TESTING THE D.M. by QUANTUM GRAVITATION THEORY BY THE MASS ASSOCIATED TO THE TURNAROUND RADIUS

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ABSTRACT

The DM by Quantum Gravitation theory (DMbQG hereafter), is a novel theory developed in the two main papers [1] and [2] Abarca, M. (2024). In the present work, in the framework of DMbQG theory it will derived the formula for the mass associated to the turnaround radius depending on the turnaround radius and it will be shown that the new expression has a deeper physical meaning regarding the same formula in the current ΛCDM paradigm.

The DMbQG theory states a direct relation between the virial radius and the turnaround radius as well as the virial mass and the mass associated to the turnaround radius.

The most important contribution of this work is placed in the chapter 2 where it has been demonstrated that the formula for the mass associated to the turnaround radius, using the DMbQG theory, is the same that the one in the current ΛCDM paradigm. However the new formula has a deeper physical meaning because its dimensionless factor is simpler regarding the one in the current ΛCDM paradigm. Also it is remarkable the fact that the both dimensionless factors depend on exclusively from Ω_m or Ω_{DE} and the both ones differ only a 5%.

The fact that in the framework of DMbQG theory has been found a better formula for the mass associated to the turnaround radius is a strong support for the DMbQG theory whose hypothesis about the origin of DM is exclusively the gravitational field.

The chapters 3, 4 and 5 are devoted to test the DMbQG theory in the clusters: Virgo, Coma and the Local Group of galaxies using recent data published about these clusters. The tests are mainly focused on the turnaround radius and its mass associated and all of them have been a complete success by the DMbQG theory.

In this second version of the paper, the mass associated to the turnaround radius formula is obtained by a different way regarding the first version. In this new method is not used any formulas with the parameter "a", achieving a more understandable process to obtain the same formula.

1. INTRODUCTION

The Lynden-Bell (1981) formula $M_0 = R_0^3 \cdot H^2 \frac{\pi^2}{8 \cdot G}$ gives the mass associated to the turnaround radius (R_0) in the case of a spherical over density with the cosmological parameter Λ= 0.

Currently, in the framework of ΛCDM the previous formula becomes

$$M_0 = R_0^3 \cdot H^2 \frac{\pi^2}{8 \cdot G} / f^2(\Omega_m) \qquad (1.1)$$

Where $f(\Omega_m) = (1-\Omega_m)^{-1} - (\Omega_m/2)(1-\Omega_m)^{-3/2} \cdot arc\ cosh[\frac{2}{\Omega_m} - 1]$

The reader can consult the previous expressions in the introduction of paper [3] Karachentsev.

Assuming $\Omega_m = 0.3$ then $f^2(0.3) = 0.6541465$ and
$M_0 = \frac{R_0^3 \cdot H^2}{G} \cdot 1.88597$ (1.2)

and assuming H= 70km/s/Mpc the dimensionless expression becomes

$M_0 / M_\odot = 2.14738 \cdot 10^{12} \cdot (\frac{R_0}{Mpc})^3$ (1.3)

where $M_\odot = 1.99 \cdot 10^{30}$ kg and $Mpc = 3.0857 \cdot 10^{22}$ m

At the present work it will obtained the same formula in the framework of DMbQG so it is needed to consult the papers [1] and [2] Abarca,M. in order to understand several formulas cited in this work.

Now it will be referred the formulas used in this work and that have been derived in the framework of the DMbQG theory. All of them have been properly demonstrated in the [1] and [2] papers.

Formulas derived from the DMbQG theory used in this work

In my papers is used the approximation $M_{VIRIAL} = M_{200}$, this is a very common approximation for most of the authors that are studying clusters of galaxies.

By definition M_{200} is the mass of the sphere whose mean density is 200 times the critic density of Universe $\rho_C = \frac{3H^2}{8\pi G}$, so it is right to get the formula

$\boldsymbol{M_{VIR} \approx M_{200} = \frac{100 H^2 R_{200}^3}{G}}$ (1.4)

being $R_{200} = R_{VIR}$ the radius of that sphere.

The following formulas have been obtained in the framework of DMbQG theory and all of them are properly cited, so the reader may look for its demonstration.

