A further specification of the Cosmological constant problem by account the two fermions in the Standard model and an effect of the reducing of vacuum by matter based on uncertainty relations

Grigory Yu. Nekrasov

Federal State University of Education 141014, Moscow region, Mytishchi, Russia

Abstract

A Cosmological constant problem has been considered in the light of the Standard model of elementary particles (SM) by account two fundamental fermions in the SM, i.e. the *u* quark and the electron and their antiparticles. All the movings of these virtual pairs and their orientations in space have been taken into account in the offered model of vacuum. This led to the more precise and accurate estimation of vacuum energy of the Universe on the discovered and examined minimal scale $\sim 1.5\times10^{-15}$m on uncertainty relations, than in the previous work. Comparison of this estimation with the calculation realized by the method and the model used in the previous work was carried out. The current calculation is the full computation, i.e. for the full scale range, which begins from 1.5×10^{-15}m and can think terminated for the effect on the maximal linear size 2.00008×10^{-11}m for the experimental data and the theoretical data on hydrogen and helium atoms correspondingly. These atoms are considered due to that they are the most common in the entire Universe. Thus, one can say that the vacuum energy near matter in the special effect of the *reducing of vacuum by matter* has been assessed more precisely as it is in the real world.

Keywords: Vacuum, Uncertainty relations, Cosmological constant, Virtual fermions, Standard model of elementary particles, Vacuum energy, Matter, Interaction of vacuum with matter, Vacuum energy density

Introduction

Like in the previous work which is called "A statement of the Cosmological constant problem and an effect of the reducing of vacuum by matter based on uncertainty relations" we can employ Heisenberg uncertainty principle to assess the vacuum energy of all empty space of the Universe and the vacuum energy near matter of the Universe. Due to the war in Ukraine and on this cause lack of resources I can compute only the following quantities: all for hydrogen and helium for *u* quark and electron, also it is worthy for the all types of neutrino to compute only the vacuum energy for empty space, i.e. without atoms or the free vacuum energy. Now these fundamental and, in the time, elementary particles do exist in the Standard model of elementary particles. All the fermions of the SM, undoubtedly, contribute in the vacuum energy according the SM itself and the quantum field theories on which the SM is built. Thus, we are going to gain the part of the full vacuum energy in this case, except the contribution of the boson fields which is responsible for the interactions, i.e. the matter contribution in vacuum energy. In this paper we are going to consider the part of only fundamental vacuums, i.e. the vacuums generated by only the fundamental particles of matter. As well known, matter can be various, namely, low-energy particles and high-energy particles. We will take into account low-energy electron and positron and heavier *u* quark and antiquark in this work. Both present in the ordinary low-energy matter. This calculation is also going to be comprehensive in the sense that all movings: forward and backward, rotations are going to be taken into account, and all changings of the sizes of wave packets in location space at the moving of them are going to be considered to make the calculation as exact as possible. And, of course, this calculation is limited in our work (present and previous) by the considered effect of the reducing of vacuum near matter. We should note that this is the one single effect form great number of quantum field effects which govern in space with matter at subatomic level. Like in the previous paper we are not going to go with the set course of the quantum field theory (QFT) in the assessment of vacuum energy, but going to chose simple and evident way, taking from QFT only empirical concept of particle-antiparticle pairs in the vacuum as a picture of it. Also we say that like in the previous work this will be built on the theory excluding the interactions of virtual particles. The exact assessment of the full fermion vacuum energy is going to be done by using uncertainty relations. We delay account of interactions of virtual particles to next paper, acting gradually.

1. The concept of discrete space, needed for finite vacuum energy

In the previous work we have defined the concept of discrete space, basing on that vacuum energy cannot be finite but must be infinite in continuous space. This is true due to that at the accounting all the scales of space, we sum energy being on each space scale. At that if the number of nonzero elements of the sum is finite, then the sum is going to be finite, and vice versa. That is why space must be discrete – for the number of energies on all the space scales is to be finite. In that work we have discussed what is motion of a body in such space, in the current work we will also be needed in concept of discrete time. Thus, we will construct discrete space-time of the special theory of relativity. It is needed to say that in the usual theory if space is discrete, then the special and the general principles of relativity have to be wrong [1]. But, as author is going to show, this is appearing to be due to not completely right approach to the problem of discrete space-time.

Let us construct discrete space and time in transformed special theory of relativity for such purpose. Let velocity v of a body be determined, according to the previous paper [2], as

$$v = \frac{l_P}{\Delta t}, \tag{1.1}$$

where $\Delta t > t_P$, here like in [2] we designated: l_P is the Planck length and t_P is the Planck time, so that, it is true

$$c = \frac{l_P}{t_P}, \tag{1.2}$$

where c is the speed of light. The author's idea can be demonstrated on the simple example of one-axis motion, i.e. motion along one axis of the Cartesian coordinate system, and it can be found out that this idea lead to the contrary conclusion of discrete and flat physical manifold than that is concluded in the article [1]. In such case, according well-known the special theory of relativity (SR), length contraction is

$$l = l'\sqrt{1-\frac{v^2}{c^2}}\,. \tag{1.3}$$

Now we designate: $l' = nl_P$ and $l = ml_P$, where n and m are integer numbers. Substituting them into (1.3), and (1.1) into (1.3), also using (1.2), we got

$$m = n\sqrt{1 - \frac{t_P^2}{\Delta t^2}} \,. \tag{1.4}$$

According (1.4), must be $m < n$. On this stage let us to take into account an axiom (1): $l_P = const$ for all frames of reference. With this aspect, indeed, there was a problem in [1], therefore we exclude it at root. Taking into account this axiom, note, the simple logic follows from (1.4): if $n = 1$, then $m < 1$, but must be always $m > 1$ and, as we can see, also, must be $n > 1$, hence, n cannot be equal to 1. In the all above we already accept that space is discrete, consisting of the Planck lengths along each dimension or Cartesian axis like in [2], now consider also discrete time, parting on the Planck times along time axis in the pseudoeuclidean four-dimensional Minkowski space-time. We quantize time interval as

$$\Delta t = \left(k+1\right) t_P \,, \tag{1.5}$$

where $k \geq 1$. Then, substituting (1.5) into (1.4), we obtain the formula

$$m = n\sqrt{1 - \frac{1}{\left(k+1\right)^2}} \,, \tag{1.6}$$

which is valid for one-dimensional length contraction. In (1.6) k is the number of indivisible time intervals, n is the number of indivisible space segments in a body in its own frame of reference, m is the number of indivisible space segments in a body in the related frame of reference. This formula already describes discrete space-time. As one can see, constant t_P is invariant at changing frames of reference, that is the containing of an axiom (2): $t_P = const$ for all frames of reference. This is in contrast with [1]: fundamental space-time quanta do not change in any situation, this thesis is laid in the basis of our theory.

Analogously, for the time dilation in such simple case we have

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \frac{v^2}{c^2}}} \,. \tag{1.7}$$

We introduce the following formulae for the time intervals in each frame of reference like (1.5)

$$\Delta t = \left(k+1\right) t_P \,, \tag{1.8}$$

$$\Delta t' = \left(k'+1\right) t_P \tag{1.9}$$

and the formula for the velocity like (1.1)

$$v = \frac{l_P}{\delta t} \,, \tag{1.10}$$

where

$$\delta t = \left(f+1\right)t_P . \qquad (1.11)$$

Here k, k' and f are integer numbers which satisfy the conditions: $k \ge 1$, $k' \ge 1$, $f \ge 1$. So that the following result is justified for the time dilation in the modified SR

$$k+1 = \frac{k'+1}{\sqrt{1-\frac{1}{\left(f+1\right)^2}}} . \qquad (1.12)$$

In validity of the conditions one can convince, if to take $k' = 1$ and $f = 1$, then one can gain

$$k = 1.3094 > 1 , \qquad (1.13)$$

so that the conditions are generally executed. Thus, we can generalize the all, that was performed above in the words: *the fundamental segments of space and time (corresponding quanta) are always invariant under Lorenz group transformations, i.e. they never change at transition of the frame of reference, but do change number of such quanta in a body or in a field (as type of matter), it can increase or decrease*. From here it follows at once that if a body has one single segment of space, then it does not change, i.e. remains constant at any speed of the body.

2. Consideration of exact admissible movings of virtual electron-positron pairs in the field of the charged center in the vacuum limitation effect

As was noted above, in this work we consider all movings of the particles in virtual pair, namely, the vacuums will be considered in the dynamics. The movings, apparently, can be in forward direction and in backward direction, i.e. to annihilation point. We take that these movings occur at the constant velocity there and back, that is they are uniform. The turn occurs instantaneously. So, we have enough information to imagine some a vacuum, let it be electron-positron vacuum for definiteness. Consider enough large limited volume of space. The vacuum is being represented the virtual electron-positron pairs which at the same time jump out from the void (empty space) and, as I said, they have the same position on the axis of moving at any instant of time. In this moment we apply Galileo's principle of relativity (Classical Relativity, CR). This means that time is absolute and flows homogeneously in every place in the Universe. All the pairs create and annihilate simultaneously. However, in reality such situation can never be, there we have chaos and non-simultaneity. This situation can be reached by the following approach. It is needed to find an average on the changing time variable on different pairs. By this approach we will succeed to shift different pairs on different values of the time variable and we will gain the required characteristic of the vacuum. As we will show below, this approach can be implemented using the distribution function. And to gain a vacuum energy – a numerical value, we need to take an average of the determined expression on the all movings of these and each one pair in space (actually, on each specially orientated axis), happening during time. So, summing up, we will need two enclosed each other average values to gain the final result, a vacuum energy.

In the calculation, as it was also said in the end of the previous paper [2], we will consider dispersion of the virtual particles wave packets. Let us at once say about this. According the carried out investigation which preceded this article, an accounting of the real dispersion of the real free fermion wave packets, as this is ordered, – from the spinor – the solution of the free Dirac equation is complicate to do because difficulty of the source of this dispersion – the integral of the spinor. To extract all possible dispersion functions which, undoubtedly, represent types of the real dispersion of matter on our level (without interaction of the virtual particles to each other) to someone would be needed a special separate research which will be very complex. Also the number of these functions is infinite because the theoretical model limitlessness of the momentum space. As a result of this circumstance the author is decided in the presented computation to account dispersion as linear function as an average dispersion on this entire infinite manifold of the dispersion functions of the wave packets. The tilt of the line characterizes an average dispersion on all dispersion functions of the real free fermion dispersion, known from the Dirac spinor.

3. Types of virtual matter that contains the Universe or vacuums

We will consider two types of the particles of the SM in the physical vacuum. Filling empty space, they appeared to be a media or a virtual matter because it consists of virtual particles. Wave packets of these particles types are overlapped, that means wave packets of the virtual particles of one single type are not overlapped but different types of virtual particles wave packets are in the same place in space between each other. Initially, according the SM, real physical vacuum consists of zero-point fluctuations of the boson fields and the corresponding quanta, as it is in the quantum field theory, and interacting with each other fermion pairs by means of these boson quanta. Fermions are performed the matter of our Universe, and exactly them we will consider, but this paper is without any consideration of their interactions. It is also needed to say that vacuum, according QFT, is sufficiently nonlinear structure and has selfinteraction. However, also we are not going to consider nonlinear component of vacuum in this paper. We delay its consideration on next paper.

As well known, particles of matter include two general families: quarks and leptons. Quarks and leptons combine with each other forming families or generations, in the time the 3 families are known. These are: (u, d, e, ν_e), (c, s, ν_μ, μ), (t, b, ν_τ, τ). Here u, d, c, s, t, b are quarks, first two are stable and building all stable and seeable matter in the Universe, others are unstable and much more heavy than first two, especially last two: t and b. The rest particles are leptons, such as: e, μ and τ are electron, muon and tau-lepton, correspondingly. Electron e together with two quarks – up and down (u, d) consist all visible matter in the Universe finally. This matter can exist at low energy and at high energy. And the other particles except the neutrinos exist only at high energies in laboratory (accelerator and collider) or in violent processes in stars and black holes. The neutrinos, such as: ν_e, ν_μ, ν_τ, called electron-neutrino, muon-neutrino and tau-neutrino correspondingly, together with mentioned above leptons form families by pairs, thus, appear the 3 families. These are: (e, ν_e), (μ, ν_μ) and (τ, ν_τ). One can unite quarks themselves also in families in a view: (u, d), (c, s) and (t, b). On the last data in research of the controversial issue on the time of writing of this article the neutrinos have small nonzero masses; the fact is going to be important for the current work.

All these fermions enter together with their own antiparticles, so that we have 12 particles and 12 antiparticles, total: 24 particles and 12 pairs. The last work has the computation that has been carried out for electron-positron pairs on the basis of the consideration of one single electron-positron pair. The current work, as already was said above, must base on this article, therefore the computation in that work is applicable for pair and not for isolated particle. Therefore for us had to be essential number 12 of the types of fermion pairs. For the further statement we need a list of masses of the particles; the masses in the listing are rounded up to two meaning digits (for quarks the range is set, in which the corresponding mass hits; via semicolon the average values for the corresponding ranges are divided, and for *u*, *d*, *s*-quarks the current masses are given, for *c*, *b*-quarks the running masses are given)

Table 1 [3, 4, 5]

Family 1		**Family 2**		**Family 3**	
Particle	***Mass***(kg)$\times 10^{-31}$	***Particle***	***Mass***(kg)$\times 10^{-31}$	***Particle***	***Mass***(kg)$\times 10^{-31}$
Electron (e)	9.11	Muon (μ)	1883.56	τ-lepton	31675.1
ν_e	<0.000036	ν_μ	<3.39	ν_τ	<324.45
u-quark	26.74 – 58.83; 42.785	s-quark	1247.86 – 2317.46; 1782.66	b-quark	73624 – 95729; 84676.5
d-quark	62.39 – 106.96; 84.675	c-quark	20678.9 – 23887.7; 22283.3	t-quark	3.015×10^6 – 3.089×10^6; 3.052×10^6

4. The theory of virtual particles

Consider the Figure 3 in the paper [2]. For this 2D geometrical scheme the given formula (2.1.7) is valid, but we need another one formula which follows from this scheme, namely

$$\tan(\varphi) = \frac{\Delta l}{2s} \tag{4.1}$$

as it is easy to see. In fact, this is different form of the formula (2.1.6). Following that paper, we equate these two formulae, in the result we get the formula for the effective scale reducing in the form

$$\Delta l = \frac{l}{1+\dfrac{r}{s}}, \tag{4.2}$$

like in the previous paper. Now we need to account the dynamics of the virtual particles in pair. This can be done by accounting time dependence in the particle's passed way in both directions, so let's take

$$s = vt, \tag{4.3}$$

so that in the forward direction at $t=0$, the start or creation point of the motion lying on the axis or even the ray will be $s=0$. To find the velocity v we use (2.1.19), noting that if velocity is constant at moving in the both directions, then it must depend only on scale l and maybe on distance from charged center r and nothing else, that is why it is good to use the theory of the previous paper, i.e. the static, not dynamical theory combining it with this new theory to extract velocity. So, substitute (2.1.19) into (4.3) and the gained expression into (4.2), then having simplified the result and having brought the gained formula a little, we obtain

$$\Delta l = \frac{l}{1 + \dfrac{\dfrac{r}{ct}}{\sqrt{1 + \dfrac{16}{3}\dfrac{m^2c^2}{\hbar^2}l^2}}}. \tag{4.4}$$

From here we extract the velocity

$$v = \frac{c}{\sqrt{1 + \dfrac{16}{3}\dfrac{m^2c^2}{\hbar^2}l^2}}. \tag{4.5}$$

As we can see, it depends only on scale l. Also, as we can see, with decreasing of l and going its value to zero, the velocity goes to speed of light c, and with increasing l and unlimited increasing of it, the velocity of the particle goes to zero value. This result is explainable because with decreasing of scale, energy of the particle will grow up, that is understood from the simple uncertainty relation (1.1) in [2], and vice versa.

Now consider formula (2.1.15) in the same work. We have found out that this formula describes the lifetime of the virtual particle or the pair, therefore to gain a half of the lifetime which is time of the motion in the one direction or in the other direction (because the speeds are equal), one needs to substitute in (2.1.15) maximal value of the reducing of scale which is already known in [2] in (2.1.21) and to divide the gained expression into $1/2$. If to do this, one can gain the following formula

$$\Delta t = \frac{l}{c}\frac{\sqrt{3 + 16\dfrac{m^2c^2}{\hbar^2}l^2}}{3 + 16\dfrac{m^2c^2}{\hbar^2}l^2} \tag{4.6}$$

and

$$T(l) = \frac{l}{2c}\frac{1}{\sqrt{3 + 16\dfrac{m^2c^2}{\hbar^2}l^2}}. \tag{4.7}$$

Combining formulae (4.5) and (4.7) with corresponding coefficients, we can get

$$\Delta t = \frac{\sqrt{3}}{3}\frac{vl}{c^2}. \tag{4.8}$$

Using (2.1.5) and (4.8) together with (4.5), we obtain intermediate result

$$v = \frac{c}{3^{1/4}}\sqrt{\frac{6s}{l}}. \tag{4.9}$$

And using here again (4.5) and simplifying, we gain final result on the current stage

$$s = \frac{\sqrt{3}}{2} \frac{l}{3 + 16 \frac{m^2 c^2}{\hbar^2} l^2}. \tag{4.10}$$

Let us now analyze this result. If

$$16 \frac{m^2 c^2}{\hbar^2} l^2 < 1, \tag{4.11}$$

then

$$s \approx \frac{\sqrt{3}}{2} \frac{l}{3} = \frac{\sqrt{3}}{6} l, \tag{4.12}$$

i.e. $s < l$. If

$$16 \frac{m^2 c^2}{\hbar^2} l^2 \geq 1, \tag{4.13}$$

then

$$s = \frac{\sqrt{3}}{2} \frac{l}{3 + \Delta_1}, \tag{4.14}$$

but $\Delta_1 \geq 1$. Let's perform (4.14) in the view

$$s = \frac{\sqrt{3}}{2} \frac{l}{\Delta_2}, \tag{4.15}$$

where $\Delta_2 \gg 1$, hence, $s < l$. That means the condition

$$s < l \tag{4.16}$$

is always true. To conclude finally on this stage, we need to do additionally some calculations.

According (2.1.5) and the well-known formula of SR, the momentum of the particle will have the view

$$\mathbf{p}_{01} = \frac{2ms}{\Delta t} \frac{1}{\sqrt{1 - \frac{4s^2}{c^2 \Delta t^2}}} \mathbf{e}_v, \tag{4.17}$$

where must be $s \neq l$, and $\mathbf{e}_v$ is the basis vector of the velocity $\mathbf{v}$, so that $|\mathbf{e}_v| = 1$. Now, let us substitute (4.7) in (4.17), then after simplifying, we gain

$$\mathbf{p}_{01} = \frac{32m^3c^3l^2s + 6\hbar^2cms}{\hbar^2 l\sqrt{3+16\frac{m^2c^2}{\hbar^2}l^2}\sqrt{1-\frac{12s^2}{l^2}-\frac{64m^2c^2}{\hbar^2}s^2}}\mathbf{e}_v . \tag{4.18}$$

This expression can be valid only at the condition

$$1-\frac{12s^2}{l^2}-\frac{64m^2c^2}{\hbar^2}s^2 > 0 , \tag{4.19}$$

so that only positive number would stand under square root and this number must not be zero. From here we come to the conclusion

$$3+16\frac{m^2c^2}{\hbar^2}l^2 < \frac{l^2}{4s^2} , \tag{4.20}$$

this is true only if $s < l$. So, now we are ready to conclude finally on this stage. According (4.16), the formula (4.18) is always valid.

