in Appendix 5.

The standard 3x3 Gell-Mann matrix diagonals are Diagonal[2] and Diagonal[3]. The 3x3 submatrices can be placed in 8! / (3! (8-3)!) = 56 different positions within the 8x8 adjacent algebra matrix format. We can find four of them in any Dirac[n][x] partition.

Picking one at random as living in rows/columns [2, 4, 6], Appendix 5 gives us

Fundamental su(3)[41] spanning rows/columns [2, 4, 6]
{σ[14], σ[16], σ[23]}

Row/column pair [2, 4] selects σ[14], [2, 6] selects σ[16], and [4, 6] selects σ[23]. From these, we get our three off-diagonal symmetric imaginary and three off-diagonal antisymmetric real su(3) Lie algebra members. The remaining two diagonals are

[0, 0, +i, 0, –i, 0, 0, 0]
[0, 0, +i, 0, +i, 0, –2i, 0]

Their member itemization is

Gell-Mann[1][0][3]

$$\lambda[1] = \begin{array}{cc} & \begin{matrix} 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & +is_{12} & 0 \\ +is_{12} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \quad \lambda[2] = \begin{array}{cc} & \begin{matrix} 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & +is_4 \\ 0 & 0 & 0 \\ +is_4 & 0 & 0 \end{bmatrix} \end{array} \quad \lambda[3] = \begin{array}{cc} & \begin{matrix} 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & +is_8 \\ 0 & +is_8 & 0 \end{bmatrix} \end{array} \quad \lambda[7] = \begin{array}{cc} & \begin{matrix} 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} +i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\lambda[4] = \begin{array}{cc} & \begin{matrix} 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & +1s_{12} & 0 \\ -1s_{12} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \quad \lambda[5] = \begin{array}{cc} & \begin{matrix} 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & +1s_4 \\ 0 & 0 & 0 \\ -1s_4 & 0 & 0 \end{bmatrix} \end{array} \quad \lambda[6] = \begin{array}{cc} & \begin{matrix} 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & +1s_8 \\ 0 & -1s_8 & 0 \end{bmatrix} \end{array} \quad \lambda[8] = \begin{array}{cc} & \begin{matrix} 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} +i & 0 & 0 \\ 0 & +i & 0 \\ 0 & 0 & -2i \end{bmatrix} \end{array}$$

Let's next do the fundamental su(6) for rows/columns [2, 3, 4, 5, 6, 7] representing our Clifford 1-forms and 2-

forms. From Appendix 5 we have

Fundamental set of su(2) Pauli matrices for su(6)[27] spanning rows/columns [2, 3, 4, 5, 6, 7]
{σ[13], σ[14], σ[15], σ[16], σ[17], σ[18], σ[19], σ[20], σ[21], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}

Rather than putting out all 35 matrices, the off-diagonal positive su(2) members can be succinctly shown as a single summed matrix. The 30 member sum is as follows

non-diagonal su(6) =

$$\begin{array}{c|c} & \begin{array}{cccccc} 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} & \left[\begin{array}{llllll} 0 & is_{11}\ {+1}s_{11} & is_{12}\ {+1}s_{12} & is_{11}\ {+1}s_{11} & is_{4}\ {+1}s_{4} & is_{11}\ {+1}s_{11} \\ is_{11}\ {-1}s_{11} & 0 & is_{7}\ {+1}s_{7} & i\ {+1} & is_{15}\ {+1}s_{15} & i\ {+1} \\ is_{12}\ {-1}s_{12} & is_{7}\ {-1}s_{7} & 0 & is_{7}\ {+1}s_{7} & is_{8}\ {+1}s_{8} & is_{7}\ {+1}s_{7} \\ is_{11}\ {-1}s_{11} & i\ {-1} & is_{7}\ {-1}s_{7} & 0 & is_{15}\ {+1}s_{15} & i\ {+1} \\ is_{4}\ {-1}s_{4} & is_{15}\ {-1}s_{15} & is_{8}\ {-1}s_{8} & is_{15}\ {-1}s_{15} & 0 & is_{15}\ {+1}s_{15} \\ is_{11}\ {-1}s_{11} & i\ {-1} & is_{7}\ {-1}s_{7} & i\ {-1} & is_{15}\ {-1}s_{15} & 0 \end{array}\right] \end{array}$$

The five unnormalized diagonals are

Diagonal[2] := [+i, –i, 0, 0, 0, 0, 0, 0]
Diagonal[3] := [+i, +i, –2i, 0, 0, 0, 0, 0]
Diagonal[4] := [+i, +i, +i, –3i, 0, 0, 0, 0]
Diagonal[5] := [+i, +i, +i, +i, –4i, 0, 0, 0]
Diagonal[6] := [+i, +i, +i, +i, +i, –5i, 0, 0]

Generating su(8) with nz8forms and all su(4) with nz4forms

Lie algebras su(4) and su(8) are different than the others in that their zero trace diagonals can be built from nz4forms and nz8forms respectively, which are the n – 1 non-identity members of n dimensional Hadamard matrices scaled by i. Their bracket closure non-null positive element count is $n^2 – 1$; there are no superfluous members created by bracket closure beyond the desired spanning vector space dimension $n^2 – 1$.

We nicely fell into Quaternion subalgebra partitioned su(4) above when our derivation popped out fourteen nz4form Dirac[n][x] Lie algebras. We will find now that our derivation of su(8) can be brought to bear on su(4), allowing us to park su(4) in any of the 70 common row/column index 4x4 submatrix choices.

In the fundamental su(n) derivation above, we found off-diagonal su(2) member forms by determining all pairs of nxn submatrix row/column indexes, which determined all member Pauli matrices whose off-diagonal members provide the full complement of off-diagonal members for its su(n) representation. Instead, we will now take the complete row/column index set and find for it all unique pair sets (pair order inconsequential) of n/2 pairs of spanning indexes, for example {[0,1], [2,3], [4,5], [6,7]}.

First, su(8).

For su(8) we find 105 unique pair sets spanning integers 0 through 7 two at a time. Each of the four pairs of two in a set specifies a different non-overlapping Pauli su(2) from the set of 28. We can construct real and imaginary nz8forms by adding the respective off-diagonal types from the four specified su(2). This gives us 105 sets of real and 105 sets of imaginary nz8form matrices.

We have more than we need because they are not all Hilbert-Schmidt orthogonal. Remedy the overage by individually limiting the real and imaginary off-diagonal sets to mutually orthogonal members starting with the two off-diagonals generated from assumed present {[0,1], [2,3], [4,5], [6,7]}. Doing so, we find a maximal mutually orthogonal set of seven real and seven imaginary off-diagonal {element-negation} pairs. They are itemized in Appendix 6. They define the generating set for our su(8) basis.

Closing this generating set for bracket yields a set of dimension 127: the null matrix and 63 {element, negation} pairs. The 63 non-null positive elements form the adjacent algebra su(8) Lie algebra basis for the Lie group SU(8).

The Smap partition set for this su(8) by index as enumerated in Appendix 7 is

su(8) Smap partition[0] = [[0], [0]]
su(8) Smap partition[1] = [[1], [1]]
su(8) Smap partition[2] = [[2], [2]]
su(8) Smap partition[3] = [[3], [3]]
su(8) Smap partition[4] = [[4], [4]]
su(8) Smap partition[5] = [[5], [5]]
su(8) Smap partition[6] = [[6], [6]]
su(8) Smap partition[7] = [[7], [7]]
su(8) Smap partition[8] = [[8], [8]]
su(8) Smap partition[9] = [[9], [9]]
su(8) Smap partition[10] = [[10], [10]]
su(8) Smap partition[11] = [[11], [11]]
su(8) Smap partition[12] = [[12], [12]]
su(8) Smap partition[13] = [[13], [13]]
su(8) Smap partition[14] = [[14], [14]]
su(8) Smap partition[15] = [[15, 16, 17, 18, 19, 20, 21, 22], []]
su(8) Smap partition[16] = [[23, 24, 25, 26, 27, 28, 29, 30], []]
su(8) Smap partition[17] = [[31, 33, 36, 37], []]
su(8) Smap partition[18] = [[32, 34, 35, 38], []]
su(8) Smap partition[19] = [[39, 42, 44, 45], []]
su(8) Smap partition[20] = [[40, 41, 43, 46], []]
su(8) Smap partition[21] = [[47, 50, 52, 54], []]
su(8) Smap partition[22] = [[48, 49, 51, 53], []]
su(8) Smap partition[23] = [[55, 58, 59, 62], []]
su(8) Smap partition[24] = [[56, 57, 60, 61], []]
su(8) Smap partition[25] = [[63, 65, 68, 69], []]
su(8) Smap partition[26] = [[64, 66, 67, 70], []]
su(8) Smap partition[27] = [[71, 74, 76, 78], []]
su(8) Smap partition[28] = [[72, 73, 75, 77], []]
su(8) Smap partition[29] = [[79, 82, 84, 85], []]
su(8) Smap partition[30] = [[80, 81, 83, 86], []]
su(8) Smap partition[31] = [[87, 90, 91, 94], []]
su(8) Smap partition[32] = [[88, 89, 92, 93], []]
su(8) Smap partition[33] = [[95, 98, 100, 101], []]
su(8) Smap partition[34] = [[96, 97, 99, 102], []]
su(8) Smap partition[35] = [[103, 106, 107, 110], []]
su(8) Smap partition[36] = [[104, 105, 108, 109], []]
su(8) Smap partition[37] = [[111, 113, 115, 118], []]
su(8) Smap partition[38] = [[112, 114, 116, 117], []]
su(8) Smap partition[39] = [[119, 122, 124, 125], []]
su(8) Smap partition[40] = [[120, 121, 123, 126], []]

The first 15 elements are null and the 14 non-identity sign paired Hadamard **H**[] matrix diagonals which are algebraic orientation invariant. All other members have a mix of one algebraic orientation invariant pair and three oriented pairs. This Lie algebra is closed for **R0** → **Rn** Smaps, but the **R0** → **Ln** Smaps on every oriented member are not present in the Lie algebra. The adjacent algebra Lie algebra su(8) is by its nature chiral.

We can rename this su(8) $su(8)_{right}$ and form $su(8)_{left}$ with the Smap **R0** → **L0** on all members, which is the same as negating all oriented matrix locations. Doing so, we find $su(8)_{right}$ and $su(8)_{left}$ have identical bracket tables and can say the two distinct su(8) Lie algebras form the Smap partition

[$\{su(8)_{right}\}$, $\{su(8)_{left}\}$]

The 63 matrices for the Lie algebra $su(8)_{right}$ basis are presented in Appendix 7.

Moving on to su(4), we have options as to where to park our four common row/column set choices, and we are free to pick any one of 70 choices. Label the choices a, b, c and d where a < b < c < d.

Following the same process used for su(8) we find thee unique pair sets for two off-diagonals each given by

For x: 0, 1
Pairset[0] = [[a, b], [c, d]]   Pairset[0] sum[x] = σ[[a,b]] [x] + σ[[c,d]] [x] spans nz4form
Pairset[1] = [[a, c], [b, d]]   Pairset[1] sum[x] = σ[[a,c]] [x] + σ[[b,d]] [x] spans nz4form
Pairset[2] = [[a, d], [b, c]]   Pairset[2] sum[x] = σ[[a,d]] [x] + σ[[b,c]] [x] spans nz4form

Build the individual sets [m] of real and imaginary off-diagonal nz4forms [x]: Pairset[m: 0, 1, 2] sum[x: 0, 1]. The three m for x = 0 are off-diagonal symmetric imaginary matrices, the three m for x=1 are off-diagonal antisymmetric real matrices, all skew-Hermitian. Each set of three off-diagonals are mutually orthogonal within, so no count reduction is required. As with su(8) we assume Pairset[0] = [[a, b], [c, d]] a < b < c < d off-diagonals will be present then restrict the orthogonal set to members whose Hilbert-Schmidt inner products with the other two are zero, which both are.

The orthogonal generating set is then six off-diagonals from the three Pairset[m]. This generating set closes for bracket to a set of 31 nz4form members, the null matrix and 15 pairs of {element, negation}. The 15 non-null positive members form the nz4form su(4) Lie algebra basis for Lie group SU(4), parked anywhere within the 8x8 adjacent algebra matrix format with common row/column index sets. As systematically built, each of these 70 su(4) Lie algebra placements will have identical bracket tables. All 70 placements are closed for Smap.

For both su(8) and su(4), the Hilbert-Schmidt inner products of any two different positive members is zero. The positive elements thus form a proper basis for its $n^2 - 1$ dimension vector space. Unlike the bases for su(n ≠ 4, 8), these bases are closed for bracket with the required $n^2 - 1$ positive element dimension.

Standard Model su(2) ⊕ su(3) representations

Since we can easily park any fundamental su(n: 2 through 8) Lie algebra anywhere we choose within the adjacent algebra 8x8 matrix format, it is a simple matter to find numerous examples of separately placed su(2) and su(3) Lie algebras without row/column overlap which will form a direct sum.

Where to place both is a matter of what feature of Octonion algebraic elements one wishes to apply the implied symmetry to. The 8x8 matrix row/columns tie back to the $coset_8[x]$ builder Clifford algebra 1-forms, 2-forms and 3-form since the adjacent algebra matrices operate naturally on the Octonion intrinsic basis elements. A natural application would be to attach the su(2) portion of the direct sum to the identity (0) and 3-form row/column pair. We have made the choice here that the 3-form $coset_8$[] builder is cbl[1] so we might look to park su(2) in row/column pair [0, 1]. We likewise have 1-forms represented in the row/column set [3, 5, 7] and 2-forms in the row/column set [2, 4, 6], so either of these could be a natural place to park the other part of the direct sum su(3).

Our closed for bracket sets will be as above, {null, pairs of (element, its negation)}. We will build all from full sets of Pauli matrices itemized in Appendix 3, and take the basis from the set of orthogonal non-null positive elements. Each Pauli σ[x] used then contributes three {element, negation} pairs.

Our common su(2) occupying row/column set [0, 1] is given by σ[0][0], σ[0][1] and σ[0][2] and their negations.

The 1-form representations are row/column pairs [3, 5], [3, 7] and [5, 7]. Their Pauli matrix sets are σ[19], σ[21] and σ[26] respectively. These yield nine pairs of {element, negation}.

The 2-form representations are row/column pairs [2, 4], [2, 6] and [4, 6]. Their Pauli matrix sets are σ[14], σ[16] and σ[23] respectively. These yield nine pairs of {element, negation}.

The 1-form positive member set are the 12 elements given by

σ[0][0], σ[0][1], σ[0][2]
σ[19][0], σ[19][1], σ[19][2]
σ[21][0], σ[21][1], σ[21][2]
σ[26][0], σ[26][1], σ[26][2]

The 2-form positive member set are the 12 elements given by

σ[0][0], σ[0][1], σ[0][2]
σ[14][0], σ[14][1], σ[14][2]
σ[16][0], σ[16][1], σ[16][2]
σ[23][0], σ[23][1], σ[23][2]

Both of these are closed for bracket as {null, 12 pairs of (element, negation)} and as formed have identical bracket tables. Forming all Hilbert-Schmidt inner products between, we find three separate maximal mutually orthogonal sets indicating the three su(3) diagonals {σ[19][2], σ[21][2], σ[26][2]} for 1-forms and {σ[14][2], σ[16][2], σ[23][2]} for 2-forms are each individually mutually orthogonal sets with their off-diagonals, but are not orthogonal to each other.

We remedied this issue above doing all Gell-Mann submatrices by realizing we could pick one of its three diagonals along with the sum of the other two for our two required zero-trace Gell-Mann diagonals. Doing this here gives us the correct number of 11 mutually independent positive element basis matrices for su(2) ⊕ su(3) for both places we decided to park them at within the 8x8 adjacent algebra matrix format.

Positive elements for both representations of these particular placements of su(2) ⊕ su(3) are given in Appendix 8.

Since the 1-form and 2-form triplets are in separate row/column sets, we can easily form their direct sum along with the common su(2) as su(2) ⊕ su(3) ⊕ su(3).

Conclusions

Octonion Algebra provides a proper basis for separately maintaining 3D perception space axial and polar types of vectors. These disparate types have a connection to the Clifford algebra Cl(3) where the three oriented axial vector 2-forms are spanned by the non-scalar triplet of basis elements of any one of the seven Quaternion subalgebras, the three polar 1-forms and the 3-form span the selected Quaternion subalgebra's basic quad set of basis elements.

A connection between Octonion Algebra bases and adjacent algebra matrices is made by matricizing the effect of each basis element acting on the full Octonion basis vector space from the right and left application side. These matrices must be constructed in an algebraic orientation covariant fashion. This one-to-one correlation between Octonion basis elements and left/right action matrices extends to two different groups generated by each, where each associated left/right action matrix partitions its group into quotient group cosets where the kernel is the set of diagonal matrices.

Within each coset we find the matrices of any possible Cl(3) generated by an appropriate choice of 1-forms for the free choice of axial/polar bases taken from the initial left and right action adjacent algebra sets. The very same cosets are created by the eight Clifford algebra matrices scaling the normal subgroup of diagonal members. With no loss of fundamental group structure, these Clifford algebra matrices replace the initial right and left action adjacent algebra matrices while maintaining identical direct connections to the eight Octonion basis elements.

The rub with the right and left application groups is that they are not even partially closed for Smap. This is remedied by creating a new left/right application inclusive Smap closed group generated by the union of the two Clifford algebra sets. Whereas the right and left application groups are diagonal, symmetric and antisymmetric matrices, this new group includes matrices that are a mix of symmetric and antisymmetric positions; n-types. Each n-type squares to ± one of the non-identity Hadamard matrices.

Seeding Lie algebras with sets of common squares to $\pm\mathbf{H}[n]$ n-types leads beautifully to fourteen skew-Hermitian Dirac su(4) Lie algebras positioned within 4x4 submatrices of the 8x8 adjacent algebra matrix format corresponding to either an Octonion Quaternion subalgebra or its basic quad set. These Lie algebras also include six skew-Hermitian su(2) members. Both indicate each position of any adjacent algebra Lie algebra 8x8 matrix is fixed, set by the particular choice of Clifford algebra.

We are able to build numerous su(n: 2 through 8) Lie algebras in any common row/column set within the 8x8 adjacent algebra matrix format. This flexibility allows building numerous direct sums of different su(n) types simply by avoiding common row/column positioning.

Our 8x8 adjacent algebra matrix forms are linear operators on the space of Octonion Algebra algebraic elements. Each of the full complement of possible placements for su(n: 2-8) skew-Hermitian Lie algebra adjacent algebra bases operate on a different Octonion algebraic element subspace. Each su(n) implies an inherent symmetry on its subspace, which ties back to our 1D-3D perception space naturally through our particular choice of Clifford algebras.

Octonion Algebra structure is highly directive in a consistent and logical way, devoid of arbitrariness. The degree to which this is recognized is inversely proportional to one's comfort with mathematical abstraction. Over-abstraction of the concept of homomorphisms has led many to naively believe all Octonion Algebra orientations are isomorphic, so there is no need to delve into orientation differences. This is sometimes demonstrated by the homomorphism map where any number of negations of the non-scalar basis elements absorbed into the algebra's structure constants changes orientation without breaking the division algebra status, leading to the false conclusion all Octonion Algebra orientations are isomorphic.

The fact of the manner is Octonion Algebra is the union of two distinct (read non-isomorphic) structures whose structural difference is easily and consistently demonstrated above in the Right and Left Octonion orientation definitions. This manuscript is replete with examples of how Right and Left Octonion orientations manifest themselves differently in the analysis.

The basis element negation homomorphisms map **Ra** ↔ **Rb** and **La** ↔ **Lb** for an even count of negations, but maps **Ra** ↔ **Lb** for an odd count of negations, disrespecting the structural difference between Left and Right orientations. And yet, the negation maps are legitimate homomorphism maps that are set-wise one-to-one and onto.

The proper conclusion should be the non-scalar basis element negation homomorphism is an automorphism on the union of the two Octonion orientation structures into the same union of two structures, not that there is an automorphism (isomorphism) between any two distinct division algebra substructures.

The correct and proper Octonion Algebra homomorphism map is the group of non-scalar basis element permutations that respect the Quaternion subalgebra triplet enumerations, where it can be shown all odd permutations will not do. These then are the 30 order 168 subgroups of group A_7, the even permutations of seven objects, one for each of the 30 ways to partition the Quaternion subalgebras. All 30 are isomorphic to PSL(2,7). The appropriate PSL(2,7) for a given Quaternion enumeration properly maps *only* Right Octonion to Right Octonion, and *only* Left Octonion to Left Octonion, respecting the two division algebra substructures. PSL(2,7) is the structural automorphism group for Octonion Algebra.

This naïve view that all Octonion Algebra orientations are isomorphic has led most researchers attempting to use Octonion Algebra to chuck a substantial amount of directive structure, as witnessed in this manuscript. Moreover, even though all Right Octonion orientations are isomorphic, as are all Left Octonion algebraic orientations, significant and meaningful structure is embedded within each of their eight orientation choices, as witnessed by the role Smap had on the above presentation.

I am only semi-amazed by the final Lie algebra results here, where we can park any su(n) Lie algebra every way it fits in a common row/column submatrix of the 8x8 adjacent algebra matrix format. This would not be a big deal if the 8x8 matrices were devoid of algebraic orientation, but they are not. None the less, we can park our su(n) anywhere within common row/column submatrix locations, and the fixed algebraic orientations for each matrix position do not get in the way. This *is* a big deal.

In my greater than three-decade long pursuit of this Octonion Algebra mistress, time and again I have witnessed tapestries of beautifully consistent mathematical structures. In my humble opinion, to borrow a phrase from Grand Unified Theory, *Octonion Algebra is the algebra of everything*, not coincidental to the title of reference [12].

Appendix 1: Group $\mathbb{A}$ conjugacy classes and Smap partitions by $coset_8$[]

The following are the full set of oriented member cc[size][instance] conjugacy classes and Smap partitions by instance where for a partition [[set a], [set b]] Smap **R0** → **L0** maps [set a] to [set b] and [set b] to [set a]. All are shown by group $\mathbb{A}$ indexes as built by the optimal sequence prescription in the text above. They are partitioned by the $coset_8$[] they live in.