1º Ratio zero gravity radius versus virial radius

In the epigraph 4.3, paper [2], it was got the ratio

$\frac{R_{ZG}}{R_{200}} = U^2$ (1.5)

being $U = \left[\frac{100}{\Omega_{DE}}\right]^{1/5}$ and assuming $\Omega_{DE} = 0.7$ then $U \approx 2.7$ and $R_{ZG} / R_{200} \approx 7.277$

2º Ratio zero velocity radius (turnaround radius) versus zero gravity radius

In the chapter 5, paper [2], was demonstrated the *zero velocity radius theorem*, that states;

$f = R_{ZV} / R_{ZG} = 0.602$ (1.6)

i.e. the ratio: turnaround radius versus the zero gravity radius is 0.602

3º Ratio zero velocity radius (turnaround radius) versus virial radius

Merging the ratio (1.6) with the ratio (1.5) it is got

$R_{ZV}/R_{200} = f\cdot U^2 = 0.602\cdot U^2 = 4.38$ (1.7)

4º Gravitating mass function

In the epigraph 4.6, paper [2], using the gravitating mass concept, defined by [4] Chernin,A and merging with the Direct mass formula, it was obtained the formula for the gravitating mass:

$$M_G(< R) = M_G(< f \cdot R_{ZG}) = \left[\sqrt{f} - f^3\right] \cdot U \cdot M_{200} \quad (1.8)$$

Where f = Radius / R_{ZG} is the dimensionless variable.

For example: As at the zero velocity radius f = R_{ZV} / R_{ZG} = 0.602 , see formula (1.6), then

$$M_G(< R_{ZV}) = M_G(< f \cdot R_{ZG}) = \left[\sqrt{f} - f^3\right] \cdot U \cdot M_{200} \quad (1.9)$$

being f= 0.602 and U≈ 2.7 then $M_G(<R_{ZV}) \approx 1.5\cdot M_{200}$ (1.10)

In the following epigraph these previous formulas will be cited frequently. The reader may study more in deep such formulas in the papers [1] and [2], where it is developed fully the DMbQG theory.

2. FORMULA FOR THE GRAVITATING MASS ASSOCIATED TO THE TURNAROUND RADIUS IN THE DMbQG THEORY

The expression (1.9) gives the gravitating mass associated to the turnaround radius i.e.

$M_G(< R_{ZV}) = M_G(< f \cdot R_{ZG}) = \left[\sqrt{f} - f^3\right] \cdot U \cdot M_{200}$ where $U = \left[\frac{100}{\Omega_{DE}}\right]^{1/5}$ and f = R_{ZV} / R_{ZG} = 0.602 by the zero velocity radius theorem, (see in the epigraph 5.4, paper [2], how the ratio f=0.602 was got assuming $\Omega_{DE} = 0.7$).

So merging (1.4) in the later expression it is obtained:

$$M_G(< R_{ZV}) = M_0 = \left[\sqrt{f} - f^3\right] \cdot U \cdot \frac{100H^2R_{200}^3}{G} \quad (2.1)$$

From the expression (1.7) $R_{200} = R_{ZV}/(f\cdot U^2)$ (2.2)

And merging (2.2) into (2.1) it is got

$$M_0 = \left[\sqrt{f} - f^3\right] \cdot U \cdot \frac{100\cdot H^2}{G}\frac{R_{ZV}^3}{f^3U^6} = \frac{H^2\cdot R_{ZV}^3}{G} \cdot \frac{\Omega_{DE}[\sqrt{f} - f^3]}{f^3} \quad (2.3)$$

Assuming Ω_{DE} =0.7, f = 0.602 and writing R_{ZV} as R_0 it is got the expression:

$$M_0 = \frac{H^2\cdot R_0^3}{G} \cdot 1.789 \quad (2.4)$$

This formula is a new crucial test for the DMbQG theory because its dimensionless factors are extremely simples: the universal Ω_{DE} , the $\sqrt{f}$ that comes from the direct mass and the f^3 that shows how the DE increases with the cubic power into the space. However the same formula in the framework of ΛCDM, see the expression (1.1), contains a quite complex function: Namely $f(\Omega_m) = (1-\Omega_m)^{-1} - (\Omega_m/2)(1-\Omega_m)^{-3/2} \cdot arc\ cosh\ [\frac{2}{\Omega_m} - 1]$

It is remarkable the fact that the both dimensionless factors:

$\frac{\pi^2}{8 \cdot f^2(\Omega_m)} = 1.886$ (current ΛCDM paradigm) and $\frac{\Omega_{DE}[\sqrt{f} - f^3]}{f^3} = 1.789$ (DMbQG theory) depend on exclusively from Ω_m or Ω_{DE} and the both ones differ only a 5%.

The formula (2.4) may be written as a dimensionless expression:

$M_0 / M_\odot = 2.037 \cdot 10^{12} \cdot (\frac{R_0}{Mpc})^3$ (2.5)

This expression is the same than (1.3) with the numerical factor only a 5% lower which is virtually negligible.