We need to make some remarks about the theory. The first, the formula for momentum of the point particle and the formula for velocity of motion of the wave packet as whole are justified, that follows from the theory lying in basis of these expressions. The second, the cause of that the first aspect is true, is that qualitatively new physics of vacuum virtual particles appears to be in the fact is that the way going by wave packet of the virtual particle as whole is always less of the linear size of the particle wave packet. Owing to the theory from which this has been withdrawn these two aspects are connected, the first can be described by the second. Now we are ready to express this understanding mathematically. Using the well-known formula of the SR for momentum of a point particle in the view

$$\mathbf{p}_{01} = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}\mathbf{e}_v , \tag{4.21}$$

and using (4.5), we can obtain the following formula

$$\mathbf{p}_{01} = \frac{\sqrt{3}}{4}\frac{\hbar}{l}\mathbf{e}_v , \tag{4.22}$$

which accords with the basic uncertainty relation for the momentum and the coordinate of the particle (1.1) in [2] with a condition that the value of the momentum is less than the value of uncertainty interval of the momentum postponed on the same axis of momentum, i.e. if

$$\Delta p_i = p_{2i} - p_{1i} , \tag{4.23}$$

then the vector inequality

$$\mathbf{p}_{01} < \mathbf{p}_2 - \mathbf{p}_1 \tag{4.24}$$

is justified. According this inequality and the fact is that $\mathbf{p}_{01}$ must hit the uncertainty interval, must be

$$\mathbf{p}_{01} > \mathbf{p}_1 . \tag{4.25}$$

Thus, in (4.22) the value of momentum and not the uncertainty, like in (1.1), stands. In the spherical polar coordinate system the basis vector of the velocity has the well-known expression via angles

$$\mathbf{e}_v = \left(\sin(\theta)\cos(\varphi), \sin(\theta)\sin(\varphi), \cos(\theta)\right), \tag{4.26}$$

where θ and φ are polar and azimuthal angles correspondingly.

5. The general mathematical theory needed for computation

As was mentioned afore, we will need in two average values: the first, we need to average all movings of the virtual particles in the pair on these movings or, that is the same, on time. It is needed to average them on the all temporal interval of the moving in one and in another direction. Let us begin from the forward direction or the one direction. The approach will be correct if the condition is true, namely: *the describing effect of the wrapping of vacuum does not impact the distribution on average values of any quantities of all virtual pairs in all atoms in the Universe*. We have the following average in the first place

$$\frac{1}{T(l)-\tau} \int_{\tau}^{T(l)} (\ldots)\, dt , \tag{5.1}$$

where $T(l)$ defines by (4.7), and there is the condition for new variable τ

$$0 \le \tau < T(l). \tag{5.2}$$

The average (5.1) is the average on time value applied for the moving of this virtual particle, it transforms the segment of the moving trajectory into a dot, i.e. it removes time variable t. But according what was said in the Section 2, all virtual pairs differ and, correspondingly, all average values for each pair also differ, therefore we need to distinguish them anyway. The best way to do this already consists in the offered average value (5.1): this difference in the initial locations of virtual particles in all pairs in the vacuum in our chosen instant of time makes τ-parameter. Actually, this parameter does it perfectly and, thus, τ-parameter is that parameter which by unambiguous and by sufficient way characterizes this unique average value of any quantity. An upper value for τ one can set by a mode

$$\tau = T(l) - \varepsilon , \tag{5.3}$$

where ε is a small enough number with dimension of time. Thus, we have one of two average values.

The second average, as was already said in the Section 2, is required an introducing the distribution function. So, we need distribution function of averages (or average values) on time on number of particles in our large volume of space. This function can be determined by the following equation

$$dN = N_f F(\tau) d\tau \,, \tag{5.4}$$

where N_f is the full number of all virtual pairs in all atoms in the Universe; N is the number of pairs which have τ-parameter in the interval

$$\tau_2 - \tau_1 = \Delta\tau \,, \tag{5.5}$$

and, finally, $F(\tau)$ is the required distribution function. Of course, number N_f can relate only to a separated volume, but globally it must describe the entire Universe. Fortunately, we already know this number from [2], it must look as

$$N_f = \sum_{n=0}^{\infty} N_a A_n \tag{5.6}$$

in the terms used in [2]. I remind them; N_a is the number of all atoms in the Universe, the sum A_n on n is the sum of all numbers of all wave packets in all ball layers in one single atom (for each numerator n this is the number of all wave packets in the given ball layer in one single atom). Thus, according the concepts of average value and distribution function, the probability of the event that this chosen pair will have τ-parameter in the interval (5.5) is going to have the view

$$P[\tau_2, \tau_1] = \int \frac{dN}{N_f} \equiv \int_{\tau_1}^{\tau_2} F(\tau) d\tau \,. \tag{5.7}$$

At that the distribution function is normalized as

$$\lim_{\varepsilon \to 0} \int_0^{T(l)-\varepsilon} F(\tau) d\tau = 1 \,. \tag{5.8}$$

If virtual pairs distribute on average values identically and uniformly, then $F(\tau)$ does not depend on τ and equals

$$F(\tau) = \frac{1}{T(l)} \,. \tag{5.9}$$

This one can easily check by substitution (5.9) in the normalization (5.8), so that one reads that (5.8) is executed, but this can be done with one single distribution function which is (5.9).

Thus, the full average value consisted of two enclosed each other averages at the condition, if we decided to use (5.9) in quality of the distribution function on the basis of the corresponding fact above, now has the view

$$\langle \ldots \rangle = \lim_{\varepsilon \to 0} \int_0^{T(l)-\varepsilon} \frac{1}{T(l)-\tau} \int_\tau^{T(l)} (\ldots) dt dN \equiv \lim_{\varepsilon \to 0} \int_0^{T(l)-\varepsilon} \frac{1}{T(l)} \left(\sum_{n=0}^{\mathrm{N1}} N_a A_n \right) \frac{1}{T(l)-\tau} \int_\tau^{T(l)} (\ldots) dt d\tau \,. \tag{5.10}$$

Here we have neglected all of scales that is larger than any atom, that is why in (5.10) in the sum the finite number of summands stands in contrast (5.6). The upper limit of the sum N1 just determines this finite number.

Here we also give the first average value for another direction or the backward direction. It has the view

$$\frac{1}{T(l)-\tau}\int_{T(l)+\tau}^{2T(l)}\left(...\Delta l_n(-t)...\delta l(t)...\right)dt, \tag{5.11}$$

where we have given at once the dependence of the integrand on the effective reducing of scale (4.4) which must have reversal dependence on time t because it thinks moving in backward direction. Here only the time dependence of the effective scale reducing is shown. We also have included in the integrand the dependence on dispersion that is expressed by the function $\delta l(t)$ which must have direct dependence on time because with flowing of time dispersion only increases. Going back to the beginning of this Section analogously the dependence in (5.11), the dependence of (5.1) must look so

$$\left(...\Delta l_n(t)...\delta l(t)...\right). \tag{5.12}$$

Here only the time dependence of the effective scale reducing is shown. That is, we must allow time to flow in direct, normal direction for the forward direction of motion of the particle and we must turn time flow direction for the backward direction of motion of the particle, that describes well and enough the backward motion in the offered model. Thus, the full average value for the backward direction at analogous the forward motion conditions will have the view

$$\langle...\rangle=\lim_{\varepsilon\to 0}\int_{0}^{T(l)-\varepsilon}\frac{1}{T(l)}\left(\sum_{n=0}^{N1}N_aA_n\right)\frac{1}{T(l)-\tau}\int_{T(l)+\tau}^{2T(l)}(...)\,dtd\tau\,. \tag{5.13}$$

In any case we must additionally integrate the first average on dN, using this expression from (5.4) with the upper and the lower limits: $\left[0,T(l)-\varepsilon\right]$, thinking that ε is enough small constant.

Now let us consider the full integrand in the both cases of the motion. It will have changed in comparison with the integrand in [2], as we already said above, on the two causes: the dynamics of the task and dispersion. The main multiplier will have the view

$$E_{v+m}^{(cell)}=\sqrt{\frac{\hbar^2c^2}{16}\frac{3}{\left(l-\Delta l_k(l,t)+\delta l(t)\right)^2}+m^2c^4}\,. \tag{5.14}$$

The number multiplier N_{fk} also will have changed due to consideration of the dynamics and dispersion:

$$N_{fk}=N_a\left(\frac{N(l,r_k,t)^2}{2}-N(l,r_k,t)+2\right), \tag{5.15}$$

where the number of the wave packets going in the one single arbitrary ball layer A_k in the second multiplier of (5.15) depends on the number of balls or wave packets placed only on the meridian slash of the one layer of the concentric ball layer packaging which has now the view

$$N(l,r_k,t)=\frac{\pi}{\arctan\left(\dfrac{l-\Delta l(l,r_k,t)+\delta l(t)}{2\sqrt{r_k^2-0.25\left(l-\Delta l(l,r_k,t)+\delta l(t)\right)^2}}\right)}. \quad (5.16)$$

Therefore the full integrand now takes the form

$$\sum_{k=0}^{n\max} N_{fk}(l,t)\sqrt{\frac{\hbar^2c^2}{16}\frac{3}{\left(l-\Delta l_k(l,t)+\delta l(t)\right)^2}+m^2c^4}\ . \quad (5.17)$$

For the backward direction we also need the modified effective scale reducing in comparison with (4.4); it has the form

$$\Delta lm=\frac{l}{1+\dfrac{rm_k}{v(l)\left(\frac{1}{2}\Delta t(l)-t\right)}}, \quad (5.18)$$

which is given taking into account the discretization of the radii of spheres layers (denoted by the k numerator) and the reversal run of time (it already has minus sign and it is thought positive). And, therefore, at accounting of the backward motion in the all formulae must be done substitutions

$$\Delta l\to\Delta lm, \quad (5.19)$$

$$r_k\to rm_k. \quad (5.20)$$

(The letter m entering in these designations means 'minus sign' which appears at the description of the backward motion.) Of course, we should not forget to integrate the gained expression on all the scales l to gain an estimation of the vacuum energy of this given vacuum. The dispersion function, as was already said in the Section 2, will look so

$$\delta l(t)=qt, \quad (5.21)$$

the tilt of the line must correspond an average function of the all real dispersion functions. In accordance with this imaginable existing average function we have chosen the value of the constant

$$q=\frac{2}{3}. \quad (5.22)$$

This choice is caused by an intuitive view that an average value of the angle $\pi/2$, in the range of which the curves of the dispersion functions must vary, is the angle $\pi/4$. Therefore we have chosen the number is close to 1 but it is smaller than unity because it is reasonable to think that the curves of the dispersion functions are nestled to the axis of the trajectory. Note that at $k=0$ $r_k = r_{\min}$, this will account in the computation given in the computer program below. Like in the article [2] the computer program is written in the "Wolfram language®".

Like that is in [2] we need the system of the equations for the radii r_k and rm_k for the forward direction and the backward direction correspondingly. In the computational view they are

$$r_{k+1} - \frac{1}{2}\left(l - \frac{l}{1+\dfrac{r_{k+1}}{v(l)t}} + \delta l(t) \right) = r_k + \frac{1}{2}\left(l - \frac{l}{1+\dfrac{r_k}{v(l)t}} + \delta l(t) \right), \tag{5.23}$$

$$rm_{k+1} - \frac{1}{2}\left(l - \frac{l}{1+\dfrac{rm_{k+1}}{v(l)\left(\frac{1}{2}\Delta t(l) - t\right)}} + \delta l\left(\frac{1}{2}\Delta t(l) + t\right) \right) = rm_k + \tag{5.24}$$

$$+\frac{1}{2}\left(l - \frac{l}{1+\dfrac{rm_k}{v(l)\left(\frac{1}{2}\Delta t(l) - t\right)}} + \delta l\left(\frac{1}{2}\Delta t(l) + t\right) \right).$$

The system of the equations appears to be when the values, beginning from zero to some maximal value, appropriate to index k and the gained different equations are written down. Actually, these equations are the quadratic equations, and we should to resolve them relatively r_{k+1} and rm_{k+1} to write fastest algorithm. The maximal value for radius, actually, is initially set (it is going in the external data of the task). And the computer program must solve as much of the equations as it is needed for the radius, depending on the minimal scale and at maximal reducing of the scale (in the maximal time, i.e. in the most distant point from the creation point of the pair), would be smaller or equal to the maximal its value, set as the external parameter. This condition is right because if it is executed, then the most number of wave packets (balls) of the minimal volume at given dispersion (the coefficient of the linear dispersion equals $2/3$) can go in the all ball layers in whole atom. I.e. in this case we must judge maximally. However, the maximal scale must not excess the maximal value of the radius of the atom, to stay in the frames of the task. Thus, the radii are the functions and they have the following dependences

$$r_k(l,t), \tag{5.25}$$

$$rm_k(l,t). \tag{5.26}$$

The maximal value of scale l can be found by solving the inequalities for l

$$r_{\min} \geq \frac{1}{2}\left|l + q\Delta t(l)\right|, \tag{5.27}$$

$$r_{\min} \geq \frac{1}{2}\left| l - \frac{l}{1 + \frac{r_{\min}}{\frac{1}{2}v(l)\Delta t(l)}} + \frac{1}{2}q\Delta t(l) \right|. \tag{5.28}$$

The solutions of these inequalities are intervals of l, therefore we need to choose maximal boundaries of the intervals and then to compare them to choose the maximal value of scale. All the computations applied below are realized by the lattice method (numerically) to add the computation in [2] and to fill its disadvantages. The lattice means *lt* flat grid, or scale-time plane consisting rectangular grid. The inequalities (5.27) and (5.28) consist of a step for time, δ of the lattice; it determines frequency of the grid and simultaneously maximal scale of the task, as it goes in these inequalities. We also must remember that integration on scale l and division into minimal length or scale – the Planck length of the all expression are needed. I.e.

$$\frac{1}{l_P}\int_{l_{\min}}^{l_{\max}} (\ldots)\,dl\,, \tag{5.29}$$

where the full expression described above must stay in the brackets.

Summing up, the estimation of the vacuum energy near matter without any interactions now takes the form

$$E_{v+m}^{(0)} \geq \frac{1}{l_P}\int_{l_{\min}}^{l_{\max}} \sum_{k=1}^{n\max} N_a \lim_{\varepsilon\to 0} \int_0^{T(l)-\varepsilon} \frac{1}{T(l)}\frac{1}{T(l)-\tau}\int_\tau^{T(l)} \left(\frac{N^2}{2} - N + 2\right) E_{v+m}^{(cell)}\,dt\,d\tau\,dl + \tag{5.30}$$

$$+\frac{1}{l_P}\int_{l_{\min}}^{l_{\max}} \sum_{k=1}^{n\max} N_a \lim_{\varepsilon\to 0} \int_0^{T(l)-\varepsilon} \frac{1}{T(l)}\frac{1}{T(l)-\tau}\int_{T(l)+\tau}^{2T(l)} \left(\frac{Nm^2}{2} - Nm + 2\right) Em_{v+m}^{(cell)}\,dt\,d\tau\,dl\,.$$

Here we omitted the dependence of $N(l, r_k, t)$ and we designated by letter m all the functions that relate to the backward motion having their corresponding modifications. The different from (5.19) and (5.20) modifications are that we must replace everywhere in the dispersion function

$$t \to \frac{1}{2}\Delta t(l) + t\,, \tag{5.31}$$

at that $t > 0$, as it is always. Let us compare the real lower boundary of the vacuum energy with the free vacuum energy of the same volume occupied by empty space in relation of real particles, but it contains virtual particles. To do this, we need to put

$$\Delta l(l, r_k, t) = 0\,, \tag{5.32}$$

$$\Delta lm(l, rm_k, t) = 0 \tag{5.33}$$

in all the formulae. I.e. there is no the vacuum limitation effect anymore. Therefore the all above formulae are going to change. We begin to perform them:

$$E_v^{(cell)} = \sqrt{\frac{\hbar^2 c^2}{16}\frac{3}{\left(l+\delta l(t)\right)^2} + m^2c^4}\ , \tag{5.34}$$

$$N(l, r_k, t) = \frac{\pi}{\arctan\left(\dfrac{l+\delta l(t)}{2\sqrt{r_k^2 - 0.25\left(l+\delta l(t)\right)^2}}\right)}, \tag{5.35}$$

$$\sum_{k=0}^{n\max'} N'_{fk}(l,t)\sqrt{\frac{\hbar^2 c^2}{16}\frac{3}{\left(l+\delta l(t)\right)^2} + m^2c^4}\ , \tag{5.36}$$

$$r_{k+1} - \frac{1}{2}\left(l+\delta l(t)\right) = r_k + \frac{1}{2}\left(l+\delta l(t)\right), \tag{5.37}$$

$$rm_{k+1} - \frac{1}{2}\left(l+\delta l\left(\frac{1}{2}\Delta t(l)+t\right)\right) = rm_k + \frac{1}{2}\left(l+\delta l\left(\frac{1}{2}\Delta t(l)+t\right)\right). \tag{5.38}$$

Here in (5.36) the mark means that the corresponding number of balls (wave packets) accounts absence of the effect, i.e. it contains the number of balls on the meridian slash in the form (5.35), and the mark that has the upper limit of the sum means difference of the number of ball layers (of the whole ball of the atom) without the reducing from the theory of the effect. In this case the inequality (5.27) remains, and the inequality (5.28) transforms in the following one

$$r_{\min} \geq \frac{1}{2}\left|l + \frac{1}{2}q\Delta t(l)\right|. \tag{5.39}$$

Thus, the estimation of the vacuum energy of the free space in atoms without nuclei corresponding to atoms which have their nuclei takes the form

$$E_v^{(a)} \geq \frac{1}{l_P}\int_{l_{\min}}^{l_{\max}}\sum_{k=1}^{n\max} N_a \lim_{\varepsilon\to 0}\int_0^{T(l)-\varepsilon}\frac{1}{T(l)}\frac{1}{T(l)-\tau}\int_\tau^{T(l)}\left(\frac{N^2}{2} - N + 2\right)E_v^{(cell)}\,dt\,d\tau\,dl + \tag{5.40}$$

$$+\frac{1}{l_P}\int_{l_{\min}}^{l_{\max}}\sum_{k=1}^{n\max} N_a \lim_{\varepsilon\to 0}\int_0^{T(l)-\varepsilon}\frac{1}{T(l)}\frac{1}{T(l)-\tau}\int_{T(l)+\tau}^{2T(l)}\left(\frac{Nm^2}{2} - Nm + 2\right)Em_v^{(cell)}\,dt\,d\tau\,dl\ ,$$

where for N we have expression (5.35), the index a in brackets means 'atoms' and

$$Em_v^{(cell)} = \sqrt{\frac{\hbar^2 c^2}{16}\frac{3}{\left(l+\delta l\left(\frac{1}{2}\Delta t(l)+t\right)\right)^2} + m^2c^4}\ . \tag{5.41}$$

Also it is needed to assess the free vacuum energy of all empty space of the Universe. This must be done because there have been introduced the dynamics and the dispersion which are absented in [2]. Formulae (2.2.12) and (2.2.13) are no longer valid in this approach. Let us imagine all empty space of the Universe as a cube, at that the volume of the space and the volume of this cube are equal. Now, we divide this enormous cube into a large number of small cubes – cells, in each of which the wave packet is. Such approach will allow estimating the free vacuum energy by our method. Thus, the full number of the wave packets in all empty space in the Universe will be

$$N_s(l,t) = \frac{V_s}{\left(l+\delta l(t)\right)^3} \tag{5.42}$$

at account of dispersion. Knowing this number, we can use our new method with average values; thus, the free vacuum energy takes the form

$$E_v^{(0)} \geq \frac{1}{l_P}\int_{l_{\min}}^{\infty}\lim_{\varepsilon\to 0}\int_0^{T(l)-\varepsilon}\frac{1}{T(l)}\frac{1}{T(l)-\tau}\int_{\tau}^{T(l)} N_s(l,t)\,E_v^{(cell)}\,dt\,d\tau\,dl + \tag{5.43}$$

$$+\frac{1}{l_P}\int_{l_{\min}}^{\infty}\lim_{\varepsilon\to 0}\int_0^{T(l)-\varepsilon}\frac{1}{T(l)}\frac{1}{T(l)-\tau}\int_{T(l)+\tau}^{2T(l)} Nm_s(l,t)\,Em_v^{(cell)}\,dt\,d\tau\,dl,$$

where $l_{\min}$ is the ultraviolet cutoff; the full number of the wave packets at the backward motion in empty space is

$$Nm_s(l,t) = \frac{V_s}{\left(l+\delta l\left(\frac{1}{2}\Delta t(l)+t\right)\right)^3}. \tag{5.44}$$

The required infinite upper limit of the integrals on l in (5.43) can be realized in practical calculation only for analytical integration, and for the numerical integration we will use it must be replaced by finite limit, actually, the same as in (5.30) – $l_{\max}$.