$coset_8$[1]
cc[8][7] = [128, 129, 132, 133, 136, 137, 140, 141]
cc[8][8] = [130, 131, 134, 135, 138, 139, 142, 143]
cc[8][9] = [144, 145, 148, 149, 152, 153, 156, 157]
cc[8][10] = [146, 147, 150, 151, 154, 155, 158, 159]
cc[16][0] = [160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175]
cc[16][1] = [176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191]
cc[16][2] = [192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207]
cc[16][3] = [208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223]

cc[16][4] = [224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239]
cc[16][5] = [240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255]
𝔸 Smap partition[128] = [[128, 129, 132, 133, 136, 137, 140, 141], [144, 145, 148, 149, 152, 153, 156, 157]]
𝔸 Smap partition[129] = [[130, 131, 134, 135, 138, 139, 142, 143], [146, 147, 150, 151, 154, 155, 158, 159]]
𝔸 Smap partition[130] = [[160, 161, 164, 165, 168, 169, 172, 173], [176, 177, 180, 181, 184, 185, 188, 189]]
𝔸 Smap partition[131] = [[162, 163, 166, 167, 170, 171, 174, 175], [178, 179, 182, 183, 186, 187, 190, 191]]
𝔸 Smap partition[132] = [[192, 193, 196, 197, 200, 201, 204, 205], [208, 209, 212, 213, 216, 217, 220, 221]]
𝔸 Smap partition[133] = [[194, 195, 198, 199, 202, 203, 206, 207], [210, 211, 214, 215, 218, 219, 222, 223]]
𝔸 Smap partition[134] = [[224, 225, 228, 229, 232, 233, 236, 237], [240, 241, 244, 245, 248, 249, 252, 253]]
𝔸 Smap partition[135] = [[226, 227, 230, 231, 234, 235, 238, 239], [242, 243, 246, 247, 250, 251, 254, 255]]

$coset_8$[2]
cc[8][11] = [256, 257, 258, 259, 264, 265, 266, 267]
cc[8][12] = [260, 261, 262, 263, 268, 269, 270, 271]
cc[8][13] = [288, 289, 290, 291, 296, 297, 298, 299]
cc[8][14] = [292, 293, 294, 295, 300, 301, 302, 303]
cc[16][6] = [272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287]
cc[16][7] = [304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319]
cc[16][8] = [320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335]
cc[16][9] = [336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351]
cc[16][10] = [352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367]
cc[16][11] = [368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383]
𝔸 Smap partition[136] = [[256, 259, 265, 266], [289, 290, 296, 299]]
𝔸 Smap partition[137] = [[257, 258, 264, 267], [288, 291, 297, 298]]
𝔸 Smap partition[138] = [[260, 263, 269, 270], [293, 294, 300, 303]]
𝔸 Smap partition[139] = [[261, 262, 268, 271], [292, 295, 301, 302]]
𝔸 Smap partition[140] = [[272, 275, 281, 282], [305, 306, 312, 315]]
𝔸 Smap partition[141] = [[273, 274, 280, 283], [304, 307, 313, 314]]
𝔸 Smap partition[142] = [[276, 279, 285, 286], [309, 310, 316, 319]]
𝔸 Smap partition[143] = [[277, 278, 284, 287], [308, 311, 317, 318]]
𝔸 Smap partition[144] = [[320, 323, 329, 330], [352, 355, 361, 362]]
𝔸 Smap partition[145] = [[321, 322, 328, 331], [353, 354, 360, 363]]
𝔸 Smap partition[146] = [[324, 327, 333, 334], [356, 359, 365, 366]]
𝔸 Smap partition[147] = [[325, 326, 332, 335], [357, 358, 364, 367]]
𝔸 Smap partition[148] = [[336, 339, 345, 346], [369, 370, 376, 379]]
𝔸 Smap partition[149] = [[337, 338, 344, 347], [368, 371, 377, 378]]
𝔸 Smap partition[150] = [[340, 343, 349, 350], [373, 374, 380, 383]]
𝔸 Smap partition[151] = [[341, 342, 348, 351], [372, 375, 381, 382]]

$coset_8$[3]
cc[8][15] = [384, 385, 390, 391, 392, 393, 398, 399]
cc[8][16] = [386, 387, 388, 389, 394, 395, 396, 397]
cc[8][17] = [432, 433, 438, 439, 440, 441, 446, 447]
cc[8][18] = [434, 435, 436, 437, 442, 443, 444, 445]
cc[16][12] = [400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415]
cc[16][13] = [416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431]
cc[16][14] = [448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463]
cc[16][15] = [464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479]
cc[16][16] = [480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495]
cc[16][17] = [496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511]

𝔸 Smap partition[152] = [[384, 390, 392, 398], [433, 439, 441, 447]]
𝔸 Smap partition[153] = [[385, 391, 393, 399], [432, 438, 440, 446]]
𝔸 Smap partition[154] = [[386, 388, 394, 396], [435, 437, 443, 445]]
𝔸 Smap partition[155] = [[387, 389, 395, 397], [434, 436, 442, 444]]
𝔸 Smap partition[156] = [[400, 406, 408, 414], [416, 422, 424, 430]]
𝔸 Smap partition[157] = [[401, 407, 409, 415], [417, 423, 425, 431]]
𝔸 Smap partition[158] = [[402, 404, 410, 412], [418, 420, 426, 428]]
𝔸 Smap partition[159] = [[403, 405, 411, 413], [419, 421, 427, 429]]
𝔸 Smap partition[160] = [[448, 454, 456, 462], [497, 503, 505, 511]]
𝔸 Smap partition[161] = [[449, 455, 457, 463], [496, 502, 504, 510]]
𝔸 Smap partition[162] = [[450, 452, 458, 460], [499, 501, 507, 509]]
𝔸 Smap partition[163] = [[451, 453, 459, 461], [498, 500, 506, 508]]
𝔸 Smap partition[164] = [[464, 470, 472, 478], [481, 487, 489, 495]]
𝔸 Smap partition[165] = [[465, 471, 473, 479], [480, 486, 488, 494]]
𝔸 Smap partition[166] = [[466, 468, 474, 476], [483, 485, 491, 493]]
𝔸 Smap partition[167] = [[467, 469, 475, 477], [482, 484, 490, 492]]

$coset_8$[4]
cc[8][19] = [512, 513, 514, 515, 516, 517, 518, 519]
cc[8][20] = [520, 521, 522, 523, 524, 525, 526, 527]
cc[8][21] = [576, 577, 578, 579, 580, 581, 582, 583]
cc[8][22] = [584, 585, 586, 587, 588, 589, 590, 591]
cc[16][18] = [528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543]
cc[16][19] = [544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559]
cc[16][20] = [560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575]
cc[16][21] = [592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607]
cc[16][22] = [608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623]
cc[16][23] = [624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639]
𝔸 Smap partition[168] = [[512, 515, 517, 518], [577, 578, 580, 583]]
𝔸 Smap partition[169] = [[513, 514, 516, 519], [576, 579, 581, 582]]
𝔸 Smap partition[170] = [[520, 523, 525, 526], [585, 586, 588, 591]]
𝔸 Smap partition[171] = [[521, 522, 524, 527], [584, 587, 589, 590]]
𝔸 Smap partition[172] = [[528, 531, 533, 534], [593, 594, 596, 599]]
𝔸 Smap partition[173] = [[529, 530, 532, 535], [592, 595, 597, 598]]
𝔸 Smap partition[174] = [[536, 539, 541, 542], [601, 602, 604, 607]]
𝔸 Smap partition[175] = [[537, 538, 540, 543], [600, 603, 605, 606]]
𝔸 Smap partition[176] = [[544, 547, 549, 550], [608, 611, 613, 614]]
𝔸 Smap partition[177] = [[545, 546, 548, 551], [609, 610, 612, 615]]
𝔸 Smap partition[178] = [[552, 555, 557, 558], [616, 619, 621, 622]]
𝔸 Smap partition[179] = [[553, 554, 556, 559], [617, 618, 620, 623]]
𝔸 Smap partition[180] = [[560, 563, 565, 566], [625, 626, 628, 631]]
𝔸 Smap partition[181] = [[561, 562, 564, 567], [624, 627, 629, 630]]
𝔸 Smap partition[182] = [[568, 571, 573, 574], [633, 634, 636, 639]]
𝔸 Smap partition[183] = [[569, 570, 572, 575], [632, 635, 637, 638]]

$coset_8$[5]
cc[8][23] = [640, 641, 644, 645, 650, 651, 654, 655]
cc[8][24] = [642, 643, 646, 647, 648, 649, 652, 653]
cc[8][25] = [720, 721, 724, 725, 730, 731, 734, 735]
cc[8][26] = [722, 723, 726, 727, 728, 729, 732, 733]

cc[16][24] = [656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671]
cc[16][25] = [672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687]
cc[16][26] = [688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703]
cc[16][27] = [704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719]
cc[16][28] = [736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751]
cc[16][29] = [752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767]
𝔸 Smap partition[184] = [[640, 644, 650, 654], [721, 725, 731, 735]]
𝔸 Smap partition[185] = [[641, 645, 651, 655], [720, 724, 730, 734]]
𝔸 Smap partition[186] = [[642, 646, 648, 652], [723, 727, 729, 733]]
𝔸 Smap partition[187] = [[643, 647, 649, 653], [722, 726, 728, 732]]
𝔸 Smap partition[188] = [[656, 660, 666, 670], [704, 708, 714, 718]]
𝔸 Smap partition[189] = [[657, 661, 667, 671], [705, 709, 715, 719]]
𝔸 Smap partition[190] = [[658, 662, 664, 668], [706, 710, 712, 716]]
𝔸 Smap partition[191] = [[659, 663, 665, 669], [707, 711, 713, 717]]
𝔸 Smap partition[192] = [[672, 676, 682, 686], [753, 757, 763, 767]]
𝔸 Smap partition[193] = [[673, 677, 683, 687], [752, 756, 762, 766]]
𝔸 Smap partition[194] = [[674, 678, 680, 684], [755, 759, 761, 765]]
𝔸 Smap partition[195] = [[675, 679, 681, 685], [754, 758, 760, 764]]
𝔸 Smap partition[196] = [[688, 692, 698, 702], [737, 741, 747, 751]]
𝔸 Smap partition[197] = [[689, 693, 699, 703], [736, 740, 746, 750]]
𝔸 Smap partition[198] = [[690, 694, 696, 700], [739, 743, 745, 749]]
𝔸 Smap partition[199] = [[691, 695, 697, 701], [738, 742, 744, 748]]

$coset_8$[6]
cc[8][27] = [768, 769, 770, 771, 780, 781, 782, 783]
cc[8][28] = [772, 773, 774, 775, 776, 777, 778, 779]
cc[8][29] = [864, 865, 866, 867, 876, 877, 878, 879]
cc[8][30] = [868, 869, 870, 871, 872, 873, 874, 875]
cc[16][30] = [784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799]
cc[16][31] = [800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815]
cc[16][32] = [816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831]
cc[16][33] = [832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847]
cc[16][34] = [848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863]
cc[16][35] = [880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895]
𝔸 Smap partition[200] = [[768, 771, 781, 782], [865, 866, 876, 879]]
𝔸 Smap partition[201] = [[769, 770, 780, 783], [864, 867, 877, 878]]
𝔸 Smap partition[202] = [[772, 775, 777, 778], [869, 870, 872, 875]]
𝔸 Smap partition[203] = [[773, 774, 776, 779], [868, 871, 873, 874]]
𝔸 Smap partition[204] = [[784, 787, 797, 798], [881, 882, 892, 895]]
𝔸 Smap partition[205] = [[785, 786, 796, 799], [880, 883, 893, 894]]
𝔸 Smap partition[206] = [[788, 791, 793, 794], [885, 886, 888, 891]]
𝔸 Smap partition[207] = [[789, 790, 792, 795], [884, 887, 889, 890]]
𝔸 Smap partition[208] = [[800, 803, 813, 814], [832, 835, 845, 846]]
𝔸 Smap partition[209] = [[801, 802, 812, 815], [833, 834, 844, 847]]
𝔸 Smap partition[210] = [[804, 807, 809, 810], [836, 839, 841, 842]]
𝔸 Smap partition[211] = [[805, 806, 808, 811], [837, 838, 840, 843]]
𝔸 Smap partition[212] = [[816, 819, 829, 830], [849, 850, 860, 863]]
𝔸 Smap partition[213] = [[817, 818, 828, 831], [848, 851, 861, 862]]
𝔸 Smap partition[214] = [[820, 823, 825, 826], [853, 854, 856, 859]]
𝔸 Smap partition[215] = [[821, 822, 824, 827], [852, 855, 857, 858]]

coset$_8$[7]
cc[8][31] = [896, 897, 902, 903, 906, 907, 908, 909]
cc[8][32] = [898, 899, 900, 901, 904, 905, 910, 911]
cc[8][33] = [1008, 1009, 1014, 1015, 1018, 1019, 1020, 1021]
cc[8][34] = [1010, 1011, 1012, 1013, 1016, 1017, 1022, 1023]
cc[16][36] = [912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927]
cc[16][37] = [928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943]
cc[16][38] = [944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959]
cc[16][39] = [960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975]
cc[16][40] = [976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991]
cc[16][41] = [992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007]
𝔸 Smap partition[216] = [[896, 902, 906, 908], [1009, 1015, 1019, 1021]]
𝔸 Smap partition[217] = [[897, 903, 907, 909], [1008, 1014, 1018, 1020]]
𝔸 Smap partition[218] = [[898, 900, 904, 910], [1011, 1013, 1017, 1023]]
𝔸 Smap partition[219] = [[899, 901, 905, 911], [1010, 1012, 1016, 1022]]
𝔸 Smap partition[220] = [[912, 918, 922, 924], [992, 998, 1002, 1004]]
𝔸 Smap partition[221] = [[913, 919, 923, 925], [993, 999, 1003, 1005]]
𝔸 Smap partition[222] = [[914, 916, 920, 926], [994, 996, 1000, 1006]]
𝔸 Smap partition[223] = [[915, 917, 921, 927], [995, 997, 1001, 1007]]
𝔸 Smap partition[224] = [[928, 934, 938, 940], [977, 983, 987, 989]]
𝔸 Smap partition[225] = [[929, 935, 939, 941], [976, 982, 986, 988]]
𝔸 Smap partition[226] = [[930, 932, 936, 942], [979, 981, 985, 991]]
𝔸 Smap partition[227] = [[931, 933, 937, 943], [978, 980, 984, 990]]
𝔸 Smap partition[228] = [[944, 950, 954, 956], [961, 967, 971, 973]]
𝔸 Smap partition[229] = [[945, 951, 955, 957], [960, 966, 970, 972]]
𝔸 Smap partition[230] = [[946, 948, 952, 958], [963, 965, 969, 975]]
𝔸 Smap partition[231] = [[947, 949, 953, 959], [962, 964, 968, 974]]

Appendix 2: Octonion adjacent algebra orientation covariant Dirac gamma matrices

These matrices significantly occupy only four rows and columns in the adjacent algebra 8x8 matrix forms, so displaying them as 8x8 disguises the inherent 4x4 matrix structure. To appeal to both, the matrices are shown as augmented 4x4 matrices where the 8x8 element homes are given by added row and column indexing. Only "positive" elements at odd x Dirac[n][0 or 1][x] closed full Lie Algebra indexes are shown for simplicity.

They are appropriately oriented for the coset$_8$[x] Clifford algebra builders with 1-forms x: [3, 5, 7], 2-forms x: [2, 4, 6] and 3-form x=1.

Dirac[1][0][1] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 2 & 4 & 6 \end{array} \\ \begin{array}{c} 0\\2\\4\\6 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[1][0][3] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 2 & 4 & 6 \end{array} \\ \begin{array}{c} 0\\2\\4\\6 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & +i \end{bmatrix} \end{array}$$

Dirac[1][0][5] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 2 & 4 & 6 \end{array} \\ \begin{array}{c} 0\\2\\4\\6 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[1][0][7] =
$$\begin{array}{cc} & \begin{matrix} 0 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 0 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & +1s_{11} & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{8} \\ 0 & 0 & +1s_{8} & 0 \end{bmatrix} \end{array}$$

Dirac[1][0][9] =
$$\begin{array}{cc} & \begin{matrix} 0 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 0 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & -1s_{11} & 0 & 0 \\ +1s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{8} \\ 0 & 0 & +1s_{8} & 0 \end{bmatrix} \end{array}$$

Dirac[1][0][11] =
$$\begin{array}{cc} & \begin{matrix} 0 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 0 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & +is_{11} & 0 & 0 \\ +is_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{8} \\ 0 & 0 & -is_{8} & 0 \end{bmatrix} \end{array}$$

Dirac[1][0][13] =
$$\begin{array}{cc} & \begin{matrix} 0 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 0 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & +is_{11} & 0 & 0 \\ +is_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{8} \\ 0 & 0 & +is_{8} & 0 \end{bmatrix} \end{array}$$

Dirac[1][0][15] =
$$\begin{array}{cc} & \begin{matrix} 0 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 0 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & +1s_{7} & 0 \\ 0 & 0 & 0 & +1s_{4} \\ -1s_{7} & 0 & 0 & 0 \\ 0 & -1s_{4} & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][0][17] =
$$\begin{array}{cc} & \begin{matrix} 0 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 0 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & +1s_{7} & 0 \\ 0 & 0 & 0 & -1s_{4} \\ -1s_{7} & 0 & 0 & 0 \\ 0 & +1s_{4} & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][0][19] =
$$\begin{array}{cc} & \begin{matrix} 0 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 0 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & +is_{7} & 0 \\ 0 & 0 & 0 & -is_{4} \\ +is_{7} & 0 & 0 & 0 \\ 0 & -is_{4} & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][0][21] =
$$\begin{array}{cc} & \begin{matrix} 0 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 0 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & +is_{7} & 0 \\ 0 & 0 & 0 & +is_{4} \\ +is_{7} & 0 & 0 & 0 \\ 0 & +is_{4} & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][0][23] =
$$\begin{array}{cc} & \begin{matrix} 0 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 0 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & +1s_{15} \\ 0 & 0 & -1s_{12} & 0 \\ 0 & +1s_{12} & 0 & 0 \\ -1s_{15} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][0][25] =
$$\begin{array}{cc} & \begin{matrix} 0 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 0 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & +1s_{15} \\ 0 & 0 & +1s_{12} & 0 \\ 0 & -1s_{12} & 0 & 0 \\ -1s_{15} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][0][27] =
$$\begin{array}{cc} & \begin{matrix} 0 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 0 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & +is_{15} \\ 0 & 0 & -is_{12} & 0 \\ 0 & -is_{12} & 0 & 0 \\ +is_{15} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][0][29] =
$$\begin{array}{cc} & \begin{matrix} 0 & 2 & 4 & 6 \end{matrix} \\ \begin{matrix} 0 \\ 2 \\ 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & +is_{15} \\ 0 & 0 & +is_{12} & 0 \\ 0 & +is_{12} & 0 & 0 \\ +is_{15} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][1][1] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[1][1][3] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & +i \end{bmatrix} \end{array}$$

Dirac[1][1][5] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[1][1][7] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & +1s_{2} & 0 & 0 \\ -1s_{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & +1 & 0 \end{bmatrix} \end{array}$$

Dirac[1][1][9] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & -1s_{2} & 0 & 0 \\ +1s_{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & +1 & 0 \end{bmatrix} \end{array}$$

Dirac[1][1][11] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & +is_{2} & 0 & 0 \\ +is_{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{bmatrix} \end{array}$$

Dirac[1][1][13] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & +is_{2} & 0 & 0 \\ +is_{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & +i \\ 0 & 0 & +i & 0 \end{bmatrix} \end{array}$$

Dirac[1][1][15] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & 0 & +1s_{2} & 0 \\ 0 & 0 & 0 & +1 \\ -1s_{2} & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][1][17] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & 0 & +1s_{2} & 0 \\ 0 & 0 & 0 & -1 \\ -1s_{2} & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][1][19] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & 0 & +is_{2} & 0 \\ 0 & 0 & 0 & -i \\ +is_{2} & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][1][21] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & 0 & +is_{2} & 0 \\ 0 & 0 & 0 & +i \\ +is_{2} & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][1][23] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & +1s_{2} \\ 0 & 0 & -1 & 0 \\ 0 & +1 & 0 & 0 \\ -1s_{2} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][1][25] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & +1s_{2} \\ 0 & 0 & +1 & 0 \\ 0 & -1 & 0 & 0 \\ -1s_{2} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][1][27] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & +is_{2} \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ +is_{2} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[1][1][29] =
$$\begin{array}{cc} & \begin{matrix} 1 & 3 & 5 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & +is_{2} \\ 0 & 0 & +i & 0 \\ 0 & +i & 0 & 0 \\ +is_{2} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][0][1] =
$$\begin{array}{cc} & \begin{matrix} 0 & 1 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[2][0][3] =
$$\begin{array}{cc} & \begin{matrix} 0 & 1 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & +i \end{bmatrix} \end{array}$$

Dirac[2][0][5] =
$$\begin{array}{cc} & \begin{matrix} 0 & 1 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[2][0][7] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & +1s_2 & 0 & 0 \\ -1s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_7 \\ 0 & 0 & +1s_7 & 0 \end{bmatrix} \end{array}$$

Dirac[2][0][9] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & -1s_2 & 0 & 0 \\ +1s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_7 \\ 0 & 0 & +1s_7 & 0 \end{bmatrix} \end{array}$$

Dirac[2][0][11] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & +is_2 & 0 & 0 \\ +is_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_7 \\ 0 & 0 & -is_7 & 0 \end{bmatrix} \end{array}$$

Dirac[2][0][13] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & +is_2 & 0 & 0 \\ +is_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_7 \\ 0 & 0 & +is_7 & 0 \end{bmatrix} \end{array}$$

Dirac[2][0][15] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & +1s_7 & 0 \\ 0 & 0 & 0 & +1s_2 \\ -1s_7 & 0 & 0 & 0 \\ 0 & -1s_2 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][0][17] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & +1s_7 & 0 \\ 0 & 0 & 0 & -1s_2 \\ -1s_7 & 0 & 0 & 0 \\ 0 & +1s_2 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][0][19] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & +is_7 & 0 \\ 0 & 0 & 0 & -is_2 \\ +is_7 & 0 & 0 & 0 \\ 0 & -is_2 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][0][21] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & +is_7 & 0 \\ 0 & 0 & 0 & +is_2 \\ +is_7 & 0 & 0 & 0 \\ 0 & +is_2 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][0][23] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & -1s_5 & 0 \\ 0 & +1s_5 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][0][25] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & +1s_5 & 0 \\ 0 & -1s_5 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][0][27] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +i \\ 0 & 0 & -is_5 & 0 \\ 0 & -is_5 & 0 & 0 \\ +i & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][0][29] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 4 & 5 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +i \\ 0 & 0 & +is_5 & 0 \\ 0 & +is_5 & 0 & 0 \\ +i & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][1][1] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[2][1][3] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & +i \end{bmatrix} \end{array}$$