To demonstrate the formula (2.3) has been used all the most important formulas previously obtained in the DMbQG theory. Namely these formulas are:

1º Ratio zero gravity radius versus virial radius: (1.5) expression.

2º Ratio zero velocity radius (turnaround radius) versus zero gravity radius: (1.6) expression.

3º Ratio zero velocity radius (turnaround radius) versus virial radius: (1.7) expression.

4º Gravitating mass function associated to the turnaround radius: (1.9) expression.

The formula (2.3) is a step forward regarding the same formula in the current ΛCDM framework because the former contains simple dimensionless factors closely related with the DMbQG theory whereas the later contains quite complex dimensionless ones, and additionally the both ones differ only a 5%.

This theoretical finding is a strong support for the DMbQG theory whose hypothesis about the origin of DM is exclusively the gravitational field.

3. TESTING THE TURNAROUND FORMULAS IN THE VIRGO CLUSTER

The data have been taken from [5] Kashibadze,O. et al. 2020. Namely the data of table 1 has been taken from the abstract and the introduction epigraphs.

Table 1 Virgo data. Source. [5] Kashibadze et al.		
Virial mass	Turnaround radius or R_{ZV}	Turnaround mass or M_0
$(6.3 \pm 0.9) \cdot 10^{14}$ $M_\odot$	7.8 ± 0.3 Mpc	$(7.4 \pm 0.9) \cdot 10^{14}$ $M_\odot$

Assuming that $M_{VIR} = M_{200}$ is a good approximation, as $M_{200} = \frac{100 H^2 R_{200}^3}{G}$ then knowing M_{200} it is right to get $R_{200} = 1.77$ Mpc so by the expression (1.7) it is right to get

$R_{ZV} = 7.75$ Mpc (3.1)

That virtually matches with the data published by Kashibadze.

On the other side, by the expression (1.10) the mass associated to the turnaround radius is

$M_0 = 1.5(6.3 \pm 0.9)\cdot 10^{14}\ M_\odot = (9.45\pm 1.35)\cdot 10^{14}\ M_\odot$ (3.2)

This result matches in the data range given by Kashibadze, although this result does not match so well than the turnaround radius.

A second way to calculate the turnaround mass in the framework of DMbQG theory is using the formula (2.5).

$$M_0/ M_\odot = 2.037 \cdot 10^{12} \cdot (\frac{R_0}{Mpc})^3 \quad (2.5)$$

Considering R_{ZV} = 7.75 Mpc calculated in the framework of DMbQG theory and using (2.5) it is got $M_0 = 9.48\cdot 10^{14}\ M_\odot$ (3.3)

The result (3.3) virtually matches with (3.2) because the formulas (1.10) and (2.5) are mathematically equivalents in the framework of DMbQG theory.

4. TESTING THE TURNAROUND FORMULAS IN THE COMA CLUSTER

4.1 THE SOURCE OF THE COMA CLUSTER DATA

The source of Coma data is [6] Benisty 2026. In this epigraph are summarized some important data such as virial radius and mass, the turnaround radius and its mass associated. In case the reader wants to look for these data in the original paper, in the following epigraphs each data is properly referred.

In his paper, the authors consider h = 0.73 so its formulas (4.3) and (4.4) stated a lower bound turnaround radius equal to 6.7 Mpc and an upper bound one equal to 11.8 Mpc.

In the Discussion epigraph, page 26, the authors give an estimate for the virial radius equal to 2.67 Mpc

In the epigraph 4.3 Virial theorem, the authors give for the virial mass the range (1.45 to 1.72)$\cdot 10^{15} M_\odot$. Also they refer that this value is consistent with other studies of M_{200} equal to $1.3\cdot 10^{15}$ $M_\odot$ obtained by weak lensing method. This statement means that the authors consider the virial mass closely related to M_{200} as is usual in clusters.

In the page 20 the equation (4.6) gives the range (0.7 to 3.9)$\cdot 10^{15} M_\odot$ for the mass associated to the turnaround radius. In the table 2 are summarized the previous Coma data.

Table 2 Coma data source: [6] Benisty 2026

Virial mass	Virial Radius	Upper bound for the Turnaround radius	Turnaround mass or M_0
1.45 to 1.72 $M_\odot$	2.67 Mpc	11.8 Mpc	(0.7 to 3.9)$\cdot 10^{15} M_\odot$

The average virial mass is $1.59\cdot 10^{15} M_\odot$ and by the expression (2.1) it is right to get R_{200}= 2.41 Mpc that differs a 10 % regarding the virial radius given by Benisty. As the DMbQG theory is developed using M_{200} and R_{200}, in this test will be used these ones.