At the next stage the wrapping vacuum coefficients have to be calculated to show that the vacuum really reduces by matter and how much that happens. We already have everything to do this. Let it be designated

$$BVM = \frac{1}{l_P}\int_{l_{\min}}^{l_{\max}}\sum_{k=1}^{n\max}N_a\lim_{\varepsilon\to 0}\int_0^{T(l)-\varepsilon}\frac{1}{T(l)}\frac{1}{T(l)-\tau}\int_{\tau}^{T(l)}\left(\frac{N^2}{2}-N+2\right)E_{v+m}^{(cell)}\,dt\,d\tau\,dl + \tag{5.45}$$

$$+\frac{1}{l_P}\int_{l_{\min}}^{l_{\max}}\sum_{k=1}^{n\max}N_a\lim_{\varepsilon\to 0}\int_0^{T(l)-\varepsilon}\frac{1}{T(l)}\frac{1}{T(l)-\tau}\int_{T(l)+\tau}^{2T(l)}\left(\frac{Nm^2}{2}-Nm+2\right)Em_{v+m}^{(cell)}\,dt\,d\tau\,dl,$$

$$BV = \frac{1}{l_P}\int_{l_{\min}}^{l_{\max}}\sum_{k=1}^{n\max}N_a\lim_{\varepsilon\to 0}\int_0^{T(l)-\varepsilon}\frac{1}{T(l)}\frac{1}{T(l)-\tau}\int_{\tau}^{T(l)}\left(\frac{N^2}{2}-N+2\right)E_v^{(cell)}\,dt\,d\tau\,dl + \tag{5.46}$$

$$+\frac{1}{l_P}\int_{l_{\min}}^{l_{\max}}\sum_{k=1}^{n\max}N_a\lim_{\varepsilon\to 0}\int_0^{T(l)-\varepsilon}\frac{1}{T(l)}\frac{1}{T(l)-\tau}\int_{T(l)+\tau}^{2T(l)}\left(\frac{Nm^2}{2}-Nm+2\right)Em_v^{(cell)}dtd\tau dl\,,$$

where 'BVM' means 'boundary vacuum plus matter' and it is the right hand side of (5.30) and 'BV' means 'boundary vacuum' and it is the right hand side of (5.39). Then the general wrapping vacuum coefficient *in this model* takes the form

$$\sigma\geq\frac{BVM}{BV}\,. \tag{5.47}$$

Actually, we divide the inequalities (5.30) and (5.40) into each other and equate the fraction of the energies in the left hand side to σ.

6. The final results of the computation

This is final stage of the work and here we provide all numerical values of every parameter of the computation and its results before presenting of the computer program itself. All the data will be resulted in the following Table 3, Table 4, Table 5 and Table 6; for the neutrinos results are given in the opposite form in comparison with the masses are, i.e. the lower limit, besides our usual estimation scheme (where all estimations are lower limits or boundaries), this is done, proceeding from the complex dependence found in the computation: the free vacuum energy of the specific matter (particle) is not always directly proportional or changes like the mass of the corresponding real particle, it can sometimes have an opposite dependence; (pm means picometer, $1\text{pm}=10^{-12}\text{m}$). The Table 2 contains the information needed for the calculation of the volume of empty space of the Universe. The computation has been carried out at the following values of the fundamental constants (1) in SI units and the values of the spatial and the temporal grids (2), needed for the numerical computation: (1) $h=6.6260755\cdot 10^{-34}$ is the Planck constant; $c=299792458$ is the speed of light in vacuum; $l_P=1.616255\cdot 10^{-35}$ is the Planck length; $V_U=3.6\cdot 10^{80}$ is the volume of the observable Universe; $N_a=7.39\cdot 10^{79}$ is the number of atoms in the Universe, also called the Eddington number; $V_s=V_U-\sum_{i=1}^{10}\left(V_{Ai}\Delta_i N_a\right)$ is the volume of the empty Universe, i.e. the volume of the empty space of the Universe, which defines via the volumes of each type of atoms V_{Ai}, the fractions of presence in number relation of each atom in the full number of atoms Δ_i and the full number of atoms in the Universe; here we considered 10 most common atoms in the Milky Way Galaxy or in the Universe in order of decreasing of their abundance, $i=(1,2,...,10)$;

Table 2 (Ten most common elements in the Universe) [6, 7, 8]

Symbol	$V_{Ai}\left(\text{m}^3\right)\times 10^{-32}$	Δ_i
$^{1}_{1}\text{H}$	6.12611	0.739
$^{4}_{2}\text{He}$	12.0599	0.24
$^{15}_{8}\text{O}$	90.059	0.0104
$^{12}_{6}\text{C}$	143.257	0.0046

$^{20}_{10}Ne$	22.5659	0.0013
$^{55}_{26}Fe$	1148.99	0.0011
$^{14}_{7}N$	114.616	0.00096
$^{28}_{14}Si$	557.109	0.00065
$^{24}_{12}Mg$	1413.3	0.00058
$^{32}_{16}S$	418.46	0.00044
Total:		0.99906

(2) $NMAX = 50$, $MMAX = 104$ are the number of segments on the scale and the time axes correspondingly; $\delta_e(l_{\max}) = 1.54803 \cdot 10^{-24}\,\mathrm{s}$, $\delta_u(l_{\max}) = 3.29625 \cdot 10^{-25}\,\mathrm{s}$ are the duration of one single time interval for the electron and the *u* quark correspondingly; $l_{\max} = 2.00008 \cdot 10^{-11}\,\mathrm{m}$, $l_{\max} = 2 \cdot 10^{-11}\,\mathrm{m}$ are the maximal values of the scale variable for the electron and the *u* quark correspondingly, and all remains the same whatever the effect is or it is no; $l_{\min} = 10^{20} l_P$ is the general minimal scale; $r_{\min} = 10^{-11}\,\mathrm{m}$ is the minimal radius of the sphere, which is the minimal boundary of the ball layer, where the limitation vacuum effect takes place in atom. The volumes of the atoms, where the effect takes place can be computed by the formula $V_A = \frac{4}{3}\pi\left(r_{\max}^3 - r_{\min}^3\right)$, and they have the following values: $V_A\left({}^1_1\mathrm{H}\right) = 6.12611 \cdot 10^{-32}\,\mathrm{m}^3$, $V_A\left({}^4_2\mathrm{He}\right) = 1.20599 \cdot 10^{-31}\,\mathrm{m}^3$. The program is separated on the six subprograms, therefore in each subprogram its own values of the spatial and temporal grids are used. In the computer program the following values for the grids have been used: $d = d_2 = d_3 = d_4 = \frac{\Delta l}{NMAX}$, and $\delta_X = \delta_{X2} = \delta_{X3} = \delta_{X4} = \frac{1}{2}\frac{\Delta t}{MMAX}$, where $\Delta l = l_{\max} - l_{\min}$ and $X = (e,u)$. The numbers in the indices and the absence of them mean belonging to one of these subprograms. More accurately one can see that in the program itself. For we have two values for the maximal scale: for the electron and for the *u* quark, there are the two considering scale range for each particle: $\Delta l = l_{\max} - l_{\min}$ and $\Delta l_2 = l_{\max 2} - l_{\min}$. Also for the other components of the program: $l_{\max 3} = l_{\max 4}$ and it can be equal $l_{\max}$ or $l_{\max 2}$. Therefore there are the few values of the small numerical constant ε, which has dimensionality of time on each stage of the computation in the computer program: $\varepsilon = T\left(l_{\min} + NMAXd\right) - \left(MMAX - 1\right)\delta$, $\varepsilon_2 = T\left(l_{\min} + NMAXd_2\right) - \left(MMAX - 1\right)\delta_{X2}$, $\varepsilon_3 = T\left(l_{\min} + NMAXd_3\right) - \left(MMAX - 1\right)\delta_{X3}$, $\varepsilon_4 = T\left(l_{\min} + NMAXd_4\right) - \left(MMAX - 1\right)\delta_{X4}$. All the values are rounded up to five meaning digits:

Table 3 (Two most common elements in the Universe) [6, 7, 8]

Symbol	$r_{\max}^{(\mathrm{Exp})}$ (pm) error $\pm$5pm, * $r_{\max}^{(\mathrm{Theor})}$ (pm)	Abundance on number in %	Abundance on number in quantity $\times 10^{75}$	Particles pairs (p means $p\bar{p}$)	$E_v^{(0)}(\mathrm{J})$ $\times 10^{133} \geq$	$E_{v+m}^{(0)}(\mathrm{J})$ $\times 10^{103} \geq$	$BV(\mathrm{J})$ $\times 10^{103}$
$^{1}_{1}H$	25	73.9	73900	*u*	1004.48	40.6758	40.6434

				e	7.07064	7.91452	7.90539
${}_{2}^{4}\mathrm{He}$	*31	24	24000	u	-	32.6181	32.5837
				e	-	5.18525	5.18142
Total:		97.9	97900		1011.55	86.3937	86.3139

These two elements were estimated spectroscopically in the Milky Way Galaxy, but if we take a condition that our Galaxy is common, typical and usual in the Universe, and according the theory of the development of the Universe [9 – 14], this is plausible, we can generalize them and the all data on the entire Universe.

Table 4 (The estimation for the neutrinos)

	ν_e	ν_μ	ν_τ	Total:
$E_{\nu}^{0)}\ \mathrm{J})\times 10^{136} \geq$	150.26	2.12968	1.30695	153.697
$2\ E_{\nu}^{0}\ \ \mathrm{J})\times 10^{133} \geq$	2.40735	2.4074	2.74955	7.5643
$E_{\nu}^{0}\ \ \mathrm{J})-$ $[2]E_{\nu}^{(0)}(\mathrm{J})\times 10^{136} \geq$	150.258	2.12727	1.3042	153.689

Table 5 (The previous computation for the elements and comparison of the two calculations)

Symbol	Particles pairs (p means $p\overline{p}$)	[2] $E_{\nu}^{0}\ \ \mathrm{J})$ $\times 10^{133} \geq$	[2] $E_{\nu+m}^{0}\ \ \mathrm{J})$ $\times 10^{103} \geq$	[2] $BV\ \ \mathrm{J})$ $\times 10^{101}$	$E_{\nu}^{0)}-[2]E_{\nu}^{0)}$ $(\mathrm{J})\times 10^{133} \geq$	$E_{\nu+m}^{0)}-[2]E_{\nu+m}^{0)}$ $(\mathrm{J})\times 10^{103} \geq$	$BV-$ $[2]BV(\mathrm{J})$ $\times 10^{103}$
${}_{1}^{1}\mathrm{H}$	u	2.41457	13.8119	5.31257	1002.07	26.8639	40.5903
	e	2.40769	3.08206	5.26895	4.66295	4.83246	7.8527
${}_{2}^{4}\mathrm{He}$	u	-	12.288	3.3964	-	20.3301	32.5497
	e	-	2.72712	3.3685	-	2.45813	5.14774
Total:		4.82226	31.9091	17.3464	1006.73	54.4846	86.1404

Table 6 (The wrapping vacuum coefficients)

Symbol	Particles pairs (p means $p\overline{p}$)	σ	$[2]\sigma$
${}_{1}^{1}\mathrm{H}$	u	1.0008	259.985
	e	1.00115	58.4948
${}_{2}^{4}\mathrm{He}$	u	1.00106	361.795
	e	1.00074	80.9595
Total:		1.00092	183.952

```
In[∘]:= Tm1 = AbsoluteTime[];
              абсолютное значение

In[∘]:= me = 9.11 × 10^-31;

In[∘]:= mμ = 1.88356 × 10^-28;

        mτ = 3.16751 × 10^-27;

        uplmνe = 3.6 × 10^-36;

In[∘]:= uplmνμ = 3.39 × 10^-31;

In[∘]:= uplmντ = 3.2445 × 10^-29;

In[∘]:= mu = 4.2785 × 10^-30;

In[∘]:= md = 8.4675 × 10^-30;

In[∘]:= ms = 1.78266 × 10^-28;

In[∘]:= mc = 2.22833 × 10^-27;

In[∘]:= mb = 8.46765 × 10^-27;

In[∘]:= mt = 3.052 × 10^-25;

In[∘]:= VU = 3.6 × 10^80;
        Na = 7.39 × 10^79;
        VA = 4/3 π (r_max^3 - r_min^3);
        m0 = me;
        c = 299 792 458;
        h = 6.6260755 × 10^-34;
        ℏ = h/(2 π);

In[∘]:= VAp_1 = 6.12611 × 10^-32;
        VAp_2 = 1.20599 × 10^-31;
        VAp_3 = 9.0059 × 10^-31;
        VAp_4 = 1.43257 × 10^-30;
        VAp_5 = 2.25659 × 10^-31;
        VAp_6 = 1.14899 × 10^-29;
        VAp_7 = 1.14616 × 10^-30;
        VAp_8 = 5.57109 × 10^-30;
        VAp_9 = 1.4133 × 10^-29;
        VAp_10 = 4.1846 × 10^-30;
```

```
In[∘]:= Δ1 = 0.739;
        Δ2 = 0.24;
        Δ3 = 0.0104;
        Δ4 = 0.0046;
        Δ5 = 0.0013;
        Δ6 = 0.0011;
        Δ7 = 0.00096;
        Δ8 = 0.00065;
        Δ9 = 0.00058;
        Δ10 = 0.00044;

In[∘]:= rmin := 10^-11

In[∘]:= lmin := 10^20 lP

In[∘]:= lP := 1.616255 × 10^-35
```

Note: lmax depends on δ and vice versa (including dependence on MMAX), see the inequalities below.

```
In[∘]:= lmax := 2.00008 × 10^-11

In[∘]:= lmax2 := 2 × 10^-11

In[∘]:= ϵ = T[lmin + NMAX d] - (MMAX - 1) δ;

In[∘]:= ϵ2 = T[lmin + NMAX d2] - (MMAX - 1) δ2;

In[∘]:= ϵ3 = T[lmin + NMAX d3] - (MMAX - 1) δ3;

In[∘]:= ϵ4 = T[lmin + NMAX d4] - (MMAX - 1) δ4;

In[∘]:= rHmax = 2.5 × 10^-11;

In[∘]:= rHemax = 3.1 × 10^-11;

In[∘]:= rOmax = 6 × 10^-11;

In[∘]:= rCmax = 7 × 10^-11;

In[∘]:= rNemax = 3.8 × 10^-11;

In[∘]:= rFemax = 1.4 × 10^-10;

In[∘]:= rNmax = 6.5 × 10^-11;

In[∘]:= rSimax = 1.1 × 10^-10;

In[∘]:= rMgmax = 1.5 × 10^-10;

In[∘]:= rSmax = 10^-10;

In[∘]:= rmax = rHmax;

In[∘]:= q := 2/3
```

$$\Delta t[l\_] := \frac{1}{c} \times \frac{\sqrt{3 + 16 \times \frac{m0^2\, c^2}{\hbar^2}\, l^2}}{3 + 16 \times \frac{m0^2\, c^2}{\hbar^2}\, l^2}$$

In[◦]:= $v[l\_] := \frac{c}{\sqrt{1 + \frac{16}{3} \times \frac{m0^2\, c^2}{\hbar^2}\, l^2}}$

In[◦]:= $\delta l[t\_] := q\, t$

In[◦]:= $\Delta L := lmax - lmin$

In[◦]:= $\Delta L2 := lmax2 - lmin$

In[◦]:= $d := \frac{\Delta L}{50}$

In[◦]:= $d2 := \frac{\Delta L2}{50}$

MMAX:=104

In[◦]:= $\delta := 1.54803 \times 10^{-24}$

In[◦]:= $\delta 2 := 1.54803 \times 10^{-24}$

In[◦]:= $\delta = \frac{1}{2} \times \frac{1}{104} \times \frac{lmin + \Delta L}{c} \times \frac{\sqrt{3 + 16 \times \frac{m0^2\, c^2}{\hbar^2}\, (lmin + \Delta L)^2}}{3 + 16 \times \frac{m0^2\, c^2}{\hbar^2}\, (lmin + \Delta L)^2}$

In[◦]:= $\delta 2 = \frac{1}{2} \times \frac{1}{104} \times \frac{lmin + \Delta L2}{c} \times \frac{\sqrt{3 + 16 \times \frac{m0^2\, c^2}{\hbar^2}\, (lmin + \Delta L2)^2}}{3 + 16 \times \frac{m0^2\, c^2}{\hbar^2}\, (lmin + \Delta L2)^2}$

In[◦]:= $NMAX = Floor\left[\frac{\Delta L}{d}\right]$
округление вн

Out[◦]=
50

In[◦]:= "NMAX2==NMAX"

In[◦]:= $MMAX = Ceiling\left[\frac{1}{2}\, \frac{1}{\delta} \times \frac{lmin + \Delta L}{c} \times \frac{\sqrt{3 + 16 \times \frac{m0^2\, c^2}{\hbar^2}\, (lmin + \Delta L)^2}}{3 + 16 \times \frac{m0^2\, c^2}{\hbar^2}\, (lmin + \Delta L)^2}\right]$
округление вверх

Out[◦]=
104

"MMAX2==MMAX"

In[◦]:= MMAX δ;