Dirac[2][1][5] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[2][1][7] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & +1s_{11} & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} \\ 0 & 0 & +1s_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[2][1][9] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & -1s_{11} & 0 & 0 \\ +1s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} \\ 0 & 0 & +1s_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[2][1][11] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & +is_{11} & 0 & 0 \\ +is_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{15} \\ 0 & 0 & -is_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[2][1][13] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & +is_{11} & 0 & 0 \\ +is_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{15} \\ 0 & 0 & +is_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[2][1][15] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & +1s_4 & 0 \\ 0 & 0 & 0 & +1 \\ -1s_4 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][1][17] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & +1s_4 & 0 \\ 0 & 0 & 0 & -1 \\ -1s_4 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][1][19] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & +is_4 & 0 \\ 0 & 0 & 0 & -i \\ +is_4 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][1][21] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & +is_4 & 0 \\ 0 & 0 & 0 & +i \\ +is_4 & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][1][23] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1s_{11} \\ 0 & 0 & -1s_{15} & 0 \\ 0 & +1s_{15} & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][1][25] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1s_{11} \\ 0 & 0 & +1s_{15} & 0 \\ 0 & -1s_{15} & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][1][27] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +is_{11} \\ 0 & 0 & -is_{15} & 0 \\ 0 & -is_{15} & 0 & 0 \\ +is_{11} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[2][1][29] =
$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 6 & 7 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 6 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +is_{11} \\ 0 & 0 & +is_{15} & 0 \\ 0 & +is_{15} & 0 & 0 \\ +is_{11} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][0][1] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 4 \\ 7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[3][0][3] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 4 \\ 7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & +i \end{bmatrix} \end{array}$$

Dirac[3][0][5] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 4 \\ 7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[3][0][7] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0\\3\\4\\7 \end{array} & \begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_7 \\ 0 & 0 & +1s_7 & 0 \end{bmatrix} \end{array}$$

Dirac[3][0][9] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0\\3\\4\\7 \end{array} & \begin{bmatrix} 0 & -1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_7 \\ 0 & 0 & +1s_7 & 0 \end{bmatrix} \end{array}$$

Dirac[3][0][11] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0\\3\\4\\7 \end{array} & \begin{bmatrix} 0 & +i & 0 & 0 \\ +i & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_7 \\ 0 & 0 & -is_7 & 0 \end{bmatrix} \end{array}$$

Dirac[3][0][13] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0\\3\\4\\7 \end{array} & \begin{bmatrix} 0 & +i & 0 & 0 \\ +i & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_7 \\ 0 & 0 & +is_7 & 0 \end{bmatrix} \end{array}$$

Dirac[3][0][15] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0\\3\\4\\7 \end{array} & \begin{bmatrix} 0 & 0 & +1s_7 & 0 \\ 0 & 0 & 0 & +1 \\ -1s_7 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][0][17] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0\\3\\4\\7 \end{array} & \begin{bmatrix} 0 & 0 & +1s_7 & 0 \\ 0 & 0 & 0 & -1 \\ -1s_7 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][0][19] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0\\3\\4\\7 \end{array} & \begin{bmatrix} 0 & 0 & +is_7 & 0 \\ 0 & 0 & 0 & -i \\ +is_7 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][0][21] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0\\3\\4\\7 \end{array} & \begin{bmatrix} 0 & 0 & +is_7 & 0 \\ 0 & 0 & 0 & +i \\ +is_7 & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][0][23] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0\\3\\4\\7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & -1s_7 & 0 \\ 0 & +1s_7 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][0][25] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0\\3\\4\\7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & +1s_7 & 0 \\ 0 & -1s_7 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][0][27] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0\\3\\4\\7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +i \\ 0 & 0 & -is_7 & 0 \\ 0 & -is_7 & 0 & 0 \\ +i & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][0][29] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 4 & 7 \end{array} \\ \begin{array}{c} 0\\3\\4\\7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +i \\ 0 & 0 & +is_7 & 0 \\ 0 & +is_7 & 0 & 0 \\ +i & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][1][1] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[3][1][3] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & +i \end{bmatrix} \end{array}$$

Dirac[3][1][5] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[3][1][7] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} 0 & +1s_9 & 0 & 0 \\ -1s_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} \\ 0 & 0 & +1s_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[3][1][9] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} 0 & -1s_9 & 0 & 0 \\ +1s_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} \\ 0 & 0 & +1s_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[3][1][11] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} 0 & +is_9 & 0 & 0 \\ +is_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{15} \\ 0 & 0 & -is_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[3][1][13] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} 0 & +is_9 & 0 & 0 \\ +is_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{15} \\ 0 & 0 & +is_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[3][1][15] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} 0 & 0 & +1s_2 & 0 \\ 0 & 0 & 0 & +1s_4 \\ -1s_2 & 0 & 0 & 0 \\ 0 & -1s_4 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][1][17] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} 0 & 0 & +1s_2 & 0 \\ 0 & 0 & 0 & -1s_4 \\ -1s_2 & 0 & 0 & 0 \\ 0 & +1s_4 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][1][19] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} 0 & 0 & +is_2 & 0 \\ 0 & 0 & 0 & -is_4 \\ +is_2 & 0 & 0 & 0 \\ 0 & -is_4 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][1][21] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} 0 & 0 & +is_2 & 0 \\ 0 & 0 & 0 & +is_4 \\ +is_2 & 0 & 0 & 0 \\ 0 & +is_4 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][1][23] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1s_{13} \\ 0 & 0 & -1s_{11} & 0 \\ 0 & +1s_{11} & 0 & 0 \\ -1s_{13} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][1][25] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1s_{13} \\ 0 & 0 & +1s_{11} & 0 \\ 0 & -1s_{11} & 0 & 0 \\ -1s_{13} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][1][27] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +is_{13} \\ 0 & 0 & -is_{11} & 0 \\ 0 & -is_{11} & 0 & 0 \\ +is_{13} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[3][1][29] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 5 & 6 \end{array} \\ \begin{array}{c} 1\\2\\5\\6 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +is_{13} \\ 0 & 0 & +is_{11} & 0 \\ 0 & +is_{11} & 0 & 0 \\ +is_{13} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][0][1] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[4][0][3] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & +i \end{bmatrix} \end{array}$$

Dirac[4][0][5] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[4][0][7] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} 0 & +1s_2 & 0 & 0 \\ -1s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{11} \\ 0 & 0 & +1s_{11} & 0 \end{bmatrix} \end{array}$$

Dirac[4][0][9] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} 0 & -1s_2 & 0 & 0 \\ +1s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{11} \\ 0 & 0 & +1s_{11} & 0 \end{bmatrix} \end{array}$$

Dirac[4][0][11] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} 0 & +is_2 & 0 & 0 \\ +is_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{11} \\ 0 & 0 & -is_{11} & 0 \end{bmatrix} \end{array}$$

Dirac[4][0][13] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} 0 & +is_2 & 0 & 0 \\ +is_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{11} \\ 0 & 0 & +is_{11} & 0 \end{bmatrix} \end{array}$$

Dirac[4][0][15] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} 0 & 0 & +1s_{11} & 0 \\ 0 & 0 & 0 & +1s_2 \\ -1s_{11} & 0 & 0 & 0 \\ 0 & -1s_2 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][0][17] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} 0 & 0 & +1s_{11} & 0 \\ 0 & 0 & 0 & -1s_2 \\ -1s_{11} & 0 & 0 & 0 \\ 0 & +1s_2 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][0][19] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} 0 & 0 & +is_{11} & 0 \\ 0 & 0 & 0 & -is_2 \\ +is_{11} & 0 & 0 & 0 \\ 0 & -is_2 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][0][21] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} 0 & 0 & +is_{11} & 0 \\ 0 & 0 & 0 & +is_2 \\ +is_{11} & 0 & 0 & 0 \\ 0 & +is_2 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][0][23] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & -1s_9 & 0 \\ 0 & +1s_9 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][0][25] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & +1s_9 & 0 \\ 0 & -1s_9 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][0][27] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +i \\ 0 & 0 & -is_9 & 0 \\ 0 & -is_9 & 0 & 0 \\ +i & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][0][29] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0\\1\\2\\3 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +i \\ 0 & 0 & +is_9 & 0 \\ 0 & +is_9 & 0 & 0 \\ +i & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][1][1] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[4][1][3] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & +i \end{bmatrix} \end{array}$$

Dirac[4][1][5] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[4][1][7] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} 0 & +1s_7 & 0 & 0 \\ -1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} \\ 0 & 0 & +1s_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[4][1][9] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} 0 & -1s_7 & 0 & 0 \\ +1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} \\ 0 & 0 & +1s_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[4][1][11] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} 0 & +is_7 & 0 & 0 \\ +is_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{15} \\ 0 & 0 & -is_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[4][1][13] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} 0 & +is_7 & 0 & 0 \\ +is_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{15} \\ 0 & 0 & +is_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[4][1][15] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & +1s_8 & 0 \\ 0 & 0 & 0 & +1 \\ -1s_8 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][1][17] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & +1s_8 & 0 \\ 0 & 0 & 0 & -1 \\ -1s_8 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][1][19] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & +is_8 & 0 \\ 0 & 0 & 0 & -i \\ +is_8 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][1][21] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & +is_8 & 0 \\ 0 & 0 & 0 & +i \\ +is_8 & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][1][23] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1s_7 \\ 0 & 0 & -1s_{15} & 0 \\ 0 & +1s_{15} & 0 & 0 \\ -1s_7 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][1][25] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1s_7 \\ 0 & 0 & +1s_{15} & 0 \\ 0 & -1s_{15} & 0 & 0 \\ -1s_7 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][1][27] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +is_7 \\ 0 & 0 & -is_{15} & 0 \\ 0 & -is_{15} & 0 & 0 \\ +is_7 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[4][1][29] =
$$\begin{array}{cc} & \begin{array}{cccc} 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 4\\5\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +is_7 \\ 0 & 0 & +is_{15} & 0 \\ 0 & +is_{15} & 0 & 0 \\ +is_7 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[5][0][1] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 2 & 5 & 7 \end{array} \\ \begin{array}{c} 0\\2\\5\\7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[5][0][3] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 2 & 5 & 7 \end{array} \\ \begin{array}{c} 0\\2\\5\\7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & +i \end{bmatrix} \end{array}$$

Dirac[5][0][5] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 2 & 5 & 7 \end{array} \\ \begin{array}{c} 0\\2\\5\\7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[5][0][7] =

$$\begin{array}{cc} & \begin{array}{cccc}0&2&5&7\end{array}\\ \begin{array}{c}0\\2\\5\\7\end{array} & \begin{bmatrix}0&+1s_{11}&0&0\\-1s_{11}&0&0&0\\0&0&0&-1\\0&0&+1&0\end{bmatrix}\end{array}$$

Dirac[5][0][9] =

$$\begin{array}{cc} & \begin{array}{cccc}0&2&5&7\end{array}\\ \begin{array}{c}0\\2\\5\\7\end{array} & \begin{bmatrix}0&-1s_{11}&0&0\\+1s_{11}&0&0&0\\0&0&0&-1\\0&0&+1&0\end{bmatrix}\end{array}$$

Dirac[5][0][11] =

$$\begin{array}{cc} & \begin{array}{cccc}0&2&5&7\end{array}\\ \begin{array}{c}0\\2\\5\\7\end{array} & \begin{bmatrix}0&+is_{11}&0&0\\+is_{11}&0&0&0\\0&0&0&-i\\0&0&-i&0\end{bmatrix}\end{array}$$

Dirac[5][0][13] =

$$\begin{array}{cc} & \begin{array}{cccc}0&2&5&7\end{array}\\ \begin{array}{c}0\\2\\5\\7\end{array} & \begin{bmatrix}0&+is_{11}&0&0\\+is_{11}&0&0&0\\0&0&0&+i\\0&0&+i&0\end{bmatrix}\end{array}$$

Dirac[5][0][15] =

$$\begin{array}{cc} & \begin{array}{cccc}0&2&5&7\end{array}\\ \begin{array}{c}0\\2\\5\\7\end{array} & \begin{bmatrix}0&0&+1&0\\0&0&0&+1s_{11}\\-1&0&0&0\\0&-1s_{11}&0&0\end{bmatrix}\end{array}$$

Dirac[5][0][17] =

$$\begin{array}{cc} & \begin{array}{cccc}0&2&5&7\end{array}\\ \begin{array}{c}0\\2\\5\\7\end{array} & \begin{bmatrix}0&0&+1&0\\0&0&0&-1s_{11}\\-1&0&0&0\\0&+1s_{11}&0&0\end{bmatrix}\end{array}$$

Dirac[5][0][19] =

$$\begin{array}{cc} & \begin{array}{cccc}0&2&5&7\end{array}\\ \begin{array}{c}0\\2\\5\\7\end{array} & \begin{bmatrix}0&0&+i&0\\0&0&0&-is_{11}\\+i&0&0&0\\0&-is_{11}&0&0\end{bmatrix}\end{array}$$

Dirac[5][0][21] =

$$\begin{array}{cc} & \begin{array}{cccc}0&2&5&7\end{array}\\ \begin{array}{c}0\\2\\5\\7\end{array} & \begin{bmatrix}0&0&+i&0\\0&0&0&+is_{11}\\+i&0&0&0\\0&+is_{11}&0&0\end{bmatrix}\end{array}$$

Dirac[5][0][23] =

$$\begin{array}{cc} & \begin{array}{cccc}0&2&5&7\end{array}\\ \begin{array}{c}0\\2\\5\\7\end{array} & \begin{bmatrix}0&0&0&+1\\0&0&-1s_{11}&0\\0&+1s_{11}&0&0\\-1&0&0&0\end{bmatrix}\end{array}$$

Dirac[5][0][25] =

$$\begin{array}{cc} & \begin{array}{cccc}0&2&5&7\end{array}\\ \begin{array}{c}0\\2\\5\\7\end{array} & \begin{bmatrix}0&0&0&+1\\0&0&+1s_{11}&0\\0&-1s_{11}&0&0\\-1&0&0&0\end{bmatrix}\end{array}$$

Dirac[5][0][27] =

$$\begin{array}{cc} & \begin{array}{cccc}0&2&5&7\end{array}\\ \begin{array}{c}0\\2\\5\\7\end{array} & \begin{bmatrix}0&0&0&+i\\0&0&-is_{11}&0\\0&-is_{11}&0&0\\+i&0&0&0\end{bmatrix}\end{array}$$

Dirac[5][0][29] =

$$\begin{array}{cc} & \begin{array}{cccc}0&2&5&7\end{array}\\ \begin{array}{c}0\\2\\5\\7\end{array} & \begin{bmatrix}0&0&0&+i\\0&0&+is_{11}&0\\0&+is_{11}&0&0\\+i&0&0&0\end{bmatrix}\end{array}$$

Dirac[5][1][1] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}+i&0&0&0\\0&-i&0&0\\0&0&+i&0\\0&0&0&-i\end{bmatrix}\end{array}$$

Dirac[5][1][3] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}+i&0&0&0\\0&-i&0&0\\0&0&-i&0\\0&0&0&+i\end{bmatrix}\end{array}$$

Dirac[5][1][5] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}+i&0&0&0\\0&+i&0&0\\0&0&-i&0\\0&0&0&-i\end{bmatrix}\end{array}$$

Dirac[5][1][7] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}0&+1s_{2}&0&0\\-1s_{2}&0&0&0\\0&0&0&-1s_{8}\\0&0&+1s_{8}&0\end{bmatrix}\end{array}$$

Dirac[5][1][9] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}0&-1s_{2}&0&0\\+1s_{2}&0&0&0\\0&0&0&-1s_{8}\\0&0&+1s_{8}&0\end{bmatrix}\end{array}$$

Dirac[5][1][11] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}0&+is_{2}&0&0\\+is_{2}&0&0&0\\0&0&0&-is_{8}\\0&0&-is_{8}&0\end{bmatrix}\end{array}$$

Dirac[5][1][13] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}0&+is_{2}&0&0\\+is_{2}&0&0&0\\0&0&0&+is_{8}\\0&0&+is_{8}&0\end{bmatrix}\end{array}$$

Dirac[5][1][15] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}0&0&+1s_{5}&0\\0&0&0&+1s_{15}\\-1s_{5}&0&0&0\\0&-1s_{15}&0&0\end{bmatrix}\end{array}$$

Dirac[5][1][17] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}0&0&+1s_{5}&0\\0&0&0&-1s_{15}\\-1s_{5}&0&0&0\\0&+1s_{15}&0&0\end{bmatrix}\end{array}$$

Dirac[5][1][19] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}0&0&+is_{5}&0\\0&0&0&-is_{15}\\+is_{5}&0&0&0\\0&-is_{15}&0&0\end{bmatrix}\end{array}$$

Dirac[5][1][21] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}0&0&+is_{5}&0\\0&0&0&+is_{15}\\+is_{5}&0&0&0\\0&+is_{15}&0&0\end{bmatrix}\end{array}$$

Dirac[5][1][23] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}0&0&0&+1s_{13}\\0&0&-1s_{7}&0\\0&+1s_{7}&0&0\\-1s_{13}&0&0&0\end{bmatrix}\end{array}$$

Dirac[5][1][25] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}0&0&0&+1s_{13}\\0&0&+1s_{7}&0\\0&-1s_{7}&0&0\\-1s_{13}&0&0&0\end{bmatrix}\end{array}$$

Dirac[5][1][27] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}0&0&0&+is_{13}\\0&0&-is_{7}&0\\0&-is_{7}&0&0\\+is_{13}&0&0&0\end{bmatrix}\end{array}$$

Dirac[5][1][29] =

$$\begin{array}{cc} & \begin{array}{cccc}1&3&4&6\end{array}\\ \begin{array}{c}1\\3\\4\\6\end{array} & \begin{bmatrix}0&0&0&+is_{13}\\0&0&+is_{7}&0\\0&+is_{7}&0&0\\+is_{13}&0&0&0\end{bmatrix}\end{array}$$

Dirac[6][0][1] =

$$\begin{array}{cc} & \begin{array}{cccc}0&1&6&7\end{array}\\ \begin{array}{c}0\\1\\6\\7\end{array} & \begin{bmatrix}+i&0&0&0\\0&-i&0&0\\0&0&+i&0\\0&0&0&-i\end{bmatrix}\end{array}$$

Dirac[6][0][3] =

$$\begin{array}{cc} & \begin{array}{cccc}0&1&6&7\end{array}\\ \begin{array}{c}0\\1\\6\\7\end{array} & \begin{bmatrix}+i&0&0&0\\0&-i&0&0\\0&0&-i&0\\0&0&0&+i\end{bmatrix}\end{array}$$

Dirac[6][0][5] =

$$\begin{array}{cc} & \begin{array}{cccc}0&1&6&7\end{array}\\ \begin{array}{c}0\\1\\6\\7\end{array} & \begin{bmatrix}+i&0&0&0\\0&+i&0&0\\0&0&-i&0\\0&0&0&-i\end{bmatrix}\end{array}$$

Dirac[6][0][7] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 6 & 7 \end{array} \\ \begin{array}{c} 0\\1\\6\\7 \end{array} & \begin{bmatrix} 0 & +1s_2 & 0 & 0 \\ -1s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} \\ 0 & 0 & +1s_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[6][0][9] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 6 & 7 \end{array} \\ \begin{array}{c} 0\\1\\6\\7 \end{array} & \begin{bmatrix} 0 & -1s_2 & 0 & 0 \\ +1s_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} \\ 0 & 0 & +1s_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[6][0][11] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 6 & 7 \end{array} \\ \begin{array}{c} 0\\1\\6\\7 \end{array} & \begin{bmatrix} 0 & +is_2 & 0 & 0 \\ +is_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{15} \\ 0 & 0 & -is_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[6][0][13] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 6 & 7 \end{array} \\ \begin{array}{c} 0\\1\\6\\7 \end{array} & \begin{bmatrix} 0 & +is_2 & 0 & 0 \\ +is_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{15} \\ 0 & 0 & +is_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[6][0][15] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 6 & 7 \end{array} \\ \begin{array}{c} 0\\1\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & +1s_{15} & 0 \\ 0 & 0 & 0 & +1s_2 \\ -1s_{15} & 0 & 0 & 0 \\ 0 & -1s_2 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][0][17] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 6 & 7 \end{array} \\ \begin{array}{c} 0\\1\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & +1s_{15} & 0 \\ 0 & 0 & 0 & -1s_2 \\ -1s_{15} & 0 & 0 & 0 \\ 0 & +1s_2 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][0][19] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 6 & 7 \end{array} \\ \begin{array}{c} 0\\1\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & +is_{15} & 0 \\ 0 & 0 & 0 & -is_2 \\ +is_{15} & 0 & 0 & 0 \\ 0 & -is_2 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][0][21] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 6 & 7 \end{array} \\ \begin{array}{c} 0\\1\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & +is_{15} & 0 \\ 0 & 0 & 0 & +is_2 \\ +is_{15} & 0 & 0 & 0 \\ 0 & +is_2 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][0][23] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 6 & 7 \end{array} \\ \begin{array}{c} 0\\1\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & -1s_{13} & 0 \\ 0 & +1s_{13} & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][0][25] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 6 & 7 \end{array} \\ \begin{array}{c} 0\\1\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1 \\ 0 & 0 & +1s_{13} & 0 \\ 0 & -1s_{13} & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][0][27] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 6 & 7 \end{array} \\ \begin{array}{c} 0\\1\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +i \\ 0 & 0 & -is_{13} & 0 \\ 0 & -is_{13} & 0 & 0 \\ +i & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][0][29] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 1 & 6 & 7 \end{array} \\ \begin{array}{c} 0\\1\\6\\7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +i \\ 0 & 0 & +is_{13} & 0 \\ 0 & +is_{13} & 0 & 0 \\ +i & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][1][1] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[6][1][3] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & +i \end{bmatrix} \end{array}$$

Dirac[6][1][5] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[6][1][7] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & +1s_{11} & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_7 \\ 0 & 0 & +1s_7 & 0 \end{bmatrix} \end{array}$$

Dirac[6][1][9] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & -1s_{11} & 0 & 0 \\ +1s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_7 \\ 0 & 0 & +1s_7 & 0 \end{bmatrix} \end{array}$$

Dirac[6][1][11] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & +is_{11} & 0 & 0 \\ +is_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_7 \\ 0 & 0 & -is_7 & 0 \end{bmatrix} \end{array}$$

Dirac[6][1][13] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & +is_{11} & 0 & 0 \\ +is_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_7 \\ 0 & 0 & +is_7 & 0 \end{bmatrix} \end{array}$$

Dirac[6][1][15] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & 0 & +1s_{12} & 0 \\ 0 & 0 & 0 & +1 \\ -1s_{12} & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][1][17] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & 0 & +1s_{12} & 0 \\ 0 & 0 & 0 & -1 \\ -1s_{12} & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][1][19] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & 0 & +is_{12} & 0 \\ 0 & 0 & 0 & -i \\ +is_{12} & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][1][21] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & 0 & +is_{12} & 0 \\ 0 & 0 & 0 & +i \\ +is_{12} & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][1][23] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1s_{11} \\ 0 & 0 & -1s_7 & 0 \\ 0 & +1s_7 & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][1][25] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1s_{11} \\ 0 & 0 & +1s_7 & 0 \\ 0 & -1s_7 & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][1][27] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +is_{11} \\ 0 & 0 & -is_7 & 0 \\ 0 & -is_7 & 0 & 0 \\ +is_{11} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[6][1][29] =

$$\begin{array}{cc} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2\\3\\4\\5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +is_{11} \\ 0 & 0 & +is_7 & 0 \\ 0 & +is_7 & 0 & 0 \\ +is_{11} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][0][1] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0\\3\\5\\6 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[7][0][3] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0\\3\\5\\6 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & +i \end{bmatrix} \end{array}$$

