Redefining the Gravitational Singularity: Resolving the Geometric Breakdown via a Finite Planck-Scale Density Bound

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We consider an alternative approach to the gravitational singularity problem by treating the core of a black hole as a saturated state with a finite density bound. In classical General Relativity, the metric diverges at $r = 0$, leading to infinite density and a breakdown of the field equations. In this paper, we replace this divergence by introducing a maximum density limit locked to the invariant Planck density ($\rho_{\max}$). By matching an internal de Sitter geometry to an external Schwarzschild metric, we find that the collapsed core stabilizes as a finite object. Evaluating this matching condition across primordial, stellar, and galactic mass scales indicates that the core volume scales dynamically ($V \neq 0$) to maintain the density plateau. This geometric mechanism offers a direct way to connect macroscopic gravity with discrete quantum limits.

I. INTRODUCTION

The classical Schwarzschild solution to Einstein's field equations provides an accurate description of macroscopic spacetime around a spherical mass [1]. However, the theory has a well-known internal problem at the spatial origin. At $r = 0$, the classical equations predict a gravitational singularity where matter collapses into a point of infinite density and zero volume ($V = 0$). In physical systems, the appearance of an infinity usually implies that the underlying model has been pushed past its limits and requires an additional boundary condition. Einstein noted that singularities indicate a limitation of the classical continuum description rather than a real physical entity.

In this work, we assume that the physical universe operates under strict boundary constraints. On the lower end, space-time possesses a clear lower bound where mass density and thermodynamic entropy vanish ($\rho = 0, S = 0$). Based on cosmological symmetry, we propose that there should also exist a corresponding upper limit on how densely matter can pack ($\rho_{\max}$). From this viewpoint, a black hole core is not an infinite divergence, but a physical state of maximum compression where matter reaches its ultimate saturation threshold.

Direct empirical observation of a black hole interior is restricted because photons cannot escape the event horizon [2]. However, we can construct a mathematical framework that incorporates the scale limits of quantum mechanics. By locking the maximum compression of matter to the invariant Planck density, the zero-volume collapse can be avoided. Instead, it is replaced by a non-zero core ($V \neq 0$) that adjusts its physical volume based on the total mass inside the system.

II. METHODS AND METRIC MATCHING FRAMEWORK

Rather than modifying the field equations with higher-derivative tensor terms, we choose to re-evaluate the physical definition of a singularity. We reject the assumption that a singularity must culminate in an infinite divergence where $\rho \to \infty$. Instead, we use a basic axiom: a physical singularity represents a state of structural geometric saturation.

We set this maximum density threshold ($\rho_{\max}$) to be equal to the invariant Planck density, which is defined by the speed of light (c), Newton's gravitational constant (G), and the reduced Planck constant ($\hbar$).

The boundary condition for the collapsing system is given by:

$$\rho \leq \rho_{\max} \tag{1}$$

Where the Planck density constant is:

$$\rho_{\max} = \frac{c^5}{\hbar G^2} \approx 5.155 \times 10^{96}\ \mathrm{kg/m^3} \tag{2}$$

To analyze this behavior within standard differential geometry, we consider a static, spherically symmetric spacetime. The line element takes the form:

$$ds^2 = -f(r)c^2dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega^2 \tag{3}$$

Outside the core region, for $r > r_{\min}$, spacetime is described by the standard external Schwarzschild metric component:

$$f_{\mathrm{out}}(r) = 1 - \frac{2GM}{c^2r} \tag{4}$$

Inside the core region, for $r < r_{\min}$, matter reaches the maximum saturation limit where the density becomes constant and equals $\rho_{\max}$. This constant density profile produces a local de Sitter geometry. The internal metric component is written as:

$$f_{\mathrm{in}}(r) = 1 - \frac{8\pi G\rho_{\max}}{3c^2}r^2 \tag{5}$$

To determine the physical radius ($r_{\min}$) where this phase transition occurs, we require the internal and external metric components to match at the boundary interface $r = r_{\min}$:

$$f_{\text{out}}(r_{\min}) = f_{\text{in}}(r_{\min}) \tag{6}$$

Substituting Eq. (4) and Eq. (5) into the matching condition yields:

$$1 - \frac{2GM}{c^2 r_{\min}} = 1 - \frac{8\pi G \rho_{\max}}{3c^2} r_{\min}^2 \tag{7}$$

Canceling the constants from both sides simplifies the expression:

$$\frac{2GM}{c^2 r_{\min}} = \frac{8\pi G \rho_{\max}}{3c^2} r_{\min}^2 \tag{8}$$

We isolate the radius term by rearranging the equation:

$$r_{\min}^3 = \frac{3M}{4\pi \rho_{\max}} \tag{9}$$

This shows that the core volume $V = \frac{4}{3}\pi r_{\min}^3$ arises directly from the metric matching procedure. Substituting the definition of the Planck density from Eq. (2) into Eq. (9) gives the final scale-invariant relation for the core radius:

$$r_{\min} = \left(\frac{3\hbar G^2 M}{4\pi c^5} \right)^{1/3} \tag{10}$$

We check the consistency of this relation by testing it against three different mass scales: a theoretical primordial black hole, a standard solar-mass black hole, and the supermassive black hole Sagittarius A*.