In[◦]:= n0 := 0

In[◦]:= $MMAXN0 = \frac{1}{2}\, \frac{1}{\delta} \times \frac{lmin}{c} \times \frac{\sqrt{3 + 16 \times \frac{m0^2\, c^2}{\hbar^2}\, lmin^2}}{3 + 16 \times \frac{m0^2\, c^2}{\hbar^2}\, lmin^2}$

Out[◦]=
1.00531

In[*]:=
```
m00 = Floor[MMAXN0]
      округление вниз
```
Out[*]=

1

In[*]:=

$$\text{Reduce}\left[r_{min} \geq \frac{1}{2}\,\text{Abs}[l + q\,\Delta t[l]],\ l,\ \text{Reals}\right]$$

привести / абсолютное значение / множество

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[*]=

$$-2.\times 10^{-11} \leq l \leq 2.\times 10^{-11}$$

$$\text{Reduce}\left[r_{min} \geq \frac{1}{2}\,\text{Abs}\left[l - \frac{l}{1 + \frac{r_{min}}{v[l]\,\frac{1}{2}\,\Delta t[l]}} + \frac{1}{2}\,q\,\Delta t[l]\right],\ l,\ \text{Reals}\right]$$

привести / абсолютное значение / множество

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[*]=

$$-1.99992\times 10^{-11} \leq l \leq 2.00008\times 10^{-11}$$

In[*]:=

$$\text{Reduce}\left[r_{max} \geq \frac{1}{2}\,\text{Abs}[l + q\,\Delta t[l]],\ l,\ \text{Reals}\right]$$

привести / абсолютное значение / множество

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[*]=

$$-5.\times 10^{-11} \leq l \leq 5.\times 10^{-11}$$

$$\text{Reduce}\left[r_{max} \geq \frac{1}{2}\,\text{Abs}\left[l - \frac{l}{1 + \frac{r_{max}}{v[l]\,\frac{1}{2}\,\Delta t[l]}} + \frac{1}{2}\,q\,\Delta t[l]\right],\ l,\ \text{Reals}\right]$$

привести / абсолютное значение / множество

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[*]=

$$-4.99997\times 10^{-11} \leq l \leq 5.00003\times 10^{-11}$$

In[*]:=
```
Do[If[i == 0, Print[1];
⋯   условный⋯ печатать
   j0 = 1;, Null];
            пустой
  If[i ≥ 1 && i ≤ 10, Print[928 i];
  условный оператор   печатать
   ji = 928 i;, Null], {i, 0, 10}]
                 пустой
```

1

928

1856

2784

3712

4640

5568

6496

7424

8352

9280

```
In[•]:= Table[r1_{0,n,m} = r_min;, {n, 0, NMAX}, {m, 0, MMAX}];
        таблица значений

In[•]:= Table[r_{0,n,m} = r_min;, {n, 0, NMAX}, {m, 0, MMAX}];
        таблица значений

In[•]:= For[a = 0, r_{a,n0,m00} ≤ r_max, a++, If[a == 0, Print["a=", 0, " ", "r_a=", N[r_min]], Null];
        цикл ДЛЯ                                  условный… печатать                      численно… пустой
          Do[l_n = lmin + n d;
          оператор цикла
           Do[t_m = m δ;
           оператор цикла
             r1_{a+1,n,m} = 1/(4 (r1_{a,n,m} + t_m v[l_n]))
               (l_n (2 r1_{a,n,m} + t_m v[l_n]) + 2 (r1_{a,n,m} + t_m v[l_n]) (r1_{a,n,m} - t_m v[l_n] + δl[t_m]) + √(8 t_m
                    v[l_n] (r1_{a,n,m} + t_m v[l_n]) (l_n r1_{a,n,m} + 2 (r1_{a,n,m} + t_m v[l_n]) (r1_{a,n,m} + δl[t_m])) +
                  (l_n (2 r1_{a,n,m} + t_m v[l_n]) + 2 (r1_{a,n,m} + t_m v[l_n]) (r1_{a,n,m} - t_m v[l_n] + δl[t_m]))^2));
             r_{a+1,n,m} = r1_{a+1,n,m};
             r1_{a,n,m} =.;
             t_m =., {m, 0, MMAX}];
           l_n =., {n, 0, NMAX}];
          If[a > 0 && (a == j_0 || a == j_1 || a == j_2 || a == j_3 || a == j_4 || a == j_5 || a == j_6 || a == j_7 || a ==
          условный оператор
               j_8 || a == j_9 || a == j_10), Print["a=", a, " ", "r_{a,n0,m00}=", r_{a,n0,m00}], Null]] // Timing
                                              печатать                                      пустой      затраченно
```

$a=0\ r_a=1.\times10^{-11}$

$a=1\ r_{a,n0,m00}=1.00016\times10^{-11}$

$a=928\ r_{a,n0,m00}=1.14998\times10^{-11}$

$a=1856\ r_{a,n0,m00}=1.29996\times10^{-11}$

$a=2784\ r_{a,n0,m00}=1.44995\times10^{-11}$

$a=3712\ r_{a,n0,m00}=1.59993\times10^{-11}$

$a=4640\ r_{a,n0,m00}=1.74992\times10^{-11}$

$a=5568\ r_{a,n0,m00}=1.8999\times10^{-11}$

$a=6496\ r_{a,n0,m00}=2.04989\times10^{-11}$

$a=7424\ r_{a,n0,m00}=2.19987\times10^{-11}$

$a=8352\ r_{a,n0,m00}=2.34986\times10^{-11}$

$a=9280\ r_{a,n0,m00}=2.49984\times10^{-11}$

Out[∘]=

{5042.39, Null}

In[∘]:=
```
s = a - 1
```

Out[∘]=

9280

In[∘]:=
```
r9280,n0,m00
```

Out[∘]=

2.49984×10^{-11}

In[∘]:=
```
Table[Rk1 = Interpolation[
  Flatten[Table[{{lmin + n1 d, m1 δ}, rk1,n1,m1}, {n1, 0, NMAX}, {m1, 0, MMAX}], 1],
  InterpolationOrder → 5];, {k1, 1, s}];
```
таблица зн··· интерполировать уплостить таблица значений порядок интерполяции

```
Rk1[l, t]
```

In[∘]:=
```
Do[Print[10 i]; fi = 10 i;, {i, 1, 5}]
```
··· печатать

10

20

30

40

50

In[∘]:=
```
Do[la1 = lmin + a1 d;
 If[a1 == 0 || a1 == 1, Print["a1=", a1, " ", "la1=", la1], If[a1 == f1 || a1 == f2 ||
    a1 == f3 || a1 == f4 || a1 == f5, Print["a1=", a1, " ", "la1=", la1], Null], Null];
 Do[tb1 = b1 δ;
```
оператор цикла условный оператор печатать условный оператор печатать пустой пустой оператор цикла

```
If[(b1 == 0 || b1 == 1 || b1 == MMAX) && (a1 == 0 || a1 == 1 || a1 == f1 || a1 == f2 ||
```
условный оператор

```
    a1 == f3 || a1 == f4 || a1 == f5), Print["b1=", b1, " ", "tb1=", tb1], Null];
```
печатать пустой

$$\mathrm{IdJ1}_{a1,b1} = \left(\frac{1}{2}\left(\frac{\pi}{\mathrm{ArcTan}\left[\frac{l_{a1}-\frac{l_{a1}\,t_{b1}}{t_{b1}+\frac{r_{min}}{v[l_{a1}]}}+\delta l[t_{b1}]}{2\sqrt{r_{min}^2-\frac{1}{4}\left(l_{a1}-\frac{l_{a1}\,t_{b1}}{t_{b1}+\frac{r_{min}}{v[l_{a1}]}}+\delta l[t_{b1}]\right)^2}}\right]}\right)\right)^2 - \frac{\pi}{\mathrm{ArcTan}\left[\frac{l_{a1}-\frac{l_{a1}\,t_{b1}}{t_{b1}+\frac{r_{min}}{v[l_{a1}]}}+\delta l[t_{b1}]}{2\sqrt{r_{min}^2-\frac{1}{4}\left(l_{a1}-\frac{l_{a1}\,t_{b1}}{t_{b1}+\frac{r_{min}}{v[l_{a1}]}}+\delta l[t_{b1}]\right)^2}}\right]} +$$

$$2\left(\sqrt{\frac{\hbar^2\,c^2}{16}\times\frac{3}{\left(l_{a1}-\frac{l_{a1}\,t_{b1}}{t_{b1}+\frac{r_{min}}{v[l_{a1}]}}+\delta l[t_{b1}]\right)^2}+m0^2\,c^4}\right) +$$

$$\sum_{k2=1}^{s}\left(\left(\frac{1}{2}\left(\frac{\pi}{\mathrm{ArcTan}\left[\frac{l_{a1}-\frac{l_{a1}\,t_{b1}}{t_{b1}+\frac{R_{k2}[l_{a1},t_{b1}]}{v[l_{a1}]}}+\delta l[t_{b1}]}{2\sqrt{(R_{k2}[l_{a1},t_{b1}])^2-\frac{1}{4}\left(l_{a1}-\frac{l_{a1}\,t_{b1}}{t_{b1}+\frac{R_{k2}[l_{a1},t_{b1}]}{v[l_{a1}]}}+\delta l[t_{b1}]\right)^2}}\right]}\right)\right)^2 -\right.$$

$$\left.\frac{\pi}{\mathrm{ArcTan}\left[\frac{l_{a1}-\frac{l_{a1}\,t_{b1}}{t_{b1}+\frac{R_{k2}[l_{a1},t_{b1}]}{v[l_{a1}]}}+\delta l[t_{b1}]}{2\sqrt{(R_{k2}[l_{a1},t_{b1}])^2-\frac{1}{4}\left(l_{a1}-\frac{l_{a1}\,t_{b1}}{t_{b1}+\frac{R_{k2}[l_{a1},t_{b1}]}{v[l_{a1}]}}+\delta l[t_{b1}]\right)^2}}\right]} + 2\right)$$

$$\left.\sqrt{\frac{\hbar^2\,c^2}{16}\times\frac{3}{\left(l_{a1}-\frac{l_{a1}\,t_{b1}}{t_{b1}+\frac{R_{k2}[l_{a1},t_{b1}]}{v[l_{a1}]}}+\delta l[t_{b1}]\right)^2}+m0^2\,c^4}\right);$$

```
tb1 =., {b1, 0, MMAX}];
], {a1, 0, NMAX}]
```

a1=0 $l_{a1}=1.61626\times10^{-15}$

b1=0 $t_{b1}=0.$

b1=1 $t_{b1}=1.54803\times10^{-24}$

b1=104 $t_{b1}=1.60995\times10^{-22}$

a1=1 $l_{a1}=4.016\times10^{-13}$

b1=0 $t_{b1}=0.$

b1=1 $t_{b1}=1.54803\times10^{-24}$

b1=104 $t_{b1}=1.60995\times10^{-22}$

a1=10 $l_{a1}=4.00145\times10^{-12}$

b1=0 $t_{b1}=0.$

b1=1 $t_{b1}=1.54803\times10^{-24}$

b1=104 $t_{b1}=1.60995\times10^{-22}$

a1=20 $l_{a1}=8.00129\times10^{-12}$

b1=0 $t_{b1}=0.$

b1=1 $t_{b1}=1.54803\times10^{-24}$

b1=104 $t_{b1}=1.60995\times10^{-22}$

a1=30 $l_{a1}=1.20011\times10^{-11}$

b1=0 $t_{b1}=0.$

b1=1 $t_{b1}=1.54803\times10^{-24}$

b1=104 $t_{b1}=1.60995\times10^{-22}$

a1=40 $l_{a1}=1.6001\times10^{-11}$

b1=0 $t_{b1}=0.$

b1=1 $t_{b1}=1.54803\times10^{-24}$

b1=104 $t_{b1}=1.60995\times10^{-22}$

a1=50 $l_{a1}=2.00008\times10^{-11}$

b1=0 $t_{b1}=0.$

b1=1 $t_{b1}=1.54803\times10^{-24}$

b1=104 $t_{b1}=1.60995\times10^{-22}$

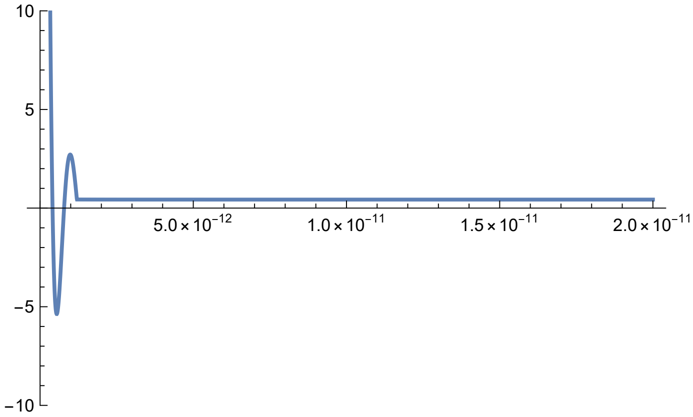
In[ ]:=
```
IdJ11 = Interpolation[
        интерполировать
  Flatten[Table[{{lmin + a11 d, b11 δ}, IdJ1_{a11,b11}}, {a11, 0, NMAX}, {b11, 0, MMAX}], 1],
  уплостить  таблица значений
  InterpolationOrder → 5];
  порядок интерполяции

IdJ11[l, t]
```

In[ ]:=
```
t := 4 δ
```

In[∘]:= Plot[IdJ11[l, t], {l, lmin, lmax}, PlotRange → {-10, 10}]
график функции | отображаемый диапазон граф

Out[∘]=



From here we can see that this computation is correct.

In[∘]:= t =.

In[∘]:= $T[l\_] := \frac{1}{2}\Delta t[l]$

"t>0"

In[∘]:= $\Delta l_{min} := \frac{l\,t}{t + \frac{r_{min}}{v[l]}}$

In[∘]:= $\Delta lm_{min} := \frac{l\left(\frac{1}{2}\Delta t[l] - t\right)}{\frac{1}{2}\Delta t[l] - t + \frac{r_{min}}{v[l]}}$

In[∘]:= $N1_{min} := \frac{\pi}{\text{ArcTan}\left[\frac{l-\Delta l_{min}+\delta l[t]}{2\sqrt{r_{min}^2-\frac{1}{4}(l-\Delta l_{min}+\delta l[t])^2}}\right]}$

In[∘]:= $N1m_{min} := \frac{\pi}{\text{ArcTan}\left[\frac{l-\Delta lm_{min}+\delta l\left[\frac{1}{2}\Delta t[l]+t\right]}{2\sqrt{r_{min}^2-\frac{1}{4}\left(l-\Delta lm_{min}+\delta l\left[\frac{1}{2}\Delta t[l]+t\right]\right)^2}}\right]}$

In[∘]:= l = lmax;

In[∘]:= $t = \frac{1}{2}\Delta t[l];$

In[∘]:= $N1_{min}$

Out[∘]=

2.00106

In[∘]:= l =.

In[∘]:= l = lmin;

In[∘]:= $N1_{min}$

Out[∘]=

39 062.6

In[∘]:= l =.

In[∘]:= t =.

In[∘]:= $l = 1.99999\times10^{-11};$

In[∘]:= $t = \frac{1}{2}\Delta t[l];$

In[∘]:= $N1m_{min}$

Out[∘]=

2.00403

It must be equal to 2.

In[∘]:= l =.

In[∘]:= l = lmin;

In[∘]:= $N1m_{min}$

Out[∘]=

38 689.2

In[∘]:= l =.

In[∘]:= t =.

The results are different due to the dispersion.

In[∘]:= $\text{Reduce}\left[r_{min}^2 - \frac{1}{4}\left(l - \Delta lm_{min} + \delta l\left[\frac{1}{2}\Delta t[l] + t\right]\right)^2 \geq 0, l\right]$
привести

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[∘]=

$-2.\times10^{-11} \leq l \leq 2.\times10^{-11}$

The extremal upper value is to be when the equality to zero is executed.

In[∘]:=
$$\text{NSolve}\left[r_{min}^2 - \frac{1}{4}\left(l - \Delta lm_{min} + \delta l\left[\frac{1}{2}\,\Delta t[l] + t\right]\right)^2 == 0,\ l,\ \text{Reals}\right]$$
численное решение уравнений | множест

Out[∘]=
$$\{\{l \to -2.10199\times10^{-11}\},\ \{l \to 2.10199\times10^{-11}\}\}$$

This result is abnormal.

In[∘]:=
$$l = 1.99999\times10^{-11};$$

In[∘]:=
$$\text{Simplify}\left[r_{min}^2 - \frac{1}{4}\left(l - \Delta lm_{min} + \delta l\left[\frac{1}{2}\,\Delta t[l] + t\right]\right)^2 \geq 0\right]$$
упростить

Out[∘]=
True

In[∘]:=
```
Do[l_a2 = lmin + a2 d;   (оператор цикла)
 If[a2 == 0 || a2 == 1, Print["a2=", a2, " ", "l_a2=", l_a2], If[a2 == f_1 || a2 == f_2 ||
 (условный оператор / печатать / условный оператор)
   a2 == f_3 || a2 == f_4 || a2 == f_5, Print["a2=", a2, " ", "l_a2=", l_a2], Null]];
   (печатать / пустой)
 Do[τ_b2 = b2 δ;   (оператор цикла)
  If[(a2 == 0 || a2 == 1) && (b2 == 0 || b2 == 1 || b2 == MMAX), Print["b2=", b2,
  (условный оператор / печатать)
   " ", "τ_b2=", τ_b2], If[(a2 == f_1 || a2 == f_2 || a2 == f_3 || a2 == f_4 || a2 == f_5) &&
   (условный оператор)
    (b2 == 0 || b2 == 1 || b2 == MMAX), Print["b2=", b2, " ", "τ_b2=", τ_b2], Null]];
    (печатать / пустой)
  J1_{a2,b2} = 1/(T[l_a2] - τ_b2) 1/T[l_a2] NIntegrate[IdJ11[l_a2, t], {t, τ_b2, T[l_a2]}];
  (квадратурное интегрирование)

  τ_b2 =., {b2, 0, MMAX}];
 l_a2 =., {a2, 0, NMAX}]
```

a2=0 l_{a2}=1.61626×10^{-15}

b2=0 τ_{b2}=0.

b2=1 τ_{b2}=1.54803×10^{-24}

b2=104 τ_{b2}=1.60995×10^{-22}

a2=1 l_{a2}=4.016×10^{-13}

b2=0 τ_{b2}=0.

b2=1 τ_{b2}=1.54803×10^{-24}

b2=104 τ_{b2}=1.60995×10^{-22}

a2=10 l_{a2}=4.00145×10^{-12}

b2=0 τ_{b2}=0.

b2=1 τ_{b2}=1.54803×10^{-24}

b2=104 τ_{b2}=1.60995×10^{-22}

a2=20 l_{a2}=8.00129×10^{-12}

b2=0 τ_{b2}=0.

b2=1 τ_{b2}=1.54803×10^{-24}

b2=104 τ_{b2}=1.60995×10^{-22}

a2=30 l_{a2}=1.20011×10^{-11}

b2=0 τ_{b2}=0.

b2=1 τ_{b2}=1.54803×10^{-24}

b2=104 τ_{b2}=1.60995×10^{-22}

a2=40 l_{a2}=1.6001×10^{-11}

b2=0 τ_{b2}=0.

b2=1 τ_{b2}=1.54803×10^{-24}

b2=104 τ_{b2}=1.60995×10^{-22}

a2=50 l_{a2}=2.00008×10^{-11}

b2=0 τ_{b2}=0.

b2=1 τ_{b2}=1.54803×10^{-24}

b2=104 τ_{b2}=1.60995×10^{-22}

In[∘]:= **δ**

Out[∘]=

1.54803×10^{-24}

In[∘]:= **T[lmin]**

Out[∘]=

1.55625×10^{-24}

```
In[∘]:= J11 = Interpolation[
            интерполировать
    Flatten[Table[{{lmin + a22 d, b22 δ}, J1a22,b22}, {a22, 0, NMAX}, {b22, 0, MMAX}], 1],
    уплостить  таблица значений
    InterpolationOrder → 5];
    порядок интерполяции
```

```
J11[l, τ]
```

In[∘]:=
```
Do[If[(a == f1 || a == f2 || a == f3 || a == f4 || a == f5), Print["a=", a], Null];
  la = lmin + a d;
  Int1a = NIntegrate[J11[la, τ], {τ, 0, T[la] - ϵ}];
  la =., {a, 0, NMAX}]
```
⋯ условный оператор   печатать   пустой

квадратурное интегрирование

a=10

a=20

a=30

a=40

a=50

In[∘]:=
```
F1 = Interpolation[Table[{lmin + a3 d, Int1a3}, {a3, 0, NMAX}], InterpolationOrder → 5];
```
интерполировать   таблица значений   порядок интерполяции

In[∘]:=
```
I1 = NIntegrate[F1[l], {l, lmin, lmax}]
```
квадратурное интегрирование

Out[∘]=

8.64509×10^{-12}

In[∘]:= $N\left[\frac{1}{lP} \text{Na I1}\right]$

численное приближение

Out[∘]=

3.95279×10^{103}

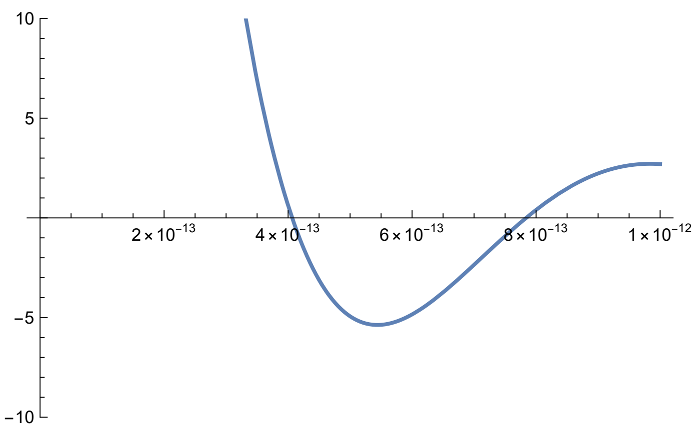
A new interesting structure is found, it must be investigated.