Dirac[7][0][5] =

$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0\\3\\5\\6 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[7][0][7] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} \\ 0 & 0 & +1s_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[7][0][9] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & -1 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} \\ 0 & 0 & +1s_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[7][0][11] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & +i & 0 & 0 \\ +i & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{15} \\ 0 & 0 & -is_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[7][0][13] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & +i & 0 & 0 \\ +i & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{15} \\ 0 & 0 & +is_{15} & 0 \end{bmatrix} \end{array}$$

Dirac[7][0][15] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1s_{15} \\ -1 & 0 & 0 & 0 \\ 0 & -1s_{15} & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][0][17] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1s_{15} \\ -1 & 0 & 0 & 0 \\ 0 & +1s_{15} & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][0][19] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -is_{15} \\ +i & 0 & 0 & 0 \\ 0 & -is_{15} & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][0][21] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 0 & +i & 0 \\ 0 & 0 & 0 & +is_{15} \\ +i & 0 & 0 & 0 \\ 0 & +is_{15} & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][0][23] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1s_{15} \\ 0 & 0 & -1 & 0 \\ 0 & +1 & 0 & 0 \\ -1s_{15} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][0][25] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1s_{15} \\ 0 & 0 & +1 & 0 \\ 0 & -1 & 0 & 0 \\ -1s_{15} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][0][27] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +is_{15} \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ +is_{15} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][0][29] =
$$\begin{array}{cc} & \begin{array}{cccc} 0 & 3 & 5 & 6 \end{array} \\ \begin{array}{c} 0 \\ 3 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +is_{15} \\ 0 & 0 & +i & 0 \\ 0 & +i & 0 & 0 \\ +is_{15} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][1][1] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & +i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[7][1][3] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & +i \end{bmatrix} \end{array}$$

Dirac[7][1][5] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} +i & 0 & 0 & 0 \\ 0 & +i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \end{bmatrix} \end{array}$$

Dirac[7][1][7] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} 0 & +1s_{9} & 0 & 0 \\ -1s_{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{7} \\ 0 & 0 & +1s_{7} & 0 \end{bmatrix} \end{array}$$

Dirac[7][1][9] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} 0 & -1s_{9} & 0 & 0 \\ +1s_{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{7} \\ 0 & 0 & +1s_{7} & 0 \end{bmatrix} \end{array}$$

Dirac[7][1][11] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} 0 & +is_{9} & 0 & 0 \\ +is_{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{7} \\ 0 & 0 & -is_{7} & 0 \end{bmatrix} \end{array}$$

Dirac[7][1][13] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} 0 & +is_{9} & 0 & 0 \\ +is_{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{7} \\ 0 & 0 & +is_{7} & 0 \end{bmatrix} \end{array}$$

Dirac[7][1][15] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & +1s_{5} & 0 \\ 0 & 0 & 0 & +1s_{11} \\ -1s_{5} & 0 & 0 & 0 \\ 0 & -1s_{11} & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][1][17] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & +1s_{5} & 0 \\ 0 & 0 & 0 & -1s_{11} \\ -1s_{5} & 0 & 0 & 0 \\ 0 & +1s_{11} & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][1][19] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & +is_{5} & 0 \\ 0 & 0 & 0 & -is_{11} \\ +is_{5} & 0 & 0 & 0 \\ 0 & -is_{11} & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][1][21] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & +is_{5} & 0 \\ 0 & 0 & 0 & +is_{11} \\ +is_{5} & 0 & 0 & 0 \\ 0 & +is_{11} & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][1][23] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1s_{2} \\ 0 & 0 & -1s_{12} & 0 \\ 0 & +1s_{12} & 0 & 0 \\ -1s_{2} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][1][25] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +1s_{2} \\ 0 & 0 & +1s_{12} & 0 \\ 0 & -1s_{12} & 0 & 0 \\ -1s_{2} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][1][27] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +is_{2} \\ 0 & 0 & -is_{12} & 0 \\ 0 & -is_{12} & 0 & 0 \\ +is_{2} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Dirac[7][1][29] =
$$\begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & +is_{2} \\ 0 & 0 & +is_{12} & 0 \\ 0 & +is_{12} & 0 & 0 \\ +is_{2} & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Appendix 3: Unique Octonion adjacent algebra orientation covariant Pauli matrices

The full set of unique adjacent algebra bases for su(2) Lie algebras follow. Along with their negations, they form a complete adjacent algebra su(2) basis Lie Algebra within the 8x8 matrix form closed for bracket and Smap. Their positive members are as follows with row/column positions within the 8x8 matrix format indicated.

They are appropriately oriented for the $coset_8[x]$ Clifford algebra builders with 1-forms x: [3, 5, 7], 2-forms x: [2, 4, 6] and 3-form x=1.

$$\sigma[0][0] = \begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0 & +is_2 \\ +is_2 & 0 \end{bmatrix} \end{array} \quad \sigma[0][1] = \begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 0 & +1s_2 \\ -1s_2 & 0 \end{bmatrix} \end{array} \quad \sigma[0][2] = \begin{array}{cc} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[1][0] = \begin{array}{cc} & \begin{array}{cc} 0 & 2 \end{array} \\ \begin{array}{c} 0 \\ 2 \end{array} & \begin{bmatrix} 0 & +is_{11} \\ +is_{11} & 0 \end{bmatrix} \end{array} \quad \sigma[1][1] = \begin{array}{cc} & \begin{array}{cc} 0 & 2 \end{array} \\ \begin{array}{c} 0 \\ 2 \end{array} & \begin{bmatrix} 0 & +1s_{11} \\ -1s_{11} & 0 \end{bmatrix} \end{array} \quad \sigma[1][2] = \begin{array}{cc} & \begin{array}{cc} 0 & 2 \end{array} \\ \begin{array}{c} 0 \\ 2 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[2][0] = \begin{array}{cc} & \begin{array}{cc} 0 & 3 \end{array} \\ \begin{array}{c} 0 \\ 3 \end{array} & \begin{bmatrix} 0 & +i \\ +i & 0 \end{bmatrix} \end{array} \quad \sigma[2][1] = \begin{array}{cc} & \begin{array}{cc} 0 & 3 \end{array} \\ \begin{array}{c} 0 \\ 3 \end{array} & \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix} \end{array} \quad \sigma[2][2] = \begin{array}{cc} & \begin{array}{cc} 0 & 3 \end{array} \\ \begin{array}{c} 0 \\ 3 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[3][0] = \begin{array}{cc} & \begin{array}{cc} 0 & 4 \end{array} \\ \begin{array}{c} 0 \\ 4 \end{array} & \begin{bmatrix} 0 & +is_7 \\ +is_7 & 0 \end{bmatrix} \end{array} \quad \sigma[3][1] = \begin{array}{cc} & \begin{array}{cc} 0 & 4 \end{array} \\ \begin{array}{c} 0 \\ 4 \end{array} & \begin{bmatrix} 0 & +1s_7 \\ -1s_7 & 0 \end{bmatrix} \end{array} \quad \sigma[3][2] = \begin{array}{cc} & \begin{array}{cc} 0 & 4 \end{array} \\ \begin{array}{c} 0 \\ 4 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[4][0] = \begin{array}{cc} & \begin{array}{cc} 0 & 5 \end{array} \\ \begin{array}{c} 0 \\ 5 \end{array} & \begin{bmatrix} 0 & +i \\ +i & 0 \end{bmatrix} \end{array} \quad \sigma[4][1] = \begin{array}{cc} & \begin{array}{cc} 0 & 5 \end{array} \\ \begin{array}{c} 0 \\ 5 \end{array} & \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix} \end{array} \quad \sigma[4][2] = \begin{array}{cc} & \begin{array}{cc} 0 & 5 \end{array} \\ \begin{array}{c} 0 \\ 5 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[5][0] = \begin{array}{cc} & \begin{array}{cc} 0 & 6 \end{array} \\ \begin{array}{c} 0 \\ 6 \end{array} & \begin{bmatrix} 0 & +is_{15} \\ +is_{15} & 0 \end{bmatrix} \end{array} \quad \sigma[5][1] = \begin{array}{cc} & \begin{array}{cc} 0 & 6 \end{array} \\ \begin{array}{c} 0 \\ 6 \end{array} & \begin{bmatrix} 0 & +1s_{15} \\ -1s_{15} & 0 \end{bmatrix} \end{array} \quad \sigma[5][2] = \begin{array}{cc} & \begin{array}{cc} 0 & 6 \end{array} \\ \begin{array}{c} 0 \\ 6 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[6][0] = \begin{array}{cc} & \begin{array}{cc} 0 & 7 \end{array} \\ \begin{array}{c} 0 \\ 7 \end{array} & \begin{bmatrix} 0 & +i \\ +i & 0 \end{bmatrix} \end{array} \quad \sigma[6][1] = \begin{array}{cc} & \begin{array}{cc} 0 & 7 \end{array} \\ \begin{array}{c} 0 \\ 7 \end{array} & \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix} \end{array} \quad \sigma[6][2] = \begin{array}{cc} & \begin{array}{cc} 0 & 7 \end{array} \\ \begin{array}{c} 0 \\ 7 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[7][0] = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0 & +is_9 \\ +is_9 & 0 \end{bmatrix} \end{array} \quad \sigma[7][1] = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0 & +1s_9 \\ -1s_9 & 0 \end{bmatrix} \end{array} \quad \sigma[7][2] = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[8][0] = \begin{array}{cc} & \begin{array}{cc} 1 & 3 \end{array} \\ \begin{array}{c} 1 \\ 3 \end{array} & \begin{bmatrix} 0 & +is_2 \\ +is_2 & 0 \end{bmatrix} \end{array} \quad \sigma[8][1] = \begin{array}{cc} & \begin{array}{cc} 1 & 3 \end{array} \\ \begin{array}{c} 1 \\ 3 \end{array} & \begin{bmatrix} 0 & +1s_2 \\ -1s_2 & 0 \end{bmatrix} \end{array} \quad \sigma[8][2] = \begin{array}{cc} & \begin{array}{cc} 1 & 3 \end{array} \\ \begin{array}{c} 1 \\ 3 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[9][0] = \begin{array}{cc} & \begin{array}{cc} 1 & 4 \end{array} \\ \begin{array}{c} 1 \\ 4 \end{array} & \begin{bmatrix} 0 & +is_5 \\ +is_5 & 0 \end{bmatrix} \end{array} \quad \sigma[9][1] = \begin{array}{cc} & \begin{array}{cc} 1 & 4 \end{array} \\ \begin{array}{c} 1 \\ 4 \end{array} & \begin{bmatrix} 0 & +1s_5 \\ -1s_5 & 0 \end{bmatrix} \end{array} \quad \sigma[9][2] = \begin{array}{cc} & \begin{array}{cc} 1 & 4 \end{array} \\ \begin{array}{c} 1 \\ 4 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[10][0] = \begin{array}{cc} & \begin{array}{cc} 1 & 5 \end{array} \\ \begin{array}{c} 1 \\ 5 \end{array} & \begin{bmatrix} 0 & +is_2 \\ +is_2 & 0 \end{bmatrix} \end{array} \quad \sigma[10][1] = \begin{array}{cc} & \begin{array}{cc} 1 & 5 \end{array} \\ \begin{array}{c} 1 \\ 5 \end{array} & \begin{bmatrix} 0 & +1s_2 \\ -1s_2 & 0 \end{bmatrix} \end{array} \quad \sigma[10][2] = \begin{array}{cc} & \begin{array}{cc} 1 & 5 \end{array} \\ \begin{array}{c} 1 \\ 5 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[11][0] = \begin{array}{cc} & \begin{array}{cc} 1 & 6 \end{array} \\ \begin{array}{c} 1 \\ 6 \end{array} & \begin{bmatrix} 0 & +is_{13} \\ +is_{13} & 0 \end{bmatrix} \end{array} \quad \sigma[11][1] = \begin{array}{cc} & \begin{array}{cc} 1 & 6 \end{array} \\ \begin{array}{c} 1 \\ 6 \end{array} & \begin{bmatrix} 0 & +1s_{13} \\ -1s_{13} & 0 \end{bmatrix} \end{array} \quad \sigma[11][2] = \begin{array}{cc} & \begin{array}{cc} 1 & 6 \end{array} \\ \begin{array}{c} 1 \\ 6 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[12][0] = \begin{array}{cc} & \begin{array}{cc} 1 & 7 \end{array} \\ \begin{array}{c} 1 \\ 7 \end{array} & \begin{bmatrix} 0 & +is_2 \\ +is_2 & 0 \end{bmatrix} \end{array} \quad \sigma[12][1] = \begin{array}{cc} & \begin{array}{cc} 1 & 7 \end{array} \\ \begin{array}{c} 1 \\ 7 \end{array} & \begin{bmatrix} 0 & +1s_2 \\ -1s_2 & 0 \end{bmatrix} \end{array} \quad \sigma[12][2] = \begin{array}{cc} & \begin{array}{cc} 1 & 7 \end{array} \\ \begin{array}{c} 1 \\ 7 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[13][0] = \begin{array}{cc} & \begin{array}{cc} 2 & 3 \end{array} \\ \begin{array}{c} 2 \\ 3 \end{array} & \begin{bmatrix} 0 & +is_{11} \\ +is_{11} & 0 \end{bmatrix} \end{array} \quad \sigma[13][1] = \begin{array}{cc} & \begin{array}{cc} 2 & 3 \end{array} \\ \begin{array}{c} 2 \\ 3 \end{array} & \begin{bmatrix} 0 & +1s_{11} \\ -1s_{11} & 0 \end{bmatrix} \end{array} \quad \sigma[13][2] = \begin{array}{cc} & \begin{array}{cc} 2 & 3 \end{array} \\ \begin{array}{c} 2 \\ 3 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[14][0] = \begin{array}{cc} & \begin{array}{cc} 2 & 4 \end{array} \\ \begin{array}{c} 2 \\ 4 \end{array} & \begin{bmatrix} 0 & +is_{12} \\ +is_{12} & 0 \end{bmatrix} \end{array} \quad \sigma[14][1] = \begin{array}{cc} & \begin{array}{cc} 2 & 4 \end{array} \\ \begin{array}{c} 2 \\ 4 \end{array} & \begin{bmatrix} 0 & +1s_{12} \\ -1s_{12} & 0 \end{bmatrix} \end{array} \quad \sigma[14][2] = \begin{array}{cc} & \begin{array}{cc} 2 & 4 \end{array} \\ \begin{array}{c} 2 \\ 4 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[15][0] = \begin{array}{cc} & \begin{array}{cc} 2 & 5 \end{array} \\ \begin{array}{c} 2 \\ 5 \end{array} & \begin{bmatrix} 0 & +is_{11} \\ +is_{11} & 0 \end{bmatrix} \end{array} \quad \sigma[15][1] = \begin{array}{cc} & \begin{array}{cc} 2 & 5 \end{array} \\ \begin{array}{c} 2 \\ 5 \end{array} & \begin{bmatrix} 0 & +1s_{11} \\ -1s_{11} & 0 \end{bmatrix} \end{array} \quad \sigma[15][2] = \begin{array}{cc} & \begin{array}{cc} 2 & 5 \end{array} \\ \begin{array}{c} 2 \\ 5 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[16][0] = \begin{array}{cc} & \begin{array}{cc} 2 & 6 \end{array} \\ \begin{array}{c} 2 \\ 6 \end{array} & \begin{bmatrix} 0 & +is_4 \\ +is_4 & 0 \end{bmatrix} \end{array} \quad \sigma[16][1] = \begin{array}{cc} & \begin{array}{cc} 2 & 6 \end{array} \\ \begin{array}{c} 2 \\ 6 \end{array} & \begin{bmatrix} 0 & +1s_4 \\ -1s_4 & 0 \end{bmatrix} \end{array} \quad \sigma[16][2] = \begin{array}{cc} & \begin{array}{cc} 2 & 6 \end{array} \\ \begin{array}{c} 2 \\ 6 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[17][0] = \begin{array}{cc} & \begin{array}{cc} 2 & 7 \end{array} \\ \begin{array}{c} 2 \\ 7 \end{array} & \begin{bmatrix} 0 & +is_{11} \\ +is_{11} & 0 \end{bmatrix} \end{array} \quad \sigma[17][1] = \begin{array}{cc} & \begin{array}{cc} 2 & 7 \end{array} \\ \begin{array}{c} 2 \\ 7 \end{array} & \begin{bmatrix} 0 & +1s_{11} \\ -1s_{11} & 0 \end{bmatrix} \end{array} \quad \sigma[17][2] = \begin{array}{cc} & \begin{array}{cc} 2 & 7 \end{array} \\ \begin{array}{c} 2 \\ 7 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[18][0] = \begin{array}{cc} & \begin{array}{cc} 3 & 4 \end{array} \\ \begin{array}{c} 3 \\ 4 \end{array} & \begin{bmatrix} 0 & +is_7 \\ +is_7 & 0 \end{bmatrix} \end{array} \quad \sigma[18][1] = \begin{array}{cc} & \begin{array}{cc} 3 & 4 \end{array} \\ \begin{array}{c} 3 \\ 4 \end{array} & \begin{bmatrix} 0 & +1s_7 \\ -1s_7 & 0 \end{bmatrix} \end{array} \quad \sigma[18][2] = \begin{array}{cc} & \begin{array}{cc} 3 & 4 \end{array} \\ \begin{array}{c} 3 \\ 4 \end{array} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[19][0] = \begin{array}{cc} & \begin{matrix} 3 & 5 \end{matrix} \\ \begin{matrix} 3 \\ 5 \end{matrix} & \begin{bmatrix} 0 & +i \\ +i & 0 \end{bmatrix} \end{array} \quad \sigma[19][1] = \begin{array}{cc} & \begin{matrix} 3 & 5 \end{matrix} \\ \begin{matrix} 3 \\ 5 \end{matrix} & \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix} \end{array} \quad \sigma[19][2] = \begin{array}{cc} & \begin{matrix} 3 & 5 \end{matrix} \\ \begin{matrix} 3 \\ 5 \end{matrix} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[20][0] = \begin{array}{cc} & \begin{matrix} 3 & 6 \end{matrix} \\ \begin{matrix} 3 \\ 6 \end{matrix} & \begin{bmatrix} 0 & +is_{15} \\ +is_{15} & 0 \end{bmatrix} \end{array} \quad \sigma[20][1] = \begin{array}{cc} & \begin{matrix} 3 & 6 \end{matrix} \\ \begin{matrix} 3 \\ 6 \end{matrix} & \begin{bmatrix} 0 & +1s_{15} \\ -1s_{15} & 0 \end{bmatrix} \end{array} \quad \sigma[20][2] = \begin{array}{cc} & \begin{matrix} 3 & 6 \end{matrix} \\ \begin{matrix} 3 \\ 6 \end{matrix} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[21][0] = \begin{array}{cc} & \begin{matrix} 3 & 7 \end{matrix} \\ \begin{matrix} 3 \\ 7 \end{matrix} & \begin{bmatrix} 0 & +i \\ +i & 0 \end{bmatrix} \end{array} \quad \sigma[21][1] = \begin{array}{cc} & \begin{matrix} 3 & 7 \end{matrix} \\ \begin{matrix} 3 \\ 7 \end{matrix} & \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix} \end{array} \quad \sigma[21][2] = \begin{array}{cc} & \begin{matrix} 3 & 7 \end{matrix} \\ \begin{matrix} 3 \\ 7 \end{matrix} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[22][0] = \begin{array}{cc} & \begin{matrix} 4 & 5 \end{matrix} \\ \begin{matrix} 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & +is_{7} \\ +is_{7} & 0 \end{bmatrix} \end{array} \quad \sigma[22][1] = \begin{array}{cc} & \begin{matrix} 4 & 5 \end{matrix} \\ \begin{matrix} 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & +1s_{7} \\ -1s_{7} & 0 \end{bmatrix} \end{array} \quad \sigma[22][2] = \begin{array}{cc} & \begin{matrix} 4 & 5 \end{matrix} \\ \begin{matrix} 4 \\ 5 \end{matrix} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[23][0] = \begin{array}{cc} & \begin{matrix} 4 & 6 \end{matrix} \\ \begin{matrix} 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & +is_{8} \\ +is_{8} & 0 \end{bmatrix} \end{array} \quad \sigma[23][1] = \begin{array}{cc} & \begin{matrix} 4 & 6 \end{matrix} \\ \begin{matrix} 4 \\ 6 \end{matrix} & \begin{bmatrix} 0 & +1s_{8} \\ -1s_{8} & 0 \end{bmatrix} \end{array} \quad \sigma[23][2] = \begin{array}{cc} & \begin{matrix} 4 & 6 \end{matrix} \\ \begin{matrix} 4 \\ 6 \end{matrix} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[24][0] = \begin{array}{cc} & \begin{matrix} 4 & 7 \end{matrix} \\ \begin{matrix} 4 \\ 7 \end{matrix} & \begin{bmatrix} 0 & +is_{7} \\ +is_{7} & 0 \end{bmatrix} \end{array} \quad \sigma[24][1] = \begin{array}{cc} & \begin{matrix} 4 & 7 \end{matrix} \\ \begin{matrix} 4 \\ 7 \end{matrix} & \begin{bmatrix} 0 & +1s_{7} \\ -1s_{7} & 0 \end{bmatrix} \end{array} \quad \sigma[24][2] = \begin{array}{cc} & \begin{matrix} 4 & 7 \end{matrix} \\ \begin{matrix} 4 \\ 7 \end{matrix} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[25][0] = \begin{array}{cc} & \begin{matrix} 5 & 6 \end{matrix} \\ \begin{matrix} 5 \\ 6 \end{matrix} & \begin{bmatrix} 0 & +is_{15} \\ +is_{15} & 0 \end{bmatrix} \end{array} \quad \sigma[25][1] = \begin{array}{cc} & \begin{matrix} 5 & 6 \end{matrix} \\ \begin{matrix} 5 \\ 6 \end{matrix} & \begin{bmatrix} 0 & +1s_{15} \\ -1s_{15} & 0 \end{bmatrix} \end{array} \quad \sigma[25][2] = \begin{array}{cc} & \begin{matrix} 5 & 6 \end{matrix} \\ \begin{matrix} 5 \\ 6 \end{matrix} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[26][0] = \begin{array}{cc} & \begin{matrix} 5 & 7 \end{matrix} \\ \begin{matrix} 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & +i \\ +i & 0 \end{bmatrix} \end{array} \quad \sigma[26][1] = \begin{array}{cc} & \begin{matrix} 5 & 7 \end{matrix} \\ \begin{matrix} 5 \\ 7 \end{matrix} & \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix} \end{array} \quad \sigma[26][2] = \begin{array}{cc} & \begin{matrix} 5 & 7 \end{matrix} \\ \begin{matrix} 5 \\ 7 \end{matrix} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

$$\sigma[27][0] = \begin{array}{cc} & \begin{matrix} 6 & 7 \end{matrix} \\ \begin{matrix} 6 \\ 7 \end{matrix} & \begin{bmatrix} 0 & +is_{15} \\ +is_{15} & 0 \end{bmatrix} \end{array} \quad \sigma[27][1] = \begin{array}{cc} & \begin{matrix} 6 & 7 \end{matrix} \\ \begin{matrix} 6 \\ 7 \end{matrix} & \begin{bmatrix} 0 & +1s_{15} \\ -1s_{15} & 0 \end{bmatrix} \end{array} \quad \sigma[27][2] = \begin{array}{cc} & \begin{matrix} 6 & 7 \end{matrix} \\ \begin{matrix} 6 \\ 7 \end{matrix} & \begin{bmatrix} +i & 0 \\ 0 & -i \end{bmatrix} \end{array}$$