III. RESULTS AND SCALE VALIDATION

Substituting real astrophysical masses into our derived scaling relation yields stable, non-zero physical core sizes. The numerical inputs and resulting core outputs are cleanly structured and centered across the page wide in Table I.

The numerical results indicate a specific scaling behavior. Even when the mass increases by billions of times, the core density does not diverge. It remains flat, locked at the Planck scale ($\rho = \rho_{\max}$). The core volume simply expands to accommodate the additional mass, showing that $V \neq 0$ is a direct mathematical consequence of setting a density ceiling within the metric framework.

TABLE I. Calculated Core Radii and Bounded Densities Across Different Mass Scales.

Object Class	**Core Radius $r_{\mathbf{min}}$ (meters)**	**Core Density ρ (kg/m^3)**
Primordial Black Hole	$\approx 1.80 \times 10^{-29}$	$\approx 5.15 \times 10^{96}$
Stellar Black Hole ($1M_\odot$)	$\approx 4.51 \times 10^{-23}$	$\approx 5.15 \times 10^{96}$
Supermassive Black Hole (Sgr A*)	$\approx 7.37 \times 10^{-21}$	$\approx 5.15 \times 10^{96}$

IV. DISCUSSION

The mathematical consistency across the mass scales indicates that setting a finite boundary for density avoids the unphysical infinities found in classical relativity. Because the radius $r_{\min}$ remains strictly greater than zero for all non-zero masses, the center of a black hole behaves as a real high-density subatomic object rather than a zero-dimensional point.

A standard critique of non-singular models is how they relate to the singularity theorems developed by Roger Penrose and Stephen Hawking [3]. These theorems prove that a singularity is unavoidable if a trapped surface forms. However, Penrose's proof assumes that matter satisfies the Strong Energy Condition (SEC), which requires that:

$$\rho + 3P \geq 0 \tag{11}$$

In our framework, a specific mechanism bypasses this condition. When matter reaches the saturation limit $\rho_{\max}$, the internal metric becomes de Sitter as shown in Eq. (5). Evaluating the stress-energy tensor from Einstein's equations for a de Sitter geometry yields a negative pressure equation of state:

$$P = -\rho_{\max} c^2 \tag{12}$$

Substituting this pressure into the Strong Energy Condition gives:

$$\rho_{\max} + 3(-\rho_{\max} c^2) \rightarrow \rho_{\max}(1 - 3c^2) < 0 \tag{13}$$

This means the core directly violates the Strong Energy Condition ($\rho + 3P < 0$). By breaking the SEC, the requirements of Penrose's theorem are no longer satisfied inside the core region. The inward collapse stops naturally because of the negative pressure state, without requiring an artificial mechanical force.

Another concern is whether an expanding core might eventually grow larger than the black hole event horizon. We can check this by comparing the scaling laws. The Schwarzschild radius (r_s)

grows linearly with mass:

$$r_s = \frac{2GM}{c^2} \propto M^1 \tag{14}$$

But our saturated core radius scales slower as a cube root:

$$r_{\min} \propto M^{1/3} \tag{15}$$

Because a linear function (M^1) grows faster than a cube root function ($M^{1/3}$), the ratio $r_{\min}/r_s$ drops toward zero for larger mass inputs. This guarantees that the core stays hidden deep inside the event horizon at all scales, avoiding the problem of exposing a naked singularity to outside observers.Conceptually, this core behavior aligns with the ideas behind the "Planck Star" models proposed in Loop Quantum Gravity (LQG) [4, 5]. It serves as a bridge where a macroscopic collapse is caught and stabilized by quantum boundaries.

V. CONCLUSION

This study shows that the gravitational singularity inside black holes can be resolved by replacing the infinite collapse model with an asymptotic density upper bound. Setting the maximum density to the Planck scale ($\rho = \rho_{\max}$) takes away the mathematical need for a zero-volume point, allowing the core volume to expand to handle new mass additions.Testing the model against primordial, solar, and supermassive masses indicates that the framework is scale-invariant and internally consistent. By removing unphysical infinities from the metric calculations, this model offers a clear, direct conceptual path that can help connect general relativity with quantum limits under a unified theory of quantum gravity. Regarding the fact that realistic astrophysical black holes possess angular momentum and spin, this initial framework was restricted to the static, spherically symmetric case to isolate the fundamental density saturation mechanism. Future iterations of this model will extend these boundary matching conditions to rotating Kerr spacetime geometries to analyze how spin parameter variations alter the core's volume expansion. .

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