In[∘]:=
```
t := 4 δ
```

In[∘]:= Plot$\left[\text{IdJ11}[l, t], \{l, \text{lmin}, 10^{-12}\}, \text{PlotRange} \to \{-10, 10\}\right]$

график функции   отображаемый диапазон граф

Out[∘]=



Following from the plot, we must write

In[∘]:= FindRoot$\left[\text{IdJ11}[l, t] == 0, \{l, 4 \times 10^{-13}\}\right]$

найти корень

Out[∘]=

$\{l \to 4.06607 \times 10^{-13} + 0.\, i\}$

In[∘]:=
```
FindRoot[IdJ11[l, t] == 0, {l, 8 × 10^-13}]
```
найти корень

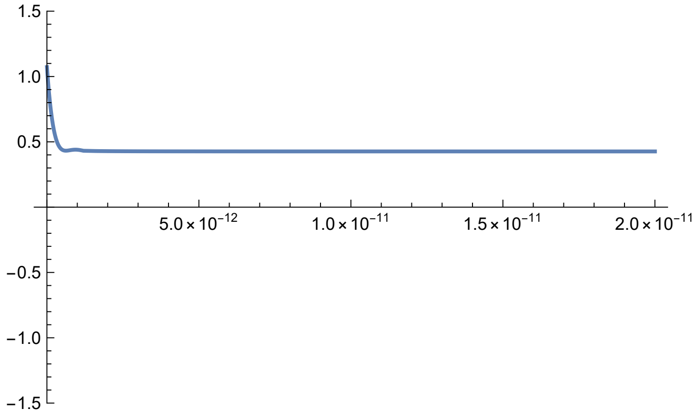
Out[∘]=

$\{l \to 7.83835 \times 10^{-13} + 0.\, i\}$

In[∘]:=
```
t =.
```

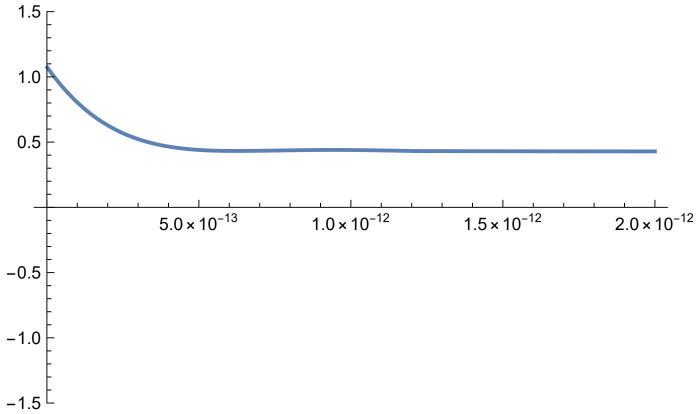
In[∘]:=
```
Plot[F1[l], {l, lmin, lmax}, PlotRange → {-1.5, 1.5}]
```
график функции   отображаемый диапазон графика

Out[∘]=



In[∘]:=
```
Plot[F1[l], {l, lmin, 2 × 10^-12}, PlotRange → {-1.5, 1.5}]
```
график функции   отображаемый диапазон графика

Out[∘]=



In[∘]:=
```
u0 := n0
```

In[∘]:=
```
w00 := m00
```

In[∘]:=
```
Table[r1m0,u,w = rmin;, {u, 0, NMAX}, {w, 0, MMAX}];
```
таблица значений

In[∘]:=
```
Table[rm0,u,w = rmin;, {u, 0, NMAX}, {w, 0, MMAX}];
```
таблица значений

```
In[*]:= For[a4 = 0, rm_{a4,u0,w00} ≤ r_max, a4++,
  If[a4 == 0, Print["a4=", 0, " ", "rm_{a4}=", N[r_min]], Null];
  Do[lm_u = lmin + u d2;
   Do[tm_w = -w δ2;
    r1m_{1+a4,u,w} = 1/(8 r1m_{a4,u,w} + 4 v[lm_u] (2 tm_w + Δt[lm_u]))
      (4 r1m_{a4,u,w}^2 + 4 r1m_{a4,u,w} (lm_u + δl[-tm_w + Δt[lm_u]/2]) + v[lm_u] (2 tm_w + Δt[lm_u]) (lm_u +
          2 δl[-tm_w + Δt[lm_u]/2] - v[lm_u] (2 tm_w + Δt[lm_u])) + Sqrt[(8 v[lm_u] (2 tm_w + Δt[lm_u])
           (2 r1m_{a4,u,w} + v[lm_u] (2 tm_w + Δt[lm_u])) (lm_u r1m_{a4,u,w} + (r1m_{a4,u,w} + δl[-tm_w +
                Δt[lm_u]/2]) (2 r1m_{a4,u,w} + v[lm_u] (2 tm_w + Δt[lm_u]))) + ((2 r1m_{a4,u,w} +
              2 δl[-tm_w + Δt[lm_u]/2] - v[lm_u] (2 tm_w + Δt[lm_u])) (2 r1m_{a4,u,w} + v[lm_u]
               (2 tm_w + Δt[lm_u])) + lm_u (4 r1m_{a4,u,w} + v[lm_u] (2 tm_w + Δt[lm_u])))^2)]);
    rm_{a4+1,u,w} = r1m_{a4+1,u,w};
    r1m_{a4,u,w} =.;
    tm_w =., {w, 0, MMAX}];
   lm_u =., {u, 0, NMAX}];
  If[a4 > 0 && (a4 == j_0 || a4 == j_1 || a4 == j_2 || a4 == j_3 ||
      a4 == j_4 || a4 == j_5 || a4 == j_6 || a4 == j_7 || a4 == j_8 || a4 == j_9 || a4 == j_10),
   Print["a4=", a4, " ", "rm_{a4,u0,w00}=", rm_{a4,u0,w00}], Null]] // Timing
```

a4=0 rm_{a4}=1. × 10^{-11}

a4=1 $rm_{a4,u0,w00}$=1.00016 × 10^{-11}

a4=928 $rm_{a4,u0,w00}$=1.14999 × 10^{-11}

a4=1856 $rm_{a4,u0,w00}$=1.29998 × 10^{-11}

a4=2784 $rm_{a4,u0,w00}$=1.44997 × 10^{-11}

a4=3712 $rm_{a4,u0,w00}$=1.59995 × 10^{-11}

a4=4640 $rm_{a4,u0,w00}$=1.74994 × 10^{-11}

a4=5568 $rm_{a4,u0,w00}$=1.89993 × 10^{-11}

a4=6496 $rm_{a4,u0,w00}$=2.04992 × 10^{-11}

a4=7424 $rm_{a4,u0,w00}$=2.19991 × 10^{-11}

a4=8352 $rm_{a4,u0,w00}$=2.3499 × 10^{-11}

a4=9280 $rm_{a4,u0,w00}$=2.49988 × 10^{-11}

Out[•]=

{9193.53, Null}

In[•]:= **sm = a4 - 1**

Out[•]=

9280

In[•]:= **$rm_{9280,u0,w00}$**

Out[•]=

2.49988 × 10^{-11}

```
In[•]:= Table[
        таблица значений
          Rm_k3 = Interpolation[Flatten[Table[{{lmin + u1 d2, -w1 δ2}, rm_{k3,u1,w1}}, {u1, 0, NMAX},
                  интерполировать  уплостить таблица значений
                {w1, 0, MMAX}], 1], InterpolationOrder → 5];, {k3, 1, sm}];
                                    порядок интерполяции

        Rm_k3[l, t]

In[•]:= Do[l_a5 = lmin + a5 d2;
        оператор цикла
          If[a5 == 0 || a5 == 1, Print["a5=", a5, " ", "l_a5=", l_a5], If[a5 == f_1 || a5 == f_2 ||
          условный оператор      печатать                              условный оператор
             a5 == f_3 || a5 == f_4 || a5 == f_5, Print["a5=", a5, " ", "l_a5=", l_a5], Null], Null];
                                                  печатать                                 пустой   пустой
          Do[t_b5 = -b5 δ2;
          оператор цикла
            If[(b5 == 0 || b5 == 1 || b5 == MMAX) && (a5 == 0 || a5 == 1 || a5 == f_1 || a5 == f_2 ||
            условный оператор
                a5 == f_3 || a5 == f_4 || a5 == f_5), Print["b5=", b5, " ", "t_b5=", t_b5], Null];
                                                      печатать                                пустой
```

$$\mathrm{IdJ2}_{a5,b5} = \left(\frac{1}{2}\left(\frac{\pi}{\mathrm{ArcTan}\left[\frac{l_{a5}-\frac{l_{a5}\left(\frac{1}{2}\Delta t[l_{a5}]+t_{b5}\right)}{\frac{1}{2}\Delta t[l_{a5}]+t_{b5}+\frac{r_{min}}{v[l_{a5}]}}+\delta l\left[\frac{1}{2}\Delta t[l_{a5}]-t_{b5}\right]}{2\sqrt{r_{min}^{2}-\frac{1}{4}\left(l_{a5}-\frac{l_{a5}\left(\frac{1}{2}\Delta t[l_{a5}]+t_{b5}\right)}{\frac{1}{2}\Delta t[l_{a5}]+t_{b5}+\frac{r_{min}}{v[l_{a5}]}}+\delta l\left[\frac{1}{2}\Delta t[l_{a5}]-t_{b5}\right]\right)^{2}}}\right]}\right.\right.^{2} -$$

$$\left.\left.\frac{\pi}{\mathrm{ArcTan}\left[\frac{l_{a5}-\frac{l_{a5}\left(\frac{1}{2}\Delta t[l_{a5}]+t_{b5}\right)}{\frac{1}{2}\Delta t[l_{a5}]+t_{b5}+\frac{r_{min}}{v[l_{a5}]}}+\delta l\left[\frac{1}{2}\Delta t[l_{a5}]-t_{b5}\right]}{2\sqrt{r_{min}^{2}-\frac{1}{4}\left(l_{a5}-\frac{l_{a5}\left(\frac{1}{2}\Delta t[l_{a5}]+t_{b5}\right)}{\frac{1}{2}\Delta t[l_{a5}]+t_{b5}+\frac{r_{min}}{v[l_{a5}]}}+\delta l\left[\frac{1}{2}\Delta t[l_{a5}]-t_{b5}\right]\right)^{2}}}\right]}+2\right)\right)$$

$$\sqrt{\frac{\hbar^{2}c^{2}}{16}\times\frac{3}{\left(l_{a5}-\frac{l_{a5}\left(\frac{1}{2}\Delta t[l_{a5}]+t_{b5}\right)}{\frac{1}{2}\Delta t[l_{a5}]+t_{b5}+\frac{r_{min}}{v[l_{a5}]}}+\delta l\left[\frac{1}{2}\Delta t[l_{a5}]-t_{b5}\right]\right)^{2}}+m\theta^{2}c^{4}} +$$

$$\sum_{k4=1}^{sm}\left(\left(\left(\frac{1}{2}\left(\frac{\pi}{\mathrm{ArcTan}\left[\frac{l_{a5}-\frac{l_{a5}\left(\frac{1}{2}\Delta t[l_{a5}]+t_{b5}\right)}{\frac{1}{2}\Delta t[l_{a5}]+t_{b5}+\frac{Rm_{k4}[l_{a5},t_{b5}]}{v[l_{a5}]}}+\delta l\left[\frac{1}{2}\Delta t[l_{a5}]-t_{b5}\right]}{2\sqrt{(Rm_{k4}[l_{a5},t_{b5}])^{2}-\frac{1}{4}\left(l_{a5}-\frac{l_{a5}\left(\frac{1}{2}\Delta t[l_{a5}]+t_{b5}\right)}{\frac{1}{2}\Delta t[l_{a5}]+t_{b5}+\frac{Rm_{k4}[l_{a5},t_{b5}]}{v[l_{a5}]}}+\delta l\left[\frac{1}{2}\Delta t[l_{a5}]-t_{b5}\right]\right)^{2}}}\right]}\right.\right.\right.\right.^{2} -$$

$$\left.\left.\frac{\pi}{\mathrm{ArcTan}\left[\frac{l_{a5}-\frac{l_{a5}\left(\frac{1}{2}\Delta t[l_{a5}]+t_{b5}\right)}{\frac{1}{2}\Delta t[l_{a5}]+t_{b5}+\frac{Rm_{k4}[l_{a5},t_{b5}]}{v[l_{a5}]}}+\delta l\left[\frac{1}{2}\Delta t[l_{a5}]-t_{b5}\right]}{2\sqrt{(Rm_{k4}[l_{a5},t_{b5}])^{2}-\frac{1}{4}\left(l_{a5}-\frac{l_{a5}\left(\frac{1}{2}\Delta t[l_{a5}]+t_{b5}\right)}{\frac{1}{2}\Delta t[l_{a5}]+t_{b5}+\frac{Rm_{k4}[l_{a5},t_{b5}]}{v[l_{a5}]}}+\delta l\left[\frac{1}{2}\Delta t[l_{a5}]-t_{b5}\right]\right)^{2}}}\right]}+2\right)\right)$$

$$\left.\sqrt{\frac{\hbar^2 c^2}{16} \times \frac{3}{\left(l_{a5} - \frac{l_{a5}\left(\frac{1}{2}\Delta t[l_{a5}] + t_{b5}\right)}{\frac{1}{2}\Delta t[l_{a5}] + t_{b5} + \frac{Rm_{k4}[l_{a5}, t_{b5}]}{v[l_{a5}]}} + \delta l\left[\frac{1}{2}\Delta t[l_{a5}] - t_{b5}\right]\right)^2} + m0^2 c^4}\right);$$

t_{b5} =., {b5, 0, MMAX}];

l_{a5} =., {a5, 0, NMAX}]

a5=0 l_{a5}=1.61626×10^{-15}

b5=0 t_{b5}=0.

b5=1 t_{b5}=-1.54803×10^{-24}

b5=104 t_{b5}=-1.60995×10^{-22}

a5=1 l_{a5}=4.01584×10^{-13}

b5=0 t_{b5}=0.

b5=1 t_{b5}=-1.54803×10^{-24}

b5=104 t_{b5}=-1.60995×10^{-22}

a5=10 l_{a5}=4.00129×10^{-12}

b5=0 t_{b5}=0.

b5=1 t_{b5}=-1.54803×10^{-24}

b5=104 t_{b5}=-1.60995×10^{-22}

a5=20 l_{a5}=8.00097×10^{-12}

b5=0 t_{b5}=0.

b5=1 t_{b5}=-1.54803×10^{-24}

b5=104 t_{b5}=-1.60995×10^{-22}

a5=30 l_{a5}=1.20006×10^{-11}

b5=0 t_{b5}=0.

b5=1 t_{b5}=-1.54803×10^{-24}

b5=104 t_{b5}=-1.60995×10^{-22}

a5=40 l_{a5}=1.60003×10^{-11}

b5=0 t_{b5}=0.

b5=1 t_{b5}=-1.54803×10^{-24}

b5=104 t_{b5}=-1.60995×10^{-22}

a5=50 l_{a5}=$2. \times 10^{-11}$

b5=0 t_{b5}=0.

b5=1 t_{b5}=-1.54803×10^{-24}

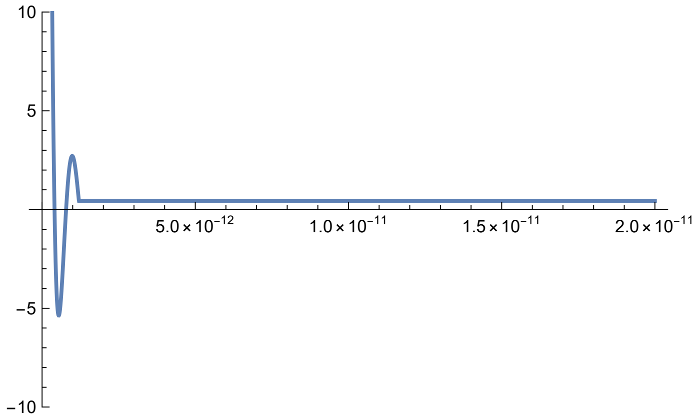
b5=104 t_{b5}=-1.60995×10^{-22}

```
In[ ]:= IdJ22 = Interpolation[Flatten[Table[{{lmin + a55 d2, -b55 δ2}, IdJ2a55,b55},
               интерполировать  уплостить таблица значений
          {a55, 0, NMAX}, {b55, 0, MMAX}], 1], InterpolationOrder → 5];
                                                порядок интерполяции

        IdJ22[l, t]

In[ ]:= t := -4 δ2

In[ ]:= Plot[IdJ22[l, t], {l, lmin, lmax2}, PlotRange → {-10, 10}]
        график функции                          отображаемый диапазон граф
Out[ ]=
```