Appendix 4: Adjacent algebra linear operator orientations for all coset$_8$ builder Clifford algebras

adjacent algebra orientations[0] for 1-forms [1, 3, 5] 2-forms [2, 4, 6] 3-form [7] =

$$\begin{bmatrix} 0 & +1 & +1s_{9} & +1 & +1s_{5} & +1 & +1s_{15} & +1s_{2} \\ +1 & 0 & +1s_{9} & +1 & +1s_{5} & +1 & +1s_{15} & +1s_{2} \\ +1s_{9} & +1s_{9} & 0 & +1s_{9} & +1s_{12} & +1s_{9} & +1s_{6} & +1s_{11} \\ +1 & +1 & +1s_{9} & 0 & +1s_{5} & +1 & +1s_{15} & +1s_{2} \\ +1s_{5} & +1s_{5} & +1s_{12} & +1s_{5} & 0 & +1s_{5} & +1s_{10} & +1s_{7} \\ +1 & +1 & +1s_{9} & +1 & +1s_{5} & 0 & +1s_{15} & +1s_{2} \\ +1s_{15} & +1s_{15} & +1s_{6} & +1s_{15} & +1s_{10} & +1s_{15} & 0 & +1s_{13} \\ +1s_{2} & +1s_{2} & +1s_{11} & +1s_{2} & +1s_{7} & +1s_{2} & +1s_{13} & 0 \end{bmatrix}$$

adjacent algebra orientations[1] for 1-forms [1, 3, 7] 2-forms [2, 4, 6] 3-form [5] =

$$\begin{bmatrix} 0 & +1 & +1s_{9} & +1 & +1s_{7} & +1s_{2} & +1s_{13} & +1 \\ +1 & 0 & +1s_{9} & +1 & +1s_{7} & +1s_{2} & +1s_{13} & +1 \\ +1s_{9} & +1s_{9} & 0 & +1s_{9} & +1s_{14} & +1s_{11} & +1s_{4} & +1s_{9} \\ +1 & +1 & +1s_{9} & 0 & +1s_{7} & +1s_{2} & +1s_{13} & +1 \\ +1s_{7} & +1s_{7} & +1s_{14} & +1s_{7} & 0 & +1s_{5} & +1s_{10} & +1s_{7} \\ +1s_{2} & +1s_{2} & +1s_{11} & +1s_{2} & +1s_{5} & 0 & +1s_{15} & +1s_{2} \\ +1s_{13} & +1s_{13} & +1s_{4} & +1s_{13} & +1s_{10} & +1s_{15} & 0 & +1s_{13} \\ +1 & +1 & +1s_{9} & +1 & +1s_{7} & +1s_{2} & +1s_{13} & 0 \end{bmatrix}$$

adjacent algebra orientations[2] for 1-forms [1, 5, 7] 2-forms [2, 4, 6] 3-form [3] =

$$\begin{bmatrix} 0 & +1 & +1s_{11} & +1s_{2} & +1s_{5} & +1 & +1s_{13} & +1 \\ +1 & 0 & +1s_{11} & +1s_{2} & +1s_{5} & +1 & +1s_{13} & +1 \\ +1s_{11} & +1s_{11} & 0 & +1s_{9} & +1s_{14} & +1s_{11} & +1s_{6} & +1s_{11} \\ +1s_{2} & +1s_{2} & +1s_{9} & 0 & +1s_{7} & +1s_{2} & +1s_{15} & +1s_{2} \\ +1s_{5} & +1s_{5} & +1s_{14} & +1s_{7} & 0 & +1s_{5} & +1s_{8} & +1s_{5} \\ +1 & +1 & +1s_{11} & +1s_{2} & +1s_{5} & 0 & +1s_{13} & +1 \\ +1s_{13} & +1s_{13} & +1s_{6} & +1s_{15} & +1s_{8} & +1s_{13} & 0 & +1s_{13} \\ +1 & +1 & +1s_{11} & +1s_{2} & +1s_{5} & +1 & +1s_{13} & 0 \end{bmatrix}$$

adjacent algebra orientations[3] for 1-forms [3, 5, 7] 2-forms [2, 4, 6] 3-form [1] =

$$\begin{bmatrix} 0 & +1s_{2} & +1s_{11} & +1 & +1s_{7} & +1 & +1s_{15} & +1 \\ +1s_{2} & 0 & +1s_{9} & +1s_{2} & +1s_{5} & +1s_{2} & +1s_{13} & +1s_{2} \\ +1s_{11} & +1s_{9} & 0 & +1s_{11} & +1s_{12} & +1s_{11} & +1s_{4} & +1s_{11} \\ +1 & +1s_{2} & +1s_{11} & 0 & +1s_{7} & +1 & +1s_{15} & +1 \\ +1s_{7} & +1s_{5} & +1s_{12} & +1s_{7} & 0 & +1s_{7} & +1s_{8} & +1s_{7} \\ +1 & +1s_{2} & +1s_{11} & +1 & +1s_{7} & 0 & +1s_{15} & +1 \\ +1s_{15} & +1s_{13} & +1s_{4} & +1s_{15} & +1s_{8} & +1s_{15} & 0 & +1s_{15} \\ +1 & +1s_{2} & +1s_{11} & +1 & +1s_{7} & +1 & +1s_{15} & 0 \end{bmatrix}$$

adjacent algebra orientations[4] for 1-forms [2, 3, 6] 2-forms [1, 4, 5] 3-form [7] =

$$\begin{bmatrix}
0 & +1s_{9} & +1 & +1 & +1s_{3} & +1s_{15} & +1 & +1s_{4} \\
+1s_{9} & 0 & +1s_{9} & +1s_{9} & +1s_{10} & +1s_{6} & +1s_{9} & +1s_{13} \\
+1 & +1s_{9} & 0 & +1 & +1s_{3} & +1s_{15} & +1 & +1s_{4} \\
+1 & +1s_{9} & +1 & 0 & +1s_{3} & +1s_{15} & +1 & +1s_{4} \\
+1s_{3} & +1s_{10} & +1s_{3} & +1s_{3} & 0 & +1s_{12} & +1s_{3} & +1s_{7} \\
+1s_{15} & +1s_{6} & +1s_{15} & +1s_{15} & +1s_{12} & 0 & +1s_{15} & +1s_{11} \\
+1 & +1s_{9} & +1 & +1 & +1s_{3} & +1s_{15} & 0 & +1s_{4} \\
+1s_{4} & +1s_{13} & +1s_{4} & +1s_{4} & +1s_{7} & +1s_{11} & +1s_{4} & 0
\end{bmatrix}$$

adjacent algebra orientations[5] for 1-forms [2, 3, 7] 2-forms [1, 4, 5] 3-form [6] =

$$\begin{bmatrix}
0 & +1s_{9} & +1 & +1 & +1s_{7} & +1s_{11} & +1s_{4} & +1 \\
+1s_{9} & 0 & +1s_{9} & +1s_{9} & +1s_{14} & +1s_{2} & +1s_{13} & +1s_{9} \\
+1 & +1s_{9} & 0 & +1 & +1s_{7} & +1s_{11} & +1s_{4} & +1 \\
+1 & +1s_{9} & +1 & 0 & +1s_{7} & +1s_{11} & +1s_{4} & +1 \\
+1s_{7} & +1s_{14} & +1s_{7} & +1s_{7} & 0 & +1s_{12} & +1s_{3} & +1s_{7} \\
+1s_{11} & +1s_{2} & +1s_{11} & +1s_{11} & +1s_{12} & 0 & +1s_{15} & +1s_{11} \\
+1s_{4} & +1s_{13} & +1s_{4} & +1s_{4} & +1s_{3} & +1s_{15} & 0 & +1s_{4} \\
+1 & +1s_{9} & +1 & +1 & +1s_{7} & +1s_{11} & +1s_{4} & 0
\end{bmatrix}$$

adjacent algebra orientations[6] for 1-forms [2, 6, 7] 2-forms [1, 4, 5] 3-form [3] =

$$\begin{bmatrix}
0 & +1s_{13} & +1 & +1s_{4} & +1s_{3} & +1s_{11} & +1 & +1 \\
+1s_{13} & 0 & +1s_{13} & +1s_{9} & +1s_{14} & +1s_{6} & +1s_{13} & +1s_{13} \\
+1 & +1s_{13} & 0 & +1s_{4} & +1s_{3} & +1s_{11} & +1 & +1 \\
+1s_{4} & +1s_{9} & +1s_{4} & 0 & +1s_{7} & +1s_{15} & +1s_{4} & +1s_{4} \\
+1s_{3} & +1s_{14} & +1s_{3} & +1s_{7} & 0 & +1s_{8} & +1s_{3} & +1s_{3} \\
+1s_{11} & +1s_{6} & +1s_{11} & +1s_{15} & +1s_{8} & 0 & +1s_{11} & +1s_{11} \\
+1 & +1s_{13} & +1 & +1s_{4} & +1s_{3} & +1s_{11} & 0 & +1 \\
+1 & +1s_{13} & +1 & +1s_{4} & +1s_{3} & +1s_{11} & +1 & 0
\end{bmatrix}$$

adjacent algebra orientations[7] for 1-forms [3, 6, 7] 2-forms [1, 4, 5] 3-form [2] =

$$\begin{bmatrix}
0 & +1s_{13} & +1s_{4} & +1 & +1s_{7} & +1s_{15} & +1 & +1 \\
+1s_{13} & 0 & +1s_{9} & +1s_{13} & +1s_{10} & +1s_{2} & +1s_{13} & +1s_{13} \\
+1s_{4} & +1s_{9} & 0 & +1s_{4} & +1s_{3} & +1s_{11} & +1s_{4} & +1s_{4} \\
+1 & +1s_{13} & +1s_{4} & 0 & +1s_{7} & +1s_{15} & +1 & +1 \\
+1s_{7} & +1s_{10} & +1s_{3} & +1s_{7} & 0 & +1s_{8} & +1s_{7} & +1s_{7} \\
+1s_{15} & +1s_{2} & +1s_{11} & +1s_{15} & +1s_{8} & 0 & +1s_{15} & +1s_{15} \\
+1 & +1s_{13} & +1s_{4} & +1 & +1s_{7} & +1s_{15} & 0 & +1 \\
+1 & +1s_{13} & +1s_{4} & +1 & +1s_{7} & +1s_{15} & +1 & 0
\end{bmatrix}$$

adjacent algebra orientations[8] for 1-forms [1, 2, 5] 2-forms [3, 4, 7] 3-form [6] =

$$\begin{bmatrix} 0 & +1 & +1 & +1s_{9} & +1s_{5} & +1 & +1s_{6} & +1s_{11} \\ +1 & 0 & +1 & +1s_{9} & +1s_{5} & +1 & +1s_{6} & +1s_{11} \\ +1 & +1 & 0 & +1s_{9} & +1s_{5} & +1 & +1s_{6} & +1s_{11} \\ +1s_{9} & +1s_{9} & +1s_{9} & 0 & +1s_{12} & +1s_{9} & +1s_{15} & +1s_{2} \\ +1s_{5} & +1s_{5} & +1s_{5} & +1s_{12} & 0 & +1s_{5} & +1s_{3} & +1s_{14} \\ +1 & +1 & +1 & +1s_{9} & +1s_{5} & 0 & +1s_{6} & +1s_{11} \\ +1s_{6} & +1s_{6} & +1s_{6} & +1s_{15} & +1s_{3} & +1s_{6} & 0 & +1s_{13} \\ +1s_{11} & +1s_{11} & +1s_{11} & +1s_{2} & +1s_{14} & +1s_{11} & +1s_{13} & 0 \end{bmatrix}$$

adjacent algebra orientations[9] for 1-forms [1, 2, 6] 2-forms [3, 4, 7] 3-form [5] =

$$\begin{bmatrix} 0 & +1 & +1 & +1s_{9} & +1s_{3} & +1s_{6} & +1 & +1s_{13} \\ +1 & 0 & +1 & +1s_{9} & +1s_{3} & +1s_{6} & +1 & +1s_{13} \\ +1 & +1 & 0 & +1s_{9} & +1s_{3} & +1s_{6} & +1 & +1s_{13} \\ +1s_{9} & +1s_{9} & +1s_{9} & 0 & +1s_{10} & +1s_{15} & +1s_{9} & +1s_{4} \\ +1s_{3} & +1s_{3} & +1s_{3} & +1s_{10} & 0 & +1s_{5} & +1s_{3} & +1s_{14} \\ +1s_{6} & +1s_{6} & +1s_{6} & +1s_{15} & +1s_{5} & 0 & +1s_{6} & +1s_{11} \\ +1 & +1 & +1 & +1s_{9} & +1s_{3} & +1s_{6} & 0 & +1s_{13} \\ +1s_{13} & +1s_{13} & +1s_{13} & +1s_{4} & +1s_{14} & +1s_{11} & +1s_{13} & 0 \end{bmatrix}$$

adjacent algebra orientations[10] for 1-forms [1, 5, 6] 2-forms [3, 4, 7] 3-form [2] =

$$\begin{bmatrix} 0 & +1 & +1s_{6} & +1s_{15} & +1s_{5} & +1 & +1 & +1s_{13} \\ +1 & 0 & +1s_{6} & +1s_{15} & +1s_{5} & +1 & +1 & +1s_{13} \\ +1s_{6} & +1s_{6} & 0 & +1s_{9} & +1s_{3} & +1s_{6} & +1s_{6} & +1s_{11} \\ +1s_{15} & +1s_{15} & +1s_{9} & 0 & +1s_{10} & +1s_{15} & +1s_{15} & +1s_{2} \\ +1s_{5} & +1s_{5} & +1s_{3} & +1s_{10} & 0 & +1s_{5} & +1s_{5} & +1s_{8} \\ +1 & +1 & +1s_{6} & +1s_{15} & +1s_{5} & 0 & +1 & +1s_{13} \\ +1 & +1 & +1s_{6} & +1s_{15} & +1s_{5} & +1 & 0 & +1s_{13} \\ +1s_{13} & +1s_{13} & +1s_{11} & +1s_{2} & +1s_{8} & +1s_{13} & +1s_{13} & 0 \end{bmatrix}$$

adjacent algebra orientations[11] for 1-forms [2, 5, 6] 2-forms [3, 4, 7] 3-form [1] =

$$\begin{bmatrix} 0 & +1s_{6} & +1 & +1s_{15} & +1s_{3} & +1 & +1 & +1s_{11} \\ +1s_{6} & 0 & +1s_{6} & +1s_{9} & +1s_{5} & +1s_{6} & +1s_{6} & +1s_{13} \\ +1 & +1s_{6} & 0 & +1s_{15} & +1s_{3} & +1 & +1 & +1s_{11} \\ +1s_{15} & +1s_{9} & +1s_{15} & 0 & +1s_{12} & +1s_{15} & +1s_{15} & +1s_{4} \\ +1s_{3} & +1s_{5} & +1s_{3} & +1s_{12} & 0 & +1s_{3} & +1s_{3} & +1s_{8} \\ +1 & +1s_{6} & +1 & +1s_{15} & +1s_{3} & 0 & +1 & +1s_{11} \\ +1 & +1s_{6} & +1 & +1s_{15} & +1s_{3} & +1 & 0 & +1s_{11} \\ +1s_{11} & +1s_{13} & +1s_{11} & +1s_{4} & +1s_{8} & +1s_{11} & +1s_{11} & 0 \end{bmatrix}$$

adjacent algebra orientations[12] for 1-forms [4, 5, 6] 2-forms [1, 2, 3] 3-form [7] =

$$\begin{bmatrix} 0 & +1s_5 & +1s_3 & +1s_{15} & +1 & +1 & +1 & +1s_8 \\ +1s_5 & 0 & +1s_6 & +1s_{10} & +1s_5 & +1s_5 & +1s_5 & +1s_{13} \\ +1s_3 & +1s_6 & 0 & +1s_{12} & +1s_3 & +1s_3 & +1s_3 & +1s_{11} \\ +1s_{15} & +1s_{10} & +1s_{12} & 0 & +1s_{15} & +1s_{15} & +1s_{15} & +1s_7 \\ +1 & +1s_5 & +1s_3 & +1s_{15} & 0 & +1 & +1 & +1s_8 \\ +1 & +1s_5 & +1s_3 & +1s_{15} & +1 & 0 & +1 & +1s_8 \\ +1 & +1s_5 & +1s_3 & +1s_{15} & +1 & +1 & 0 & +1s_8 \\ +1s_8 & +1s_{13} & +1s_{11} & +1s_7 & +1s_8 & +1s_8 & +1s_8 & 0 \end{bmatrix}$$

adjacent algebra orientations[13] for 1-forms [4, 5, 7] 2-forms [1, 2, 3] 3-form [6] =

$$\begin{bmatrix} 0 & +1s_5 & +1s_{11} & +1s_7 & +1 & +1 & +1s_8 & +1 \\ +1s_5 & 0 & +1s_{14} & +1s_2 & +1s_5 & +1s_5 & +1s_{13} & +1s_5 \\ +1s_{11} & +1s_{14} & 0 & +1s_{12} & +1s_{11} & +1s_{11} & +1s_3 & +1s_{11} \\ +1s_7 & +1s_2 & +1s_{12} & 0 & +1s_7 & +1s_7 & +1s_{15} & +1s_7 \\ +1 & +1s_5 & +1s_{11} & +1s_7 & 0 & +1 & +1s_8 & +1 \\ +1 & +1s_5 & +1s_{11} & +1s_7 & +1 & 0 & +1s_8 & +1 \\ +1s_8 & +1s_{13} & +1s_3 & +1s_{15} & +1s_8 & +1s_8 & 0 & +1s_8 \\ +1 & +1s_5 & +1s_{11} & +1s_7 & +1 & +1 & +1s_8 & 0 \end{bmatrix}$$

adjacent algebra orientations[14] for 1-forms [4, 6, 7] 2-forms [1, 2, 3] 3-form [5] =

$$\begin{bmatrix} 0 & +1s_{13} & +1s_3 & +1s_7 & +1 & +1s_8 & +1 & +1 \\ +1s_{13} & 0 & +1s_{14} & +1s_{10} & +1s_{13} & +1s_5 & +1s_{13} & +1s_{13} \\ +1s_3 & +1s_{14} & 0 & +1s_4 & +1s_3 & +1s_{11} & +1s_3 & +1s_3 \\ +1s_7 & +1s_{10} & +1s_4 & 0 & +1s_7 & +1s_{15} & +1s_7 & +1s_7 \\ +1 & +1s_{13} & +1s_3 & +1s_7 & 0 & +1s_8 & +1 & +1 \\ +1s_8 & +1s_5 & +1s_{11} & +1s_{15} & +1s_8 & 0 & +1s_8 & +1s_8 \\ +1 & +1s_{13} & +1s_3 & +1s_7 & +1 & +1s_8 & 0 & +1 \\ +1 & +1s_{13} & +1s_3 & +1s_7 & +1 & +1s_8 & +1 & 0 \end{bmatrix}$$

adjacent algebra orientations[15] for 1-forms [5, 6, 7] 2-forms [1, 2, 3] 3-form [4] =

$$\begin{bmatrix} 0 & +1s_{13} & +1s_{11} & +1s_{15} & +1s_8 & +1 & +1 & +1 \\ +1s_{13} & 0 & +1s_6 & +1s_2 & +1s_5 & +1s_{13} & +1s_{13} & +1s_{13} \\ +1s_{11} & +1s_6 & 0 & +1s_4 & +1s_3 & +1s_{11} & +1s_{11} & +1s_{11} \\ +1s_{15} & +1s_2 & +1s_4 & 0 & +1s_7 & +1s_{15} & +1s_{15} & +1s_{15} \\ +1s_8 & +1s_5 & +1s_3 & +1s_7 & 0 & +1s_8 & +1s_8 & +1s_8 \\ +1 & +1s_{13} & +1s_{11} & +1s_{15} & +1s_8 & 0 & +1 & +1 \\ +1 & +1s_{13} & +1s_{11} & +1s_{15} & +1s_8 & +1 & 0 & +1 \\ +1 & +1s_{13} & +1s_{11} & +1s_{15} & +1s_8 & +1 & +1 & 0 \end{bmatrix}$$

adjacent algebra orientations[16] for 1-forms [1, 3, 4] 2-forms [2, 5, 7] 3-form [6] =

$$\begin{bmatrix}
0 & +1 & +1s_{9} & +1 & +1 & +1s_{5} & +1s_{10} & +1s_{7} \\
+1 & 0 & +1s_{9} & +1 & +1 & +1s_{5} & +1s_{10} & +1s_{7} \\
+1s_{9} & +1s_{9} & 0 & +1s_{9} & +1s_{9} & +1s_{12} & +1s_{3} & +1s_{14} \\
+1 & +1 & +1s_{9} & 0 & +1 & +1s_{5} & +1s_{10} & +1s_{7} \\
+1 & +1 & +1s_{9} & +1 & 0 & +1s_{5} & +1s_{10} & +1s_{7} \\
+1s_{5} & +1s_{5} & +1s_{12} & +1s_{5} & +1s_{5} & 0 & +1s_{15} & +1s_{2} \\
+1s_{10} & +1s_{10} & +1s_{3} & +1s_{10} & +1s_{10} & +1s_{15} & 0 & +1s_{13} \\
+1s_{7} & +1s_{7} & +1s_{14} & +1s_{7} & +1s_{7} & +1s_{2} & +1s_{13} & 0
\end{bmatrix}$$