4.2 TURNAROUND RADIUS AND MASS

By the expression (1.7) it is right to get $R_{ZV} = 4.38\cdot R_{200} = 4.38\cdot 2.41$Mpc =10.55 Mpc

R_{ZV} =10.55 Mpc (4.1)

that it is 1.3 Mpc lower than the turnaround upper bound radius.

By the expression (1.10) $M_0 = 1.5\cdot 1.59\cdot 10^{15}\ M_\odot = 2.38\cdot 10^{15}\ M_\odot$

$M_0 = 2.38\cdot 10^{15}\ M_\odot$ (4.2)

that is just the average value of the mass range $(0.7 \text{ to } 3.9)\cdot 10^{15} M_\odot$ given by Benisty.

A second way to calculate the turnaround mass in the framework of DMbQG theory is using the formula (2.5).

$M_0/M_\odot = 2.037 \cdot 10^{12} \cdot (\frac{R_0}{Mpc})^3$ (2.5)

Considering R_{ZV} = 10.55 Mpc calculated in the framework of DMbQG theory and using (2.5) it is got

$M_0 = 2.39\cdot 10^{15}\ M_\odot$ (4.3)

This result virtually matches with (4.2) because the formulas (1.10) and (2.5) are mathematically equivalents in the framework of DMbQG theory.

5. TESTING THE DMBQG THEORY IN THE LOCAL GROUP OF GALAXIES

In this epigraph it will be used two different data sources for the Local Group. Namely the papers are [7] Shaya,E et al.(2017) and [8] Santos-Santos et al.(2023).

5.1 TESTING THE TOTAL MASS OF THE LOCAL GROUP UP TO 770 KPC

In the abstract of [7] Shaya,E et al. paper, the authors affirm that the mass for the MW plus M31 is equal to $M_{LG} = (5.15\pm0.35)\cdot 10^{12} M_\odot$

In the paper [1] Abarca, the DMbQG theory is introduced and developed fully. The DM theory is initially developed into the halo region of the galaxies and the paper ended showing a way to extend the theory to the clusters. According this theory each cluster of galaxies has a specific parameter a^2 which may be obtained by the addition of the specific parameter associated to each galaxy. For example in the table 19, the LG parameter a^2 is $4.28\cdot 10^{21}\ m^{2.5}/s^2$, obtained by the addition of the parameters associated to MW, M31, M33 and LMC.

The DMbQG theory stated that the total mass is unbounded and the total mass at a specific radius is calculated by the Direct mass: $M_{TOTAL}(r) = \frac{a^2\cdot\sqrt{r}}{G}$ (5.1)

In the last column of the table 20 of paper [1] Abarca, it is calculated the total mass at different distances for the L.G. using the value for the parameter $a^2 = 4.28\cdot 10^{21}\ m^{2.5}/s^2$. Namely for a radius equal to 770 kpc (distance between MW and M31) the total mass is equal to $4.97\cdot 10^{12}$ $M_\odot$ that match fully with the data provided by [7] Shaya,E.

Even at this scale the DE is not negligible regarding the DM and may be quantified around a 6% so it is not significant.

In DMbQG theory each galaxy has a specific parameter a^2 and may be added in order to study the DM into the external region of the cluster, assuming spherical symmetry. It is clear that the spherical symmetry for the LG is an approximation too rough. However it will be shown in the following epigraph that the turnaround radius calculated by the theory match very well with the measures published.

5.2 TESTING THE DMbQG THEORY BY THE L.G. TURNAROUND RADIUS

In the abstract of paper [8] Santos-Santos et al. the authors stated that the LG turnaround radius is 1.25 Mpc. In this epigraph it will be compared that measure with its theoretical value in the framework of DMbQG theory.

In the chapter 13 of paper [1] Abarca, it is obtained the formula (13.8) for the zero gravity radius R_{ZG}, a concept introduced by [4] Chernin, A., but calculated specifically in the framework of the DMbQG theory. By the formula (13.8), in the table 22 is shown how the R_{ZG} for the LG is 2.19 Mpc. Do not mislead zero gravity radius with turnaround radius, called zero velocity radius as well.

By the zero velocity radius theorem, formula (1.6), $R_{ZV} = 0.602 \cdot R_{ZG}$, so the $R_{TA} = 0.6 \cdot 2.19 = 1.3$ Mpc that match almost mathematically with the result published by [8] Santos-Santos $R_{TA}=1.25$ Mpc .