From here we can see that this computation is correct.

```
In[ ]:= t =.

In[ ]:= Do[la6 = lmin + a6 d2;
        оператор цикла
         If[a6 == 0 || a6 == 1, Print["a6=", a6, " ", "la6=", la6], If[a6 == f1 || a6 == f2 ||
         условный оператор       печатать                             условный оператор
            a6 == f3 || a6 == f4 || a6 == f5, Print["a6=", a6, " ", "la6=", la6], Null]];
                                              печатать                              пустой
         Do[τb6 = b6 δ2;
         оператор цикла
          If[(a6 == 0 || a6 == 1) && (b6 == 0 || b6 == 1 || b6 == MMAX), Print["b6=", b6,
          условный оператор                                               печатать
            " ", "τb6=", τb6], If[(a6 == f1 || a6 == f2 || a6 == f3 || a6 == f4 || a6 == f5) &&
                               условный оператор
             (b6 == 0 || b6 == 1 || b6 == MMAX), Print["b6=", b6, " ", "τb6=", τb6], Null]];
                                                 печатать                          пустой
          J2a6,b6 = 1/(T[la6] - τb6) 1/T[la6] NIntegrate[IdJ22[la6, t], {t, T[la6] + τb6, 2 T[la6]}];
                                              квадратурное интегрирование

          τb6 =., {b6, 0, MMAX}];
         la6 =., {a6, 0, NMAX}]
```

```
a6=0 la6=1.61626×10^-15
b6=0 τb6=0.
b6=1 τb6=1.54803×10^-24
b6=104 τb6=1.60995×10^-22
a6=1 la6=4.01584×10^-13
b6=0 τb6=0.
b6=1 τb6=1.54803×10^-24
b6=104 τb6=1.60995×10^-22
a6=10 la6=4.00129×10^-12
b6=0 τb6=0.
b6=1 τb6=1.54803×10^-24
b6=104 τb6=1.60995×10^-22
a6=20 la6=8.00097×10^-12
b6=0 τb6=0.
b6=1 τb6=1.54803×10^-24
b6=104 τb6=1.60995×10^-22
a6=30 la6=1.20006×10^-11
b6=0 τb6=0.
b6=1 τb6=1.54803×10^-24
b6=104 τb6=1.60995×10^-22
a6=40 la6=1.60003×10^-11
b6=0 τb6=0.
b6=1 τb6=1.54803×10^-24
b6=104 τb6=1.60995×10^-22
a6=50 la6=2.×10^-11
b6=0 τb6=0.
b6=1 τb6=1.54803×10^-24
b6=104 τb6=1.60995×10^-22
```

```
In[◦]:= J22 = Interpolation[
              интерполировать
   Flatten[Table[{{lmin + a66 d2, b66 δ2}, J2_{a66,b66}}, {a66, 0, NMAX}, {b66, 0, MMAX}], 1],
   уплостить таблица значений
   InterpolationOrder → 5];
   порядок интерполяции

J22[l, τ]

In[◦]:= Do[If[(am == f1 || am == f2 || am == f3 || am == f4 || am == f5), Print["am=", am], Null];
        условный оператор                                                 печатать       пустой
  l_am = lmin + am d2;
  Int2_am = NIntegrate[J22[l_am, τ], {τ, 0, T[l_am] - ϵ2}];
            квадратурное интегрирование
  l_am =., {am, 0, NMAX}]
```

am=10

am=20

am=30

am=40

am=50

```
In[∘]:= F2 = Interpolation[Table[{lmin + a7 d2, Int2a7}, {a7, 0, NMAX}], InterpolationOrder → 5];
        интерполировать  таблица значений                         порядок интерполяции
```

```
In[∘]:= I2 = NIntegrate[F2[l], {l, lmin, lmax2}]
        квадратурное интегрирование
```

Out[∘]=

8.65711×10^{-12}

```
In[∘]:= N[1/lP Na I2]
        численное приближение
```

Out[∘]=

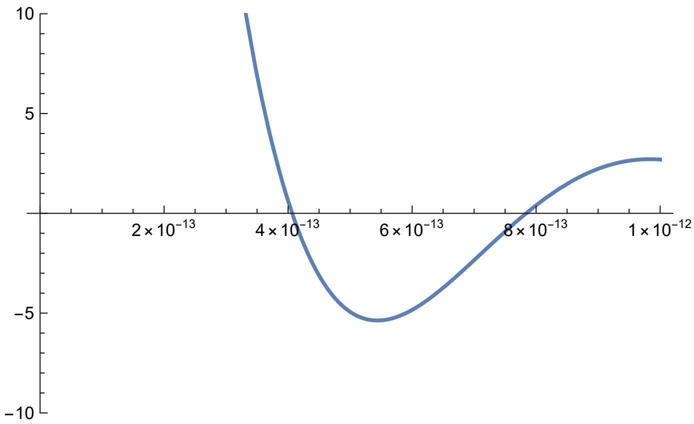
3.95829×10^{103}

A new interesting structure is found, it must be investigated.

```
In[∘]:= t := -4 δ2
```

```
In[∘]:= Plot[IdJ22[l, t], {l, lmin, 10^-12}, PlotRange → {-10, 10}]
        график функции                          отображаемый диапазон граф
```

Out[∘]=



Following from the plot, we must write

```
In[∘]:= FindRoot[IdJ22[l, t] == 0, {l, 4 × 10^-13}]
        найти корень
```

Out[∘]=

$\{l \to 4.06592 \times 10^{-13}\}$

```
In[∘]:= FindRoot[IdJ22[l, t] == 0, {l, 8 × 10^-13}]
        найти корень
```

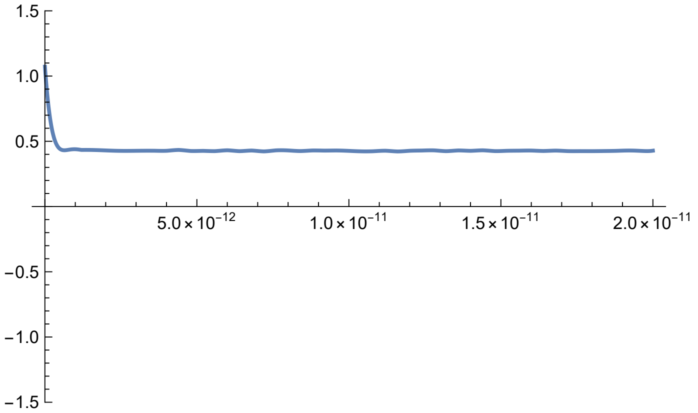
Out[∘]=

$\{l \to 7.83798 \times 10^{-13}\}$

```
In[∘]:= t =.
```

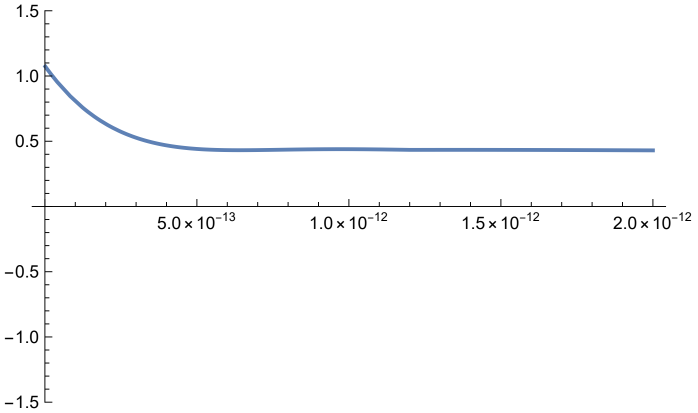
In[∘]:=
```
Plot[F2[l], {l, lmin, lmax2}, PlotRange → {-1.5, 1.5}]
```
график функции отображаемый диапазон графика

Out[∘]=



In[∘]:=
```
Plot[F2[l], {l, lmin, 2 × 10^-12}, PlotRange → {-1.5, 1.5}]
```
график функции отображаемый диапазон графика

Out[∘]=



In[∘]:=
```
I0 = I1 + I2
```

Out[∘]=

1.73022×10^{-11}

In[∘]:=

$N\left[\frac{1}{lP} \Delta_1 \, Na \, I0\right]$

численное приближение

Out[∘]=

5.84629×10^{103}

$N\left[\frac{1}{lP} \Delta_2 \, Na \, I0\right]$

численное приближение

In[∘]:=
```
Tm2 = AbsoluteTime[];
```
абсолютное значение

In[∘]:=
```
ComputationTime = Tm2 - Tm1
```

Out[∘]=

40 279.5239986

The free vacuum energy

```
In[•]:= Vs := VU - Sum[(VAp_i Δ_i Na), {i, 1, 10}]
```

The forward motion

```
In[•]:= Do[l_α1 = lmin + α1 d;
оператор цикла
  If[α1 == 0 || α1 == 1, Print["α1=", α1, " ", "l_α1=", l_α1], If[α1 == f_1 || α1 == f_2 ||
  условный оператор      печатать                                  условный оператор
     α1 == f_3 || α1 == f_4 || α1 == f_5, Print["α1=", α1, " ", "l_α1=", l_α1], Null]];
                                          печатать                             пустой
  Do[τ_β1 = β1 δ;
  оператор цикла
   If[(α1 == 0 || α1 == 1) && (β1 == 0 || β1 == 1 || β1 == MMAX), Print["β1=", β1,
   условный оператор                                                печатать
     " ", "τ_β1=", τ_β1], If[(α1 == f_1 || α1 == f_2 || α1 == f_3 || α1 == f_4 || α1 == f_5) &&
                          условный оператор
      (β1 == 0 || β1 == 1 || β1 == MMAX), Print["β1=", β1, " ", "τ_β1=", τ_β1], Null]];
                                           печатать                           пустой
   J3_{α1,β1} = 1/(T[l_α1] - τ_β1) NIntegrate[Vs/(l_α1 + δl[t])^3 Sqrt[ħ^2 c^2/16 × 3/(l_α1 + δl[t])^2 + m0^2 c^4],
                                   квадратурное интегрирование
      {t, τ_β1, T[l_α1]}] × 1/T[l_α1];
   τ_β1 =., {β1, 0, MMAX}];
  l_α1 =., {α1, 0, NMAX}]
```

α1=0 $l_{\alpha1}$=1.61626×10^{-15}

β1=0 $\tau_{\beta1}$=0.

β1=1 $\tau_{\beta1}$=1.54803×10^{-24}

β1=104 $\tau_{\beta1}$=1.60995×10^{-22}

α1=1 $l_{\alpha1}$=4.016×10^{-13}

β1=0 $\tau_{\beta1}$=0.

β1=1 $\tau_{\beta1}$=1.54803×10^{-24}

β1=104 $\tau_{\beta1}$=1.60995×10^{-22}

α1=10 $l_{\alpha1}$=4.00145×10^{-12}

β1=0 $\tau_{\beta1}$=0.

β1=1 $\tau_{\beta1}$=1.54803×10^{-24}

β1=104 $\tau_{\beta1}$=1.60995×10^{-22}

α1=20 $l_{\alpha1}$=8.00129×10^{-12}

β1=0 $\tau_{\beta1}$=0.

β1=1 $\tau_{\beta1}$=1.54803×10^{-24}

β1=104 $\tau_{\beta1}$=1.60995×10^{-22}

α1=30 $l_{\alpha1}$=1.20011×10^{-11}

β1=0 $\tau_{\beta1}$=0.

β1=1 $\tau_{\beta1}$=1.54803×10^{-24}

β1=104 $\tau_{\beta1}$=1.60995×10^{-22}

α1=40 $l_{\alpha1}$=1.6001×10^{-11}

β1=0 $\tau_{\beta1}$=0.

β1=1 $\tau_{\beta1}$=1.54803×10^{-24}

β1=104 $\tau_{\beta1}$=1.60995×10^{-22}

α1=50 $l_{\alpha1}$=2.00008×10^{-11}

β1=0 $\tau_{\beta1}$=0.

β1=1 $\tau_{\beta1}$=1.54803×10^{-24}

β1=104 $\tau_{\beta1}$=1.60995×10^{-22}

```
In[◦]:= J33 = Interpolation[
           интерполировать
   Flatten[Table[{{lmin + α11 d, β11 δ}, J3[α11,β11]}, {α11, 0, NMAX}, {β11, 0, MMAX}], 1],
   уплостить таблица значений
   InterpolationOrder → 5];
   порядок интерполяции

J33[l, τ]
```

```
In[◦]:= Do[If[(α == f1 || α == f2 || α == f3 || α == f4 || α == f5), Print["α=", α], Null];
        ··· условный оператор                                   печатать        пустой
  l[α] = lmin + α d;
  Int3[α] = NIntegrate[J33[l[α], τ], {τ, 0, T[l[α]] - ϵ}];
            квадратурное интегрирование
  l[α] =., {α, 0, NMAX}]
```

α=10

α=20

α=30

α=40

α=50

In[∘]:=
```
F3 = Interpolation[Table[{lmin + α3 d, Int3α3}, {α3, 0, NMAX}], InterpolationOrder → 5];
```
интерполировать таблица значений порядок интерполяции

In[∘]:=
```
I3 = NIntegrate[F3[l], {l, lmin, lmax}]
```
квадратурное интегрирование

Out[∘]=

5.11251×10^{98}

In[∘]:=
$N\left[\frac{1}{1P} I3\right]$

численное приближение

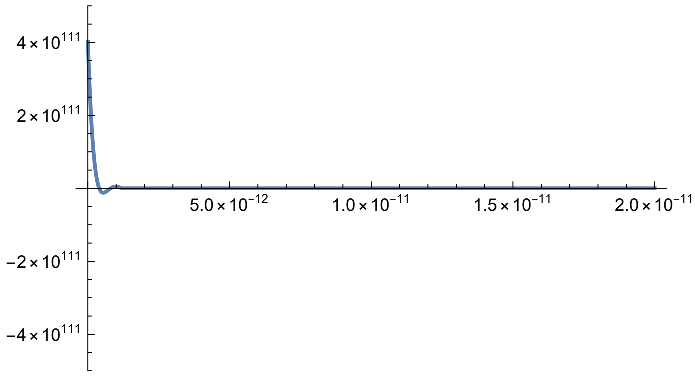
Out[∘]=

3.16318×10^{133}

It is needed to check for a new interesting structure may be found, it must be investigated.

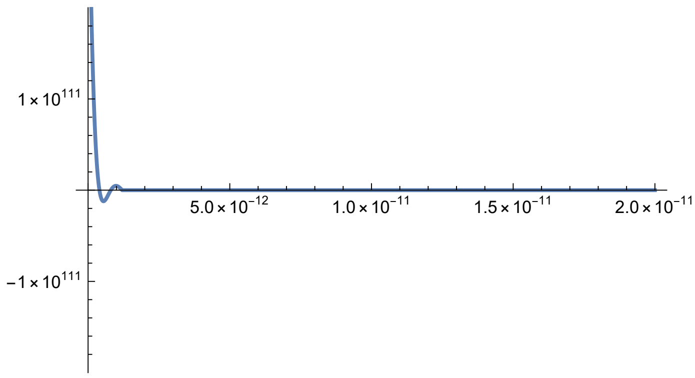
In[∘]:=
```
Plot[F3[l], {l, lmin, lmax}, PlotRange → {-5 × 10^111, 5 × 10^111}]
```
график функции отображаемый диапазон графика

Out[∘]=



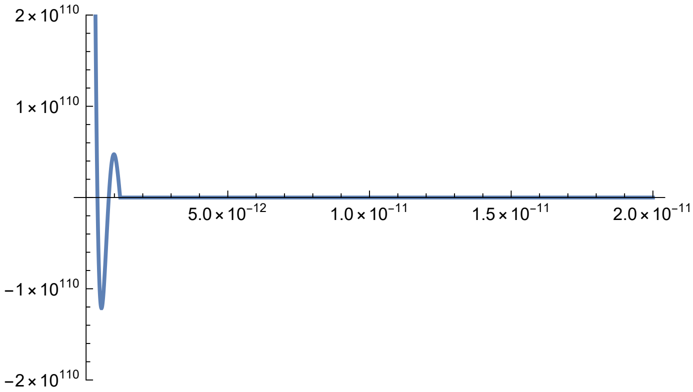
In[∘]:=
```
Plot[F3[l], {l, lmin, lmax}, PlotRange → {-2 × 10^111, 2 × 10^111}]
```
график функции отображаемый диапазон графика

Out[∘]=



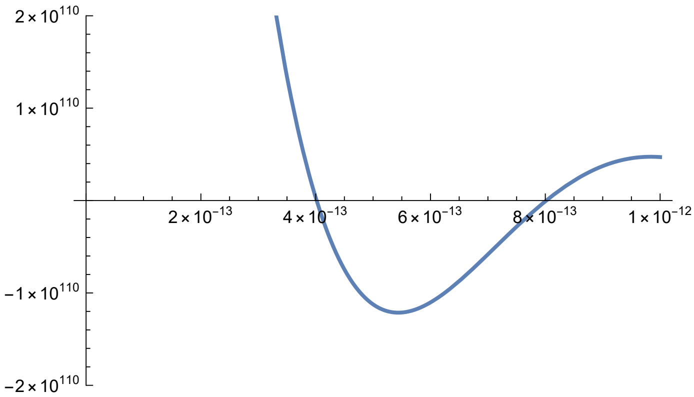
In[◦]:=
```
Plot[F3[l], {l, lmin, lmax}, PlotRange → {-2 × 10^110, 2 × 10^110}]
```
график функции | отображаемый диапазон графика

Out[◦]=



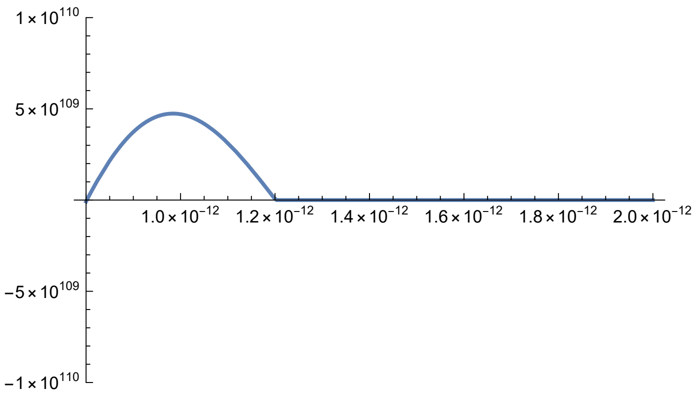
In[◦]:=
```
Plot[F3[l], {l, lmin, 10^-12}, PlotRange → {-2 × 10^110, 2 × 10^110}]
```
график функции | отображаемый диапазон графика

Out[◦]=



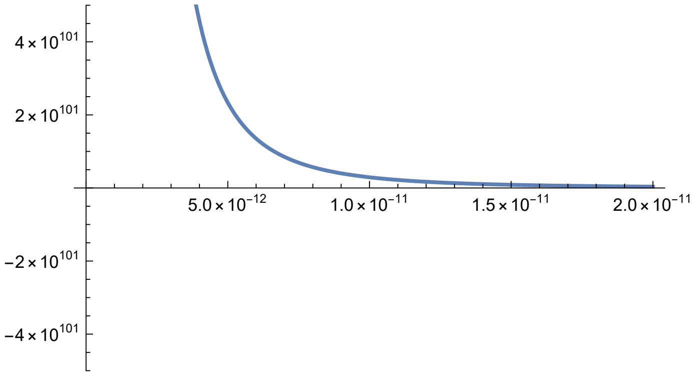
In[◦]:=
```
Plot[F3[l], {l, 8 × 10^-13, 2 × 10^-12}, PlotRange → {-10^110, 10^110}]
```
график функции | отображаемый диапазон графика