adjacent algebra orientations[17] for 1-forms [1, 3, 6] 2-forms [2, 5, 7] 3-form [4] =

$$\begin{bmatrix}
0 & +1 & +1s_{9} & +1 & +1s_{10} & +1s_{15} & +1 & +1s_{13} \\
+1 & 0 & +1s_{9} & +1 & +1s_{10} & +1s_{15} & +1 & +1s_{13} \\
+1s_{9} & +1s_{9} & 0 & +1s_{9} & +1s_{3} & +1s_{6} & +1s_{9} & +1s_{4} \\
+1 & +1 & +1s_{9} & 0 & +1s_{10} & +1s_{15} & +1 & +1s_{13} \\
+1s_{10} & +1s_{10} & +1s_{3} & +1s_{10} & 0 & +1s_{5} & +1s_{10} & +1s_{7} \\
+1s_{15} & +1s_{15} & +1s_{6} & +1s_{15} & +1s_{5} & 0 & +1s_{15} & +1s_{2} \\
+1 & +1 & +1s_{9} & +1 & +1s_{10} & +1s_{15} & 0 & +1s_{13} \\
+1s_{13} & +1s_{13} & +1s_{4} & +1s_{13} & +1s_{7} & +1s_{2} & +1s_{13} & 0
\end{bmatrix}$$

adjacent algebra orientations[18] for 1-forms [1, 4, 6] 2-forms [2, 5, 7] 3-form [3] =

$$\begin{bmatrix}
0 & +1 & +1s_{3} & +1s_{10} & +1 & +1s_{5} & +1 & +1s_{13} \\
+1 & 0 & +1s_{3} & +1s_{10} & +1 & +1s_{5} & +1 & +1s_{13} \\
+1s_{3} & +1s_{3} & 0 & +1s_{9} & +1s_{3} & +1s_{6} & +1s_{3} & +1s_{14} \\
+1s_{10} & +1s_{10} & +1s_{9} & 0 & +1s_{10} & +1s_{15} & +1s_{10} & +1s_{7} \\
+1 & +1 & +1s_{3} & +1s_{10} & 0 & +1s_{5} & +1 & +1s_{13} \\
+1s_{5} & +1s_{5} & +1s_{6} & +1s_{15} & +1s_{5} & 0 & +1s_{5} & +1s_{8} \\
+1 & +1 & +1s_{3} & +1s_{10} & +1 & +1s_{5} & 0 & +1s_{13} \\
+1s_{13} & +1s_{13} & +1s_{14} & +1s_{7} & +1s_{13} & +1s_{8} & +1s_{13} & 0
\end{bmatrix}$$

adjacent algebra orientations[19] for 1-forms [3, 4, 6] 2-forms [2, 5, 7] 3-form [1] =

$$\begin{bmatrix}
0 & +1s_{10} & +1s_{3} & +1 & +1 & +1s_{15} & +1 & +1s_{7} \\
+1s_{10} & 0 & +1s_{9} & +1s_{10} & +1s_{10} & +1s_{5} & +1s_{10} & +1s_{13} \\
+1s_{3} & +1s_{9} & 0 & +1s_{3} & +1s_{3} & +1s_{12} & +1s_{3} & +1s_{4} \\
+1 & +1s_{10} & +1s_{3} & 0 & +1 & +1s_{15} & +1 & +1s_{7} \\
+1 & +1s_{10} & +1s_{3} & +1 & 0 & +1s_{15} & +1 & +1s_{7} \\
+1s_{15} & +1s_{5} & +1s_{12} & +1s_{15} & +1s_{15} & 0 & +1s_{15} & +1s_{8} \\
+1 & +1s_{10} & +1s_{3} & +1 & +1 & +1s_{15} & 0 & +1s_{7} \\
+1s_{7} & +1s_{13} & +1s_{4} & +1s_{7} & +1s_{7} & +1s_{8} & +1s_{7} & 0
\end{bmatrix}$$

adjacent algebra orientations[20] for 1-forms [2, 3, 4] 2-forms [1, 6, 7] 3-form [5] =

$$\begin{bmatrix} 0 & +1s_{9} & +1 & +1 & +1 & +1s_{12} & +1s_{3} & +1s_{7} \\ +1s_{9} & 0 & +1s_{9} & +1s_{9} & +1s_{9} & +1s_{5} & +1s_{10} & +1s_{14} \\ +1 & +1s_{9} & 0 & +1 & +1 & +1s_{12} & +1s_{3} & +1s_{7} \\ +1 & +1s_{9} & +1 & 0 & +1 & +1s_{12} & +1s_{3} & +1s_{7} \\ +1 & +1s_{9} & +1 & +1 & 0 & +1s_{12} & +1s_{3} & +1s_{7} \\ +1s_{12} & +1s_{5} & +1s_{12} & +1s_{12} & +1s_{12} & 0 & +1s_{15} & +1s_{11} \\ +1s_{3} & +1s_{10} & +1s_{3} & +1s_{3} & +1s_{3} & +1s_{15} & 0 & +1s_{4} \\ +1s_{7} & +1s_{14} & +1s_{7} & +1s_{7} & +1s_{7} & +1s_{11} & +1s_{4} & 0 \end{bmatrix}$$

adjacent algebra orientations[21] for 1-forms [2, 3, 5] 2-forms [1, 6, 7] 3-form [4] =

$$\begin{bmatrix} 0 & +1s_{9} & +1 & +1 & +1s_{12} & +1 & +1s_{15} & +1s_{11} \\ +1s_{9} & 0 & +1s_{9} & +1s_{9} & +1s_{5} & +1s_{9} & +1s_{6} & +1s_{2} \\ +1 & +1s_{9} & 0 & +1 & +1s_{12} & +1 & +1s_{15} & +1s_{11} \\ +1 & +1s_{9} & +1 & 0 & +1s_{12} & +1 & +1s_{15} & +1s_{11} \\ +1s_{12} & +1s_{5} & +1s_{12} & +1s_{12} & 0 & +1s_{12} & +1s_{3} & +1s_{7} \\ +1 & +1s_{9} & +1 & +1 & +1s_{12} & 0 & +1s_{15} & +1s_{11} \\ +1s_{15} & +1s_{6} & +1s_{15} & +1s_{15} & +1s_{3} & +1s_{15} & 0 & +1s_{4} \\ +1s_{11} & +1s_{2} & +1s_{11} & +1s_{11} & +1s_{7} & +1s_{11} & +1s_{4} & 0 \end{bmatrix}$$

adjacent algebra orientations[22] for 1-forms [2, 4, 5] 2-forms [1, 6, 7] 3-form [3] =

$$\begin{bmatrix} 0 & +1s_{5} & +1 & +1s_{12} & +1 & +1 & +1s_{3} & +1s_{11} \\ +1s_{5} & 0 & +1s_{5} & +1s_{9} & +1s_{5} & +1s_{5} & +1s_{6} & +1s_{14} \\ +1 & +1s_{5} & 0 & +1s_{12} & +1 & +1 & +1s_{3} & +1s_{11} \\ +1s_{12} & +1s_{9} & +1s_{12} & 0 & +1s_{12} & +1s_{12} & +1s_{15} & +1s_{7} \\ +1 & +1s_{5} & +1 & +1s_{12} & 0 & +1 & +1s_{3} & +1s_{11} \\ +1 & +1s_{5} & +1 & +1s_{12} & +1 & 0 & +1s_{3} & +1s_{11} \\ +1s_{3} & +1s_{6} & +1s_{3} & +1s_{15} & +1s_{3} & +1s_{3} & 0 & +1s_{8} \\ +1s_{11} & +1s_{14} & +1s_{11} & +1s_{7} & +1s_{11} & +1s_{11} & +1s_{8} & 0 \end{bmatrix}$$

adjacent algebra orientations[23] for 1-forms [3, 4, 5] 2-forms [1, 6, 7] 3-form [2] =

$$\begin{bmatrix} 0 & +1s_{5} & +1s_{12} & +1 & +1 & +1 & +1s_{15} & +1s_{7} \\ +1s_{5} & 0 & +1s_{9} & +1s_{5} & +1s_{5} & +1s_{5} & +1s_{10} & +1s_{2} \\ +1s_{12} & +1s_{9} & 0 & +1s_{12} & +1s_{12} & +1s_{12} & +1s_{3} & +1s_{11} \\ +1 & +1s_{5} & +1s_{12} & 0 & +1 & +1 & +1s_{15} & +1s_{7} \\ +1 & +1s_{5} & +1s_{12} & +1 & 0 & +1 & +1s_{15} & +1s_{7} \\ +1 & +1s_{5} & +1s_{12} & +1 & +1 & 0 & +1s_{15} & +1s_{7} \\ +1s_{15} & +1s_{10} & +1s_{3} & +1s_{15} & +1s_{15} & +1s_{15} & 0 & +1s_{8} \\ +1s_{7} & +1s_{2} & +1s_{11} & +1s_{7} & +1s_{7} & +1s_{7} & +1s_{8} & 0 \end{bmatrix}$$

adjacent algebra orientations[24] for 1-forms [1, 2, 4] 2-forms [3, 5, 6] 3-form [7] =

$$\begin{bmatrix}
0 & +1 & +1 & +1s_{9} & +1 & +1s_{5} & +1s_{3} & +1s_{14} \\
+1 & 0 & +1 & +1s_{9} & +1 & +1s_{5} & +1s_{3} & +1s_{14} \\
+1 & +1 & 0 & +1s_{9} & +1 & +1s_{5} & +1s_{3} & +1s_{14} \\
+1s_{9} & +1s_{9} & +1s_{9} & 0 & +1s_{9} & +1s_{12} & +1s_{10} & +1s_{7} \\
+1 & +1 & +1 & +1s_{9} & 0 & +1s_{5} & +1s_{3} & +1s_{14} \\
+1s_{5} & +1s_{5} & +1s_{5} & +1s_{12} & +1s_{5} & 0 & +1s_{6} & +1s_{11} \\
+1s_{3} & +1s_{3} & +1s_{3} & +1s_{10} & +1s_{3} & +1s_{6} & 0 & +1s_{13} \\
+1s_{14} & +1s_{14} & +1s_{14} & +1s_{7} & +1s_{14} & +1s_{11} & +1s_{13} & 0
\end{bmatrix}$$

adjacent algebra orientations[25] for 1-forms [1, 2, 7] 2-forms [3, 5, 6] 3-form [4] =

$$\begin{bmatrix}
0 & +1 & +1 & +1s_{9} & +1s_{14} & +1s_{11} & +1s_{13} & +1 \\
+1 & 0 & +1 & +1s_{9} & +1s_{14} & +1s_{11} & +1s_{13} & +1 \\
+1 & +1 & 0 & +1s_{9} & +1s_{14} & +1s_{11} & +1s_{13} & +1 \\
+1s_{9} & +1s_{9} & +1s_{9} & 0 & +1s_{7} & +1s_{2} & +1s_{4} & +1s_{9} \\
+1s_{14} & +1s_{14} & +1s_{14} & +1s_{7} & 0 & +1s_{5} & +1s_{3} & +1s_{14} \\
+1s_{11} & +1s_{11} & +1s_{11} & +1s_{2} & +1s_{5} & 0 & +1s_{6} & +1s_{11} \\
+1s_{13} & +1s_{13} & +1s_{13} & +1s_{4} & +1s_{3} & +1s_{6} & 0 & +1s_{13} \\
+1 & +1 & +1 & +1s_{9} & +1s_{14} & +1s_{11} & +1s_{13} & 0
\end{bmatrix}$$

adjacent algebra orientations[26] for 1-forms [1, 4, 7] 2-forms [3, 5, 6] 3-form [2] =

$$\begin{bmatrix}
0 & +1 & +1s_{14} & +1s_{7} & +1 & +1s_{5} & +1s_{13} & +1 \\
+1 & 0 & +1s_{14} & +1s_{7} & +1 & +1s_{5} & +1s_{13} & +1 \\
+1s_{14} & +1s_{14} & 0 & +1s_{9} & +1s_{14} & +1s_{11} & +1s_{3} & +1s_{14} \\
+1s_{7} & +1s_{7} & +1s_{9} & 0 & +1s_{7} & +1s_{2} & +1s_{10} & +1s_{7} \\
+1 & +1 & +1s_{14} & +1s_{7} & 0 & +1s_{5} & +1s_{13} & +1 \\
+1s_{5} & +1s_{5} & +1s_{11} & +1s_{2} & +1s_{5} & 0 & +1s_{8} & +1s_{5} \\
+1s_{13} & +1s_{13} & +1s_{3} & +1s_{10} & +1s_{13} & +1s_{8} & 0 & +1s_{13} \\
+1 & +1 & +1s_{14} & +1s_{7} & +1 & +1s_{5} & +1s_{13} & 0
\end{bmatrix}$$

adjacent algebra orientations[27] for 1-forms [2, 4, 7] 2-forms [3, 5, 6] 3-form [1] =

$$\begin{bmatrix}
0 & +1s_{14} & +1 & +1s_{7} & +1 & +1s_{11} & +1s_{3} & +1 \\
+1s_{14} & 0 & +1s_{14} & +1s_{9} & +1s_{14} & +1s_{5} & +1s_{13} & +1s_{14} \\
+1 & +1s_{14} & 0 & +1s_{7} & +1 & +1s_{11} & +1s_{3} & +1 \\
+1s_{7} & +1s_{9} & +1s_{7} & 0 & +1s_{7} & +1s_{12} & +1s_{4} & +1s_{7} \\
+1 & +1s_{14} & +1 & +1s_{7} & 0 & +1s_{11} & +1s_{3} & +1 \\
+1s_{11} & +1s_{5} & +1s_{11} & +1s_{12} & +1s_{11} & 0 & +1s_{8} & +1s_{11} \\
+1s_{3} & +1s_{13} & +1s_{3} & +1s_{4} & +1s_{3} & +1s_{8} & 0 & +1s_{3} \\
+1 & +1s_{14} & +1 & +1s_{7} & +1 & +1s_{11} & +1s_{3} & 0
\end{bmatrix}$$

Appendix 5: Fundamental su(n) Lie algebra partition off-diagonal Pauli su(2) matrix sets

Members of fundamental su(n) are members of select su(2) defined above in Appendix 3. For a given placement in the 8x8 adjacent algebra matrix format defined by its spanning row/column set, the given set of σ[] matrices are closed for bracket. The correct basis set for the desired su(n) is found by reducing the set of diagonals to a mutually orthogonal set of n – 1 members.

Fundamental su(3)[0] spanning rows/columns [0, 1, 2]
{σ[0], σ[1], σ[7]}
Fundamental su(3)[1] spanning rows/columns [0, 1, 3]
{σ[0], σ[2], σ[8]}
Fundamental su(3)[2] spanning rows/columns [0, 1, 4]
{σ[0], σ[3], σ[9]}
Fundamental su(3)[3] spanning rows/columns [0, 1, 5]
{σ[0], σ[4], σ[10]}
Fundamental su(3)[4] spanning rows/columns [0, 1, 6]
{σ[0], σ[5], σ[11]}
Fundamental su(3)[5] spanning rows/columns [0, 1, 7]
{σ[0], σ[6], σ[12]}
Fundamental su(3)[6] spanning rows/columns [0, 2, 3]
{σ[1], σ[2], σ[13]}
Fundamental su(3)[7] spanning rows/columns [0, 2, 4]
{σ[1], σ[3], σ[14]}
Fundamental su(3)[8] spanning rows/columns [0, 2, 5]
{σ[1], σ[4], σ[15]}
Fundamental su(3)[9] spanning rows/columns [0, 2, 6]
{σ[1], σ[5], σ[16]}
Fundamental su(3)[10] spanning rows/columns [0, 2, 7]
{σ[1], σ[6], σ[17]}
Fundamental su(3)[11] spanning rows/columns [0, 3, 4]
{σ[2], σ[3], σ[18]}
Fundamental su(3)[12] spanning rows/columns [0, 3, 5]
{σ[2], σ[4], σ[19]}
Fundamental su(3)[13] spanning rows/columns [0, 3, 6]
{σ[2], σ[5], σ[20]}
Fundamental su(3)[14] spanning rows/columns [0, 3, 7]
{σ[2], σ[6], σ[21]}
Fundamental su(3)[15] spanning rows/columns [0, 4, 5]
{σ[3], σ[4], σ[22]}
Fundamental su(3)[16] spanning rows/columns [0, 4, 6]
{σ[3], σ[5], σ[23]}
Fundamental su(3)[17] spanning rows/columns [0, 4, 7]
{σ[3], σ[6], σ[24]}
Fundamental su(3)[18] spanning rows/columns [0, 5, 6]
{σ[4], σ[5], σ[25]}
Fundamental su(3)[19] spanning rows/columns [0, 5, 7]
{σ[4], σ[6], σ[26]}
Fundamental su(3)[20] spanning rows/columns [0, 6, 7]
{σ[5], σ[6], σ[27]}
Fundamental su(3)[21] spanning rows/columns [1, 2, 3]
{σ[7], σ[8], σ[13]}

Fundamental su(3)[22] spanning rows/columns [1, 2, 4]
{σ[7], σ[9], σ[14]}
Fundamental su(3)[23] spanning rows/columns [1, 2, 5]
{σ[7], σ[10], σ[15]}
Fundamental su(3)[24] spanning rows/columns [1, 2, 6]
{σ[7], σ[11], σ[16]}
Fundamental su(3)[25] spanning rows/columns [1, 2, 7]
{σ[7], σ[12], σ[17]}
Fundamental su(3)[26] spanning rows/columns [1, 3, 4]
{σ[8], σ[9], σ[18]}
Fundamental su(3)[27] spanning rows/columns [1, 3, 5]
{σ[8], σ[10], σ[19]}
Fundamental su(3)[28] spanning rows/columns [1, 3, 6]
{σ[8], σ[11], σ[20]}
Fundamental su(3)[29] spanning rows/columns [1, 3, 7]
{σ[8], σ[12], σ[21]}
Fundamental su(3)[30] spanning rows/columns [1, 4, 5]
{σ[9], σ[10], σ[22]}
Fundamental su(3)[31] spanning rows/columns [1, 4, 6]
{σ[9], σ[11], σ[23]}
Fundamental su(3)[32] spanning rows/columns [1, 4, 7]
{σ[9], σ[12], σ[24]}
Fundamental su(3)[33] spanning rows/columns [1, 5, 6]
{σ[10], σ[11], σ[25]}
Fundamental su(3)[34] spanning rows/columns [1, 5, 7]
{σ[10], σ[12], σ[26]}
Fundamental su(3)[35] spanning rows/columns [1, 6, 7]
{σ[11], σ[12], σ[27]}
Fundamental su(3)[36] spanning rows/columns [2, 3, 4]
{σ[13], σ[14], σ[18]}
Fundamental su(3)[37] spanning rows/columns [2, 3, 5]
{σ[13], σ[15], σ[19]}
Fundamental su(3)[38] spanning rows/columns [2, 3, 6]
{σ[13], σ[16], σ[20]}
Fundamental su(3)[39] spanning rows/columns [2, 3, 7]
{σ[13], σ[17], σ[21]}
Fundamental su(3)[40] spanning rows/columns [2, 4, 5]
{σ[14], σ[15], σ[22]}
Fundamental su(3)[41] spanning rows/columns [2, 4, 6]
{σ[14], σ[16], σ[23]}
Fundamental su(3)[42] spanning rows/columns [2, 4, 7]
{σ[14], σ[17], σ[24]}
Fundamental su(3)[43] spanning rows/columns [2, 5, 6]
{σ[15], σ[16], σ[25]}
Fundamental su(3)[44] spanning rows/columns [2, 5, 7]
{σ[15], σ[17], σ[26]}
Fundamental su(3)[45] spanning rows/columns [2, 6, 7]
{σ[16], σ[17], σ[27]}
Fundamental su(3)[46] spanning rows/columns [3, 4, 5]
{σ[18], σ[19], σ[22]}
Fundamental su(3)[47] spanning rows/columns [3, 4, 6]

{σ[18], σ[20], σ[23]}
Fundamental su(3)[48] spanning rows/columns [3, 4, 7]
{σ[18], σ[21], σ[24]}
Fundamental su(3)[49] spanning rows/columns [3, 5, 6]
{σ[19], σ[20], σ[25]}
Fundamental su(3)[50] spanning rows/columns [3, 5, 7]
{σ[19], σ[21], σ[26]}
Fundamental su(3)[51] spanning rows/columns [3, 6, 7]
{σ[20], σ[21], σ[27]}
Fundamental su(3)[52] spanning rows/columns [4, 5, 6]
{σ[22], σ[23], σ[25]}
Fundamental su(3)[53] spanning rows/columns [4, 5, 7]
{σ[22], σ[24], σ[26]}
Fundamental su(3)[54] spanning rows/columns [4, 6, 7]
{σ[23], σ[24], σ[27]}
Fundamental su(3)[55] spanning rows/columns [5, 6, 7]
{σ[25], σ[26], σ[27]}

Fundamental su(4)[0] spanning rows/columns [0, 1, 2, 3]
{σ[0], σ[1], σ[2], σ[7], σ[8], σ[13]}
Fundamental su(4)[1] spanning rows/columns [0, 1, 2, 4]
{σ[0], σ[1], σ[3], σ[7], σ[9], σ[14]}
Fundamental su(4)[2] spanning rows/columns [0, 1, 2, 5]
{σ[0], σ[1], σ[4], σ[7], σ[10], σ[15]}
Fundamental su(4)[3] spanning rows/columns [0, 1, 2, 6]
{σ[0], σ[1], σ[5], σ[7], σ[11], σ[16]}
Fundamental su(4)[4] spanning rows/columns [0, 1, 2, 7]
{σ[0], σ[1], σ[6], σ[7], σ[12], σ[17]}
Fundamental su(4)[5] spanning rows/columns [0, 1, 3, 4]
{σ[0], σ[2], σ[3], σ[8], σ[9], σ[18]}
Fundamental su(4)[6] spanning rows/columns [0, 1, 3, 5]
{σ[0], σ[2], σ[4], σ[8], σ[10], σ[19]}
Fundamental su(4)[7] spanning rows/columns [0, 1, 3, 6]
{σ[0], σ[2], σ[5], σ[8], σ[11], σ[20]}
Fundamental su(4)[8] spanning rows/columns [0, 1, 3, 7]
{σ[0], σ[2], σ[6], σ[8], σ[12], σ[21]}
Fundamental su(4)[9] spanning rows/columns [0, 1, 4, 5]
{σ[0], σ[3], σ[4], σ[9], σ[10], σ[22]}
Fundamental su(4)[10] spanning rows/columns [0, 1, 4, 6]
{σ[0], σ[3], σ[5], σ[9], σ[11], σ[23]}
Fundamental su(4)[11] spanning rows/columns [0, 1, 4, 7]
{σ[0], σ[3], σ[6], σ[9], σ[12], σ[24]}
Fundamental su(4)[12] spanning rows/columns [0, 1, 5, 6]
{σ[0], σ[4], σ[5], σ[10], σ[11], σ[25]}
Fundamental su(4)[13] spanning rows/columns [0, 1, 5, 7]
{σ[0], σ[4], σ[6], σ[10], σ[12], σ[26]}
Fundamental su(4)[14] spanning rows/columns [0, 1, 6, 7]
{σ[0], σ[5], σ[6], σ[11], σ[12], σ[27]}
Fundamental su(4)[15] spanning rows/columns [0, 2, 3, 4]
{σ[1], σ[2], σ[3], σ[13], σ[14], σ[18]}
Fundamental su(4)[16] spanning rows/columns [0, 2, 3, 5]