Additionally, also in the abstract of the paper [8] Santos-Santos et al. the authors affirm that the pure Hubble flow (i.e. where the radial velocity becomes comparable to H·d) is not reached out to at least 3 Mpc.

This result may be explained by the concept of zero gravitating radius (2.2 Mpc for the LG), because at such distance the total mass (DM plus baryonic matter) is counterbalanced by the DE obtaining a gravitating mass equal to zero and consequently at 2.2 Mpc the galaxies cannot have an adequate Hubble flow so it is needed an extra distance to reach the Hubble flow regime.

5.3 TURNAROUND RADIUS AND ITS MASS VERSUS R_{200} AND M_{200}

In the paper [2], from the expression (3.4) of that paper, it is possible to clear up M_{200}

$$M_{200} = \frac{a^{12/5}}{G \cdot (10 \cdot H)^{2/5}} \quad (5.2)$$

Merging the expression (1.4) $\boldsymbol{M_{200}} = \frac{\boldsymbol{100 H^2 R_{200}^3}}{\boldsymbol{G}}$ and (5.2) it is possible to clear up R_{200}

$$R_{200} = \left[\frac{a^2}{100 \cdot H^2}\right]^{2/5} \quad (5.3)$$

Assuming that for the L.G. $a^2 = 4.28 \cdot 10^{21}$ $m^{2.5}/s^2$ then $M_{200} = 3.1 \cdot 10^{12}$ $M_\odot$ and $R_{200} = 301$ kpc By the ratio $R_{ZV}/R_{200} = 4.38$, formula (1.7), it is possible to test newly the turnaround radius, $R_{ZV} = 301\text{kpc} \cdot 4.38 = 1.32$ Mpc and by the formula (1.10) the turnaround mass:

$$M_0 = 1.5 \cdot M_{200} = 4.65 \cdot 10^{12}\ M_\odot \quad (5.4)$$

Notice how for a radius equal to 1.32 Mpc the DE is not negligible and the gravitating mass associated to R_{ZV} is lower than the total mass estimated for the LG when it is studied the gravitational interaction MW+LMC between M31+M33 where the total mass is equal to 4.97 $\cdot 10^{12}$ $M_{\odot}$.

The second method to calculate the turnaround mass is the formula (2.5) that for R_{ZV} =1.32 Mpc is $M_0 = 4.68 \cdot 10^{12}$ $M_{\odot}$ (5.5)

The results (5.4) and (5.5) are virtually equals because the formulas (1.10) and (2.5) are mathematically equivalents in the framework of DMbQG theory.

6. CONCLUDING REMARKS

This work, specially the chapter 2 is a new and crucial test for the novel DMbQG theory. The hypothesis of this theory is that the DM is generated by the own gravitational field and the total mass (baryonic plus DM), under spherical symmetry, grows up with the square root of the radius, its formula is called Direct mass. The theory is fully developed in the papers [1] and [2].

In the chapter 2 is placed the most important finding of this work where it has been demonstrated that the formula for the mass associated to the turnaround radius, using the DMbQG theory, is the same that the one in the current ΛCDM paradigm. Its dimensionless factors differ only a 5%. However the new formula has a deeper physical meaning because its dimensionless factor is simpler regarding the one in the other formula. Namely the factors are: $\frac{\Omega_{DE}[\sqrt{f} - f^3]}{f^3} = 1.789$, where $\Omega_{DE} = 0.7$ and $f = R_{ZV} / R_{ZG} \approx 0.602$ in the DMbQG theory and $\frac{\pi^2}{8 \cdot f^2(\Omega_m)} = 1.886$ in the current ΛCDM paradigm, where the expression $f^2(\Omega_m)$ is a quite complex function that assuming $\Omega_m = 0.3$ then $f^2(0.3) = 0.6541465$.

It is remarkable the fact that the both dimensionless factors depend on exclusively from Ω_m or Ω_{DE} and the both ones differ only a 5%.

The fact that in the framework of DMbQG theory has been found a better formula for the mass associated to the turnaround radius is a strong support for the DMbQG theory whose hypothesis about the origin of DM is exclusively the gravitational field.

The chapters 3, 4 and 5 are devoted to test the DMbQG theory in the clusters, Virgo, Coma and the Local Group of galaxies using recent data published about these clusters. The tests are mainly focused on the turnaround radius and its mass associated and all of them have been a complete success by the DMbQG theory.

7. BIBLIOGRAPHY

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