Out[◦]=



In[ ]:=
```
Plot[F3[l], {l, 10^-12, lmax}, PlotRange → {-5 × 10^101, 5 × 10^101}]
```
график функции | отображаемый диапазон графика

Out[ ]=



Following from the plots, we must write

In[ ]:=
```
FindRoot[F3[l] == 0, {l, 4 × 10^-13}]
```
найти корень

FindRoot: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

Out[ ]=

$\{l \to 4.016 \times 10^{-13}\}$

In[ ]:=
```
FindRoot[F3[l] == 0, {l, 8 × 10^-13}]
```
найти корень

FindRoot: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

Out[ ]=

$\{l \to 8.01583 \times 10^{-13}\}$

In[ ]:=
```
Ir3 = NIntegrate[F3[l], {l, lmin, 4.016 × 10^-13}] -
  NIntegrate[F3[l], {l, 4.016 × 10^-13, 8.01583 × 10^-13}] +
  NIntegrate[F3[l], {l, 8.01583 × 10^-13, lmax}]
```
квадратурное интегрирование

квадратурное интегрирование

квадратурное интегрирование

Out[ ]=

5.71398×10^{98}

In[ ]:=

$N\left[\frac{1}{lP} Ir3\right]$

численное приближение

Out[ ]=

3.53532×10^{133}

The backward motion

```
In[•]:= Do[l_α2 = lmin + α2 d;
        оператор цикла
  If[α2 == 0 || α2 == 1, Print["α2=", α2, " ", "l_α2=", l_α2], If[α2 == f1 || α2 == f2 ||
  условный оператор      печатать                            условный оператор
     α2 == f3 || α2 == f4 || α2 == f5, Print["α2=", α2, " ", "l_α2=", l_α2], Null]];
                                       печатать                               пустой
  Do[τ_β2 = -β2 δ;
  оператор цикла
   If[(α2 == 0 || α2 == 1) && (β2 == 0 || β2 == 1 || β2 == MMAX), Print["β2=", β2,
   условный оператор                                               печатать
     " ", "τ_β2=", τ_β2], If[(α2 == f1 || α2 == f2 || α2 == f3 || α2 == f4 || α2 == f5) &&
                          условный оператор
      (β2 == 0 || β2 == 1 || β2 == MMAX), Print["β2=", β2, " ", "τ_β2=", τ_β2], Null]];
                                           печатать                              пустой
```

$$J4_{\alpha2,\beta2} = \frac{1}{T[l_{\alpha2}] + \tau_{\beta2}} \text{NIntegrate}\left[\frac{Vs}{\left(l_{\alpha2} + \delta l\left[\frac{1}{2}\Delta t[l_{\alpha2}] - t\right]\right)^3}\right.$$

квадратурное интегрирование

$$\left.\sqrt{\frac{\hbar^2 c^2}{16} \times \frac{3}{\left(l_{\alpha2} + \delta l\left[\frac{1}{2}\Delta t[l_{\alpha2}] - t\right]\right)^2} + m0^2 c^4}, \{t, T[l_{\alpha2}] - \tau_{\beta2}, 2\,T[l_{\alpha2}]\}\right] \times \frac{1}{T[l_{\alpha2}]};$$

```
   τ_β2 =., {β2, 0, MMAX}];
  l_α2 =., {α2, 0, NMAX}]
```

$\alpha 2=0$ $l_{\alpha 2}=1.61626\times 10^{-15}$

$\beta 2=0$ $\tau_{\beta 2}=0.$

$\beta 2=1$ $\tau_{\beta 2}=-1.54803\times 10^{-24}$

$\beta 2=104$ $\tau_{\beta 2}=-1.60995\times 10^{-22}$

$\alpha 2=1$ $l_{\alpha 2}=4.016\times 10^{-13}$

$\beta 2=0$ $\tau_{\beta 2}=0.$

$\beta 2=1$ $\tau_{\beta 2}=-1.54803\times 10^{-24}$

$\beta 2=104$ $\tau_{\beta 2}=-1.60995\times 10^{-22}$

$\alpha 2=10$ $l_{\alpha 2}=4.00145\times 10^{-12}$

$\beta 2=0$ $\tau_{\beta 2}=0.$

$\beta 2=1$ $\tau_{\beta 2}=-1.54803\times 10^{-24}$

$\beta 2=104$ $\tau_{\beta 2}=-1.60995\times 10^{-22}$

$\alpha 2=20$ $l_{\alpha 2}=8.00129\times 10^{-12}$

$\beta 2=0$ $\tau_{\beta 2}=0.$

$\beta 2=1$ $\tau_{\beta 2}=-1.54803\times 10^{-24}$

$\beta 2=104$ $\tau_{\beta 2}=-1.60995\times 10^{-22}$

$\alpha 2=30$ $l_{\alpha 2}=1.20011\times 10^{-11}$

$\beta 2=0$ $\tau_{\beta 2}=0.$

$\beta 2=1$ $\tau_{\beta 2}=-1.54803\times 10^{-24}$

$\beta 2=104$ $\tau_{\beta 2}=-1.60995\times 10^{-22}$

$\alpha 2=40$ $l_{\alpha 2}=1.6001\times 10^{-11}$

$\beta 2=0$ $\tau_{\beta 2}=0.$

$\beta 2=1$ $\tau_{\beta 2}=-1.54803\times 10^{-24}$

$\beta 2=104$ $\tau_{\beta 2}=-1.60995\times 10^{-22}$

$\alpha 2=50$ $l_{\alpha 2}=2.00008\times 10^{-11}$

$\beta 2=0$ $\tau_{\beta 2}=0.$

$\beta 2=1$ $\tau_{\beta 2}=-1.54803\times 10^{-24}$

$\beta 2=104$ $\tau_{\beta 2}=-1.60995\times 10^{-22}$

```
In[]:= J44 = Interpolation[
          интерполировать
   Flatten[Table[{{lmin + α22 d, -β22 δ}, J4α22,β22}, {α22, 0, NMAX}, {β22, 0, MMAX}], 1],
   уплостить таблица значений
   InterpolationOrder → 5];
   порядок интерполяции

J44[l, τ]
```

```
In[]:= Do[If[(αm == f1 || αm == f2 || αm == f3 || αm == f4 || αm == f5), Print["αm=", αm], Null];
       условный оператор                                       печатать        пустой
  lαm = lmin + αm d;
  Int4αm = NIntegrate[J44[lαm, τ], {τ, -(T[lαm] - ϵ), 0}];
                 квадратурное интегрирование
  lαm =., {αm, 0, NMAX}]
```

αm=10

αm=20

αm=30

αm=40

αm=50

In[∘]:=
```
F4 = Interpolation[Table[{lmin + α4 d, Int4α4}, {α4, 0, NMAX}], InterpolationOrder → 5];
```
интерполировать · таблица значений · порядок интерполяции

In[∘]:=
```
I4 = NIntegrate[F4[l], {l, lmin, lmax}]
```
квадратурное интегрирование

Out[∘]=

5.11251×10^{98}

In[∘]:= $N\left[\frac{1}{lP}\ I4\right]$

численное приближение

Out[∘]=

3.16318×10^{133}

In[∘]:=
```
I00 = I3 + I4
```

Out[∘]=

1.0225×10^{99}

In[∘]:= $N\left[\frac{1}{lP}\ I00\right]$

численное приближение

Out[∘]=

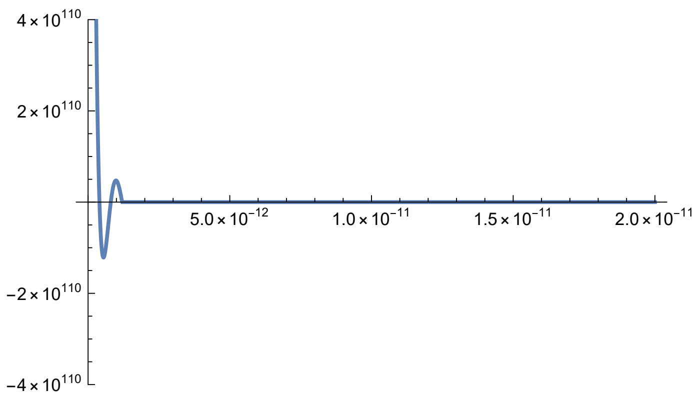
6.32636×10^{133}

It is needed to check for a new interesting structure may be found, it must be investigated.

In[∘]:= Plot[F4[l], {l, lmin, lmax}, PlotRange → {−4 × 10^110, 4 × 10^110}]

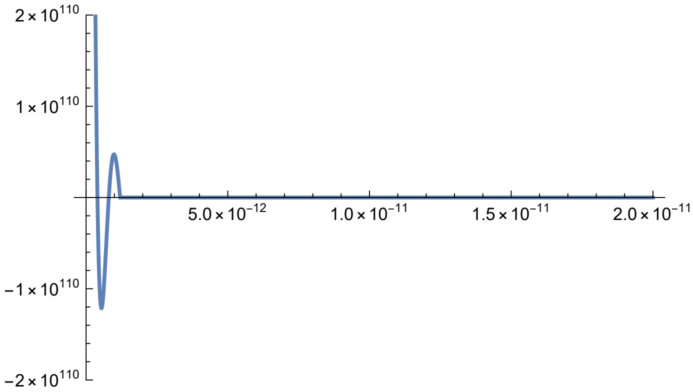
график функции · отображаемый диапазон графика

Out[∘]=



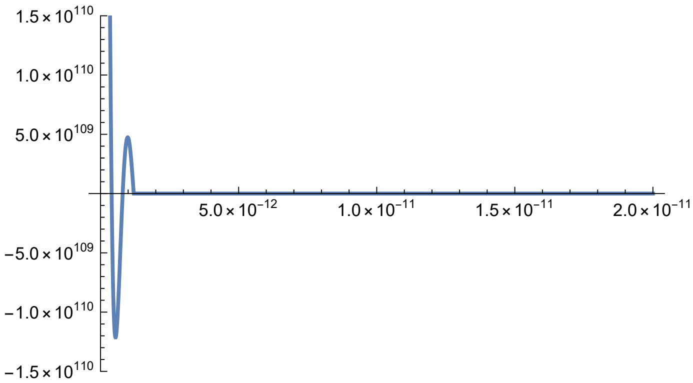
In[•]:=
```
Plot[F4[l], {l, lmin, lmax}, PlotRange → {-2 × 10^110, 2 × 10^110}]
```
график функции　　отображаемый диапазон графика

Out[•]=



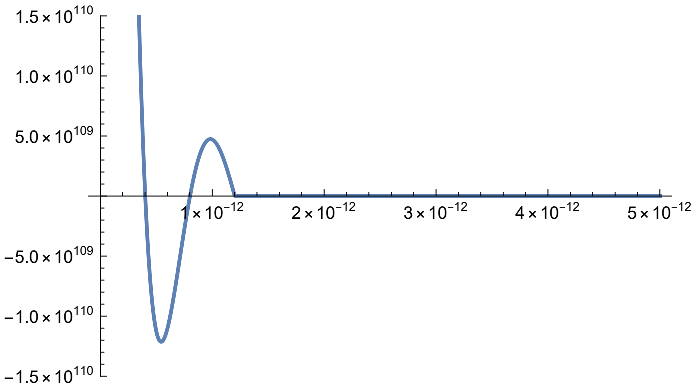
In[•]:=
```
Plot[F4[l], {l, lmin, lmax}, PlotRange → {-1.5 × 10^110, 1.5 × 10^110}]
```
график функции　　отображаемый диапазон графика

Out[•]=



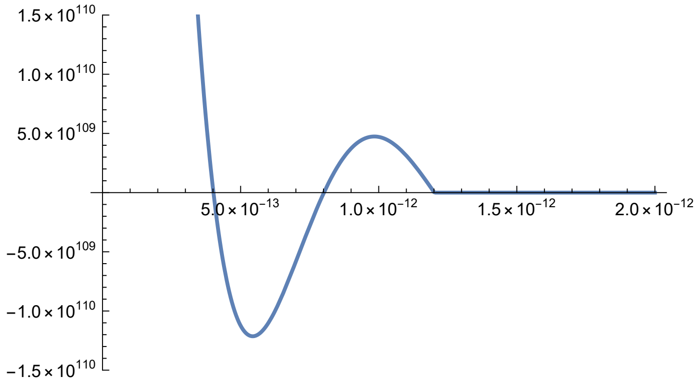
In[•]:=
```
Plot[F4[l], {l, lmin, 5 × 10^-12}, PlotRange → {-1.5 × 10^110, 1.5 × 10^110}]
```
график функции　　отображаемый диапазон графика

Out[•]=



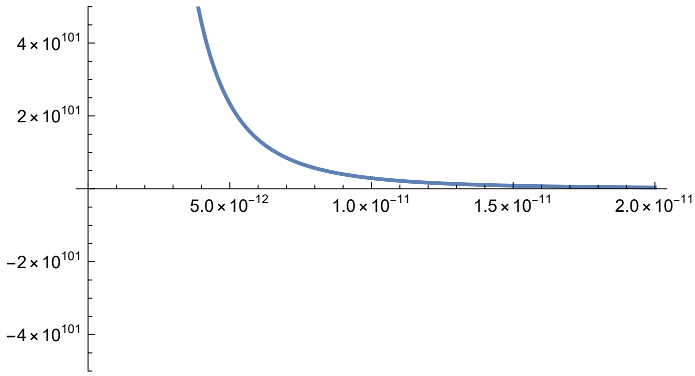
In[∘]:=
```
Plot[F4[l], {l, lmin, 2×10^-12}, PlotRange → {-1.5×10^110, 1.5×10^110}]
```
график функции · отображаемый диапазон графика

Out[∘]=



In[∘]:=
```
Plot[F4[l], {l, 10^-12, lmax}, PlotRange → {-5×10^101, 5×10^101}]
```
график функции · отображаемый диапазон графика

Out[∘]=



Following from the plots, we must write

In[∘]:=
```
FindRoot[F4[l] == 0, {l, 4×10^-13}]
```
найти корень

FindRoot: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

Out[∘]=

$\{l \to 4.016 \times 10^{-13}\}$

In[∘]:=
```
FindRoot[F4[l] == 0, {l, 8×10^-13}]
```
найти корень

FindRoot: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

Out[∘]=

$\{l \to 8.01583 \times 10^{-13}\}$

```
In[•]:= Ir4 = NIntegrate[F4[l], {l, lmin, 4.016 × 10^-13}] -
              квадратурное интегрирование
          NIntegrate[F4[l], {l, 4.016 × 10^-13, 8.01583 × 10^-13}] +
          квадратурное интегрирование
          NIntegrate[F4[l], {l, 8.01583 × 10^-13, lmax}]
          квадратурное интегрирование
Out[•]=
        5.71398 × 10^98

In[•]:= N[1/lP Ir4]
        численное приближение
Out[•]=
        3.53532 × 10^133

In[•]:= Ir00 = Ir3 + Ir4
Out[•]=
        1.1428 × 10^99

In[•]:= N[1/lP Ir00]
        численное приближение
Out[•]=
        7.07064 × 10^133

In[•]:= Tm2 = AbsoluteTime[];
              абсолютное значение

In[•]:= ComputationTime = Tm2 - Tm1
Out[•]=
        41.6397715
```

Below is the begining of the computation of the free vacuum energy in atoms without the limitation vacuum effect.

These are linear equations.

This is fastest algorithm, as the examination has shown.

In[∘]:= $\text{lmax3} := 2 \times 10^{-11}$

In[∘]:= $\Delta\text{L3} := \text{lmax3} - \text{lmin}$

$$\text{d3} := \frac{\Delta\text{L}}{50}$$

In[∘]:= $\delta3 := 1.54803 \times 10^{-24}$

$$\delta3 = \frac{1}{2} \times \frac{1}{104} \times \frac{\text{lmin} + \Delta\text{L3}}{c} \times \frac{1}{\sqrt{3 + 16 \times \frac{m0^2\, c^2}{\hbar^2}\ (\text{lmin} + \Delta\text{L3})^2}}$$

"NMAX3==NMAX"

"MMAX3==MMAX"

In[∘]:= $\text{lmax4} := 2 \times 10^{-11}$

In[∘]:= $\Delta\text{L4} := \text{lmax4} - \text{lmin}$

In[∘]:= $\text{d4} := \frac{\Delta\text{L4}}{50}$

In[∘]:= $\delta4 := 1.54803 \times 10^{-24}$

$$\delta4 = \frac{1}{2} \times \frac{1}{104} \times \frac{\text{lmin} + \Delta\text{L4}}{c} \times \frac{1}{\sqrt{3 + 16 \times \frac{m0^2\, c^2}{\hbar^2}\ (\text{lmin} + \Delta\text{L4})^2}}$$

"NMAX4==NMAX"

"MMAX4==MMAX"

In[∘]:= Table[$r2_{0,x1,y1} = r_{min}$;, {y1, 0, MMAX}, {x1, 0, NMAX}];
таблица значений

In[∘]:= Table[$ra_{0,x1,y1} = r_{min}$;, {y1, 0, MMAX}, {x1, 0, NMAX}];
таблица значений

```
For[s1 = 0, ra_{s1,n0,m00} ≤ r_max, s1++,
цикл ДЛЯ
  If[s1 == 0, Print["s1=", 0, " ", "ra_{s1}=", N[r_min]], Null];
  условный ⋯ печатать                          численно⋯ пустой
  Do[l_{x1} = lmin + x1 d3;
  оператор цикла
   Do[t_{y1} = y1 δ3;
   оператор цикла
    r2_{s1+1,x1,y1} = r2_{s1,x1,y1} + l_{x1} + δl[t_{y1}];
    ra_{s1+1,x1,y1} = r2_{s1+1,x1,y1};
    r2_{s1,x1,y1} =.;
    t_{y1} =., {y1, 0, MMAX}];
   l_{x1} =., {x1, 0, NMAX}];
  If[s1 > 0 && (s1 == j_0 || s1 == j_1 || s1 == j_2 || s1 == j_3 ||
  условный оператор
      s1 == j_4 || s1 == j_5 || s1 == j_6 || s1 == j_7 || s1 == j_8 || s1 == j_9 || s1 == j_10),
   Print["s1=", s1, " ", "ra_{s1,n0,m00}=", ra_{s1,n0,m00}], Null]] // Timing
   печатать                                            пустой      затраченное время
```

s1=0 ra_{s1}=1. × 10^{-11}

s1=1 $ra_{s1,n0,m00}$=1.00016 × 10^{-11}

s1=928 $ra_{s1,n0,m00}$=1.14999 × 10^{-11}

s1=1856 $ra_{s1,n0,m00}$=1.29998 × 10^{-11}

s1=2784 $ra_{s1,n0,m00}$=1.44997 × 10^{-11}

s1=3712 $ra_{s1,n0,m00}$=1.59995 × 10^{-11}

s1=4640 $ra_{s1,n0,m00}$=1.74994 × 10^{-11}

s1=5568 $ra_{s1,n0,m00}$=1.89993 × 10^{-11}

s1=6496 $ra_{s1,n0,m00}$=2.04992 × 10^{-11}

s1=7424 $ra_{s1,n0,m00}$=2.19991 × 10^{-11}

s1=8352 $ra_{s1,n0,m00}$=2.3499 × 10^{-11}

s1=9280 $ra_{s1,n0,m00}$=2.49988 × 10^{-11}

Out[∘]=

{1280.97, Null}

```
In[∘]:= p1 = s1 - 1;
```

```
In[∘]:= Table[
        таблица значений
          ra_{k5} = Interpolation[Flatten[Table[{{lmin + x11 d3, y11 δ3}, ra_{k5,x11,y11}}, {x11, 0, NMAX},
                    интерполировать  уплостить  таблица значений
              {y11, 0, MMAX}], 1], InterpolationOrder → 5];, {k5, 1, p1}];
                                   порядок интерполяции

        ra_{k5}[l, t]
```

```
In[∘]:= Table[r2m_{0,x2,y2} = r_min;, {y2, 0, MMAX}, {x2, 0, NMAX}];
        таблица значений
```

```
In[∘]:= Table[ram_{0,x2,y2} = r_min;, {y2, 0, MMAX}, {x2, 0, NMAX}];
        таблица значений
```