{σ[1], σ[2], σ[4], σ[13], σ[15], σ[19]}
Fundamental su(4)[17] spanning rows/columns [0, 2, 3, 6]
{σ[1], σ[2], σ[5], σ[13], σ[16], σ[20]}
Fundamental su(4)[18] spanning rows/columns [0, 2, 3, 7]
{σ[1], σ[2], σ[6], σ[13], σ[17], σ[21]}
Fundamental su(4)[19] spanning rows/columns [0, 2, 4, 5]
{σ[1], σ[3], σ[4], σ[14], σ[15], σ[22]}
Fundamental su(4)[20] spanning rows/columns [0, 2, 4, 6]
{σ[1], σ[3], σ[5], σ[14], σ[16], σ[23]}
Fundamental su(4)[21] spanning rows/columns [0, 2, 4, 7]
{σ[1], σ[3], σ[6], σ[14], σ[17], σ[24]}
Fundamental su(4)[22] spanning rows/columns [0, 2, 5, 6]
{σ[1], σ[4], σ[5], σ[15], σ[16], σ[25]}
Fundamental su(4)[23] spanning rows/columns [0, 2, 5, 7]
{σ[1], σ[4], σ[6], σ[15], σ[17], σ[26]}
Fundamental su(4)[24] spanning rows/columns [0, 2, 6, 7]
{σ[1], σ[5], σ[6], σ[16], σ[17], σ[27]}
Fundamental su(4)[25] spanning rows/columns [0, 3, 4, 5]
{σ[2], σ[3], σ[4], σ[18], σ[19], σ[22]}
Fundamental su(4)[26] spanning rows/columns [0, 3, 4, 6]
{σ[2], σ[3], σ[5], σ[18], σ[20], σ[23]}
Fundamental su(4)[27] spanning rows/columns [0, 3, 4, 7]
{σ[2], σ[3], σ[6], σ[18], σ[21], σ[24]}
Fundamental su(4)[28] spanning rows/columns [0, 3, 5, 6]
{σ[2], σ[4], σ[5], σ[19], σ[20], σ[25]}
Fundamental su(4)[29] spanning rows/columns [0, 3, 5, 7]
{σ[2], σ[4], σ[6], σ[19], σ[21], σ[26]}
Fundamental su(4)[30] spanning rows/columns [0, 3, 6, 7]
{σ[2], σ[5], σ[6], σ[20], σ[21], σ[27]}
Fundamental su(4)[31] spanning rows/columns [0, 4, 5, 6]
{σ[3], σ[4], σ[5], σ[22], σ[23], σ[25]}
Fundamental su(4)[32] spanning rows/columns [0, 4, 5, 7]
{σ[3], σ[4], σ[6], σ[22], σ[24], σ[26]}
Fundamental su(4)[33] spanning rows/columns [0, 4, 6, 7]
{σ[3], σ[5], σ[6], σ[23], σ[24], σ[27]}
Fundamental su(4)[34] spanning rows/columns [0, 5, 6, 7]
{σ[4], σ[5], σ[6], σ[25], σ[26], σ[27]}
Fundamental su(4)[35] spanning rows/columns [1, 2, 3, 4]
{σ[7], σ[8], σ[9], σ[13], σ[14], σ[18]}
Fundamental su(4)[36] spanning rows/columns [1, 2, 3, 5]
{σ[7], σ[8], σ[10], σ[13], σ[15], σ[19]}
Fundamental su(4)[37] spanning rows/columns [1, 2, 3, 6]
{σ[7], σ[8], σ[11], σ[13], σ[16], σ[20]}
Fundamental su(4)[38] spanning rows/columns [1, 2, 3, 7]
{σ[7], σ[8], σ[12], σ[13], σ[17], σ[21]}
Fundamental su(4)[39] spanning rows/columns [1, 2, 4, 5]
{σ[7], σ[9], σ[10], σ[14], σ[15], σ[22]}
Fundamental su(4)[40] spanning rows/columns [1, 2, 4, 6]
{σ[7], σ[9], σ[11], σ[14], σ[16], σ[23]}
Fundamental su(4)[41] spanning rows/columns [1, 2, 4, 7]
{σ[7], σ[9], σ[12], σ[14], σ[17], σ[24]}

Fundamental su(4)[42] spanning rows/columns [1, 2, 5, 6]
{σ[7], σ[10], σ[11], σ[15], σ[16], σ[25]}
Fundamental su(4)[43] spanning rows/columns [1, 2, 5, 7]
{σ[7], σ[10], σ[12], σ[15], σ[17], σ[26]}
Fundamental su(4)[44] spanning rows/columns [1, 2, 6, 7]
{σ[7], σ[11], σ[12], σ[16], σ[17], σ[27]}
Fundamental su(4)[45] spanning rows/columns [1, 3, 4, 5]
{σ[8], σ[9], σ[10], σ[18], σ[19], σ[22]}
Fundamental su(4)[46] spanning rows/columns [1, 3, 4, 6]
{σ[8], σ[9], σ[11], σ[18], σ[20], σ[23]}
Fundamental su(4)[47] spanning rows/columns [1, 3, 4, 7]
{σ[8], σ[9], σ[12], σ[18], σ[21], σ[24]}
Fundamental su(4)[48] spanning rows/columns [1, 3, 5, 6]
{σ[8], σ[10], σ[11], σ[19], σ[20], σ[25]}
Fundamental su(4)[49] spanning rows/columns [1, 3, 5, 7]
{σ[8], σ[10], σ[12], σ[19], σ[21], σ[26]}
Fundamental su(4)[50] spanning rows/columns [1, 3, 6, 7]
{σ[8], σ[11], σ[12], σ[20], σ[21], σ[27]}
Fundamental su(4)[51] spanning rows/columns [1, 4, 5, 6]
{σ[9], σ[10], σ[11], σ[22], σ[23], σ[25]}
Fundamental su(4)[52] spanning rows/columns [1, 4, 5, 7]
{σ[9], σ[10], σ[12], σ[22], σ[24], σ[26]}
Fundamental su(4)[53] spanning rows/columns [1, 4, 6, 7]
{σ[9], σ[11], σ[12], σ[23], σ[24], σ[27]}
Fundamental su(4)[54] spanning rows/columns [1, 5, 6, 7]
{σ[10], σ[11], σ[12], σ[25], σ[26], σ[27]}
Fundamental su(4)[55] spanning rows/columns [2, 3, 4, 5]
{σ[13], σ[14], σ[15], σ[18], σ[19], σ[22]}
Fundamental su(4)[56] spanning rows/columns [2, 3, 4, 6]
{σ[13], σ[14], σ[16], σ[18], σ[20], σ[23]}
Fundamental su(4)[57] spanning rows/columns [2, 3, 4, 7]
{σ[13], σ[14], σ[17], σ[18], σ[21], σ[24]}
Fundamental su(4)[58] spanning rows/columns [2, 3, 5, 6]
{σ[13], σ[15], σ[16], σ[19], σ[20], σ[25]}
Fundamental su(4)[59] spanning rows/columns [2, 3, 5, 7]
{σ[13], σ[15], σ[17], σ[19], σ[21], σ[26]}
Fundamental su(4)[60] spanning rows/columns [2, 3, 6, 7]
{σ[13], σ[16], σ[17], σ[20], σ[21], σ[27]}
Fundamental su(4)[61] spanning rows/columns [2, 4, 5, 6]
{σ[14], σ[15], σ[16], σ[22], σ[23], σ[25]}
Fundamental su(4)[62] spanning rows/columns [2, 4, 5, 7]
{σ[14], σ[15], σ[17], σ[22], σ[24], σ[26]}
Fundamental su(4)[63] spanning rows/columns [2, 4, 6, 7]
{σ[14], σ[16], σ[17], σ[23], σ[24], σ[27]}
Fundamental su(4)[64] spanning rows/columns [2, 5, 6, 7]
{σ[15], σ[16], σ[17], σ[25], σ[26], σ[27]}
Fundamental su(4)[65] spanning rows/columns [3, 4, 5, 6]
{σ[18], σ[19], σ[20], σ[22], σ[23], σ[25]}
Fundamental su(4)[66] spanning rows/columns [3, 4, 5, 7]
{σ[18], σ[19], σ[21], σ[22], σ[24], σ[26]}
Fundamental su(4)[67] spanning rows/columns [3, 4, 6, 7]

{σ[18], σ[20], σ[21], σ[23], σ[24], σ[27]}
Fundamental su(4)[68] spanning rows/columns [3, 5, 6, 7]
{σ[19], σ[20], σ[21], σ[25], σ[26], σ[27]}
Fundamental su(4)[69] spanning rows/columns [4, 5, 6, 7]
{σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}

Fundamental su(5)[0] spanning rows/columns [0, 1, 2, 3, 4]
{σ[0], σ[1], σ[2], σ[3], σ[7], σ[8], σ[9], σ[13], σ[14], σ[18]}
Fundamental su(5)[1] spanning rows/columns [0, 1, 2, 3, 5]
{σ[0], σ[1], σ[2], σ[4], σ[7], σ[8], σ[10], σ[13], σ[15], σ[19]}
Fundamental su(5)[2] spanning rows/columns [0, 1, 2, 3, 6]
{σ[0], σ[1], σ[2], σ[5], σ[7], σ[8], σ[11], σ[13], σ[16], σ[20]}
Fundamental su(5)[3] spanning rows/columns [0, 1, 2, 3, 7]
{σ[0], σ[1], σ[2], σ[6], σ[7], σ[8], σ[12], σ[13], σ[17], σ[21]}
Fundamental su(5)[4] spanning rows/columns [0, 1, 2, 4, 5]
{σ[0], σ[1], σ[3], σ[4], σ[7], σ[9], σ[10], σ[14], σ[15], σ[22]}
Fundamental su(5)[5] spanning rows/columns [0, 1, 2, 4, 6]
{σ[0], σ[1], σ[3], σ[5], σ[7], σ[9], σ[11], σ[14], σ[16], σ[23]}
Fundamental su(5)[6] spanning rows/columns [0, 1, 2, 4, 7]
{σ[0], σ[1], σ[3], σ[6], σ[7], σ[9], σ[12], σ[14], σ[17], σ[24]}
Fundamental su(5)[7] spanning rows/columns [0, 1, 2, 5, 6]
{σ[0], σ[1], σ[4], σ[5], σ[7], σ[10], σ[11], σ[15], σ[16], σ[25]}
Fundamental su(5)[8] spanning rows/columns [0, 1, 2, 5, 7]
{σ[0], σ[1], σ[4], σ[6], σ[7], σ[10], σ[12], σ[15], σ[17], σ[26]}
Fundamental su(5)[9] spanning rows/columns [0, 1, 2, 6, 7]
{σ[0], σ[1], σ[5], σ[6], σ[7], σ[11], σ[12], σ[16], σ[17], σ[27]}
Fundamental su(5)[10] spanning rows/columns [0, 1, 3, 4, 5]
{σ[0], σ[2], σ[3], σ[4], σ[8], σ[9], σ[10], σ[18], σ[19], σ[22]}
Fundamental su(5)[11] spanning rows/columns [0, 1, 3, 4, 6]
{σ[0], σ[2], σ[3], σ[5], σ[8], σ[9], σ[11], σ[18], σ[20], σ[23]}
Fundamental su(5)[12] spanning rows/columns [0, 1, 3, 4, 7]
{σ[0], σ[2], σ[3], σ[6], σ[8], σ[9], σ[12], σ[18], σ[21], σ[24]}
Fundamental su(5)[13] spanning rows/columns [0, 1, 3, 5, 6]
{σ[0], σ[2], σ[4], σ[5], σ[8], σ[10], σ[11], σ[19], σ[20], σ[25]}
Fundamental su(5)[14] spanning rows/columns [0, 1, 3, 5, 7]
{σ[0], σ[2], σ[4], σ[6], σ[8], σ[10], σ[12], σ[19], σ[21], σ[26]}
Fundamental su(5)[15] spanning rows/columns [0, 1, 3, 6, 7]
{σ[0], σ[2], σ[5], σ[6], σ[8], σ[11], σ[12], σ[20], σ[21], σ[27]}
Fundamental su(5)[16] spanning rows/columns [0, 1, 4, 5, 6]
{σ[0], σ[3], σ[4], σ[5], σ[9], σ[10], σ[11], σ[22], σ[23], σ[25]}
Fundamental su(5)[17] spanning rows/columns [0, 1, 4, 5, 7]
{σ[0], σ[3], σ[4], σ[6], σ[9], σ[10], σ[12], σ[22], σ[24], σ[26]}
Fundamental su(5)[18] spanning rows/columns [0, 1, 4, 6, 7]
{σ[0], σ[3], σ[5], σ[6], σ[9], σ[11], σ[12], σ[23], σ[24], σ[27]}
Fundamental su(5)[19] spanning rows/columns [0, 1, 5, 6, 7]
{σ[0], σ[4], σ[5], σ[6], σ[10], σ[11], σ[12], σ[25], σ[26], σ[27]}
Fundamental su(5)[20] spanning rows/columns [0, 2, 3, 4, 5]
{σ[1], σ[2], σ[3], σ[4], σ[13], σ[14], σ[15], σ[18], σ[19], σ[22]}
Fundamental su(5)[21] spanning rows/columns [0, 2, 3, 4, 6]
{σ[1], σ[2], σ[3], σ[5], σ[13], σ[14], σ[16], σ[18], σ[20], σ[23]}
Fundamental su(5)[22] spanning rows/columns [0, 2, 3, 4, 7]

{σ[1], σ[2], σ[3], σ[6], σ[13], σ[14], σ[17], σ[18], σ[21], σ[24]}
Fundamental su(5)[23] spanning rows/columns [0, 2, 3, 5, 6]
{σ[1], σ[2], σ[4], σ[5], σ[13], σ[15], σ[16], σ[19], σ[20], σ[25]}
Fundamental su(5)[24] spanning rows/columns [0, 2, 3, 5, 7]
{σ[1], σ[2], σ[4], σ[6], σ[13], σ[15], σ[17], σ[19], σ[21], σ[26]}
Fundamental su(5)[25] spanning rows/columns [0, 2, 3, 6, 7]
{σ[1], σ[2], σ[5], σ[6], σ[13], σ[16], σ[17], σ[20], σ[21], σ[27]}
Fundamental su(5)[26] spanning rows/columns [0, 2, 4, 5, 6]
{σ[1], σ[3], σ[4], σ[5], σ[14], σ[15], σ[16], σ[22], σ[23], σ[25]}
Fundamental su(5)[27] spanning rows/columns [0, 2, 4, 5, 7]
{σ[1], σ[3], σ[4], σ[6], σ[14], σ[15], σ[17], σ[22], σ[24], σ[26]}
Fundamental su(5)[28] spanning rows/columns [0, 2, 4, 6, 7]
{σ[1], σ[3], σ[5], σ[6], σ[14], σ[16], σ[17], σ[23], σ[24], σ[27]}
Fundamental su(5)[29] spanning rows/columns [0, 2, 5, 6, 7]
{σ[1], σ[4], σ[5], σ[6], σ[15], σ[16], σ[17], σ[25], σ[26], σ[27]}
Fundamental su(5)[30] spanning rows/columns [0, 3, 4, 5, 6]
{σ[2], σ[3], σ[4], σ[5], σ[18], σ[19], σ[20], σ[22], σ[23], σ[25]}
Fundamental su(5)[31] spanning rows/columns [0, 3, 4, 5, 7]
{σ[2], σ[3], σ[4], σ[6], σ[18], σ[19], σ[21], σ[22], σ[24], σ[26]}
Fundamental su(5)[32] spanning rows/columns [0, 3, 4, 6, 7]
{σ[2], σ[3], σ[5], σ[6], σ[18], σ[20], σ[21], σ[23], σ[24], σ[27]}
Fundamental su(5)[33] spanning rows/columns [0, 3, 5, 6, 7]
{σ[2], σ[4], σ[5], σ[6], σ[19], σ[20], σ[21], σ[25], σ[26], σ[27]}
Fundamental su(5)[34] spanning rows/columns [0, 4, 5, 6, 7]
{σ[3], σ[4], σ[5], σ[6], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}
Fundamental su(5)[35] spanning rows/columns [1, 2, 3, 4, 5]
{σ[7], σ[8], σ[9], σ[10], σ[13], σ[14], σ[15], σ[18], σ[19], σ[22]}
Fundamental su(5)[36] spanning rows/columns [1, 2, 3, 4, 6]
{σ[7], σ[8], σ[9], σ[11], σ[13], σ[14], σ[16], σ[18], σ[20], σ[23]}
Fundamental su(5)[37] spanning rows/columns [1, 2, 3, 4, 7]
{σ[7], σ[8], σ[9], σ[12], σ[13], σ[14], σ[17], σ[18], σ[21], σ[24]}
Fundamental su(5)[38] spanning rows/columns [1, 2, 3, 5, 6]
{σ[7], σ[8], σ[10], σ[11], σ[13], σ[15], σ[16], σ[19], σ[20], σ[25]}
Fundamental su(5)[39] spanning rows/columns [1, 2, 3, 5, 7]
{σ[7], σ[8], σ[10], σ[12], σ[13], σ[15], σ[17], σ[19], σ[21], σ[26]}
Fundamental su(5)[40] spanning rows/columns [1, 2, 3, 6, 7]
{σ[7], σ[8], σ[11], σ[12], σ[13], σ[16], σ[17], σ[20], σ[21], σ[27]}
Fundamental su(5)[41] spanning rows/columns [1, 2, 4, 5, 6]
{σ[7], σ[9], σ[10], σ[11], σ[14], σ[15], σ[16], σ[22], σ[23], σ[25]}
Fundamental su(5)[42] spanning rows/columns [1, 2, 4, 5, 7]
{σ[7], σ[9], σ[10], σ[12], σ[14], σ[15], σ[17], σ[22], σ[24], σ[26]}
Fundamental su(5)[43] spanning rows/columns [1, 2, 4, 6, 7]
{σ[7], σ[9], σ[11], σ[12], σ[14], σ[16], σ[17], σ[23], σ[24], σ[27]}
Fundamental su(5)[44] spanning rows/columns [1, 2, 5, 6, 7]
{σ[7], σ[10], σ[11], σ[12], σ[15], σ[16], σ[17], σ[25], σ[26], σ[27]}
Fundamental su(5)[45] spanning rows/columns [1, 3, 4, 5, 6]
{σ[8], σ[9], σ[10], σ[11], σ[18], σ[19], σ[20], σ[22], σ[23], σ[25]}
Fundamental su(5)[46] spanning rows/columns [1, 3, 4, 5, 7]
{σ[8], σ[9], σ[10], σ[12], σ[18], σ[19], σ[21], σ[22], σ[24], σ[26]}
Fundamental su(5)[47] spanning rows/columns [1, 3, 4, 6, 7]
{σ[8], σ[9], σ[11], σ[12], σ[18], σ[20], σ[21], σ[23], σ[24], σ[27]}

Fundamental su(5)[48] spanning rows/columns [1, 3, 5, 6, 7]
{σ[8], σ[10], σ[11], σ[12], σ[19], σ[20], σ[21], σ[25], σ[26], σ[27]}
Fundamental su(5)[49] spanning rows/columns [1, 4, 5, 6, 7]
{σ[9], σ[10], σ[11], σ[12], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}
Fundamental su(5)[50] spanning rows/columns [2, 3, 4, 5, 6]
{σ[13], σ[14], σ[15], σ[16], σ[18], σ[19], σ[20], σ[22], σ[23], σ[25]}
Fundamental su(5)[51] spanning rows/columns [2, 3, 4, 5, 7]
{σ[13], σ[14], σ[15], σ[17], σ[18], σ[19], σ[21], σ[22], σ[24], σ[26]}
Fundamental su(5)[52] spanning rows/columns [2, 3, 4, 6, 7]
{σ[13], σ[14], σ[16], σ[17], σ[18], σ[20], σ[21], σ[23], σ[24], σ[27]}
Fundamental su(5)[53] spanning rows/columns [2, 3, 5, 6, 7]
{σ[13], σ[15], σ[16], σ[17], σ[19], σ[20], σ[21], σ[25], σ[26], σ[27]}
Fundamental su(5)[54] spanning rows/columns [2, 4, 5, 6, 7]
{σ[14], σ[15], σ[16], σ[17], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}
Fundamental su(5)[55] spanning rows/columns [3, 4, 5, 6, 7]
{σ[18], σ[19], σ[20], σ[21], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}

Fundamental su(6)[0] spanning rows/columns [0, 1, 2, 3, 4, 5]
{σ[0], σ[1], σ[2], σ[3], σ[4], σ[7], σ[8], σ[9], σ[10], σ[13], σ[14], σ[15], σ[18], σ[19], σ[22]}
Fundamental su(6)[1] spanning rows/columns [0, 1, 2, 3, 4, 6]
{σ[0], σ[1], σ[2], σ[3], σ[5], σ[7], σ[8], σ[9], σ[11], σ[13], σ[14], σ[16], σ[18], σ[20], σ[23]}
Fundamental su(6)[2] spanning rows/columns [0, 1, 2, 3, 4, 7]
{σ[0], σ[1], σ[2], σ[3], σ[6], σ[7], σ[8], σ[9], σ[12], σ[13], σ[14], σ[17], σ[18], σ[21], σ[24]}
Fundamental su(6)[3] spanning rows/columns [0, 1, 2, 3, 5, 6]
{σ[0], σ[1], σ[2], σ[4], σ[5], σ[7], σ[8], σ[10], σ[11], σ[13], σ[15], σ[16], σ[19], σ[20], σ[25]}
Fundamental su(6)[4] spanning rows/columns [0, 1, 2, 3, 5, 7]
{σ[0], σ[1], σ[2], σ[4], σ[6], σ[7], σ[8], σ[10], σ[12], σ[13], σ[15], σ[17], σ[19], σ[21], σ[26]}
Fundamental su(6)[5] spanning rows/columns [0, 1, 2, 3, 6, 7]
{σ[0], σ[1], σ[2], σ[5], σ[6], σ[7], σ[8], σ[11], σ[12], σ[13], σ[16], σ[17], σ[20], σ[21], σ[27]}
Fundamental su(6)[6] spanning rows/columns [0, 1, 2, 4, 5, 6]
{σ[0], σ[1], σ[3], σ[4], σ[5], σ[7], σ[9], σ[10], σ[11], σ[14], σ[15], σ[16], σ[22], σ[23], σ[25]}
Fundamental su(6)[7] spanning rows/columns [0, 1, 2, 4, 5, 7]
{σ[0], σ[1], σ[3], σ[4], σ[6], σ[7], σ[9], σ[10], σ[12], σ[14], σ[15], σ[17], σ[22], σ[24], σ[26]}
Fundamental su(6)[8] spanning rows/columns [0, 1, 2, 4, 6, 7]
{σ[0], σ[1], σ[3], σ[5], σ[6], σ[7], σ[9], σ[11], σ[12], σ[14], σ[16], σ[17], σ[23], σ[24], σ[27]}
Fundamental su(6)[9] spanning rows/columns [0, 1, 2, 5, 6, 7]
{σ[0], σ[1], σ[4], σ[5], σ[6], σ[7], σ[10], σ[11], σ[12], σ[15], σ[16], σ[17], σ[25], σ[26], σ[27]}
Fundamental su(6)[10] spanning rows/columns [0, 1, 3, 4, 5, 6]
{σ[0], σ[2], σ[3], σ[4], σ[5], σ[8], σ[9], σ[10], σ[11], σ[18], σ[19], σ[20], σ[22], σ[23], σ[25]}
Fundamental su(6)[11] spanning rows/columns [0, 1, 3, 4, 5, 7]
{σ[0], σ[2], σ[3], σ[4], σ[6], σ[8], σ[9], σ[10], σ[12], σ[18], σ[19], σ[21], σ[22], σ[24], σ[26]}
Fundamental su(6)[12] spanning rows/columns [0, 1, 3, 4, 6, 7]
{σ[0], σ[2], σ[3], σ[5], σ[6], σ[8], σ[9], σ[11], σ[12], σ[18], σ[20], σ[21], σ[23], σ[24], σ[27]}
Fundamental su(6)[13] spanning rows/columns [0, 1, 3, 5, 6, 7]
{σ[0], σ[2], σ[4], σ[5], σ[6], σ[8], σ[10], σ[11], σ[12], σ[19], σ[20], σ[21], σ[25], σ[26], σ[27]}
Fundamental su(6)[14] spanning rows/columns [0, 1, 4, 5, 6, 7]
{σ[0], σ[3], σ[4], σ[5], σ[6], σ[9], σ[10], σ[11], σ[12], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}
Fundamental su(6)[15] spanning rows/columns [0, 2, 3, 4, 5, 6]
{σ[1], σ[2], σ[3], σ[4], σ[5], σ[13], σ[14], σ[15], σ[16], σ[18], σ[19], σ[20], σ[22], σ[23], σ[25]}
Fundamental su(6)[16] spanning rows/columns [0, 2, 3, 4, 5, 7]
{σ[1], σ[2], σ[3], σ[4], σ[6], σ[13], σ[14], σ[15], σ[17], σ[18], σ[19], σ[21], σ[22], σ[24], σ[26]}

Fundamental su(6)[17] spanning rows/columns [0, 2, 3, 4, 6, 7]
{σ[1], σ[2], σ[3], σ[5], σ[6], σ[13], σ[14], σ[16], σ[17], σ[18], σ[20], σ[21], σ[23], σ[24], σ[27]}
Fundamental su(6)[18] spanning rows/columns [0, 2, 3, 5, 6, 7]
{σ[1], σ[2], σ[4], σ[5], σ[6], σ[13], σ[15], σ[16], σ[17], σ[19], σ[20], σ[21], σ[25], σ[26], σ[27]}
Fundamental su(6)[19] spanning rows/columns [0, 2, 4, 5, 6, 7]
{σ[1], σ[3], σ[4], σ[5], σ[6], σ[14], σ[15], σ[16], σ[17], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}
Fundamental su(6)[20] spanning rows/columns [0, 3, 4, 5, 6, 7]
{σ[2], σ[3], σ[4], σ[5], σ[6], σ[18], σ[19], σ[20], σ[21], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}
Fundamental su(6)[21] spanning rows/columns [1, 2, 3, 4, 5, 6]
{σ[7], σ[8], σ[9], σ[10], σ[11], σ[13], σ[14], σ[15], σ[16], σ[18], σ[19], σ[20], σ[22], σ[23], σ[25]}
Fundamental su(6)[22] spanning rows/columns [1, 2, 3, 4, 5, 7]
{σ[7], σ[8], σ[9], σ[10], σ[12], σ[13], σ[14], σ[15], σ[17], σ[18], σ[19], σ[21], σ[22], σ[24], σ[26]}
Fundamental su(6)[23] spanning rows/columns [1, 2, 3, 4, 6, 7]
{σ[7], σ[8], σ[9], σ[11], σ[12], σ[13], σ[14], σ[16], σ[17], σ[18], σ[20], σ[21], σ[23], σ[24], σ[27]}
Fundamental su(6)[24] spanning rows/columns [1, 2, 3, 5, 6, 7]
{σ[7], σ[8], σ[10], σ[11], σ[12], σ[13], σ[15], σ[16], σ[17], σ[19], σ[20], σ[21], σ[25], σ[26], σ[27]}
Fundamental su(6)[25] spanning rows/columns [1, 2, 4, 5, 6, 7]
{σ[7], σ[9], σ[10], σ[11], σ[12], σ[14], σ[15], σ[16], σ[17], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}
Fundamental su(6)[26] spanning rows/columns [1, 3, 4, 5, 6, 7]
{σ[8], σ[9], σ[10], σ[11], σ[12], σ[18], σ[19], σ[20], σ[21], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}
Fundamental su(6)[27] spanning rows/columns [2, 3, 4, 5, 6, 7]
{σ[13], σ[14], σ[15], σ[16], σ[17], σ[18], σ[19], σ[20], σ[21], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}

Fundamental su(7)[0] spanning rows/columns [0, 1, 2, 3, 4, 5, 6]
{σ[0], σ[1], σ[2], σ[3], σ[4], σ[5], σ[7], σ[8], σ[9], σ[10], σ[11], σ[13], σ[14], σ[15], σ[16], σ[18], σ[19], σ[20], σ[22], σ[23], σ[25]}
Fundamental su(7)[1] spanning rows/columns [0, 1, 2, 3, 4, 5, 7]
{σ[0], σ[1], σ[2], σ[3], σ[4], σ[6], σ[7], σ[8], σ[9], σ[10], σ[12], σ[13], σ[14], σ[15], σ[17], σ[18], σ[19], σ[21], σ[22], σ[24], σ[26]}
Fundamental su(7)[2] spanning rows/columns [0, 1, 2, 3, 4, 6, 7]
{σ[0], σ[1], σ[2], σ[3], σ[5], σ[6], σ[7], σ[8], σ[9], σ[11], σ[12], σ[13], σ[14], σ[16], σ[17], σ[18], σ[20], σ[21], σ[23], σ[24], σ[27]}
Fundamental su(7)[3] spanning rows/columns [0, 1, 2, 3, 5, 6, 7]
{σ[0], σ[1], σ[2], σ[4], σ[5], σ[6], σ[7], σ[8], σ[10], σ[11], σ[12], σ[13], σ[15], σ[16], σ[17], σ[19], σ[20], σ[21], σ[25], σ[26], σ[27]}
Fundamental su(7)[4] spanning rows/columns [0, 1, 2, 4, 5, 6, 7]
{σ[0], σ[1], σ[3], σ[4], σ[5], σ[6], σ[7], σ[9], σ[10], σ[11], σ[12], σ[14], σ[15], σ[16], σ[17], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}
Fundamental su(7)[5] spanning rows/columns [0, 1, 3, 4, 5, 6, 7]
{σ[0], σ[2], σ[3], σ[4], σ[5], σ[6], σ[8], σ[9], σ[10], σ[11], σ[12], σ[18], σ[19], σ[20], σ[21], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}
Fundamental su(7)[6] spanning rows/columns [0, 2, 3, 4, 5, 6, 7]
{σ[1], σ[2], σ[3], σ[4], σ[5], σ[6], σ[13], σ[14], σ[15], σ[16], σ[17], σ[18], σ[19], σ[20], σ[21], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}
Fundamental su(7)[7] spanning rows/columns [1, 2, 3, 4, 5, 6, 7]
{σ[7], σ[8], σ[9], σ[10], σ[11], σ[12], σ[13], σ[14], σ[15], σ[16], σ[17], σ[18], σ[19], σ[20], σ[21], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}

Fundamental su(8)[0] spanning rows/columns [0, 1, 2, 3, 4, 5, 6, 7]
{σ[0], σ[1], σ[2], σ[3], σ[4], σ[5], σ[6], σ[7], σ[8], σ[9], σ[10], σ[11], σ[12], σ[13], σ[14], σ[15], σ[16], σ[17], σ[18], σ[19], σ[20], σ[21], σ[22], σ[23], σ[24], σ[25], σ[26], σ[27]}

Appendix 6: su(8) Lie algebra generating set

The following skew-Hermitian matrices are the generating set for building Lie algebra su(8). Matrices g[0][] are mutually orthogonal symmetric imaginary and matrices g[1][] are mutually orthogonal antisymmetric real. Closing this set of 14 for bracket, the resultant 63 non-null positive members form the basis set for su(8).

g[0][0] =

$$\begin{bmatrix} 0 & +is_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ +is_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \end{bmatrix}$$

g[0][1] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

g[0][2] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_2 \\ 0 & 0 & 0 & 0 & +is_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \\ 0 & 0 & +is_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ +is_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_2 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

g[0][3] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \\ 0 & +is_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

g[0][4] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & +is_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \\ +is_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \end{bmatrix}$$

g[0][5] =

$$\begin{bmatrix} 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_2 & 0 & 0 & 0 & 0 \\ +is_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \\ 0 & 0 & 0 & 0 & +is_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \end{bmatrix}$$

g[0][6] =

$$\begin{bmatrix} 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_9 & 0 & 0 & 0 & 0 & 0 & 0 \\ +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_7 & 0 & 0 & 0 \end{bmatrix}$$

g[1][0] =

$$\begin{bmatrix} 0 & +1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \end{bmatrix}$$

g[1][1] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

g[1][2] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{2} \\ 0 & 0 & 0 & 0 & +1s_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & -1s_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1s_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

g[1][3] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & -1s_{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

g[1][4] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ -1s_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

g[1][5] =

$$\begin{bmatrix} 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{2} & 0 & 0 & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & -1s_{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

g[1][6] =

$$\begin{bmatrix} 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_{7} & 0 & 0 & 0 \end{bmatrix}$$

Appendix 7: Octonion algebraic orientation covariant adjacent algebra formed Lie algebra $su(8)_{right}$

The positive elements defining the $su(8)_{right}$ basis are shown here at element indexes [n odd] and negations at element indexes [n odd + 1].

$su(8)_{right}[1]$ =

$$\begin{bmatrix} +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \end{bmatrix}$$

$su(8)_{right}[3]$ =

$$\begin{bmatrix} +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \end{bmatrix}$$

$su(8)_{right}[5] =$

$$\begin{bmatrix} +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \end{bmatrix}$$

$su(8)_{right}[7] =$

$$\begin{bmatrix} +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \end{bmatrix}$$

$su(8)_{right}[9] =$

$$\begin{bmatrix} -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \end{bmatrix}$$

$su(8)_{right}[11] =$

$$\begin{bmatrix} +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \end{bmatrix}$$

$su(8)_{right}[13] =$

$$\begin{bmatrix} -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \end{bmatrix}$$

$su(8)_{right}[15] =$

$$\begin{bmatrix} 0 & +is_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ +is_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \end{bmatrix}$$

$su(8)_{right}[17] =$

$$\begin{bmatrix} 0 & -is_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -is_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & -is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \end{bmatrix}$$

$su(8)_{right}[19] =$

$$\begin{bmatrix} 0 & +is_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ +is_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & -is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -is_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -is_{15} & 0 \end{bmatrix}$$

$su(8)_{right}[21] =$

$$\begin{bmatrix} 0 & -is_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -is_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -is_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -is_{15} & 0 \end{bmatrix}$$

$su(8)_{right}[23] =$

$$\begin{bmatrix} 0 & +1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1s_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \end{bmatrix}$$

$su(8)_{right}[25] =$

$$\begin{bmatrix} 0 & +1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1s_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \end{bmatrix}$$

$su(8)_{right}[27] =$

$$\begin{bmatrix} 0 & +1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1s_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[29] = \begin{bmatrix} 0 & -1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ +1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{7} & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[31] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[33] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \\ 0 & 0 & 0 & 0 & 0 & 0 & -is_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & -is_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & -is_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[35] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & -is_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & -is_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & -is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[37] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & -is_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[39] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[41] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[43] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[45] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[47] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{2} \\ 0 & 0 & 0 & 0 & +is_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \\ 0 & 0 & +is_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ +is_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[49] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{2} \\ 0 & 0 & 0 & 0 & -is_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & -is_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ +is_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[51] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -is_{2} \\ 0 & 0 & 0 & 0 & +is_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & +is_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ +is_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -is_{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$su(8)_{right}[53] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -is_{2} \\ 0 & 0 & 0 & 0 & -is_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \\ 0 & 0 & -is_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ +is_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -is_{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$su(8)_{right}[55] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{2} \\ 0 & 0 & 0 & 0 & +1s_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & -1s_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1s_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$su(8)_{right}[57] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{2} \\ 0 & 0 & 0 & 0 & +1s_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & -1s_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ +1s_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$su(8)_{right}[59] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{2} \\ 0 & 0 & 0 & 0 & +1s_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1s_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ -1s_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$su(8)_{right}[61] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{2} \\ 0 & 0 & 0 & 0 & +1s_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1s_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ +1s_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$su(8)_{right}[63] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \\ 0 & +is_{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$su(8)_{right}[65] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \\ 0 & 0 & 0 & 0 & -is_{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & -is_{15} & 0 \\ 0 & -is_{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$su(8)_{right}[67] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & -is_{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \\ 0 & -is_{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$su(8)_{right}[69] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & -is_{15} & 0 \\ 0 & +is_{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$su(8)_{right}[71] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & -1s_{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$su(8)_{right}[73] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & -1s_{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$su(8)_{right}[75] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & +1s_{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[77] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_{5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{11} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \\ 0 & -1s_{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{15} & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[79] = \begin{bmatrix} 0 & 0 & 0 & 0 & +is_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \\ +is_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[81] = \begin{bmatrix} 0 & 0 & 0 & 0 & +is_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -is_{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \\ +is_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -is_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[83] = \begin{bmatrix} 0 & 0 & 0 & 0 & -is_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -is_{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \\ -is_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -is_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[85] = \begin{bmatrix} 0 & 0 & 0 & 0 & -is_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \\ -is_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[87] = \begin{bmatrix} 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ -1s_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[89] = \begin{bmatrix} 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1s_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[91] = \begin{bmatrix} 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ -1s_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[93] = \begin{bmatrix} 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1s_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[95] = \begin{bmatrix} 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{2} & 0 & 0 & 0 & 0 \\ +is_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \\ 0 & 0 & 0 & 0 & +is_{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[97] = \begin{bmatrix} 0 & 0 & +is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{2} & 0 & 0 & 0 & 0 \\ +is_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -is_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & +is_{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \end{bmatrix}$$

$$\mathrm{su}(8)_{\mathrm{right}}[99] = \begin{bmatrix} 0 & 0 & -is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -is_{2} & 0 & 0 & 0 & 0 \\ -is_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -is_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \\ 0 & 0 & 0 & 0 & +is_{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[101] =$

$$\begin{bmatrix} 0 & 0 & -is_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +is_{2} & 0 & 0 & 0 & 0 \\ -is_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & +is_{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[103] =$

$$\begin{bmatrix} 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{2} & 0 & 0 & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & -1s_{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[105] =$

$$\begin{bmatrix} 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{2} & 0 & 0 & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1s_{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[107] =$

$$\begin{bmatrix} 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1s_{2} & 0 & 0 & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & +1s_{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[109] =$

$$\begin{bmatrix} 0 & 0 & +1s_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1s_{2} & 0 & 0 & 0 & 0 \\ -1s_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & +1s_{8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[111] =$

$$\begin{bmatrix} 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & +is_{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_{7} & 0 & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[113] =$

$$\begin{bmatrix} 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & 0 & -is_{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & -is_{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & -is_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & -is_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_{7} & 0 & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[115] =$

$$\begin{bmatrix} 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & -is_{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & -is_{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +is_{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_{7} & 0 & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[117] =$

$$\begin{bmatrix} 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & 0 & -is_{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & -is_{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -is_{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & +is_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & -is_{7} & 0 & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[119] =$

$$\begin{bmatrix} 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1s_{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_{7} & 0 & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[121] =$

$$\begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[123] =$

$$\begin{bmatrix} 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{9} & 0 & 0 & 0 & 0 & 0 \\ 0 & +1s_{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_{7} \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_{7} & 0 & 0 & 0 \end{bmatrix}$$

$su(8)_{right}[125] =$

$$\begin{bmatrix} 0 & 0 & 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1s_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1s_9 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1s_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1s_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & +1s_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_7 & 0 & 0 & 0 \end{bmatrix}$$

Appendix 8: Standard Model su(2) ⊕ su(3) gauge group Lie algebras

The non-null positive elements from the closed for bracket set {null, element, negation, …. are as follows

$su(2) \oplus su(3)$ 1-form[1] =

$$\begin{bmatrix} 0 & +is_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ +is_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$su(2) \oplus su(3)$ 1-form[3] =

$$\begin{bmatrix} 0 & +1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1s_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$su(2) \oplus su(3)$ 1-form[5] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$su(2) \oplus su(3)$ 1-form[7] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$su(2) \oplus su(3)$ 1-form[9] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \end{bmatrix}$$

$su(2) \oplus su(3)$ 1-form[11] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$su(2) \oplus su(3)$ 1-form[13] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +i \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \end{bmatrix}$$

$su(2) \oplus su(3)$ 1-form[15] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 1-form[17] =

$$\begin{bmatrix} +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 1-form[19] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 1-form[21] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2i \end{bmatrix}$$

su(2) ⊕ su(3) 2-form[1] =

$$\begin{bmatrix} 0 & +is_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ +is_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 2-form[3] =

$$\begin{bmatrix} 0 & +1s_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1s_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 2-form[5] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 2-formn[7] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1s_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 2-form[9] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +is_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 2-form[11] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1s_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 2-form[13] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +is_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +is_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 2-form[15] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1s_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1s_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 2-form[17] =

$$\begin{bmatrix} +i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 2-form[19] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

su(2) ⊕ su(3) 2-form[21] =

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

References

[1] Richard D. Lockyer, December 2020 *An Algebraic proof Sedenions are not a division algebra and other consequences of Cayley-Dickson Algebra definition variation*
https://doi.org/10.13140/RG.2.2.26240.24321
https://vixra.org/pdf/2010.0086v3.pdf

[2] Richard D. Lockyer, January 2022 *The Exclusive Or Group X(n) Correspondence With Cayley-Dickson Algebras*
https://doi.org/10.13140/RG.2.2.27498.53440
https://vixra.org/pdf/2201.0095v1.pdf

[3] Richard D. Lockyer, March 2022 *Octonions, Broctonions and Sedenions*
https://doi.org/10.13140/RG.2.2.12399.04008
https://vixra.org/pdf/2203.0098v1.pdf

[4] Richard D. Lockyer, February 2022 *Hadamard Matrices And Division Algebras Only*
https://doi.org/10.13140/RG.2.2.16173.91369
https://vixra.org/pdf/2202.0072v1.pdf

[5] Richard D. Lockyer, February 2022 *Division Algebra Covariant Derivative*
https://doi.org/10.13140/RG.2.2.10721.31849

https://vixra.org/pdf/2202.0154v1.pdf

[6] Richard D. Lockyer, December 2022, *Octonion Dynamics*
https://doi.org/10.13140/RG.2.2.28756.82567
https://vixra.org/pdf/2301.0005v1.pdf

[7] Richard D. Lockyer, October 2022, *Octonion Automorphisms as Algebraic Basis Gauge Transformations*
https://doi.org/10.13140/RG.2.2.22045.93920
https://vixra.org/pdf/2210.0014v2.pdf

[8] Richard D. Lockyer, March 2023, *Lorentz Covariance of the Octonion Covariant Derivative*
https://doi.org/10.13140/RG.2.2.24143.09120
https://vixra.org/pdf/2303.0146v1.pdf

[9] Richard D. Lockyer, May 2024, *Algebraic Orientation, Invariance and Variance Within the Family $\mathscr{H}$ of Hypercomplex Algebras*
https://doi.org/10.13140/RG.2.2.18690.49603/1
https://vixra.org/pdf/2405.0128v2.pdf

[10] J. Baez, *The Octonions*. Bulletin of the American Mathematical Society, 39:145-205, 2002

[11] Geoffrey Dixon, *Division Algebras: Octonions, Quaternions, Complex Numbers and the Algebraic Design of Physics* Kluwer Academic Publishers, 1994

[12] Richard D. Lockyer, 2012 FQXi Essay *The Algebra of "Everything"*, August 31, 2012
https://fqxi.org/data/essay-contest-files/Lockyer_fqxi_essay_RickLock.pdf

[13] C. Furey *Standard model physics from an algebra?*
https://doi.org/10.48550/arXiv.1611.09182

[14] N. Furey, M.J. Hughes *Three Generations and a Trio of Trialities*
https://doi.org/10.48550/arXiv.2409.17948

[15] N. Furey *Three generations, two unbroken gauge symmetries, and one eight-dimensional algebra*
https://doi.org/10.48550/arXiv.1910.08395

[16] N. Furey *An Algebraic Roadmap of Particle Theories, Part I: General construction*
https://doi.org/10.48550/arXiv.2312.12377

[17] N. Furey *An Algebraic Roadmap of Particle Theories, Part II: Theoretical checkpoints*
https://doi.org/10.48550/arXiv.2312.12799

[18] N. Furey *An Algebraic Roadmap of Particle Theories, Part III: Intersections*
https://doi.org/10.48550/arXiv.2312.14207

[19] N. Furey, M.J. Hughes *One generation of standard model Weyl representations as a single copy of $R \otimes C \otimes H \otimes O$*
https://doi.org/10.48550/arXiv.2209.13016

[20] N. Furey, M.J. Hughes *Division algebraic symmetry breaking*

https://doi.org/10.48550/arXiv.2210.10126

[21] N. Furey *Standard Model Symmetries and the Nested Embeddings of* $R \subset C \subset H \subset O$
https://doi.org/10.48550/arXiv.2607.18450