In[∘]:=
```
For[s2 = 0, ram_{s2,n0,m00} ≤ r_max, s2++,
цикл ДЛЯ
  If[s2 == 0, Print["s2=", 0, " ", "ram_s2=", N[r_min]], Null];
  условный ⋯ печатать                          численно⋯ пустой
  Do[l_x2 = lmin + x2 d4;
  оператор цикла
   Do[t_y2 = y2 δ4;
   оператор цикла
    r2m_{s2+1,x2,y2} = r2m_{s2,x2,y2} + l_x2 + δl[1/2 Δt[l_x2] + t_y2];
    ram_{s2+1,x2,y2} = r2m_{s2+1,x2,y2};
    r2m_{s2,x2,y2} =.;
    t_y2 =., {y2, 0, MMAX}];
   l_x2 =., {x2, 0, NMAX}];
  If[s2 > 0 && (s2 == j_0 || s2 == j_1 || s2 == j_2 || s2 == j_3 ||
  условный оператор
      s2 == j_4 || s2 == j_5 || s2 == j_6 || s2 == j_7 || s2 == j_8 || s2 == j_9 || s2 == j_10),
   Print["s2=", s2, " ", "ram_{s2,n0,m00}=", ram_{s2,n0,m00}], Null]] // Timing
   печатать                                                   пустой      затраченное время
```

s2=0 ram$_{s2}$=$1.\times10^{-11}$

s2=1 ram$_{s2,n0,m00}$=1.00016×10^{-11}

s2=928 ram$_{s2,n0,m00}$=1.14999×10^{-11}

s2=1856 ram$_{s2,n0,m00}$=1.29998×10^{-11}

s2=2784 ram$_{s2,n0,m00}$=1.44997×10^{-11}

s2=3712 ram$_{s2,n0,m00}$=1.59995×10^{-11}

s2=4640 ram$_{s2,n0,m00}$=1.74994×10^{-11}

s2=5568 ram$_{s2,n0,m00}$=1.89993×10^{-11}

s2=6496 ram$_{s2,n0,m00}$=2.04992×10^{-11}

s2=7424 ram$_{s2,n0,m00}$=2.19991×10^{-11}

s2=8352 ram$_{s2,n0,m00}$=2.3499×10^{-11}

s2=9280 ram$_{s2,n0,m00}$=2.49988×10^{-11}

Out[∘]=

{1798.58, Null}

In[∘]:=
```
p2 = s2 - 1;
```

In[∘]:=
```
Table[ram_k6 = Interpolation[Flatten[Table[{{lmin + x22 d4, y22 δ4}, ram_{k6,x22,y22}},
таблица знач⋯ интерполировать  уплостить таблица значений
        {x22, 0, NMAX}, {y22, 0, MMAX}], 1], InterpolationOrder → 5];, {k6, 1, p2}];
                                              порядок интерполяции

ram_k6[l, t]
```

In[∘]:= Reduce$\left[r_{min} \geq \frac{1}{2} \text{Abs}\left[l + \frac{1}{2} q \Delta t[l]\right], l, \text{Reals}\right]$
привести абсолютное значение множест

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[∘]=
$-2. \times 10^{-11} \leq l \leq 2. \times 10^{-11}$

In[∘]:= Reduce$\left[r_{min} \geq \frac{1}{2} \text{Abs}[l + q \Delta t[l]], l, \text{Reals}\right]$
привести абсолютное значение множест

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[∘]=
$-2. \times 10^{-11} \leq l \leq 2. \times 10^{-11}$

In[∘]:= Reduce$\left[r_{max} \geq \frac{1}{2} \text{Abs}\left[l + \frac{1}{2} q \Delta t[l]\right], l, \text{Reals}\right]$
привести абсолютное значение множест

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[∘]=
$-5. \times 10^{-11} \leq l \leq 5. \times 10^{-11}$

In[∘]:= Reduce$\left[r_{max} \geq \frac{1}{2} \text{Abs}[l + q \Delta t[l]], l, \text{Reals}\right]$
привести абсолютное значение множест

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[∘]=
$-5. \times 10^{-11} \leq l \leq 5. \times 10^{-11}$

From here the maximal scale for the calculations without the limitation vacuum effect can be found.

The forward direction

In[∘]:=
```
sa = s1 - 1;
```

In[ ]:= Do[l_{sa1} = lmin + sa1 d3;
оператор цикла

If[sa1 == 0 || sa1 == 1, Print["sa1=", sa1, " ", "l_{sa1}=", l_{sa1}], If[sa1 == f_1 || sa1 == f_2 ||
условный оператор | печатать | условный оператор

sa1 == f_3 || sa1 == f_4 || sa1 == f_5, Print["sa1=", sa1, " ", "l_{sa1}=", l_{sa1}], Null], Null];
печатать | пустой | пустой

Do[t_{sb1} = sb1 δ3;
оператор цикла

If[(sb1 == 0 || sb1 == 1 || sb1 == MMAX) && (sa1 == 0 || sa1 == 1 || sa1 == f_1 || sa1 == f_2 ||
условный оператор

sa1 == f_3 || sa1 == f_4 || sa1 == f_5), Print["sb1=", sb1, " ", "t_{sb1}=", t_{sb1}], Null];
печатать | пустой

$$IdJ1a_{sa1,sb1} = \left(\frac{1}{2}\left(\frac{\pi}{\text{ArcTan}\left[\frac{l_{sa1}+\delta l[t_{sb1}]}{2\sqrt{r_{min}^2-\frac{1}{4}(l_{sa1}+\delta l[t_{sb1}])^2}}\right]}\right)^2 - \frac{\pi}{\text{ArcTan}\left[\frac{l_{sa1}+\delta l[t_{sb1}]}{2\sqrt{r_{min}^2-\frac{1}{4}(l_{sa1}+\delta l[t_{sb1}])^2}}\right]} + 2\right)$$

$$\sqrt{\frac{\hbar^2 c^2}{16}\times\frac{3}{(l_{sa1}+\delta l[t_{sb1}])^2} + m0^2 c^4} +$$

$$\sum_{k7=1}^{sa}\left(\left(\frac{1}{2}\left(\frac{\pi}{\text{ArcTan}\left[\frac{l_{sa1}+\delta l[t_{sb1}]}{2\sqrt{(ra_{k7}[l_{sa1},t_{sb1}])^2-\frac{1}{4}(l_{sa1}+\delta l[t_{sb1}])^2}}\right]}\right)^2 - \right.\right.$$

$$\left.\left.\frac{\pi}{\text{ArcTan}\left[\frac{l_{sa1}+\delta l[t_{sb1}]}{2\sqrt{(ra_{k7}[l_{sa1},t_{sb1}])^2-\frac{1}{4}(l_{sa1}+\delta l[t_{sb1}])^2}}\right]} + 2\right)\sqrt{\frac{\hbar^2 c^2}{16}\times\frac{3}{(l_{sa1}+\delta l[t_{sb1}])^2} + m0^2 c^4}\right);$$

t_{sb1} =., {sb1, 0, MMAX}];

l_{sa1} =., {sa1, 0, NMAX}]

sa1=0 l_{sa1}=1.61626×10^{-15}

sb1=0 t_{sb1}=0.

sb1=1 t_{sb1}=1.54803×10^{-24}

sb1=104 t_{sb1}=1.60995×10^{-22}

sa1=1 l_{sa1}=4.01584×10^{-13}

sb1=0 t_{sb1}=0.

sb1=1 t_{sb1}=1.54803×10^{-24}

sb1=104 t_{sb1}=1.60995×10^{-22}

sa1=10 l_{sa1}=4.00129×10^{-12}

sb1=0 t_{sb1}=0.

sb1=1 t_{sb1}=1.54803×10^{-24}

sb1=104 t_{sb1}=1.60995×10^{-22}

sa1=20 l_{sa1}=8.00097×10^{-12}

sb1=0 t_{sb1}=0.

sb1=1 t_{sb1}=1.54803×10^{-24}

sb1=104 t_{sb1}=1.60995×10^{-22}

sa1=30 l_{sa1}=1.20006×10^{-11}

sb1=0 t_{sb1}=0.

sb1=1 t_{sb1}=1.54803×10^{-24}

sb1=104 t_{sb1}=1.60995×10^{-22}

sa1=40 l_{sa1}=1.60003×10^{-11}

sb1=0 t_{sb1}=0.

sb1=1 t_{sb1}=1.54803×10^{-24}

sb1=104 t_{sb1}=1.60995×10^{-22}

sa1=50 l_{sa1}=$2.\times10^{-11}$

sb1=0 t_{sb1}=0.

sb1=1 t_{sb1}=1.54803×10^{-24}

sb1=104 t_{sb1}=1.60995×10^{-22}

```
In[]:= IdJ11a = Interpolation[Flatten[Table[{{lmin + sa11 d3, sb11 δ3}, IdJ1a_{sa11,sb11}},
              интерполировать  уплостить таблица значений
          {sa11, 0, NMAX}, {sb11, 0, MMAX}], 1], InterpolationOrder → 5];
                                                 порядок интерполяции

IdJ11a[l, t]
```

```
In[●]:= Do[l_aa1 = lmin + aa1 d3;
       оператор цикла
  If[aa1 == 0 || aa1 == 1, Print["aa1=", aa1, " ", "l_aa1=", l_aa1], If[aa1 == f_1 || aa1 == f_2 ||
  условный оператор        печатать                                  условный оператор
     aa1 == f_3 || aa1 == f_4 || aa1 == f_5, Print["aa1=", aa1, " ", "l_aa1=", l_aa1], Null]];
                                              печатать                                 пустой
  Do[τ_bb1 = bb1 δ3;
  оператор цикла
   If[(aa1 == 0 || aa1 == 1) && (bb1 == 0 || bb1 == 1 || bb1 == MMAX), Print["bb1=", bb1, " ",
   условный оператор                                                    печатать
     "τ_bb1=", τ_bb1], If[(aa1 == f_1 || aa1 == f_2 || aa1 == f_3 || aa1 == f_4 || aa1 == f_5) &&
                       условный оператор
      (bb1 == 0 || bb1 == 1 || bb1 == MMAX), Print["bb1=", bb1, " ", "τ_bb1=", τ_bb1], Null]];
                                              печатать                                пустой
   J1a_{aa1,bb1} = 1/(T[l_aa1] - τ_bb1) 1/T[l_aa1] NIntegrate[IdJ11a[l_aa1, t], {t, τ_bb1, T[l_aa1]}];
                                                   квадратурное интегрирование
   τ_bb1 =., {bb1, 0, MMAX}];
  l_aa1 =., {aa1, 0, NMAX}]
```

aa1=0 l_{aa1}=1.61626×10^-15

bb1=0 τ_{bb1}=0.

bb1=1 τ_{bb1}=1.54803×10^-24

bb1=104 τ_{bb1}=1.60995×10^-22

aa1=1 l_{aa1}=4.01584×10^-13

bb1=0 τ_{bb1}=0.

bb1=1 τ_{bb1}=1.54803×10^-24

bb1=104 τ_{bb1}=1.60995×10^-22

aa1=10 l_{aa1}=4.00129×10^-12

bb1=0 τ_{bb1}=0.

bb1=1 τ_{bb1}=1.54803×10^-24

bb1=104 τ_{bb1}=1.60995×10^-22

aa1=20 l_{aa1}=8.00097×10^-12

bb1=0 τ_{bb1}=0.

bb1=1 τ_{bb1}=1.54803×10^-24

bb1=104 τ_{bb1}=1.60995×10^-22

aa1=30 l_{aa1}=1.20006×10^-11

bb1=0 τ_{bb1}=0.

bb1=1 τ_{bb1}=1.54803×10^-24

bb1=104 τ_{bb1}=1.60995×10^-22

aa1=40 l_{aa1}=1.60003×10^-11

bb1=0 τ_{bb1}=0.

bb1=1 τ_{bb1}=1.54803×10^-24

bb1=104 τ_{bb1}=1.60995×10^-22

aa1=50 l_{aa1}=2.×10^-11

bb1=0 τ_{bb1}=0.

bb1=1 τ_{bb1}=1.54803×10^-24

bb1=104 τ_{bb1}=1.60995×10^-22

```
In[∘]:= J11a = Interpolation[Flatten[Table[{{lmin + aa11 d3, bb11 δ3}, J1a[aa11,bb11]},
            интерполировать   уплотнить таблица значений
        {aa11, 0, NMAX}, {bb11, 0, MMAX}], 1], InterpolationOrder → 5];
                                                порядок интерполяции

J11a[l, τ]

In[∘]:= Do[If[(aa == f1 || aa == f2 || aa == f3 || aa == f4 || aa == f5), Print["aa=", aa], Null];
        условный оператор                                                   печатать     пустой
  l[aa] = lmin + aa d3;
  Int1a[aa] = NIntegrate[J11a[l[aa], τ], {τ, 0, T[l[aa]] - ϵ3}];
                квадратурное интегрирование
  l[aa] =., {aa, 0, NMAX}]
```

aa=10

aa=20

aa=30

aa=40

aa=50

In[◦]:=
```
F1a =
  Interpolation[Table[{lmin + a8 d3, Int1a_a8}, {a8, 0, NMAX}], InterpolationOrder → 5];
  интерполировать  таблица значений                          порядок интерполяции
```

In[◦]:=
```
I1a = NIntegrate[F1a[l], {l, lmin, lmax3}]
      квадратурное интегрирование
```

Out[◦]=

8.64476×10^{-12}

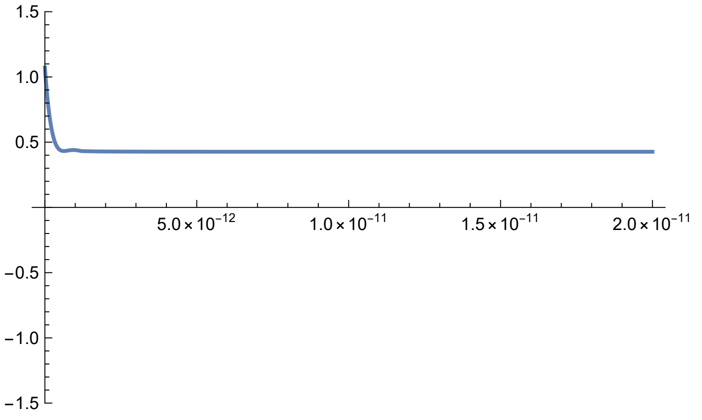
In[◦]:= $N\left[\frac{1}{lP} Na\ I1a\right]$
численное приближение

Out[◦]=

3.95264×10^{103}

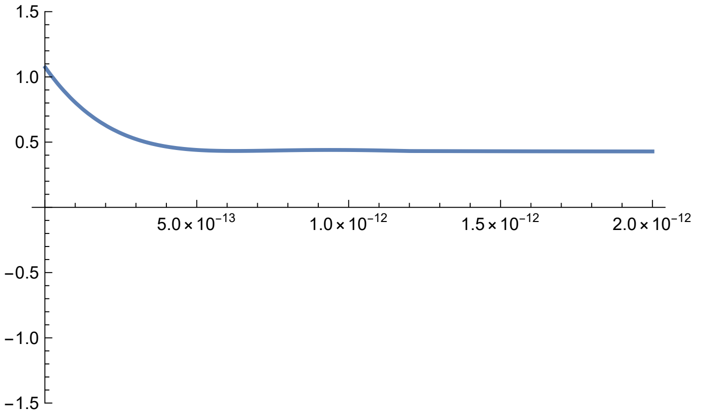
In[◦]:=
```
Plot[F1a[l], {l, lmin, lmax3}, PlotRange → {-1.5, 1.5}]
график функции                 отображаемый диапазон графика
```

Out[◦]=



In[◦]:=
```
Plot[F1a[l], {l, lmin, 2×10^-12}, PlotRange → {-1.5, 1.5}]
график функции                       отображаемый диапазон графика
```

Out[◦]=



The backward direction

```
In[•]:= sam = s2 - 1;

In[•]:= Do[l_sa2 = lmin + sa2 d4;
        оператор цикла

  If[sa2 == 0 || sa2 == 1, Print["sa2=", sa2, " ", "l_sa2=", l_sa2], If[sa2 == f_1 || sa2 == f_2 ||
  условный оператор          печатать                               условный оператор
     sa2 == f_3 || sa2 == f_4 || sa2 == f_5, Print["sa2=", sa2, " ", "l_sa2=", l_sa2], Null], Null];
                                             печатать                                  пустой  пустой

  Do[t_sb2 = sb2 δ4;
  оператор цикла

   If[(sb2 == 0 || sb2 == 1 || sb2 == MMAX) && (sa2 == 0 || sa2 == 1 || sa2 == f_1 || sa2 == f_2 ||
   условный оператор
      sa2 == f_3 || sa2 == f_4 || sa2 == f_5), Print["sb2=", sb2, " ", "t_sb2=", t_sb2], Null];
                                               печатать                                 пустой
```

$$\text{IdJ2a}_{sa2,sb2} = \left(\frac{1}{2}\left(\frac{\pi}{\text{ArcTan}\left[\frac{l_{sa2}+\delta l\left[\frac{1}{2}\,\Delta t[l_{sa2}]+t_{sb2}\right]}{2\sqrt{r_{min}^2-\frac{1}{4}\left(l_{sa2}+\delta l\left[\frac{1}{2}\,\Delta t[l_{sa2}]+t_{sb2}\right]\right)^2}}\right]}\right)^2 - \frac{\pi}{\text{ArcTan}\left[\frac{l_{sa2}+\delta l\left[\frac{1}{2}\,\Delta t[l_{sa2}]+t_{sb2}\right]}{2\sqrt{r_{min}^2-\frac{1}{4}\left(l_{sa2}+\delta l\left[\frac{1}{2}\,\Delta t[l_{sa2}]+t_{sb2}\right]\right)^2}}\right]} + 2\right)$$

$$\sqrt{\frac{\hbar^2 c^2}{16}\times\frac{3}{\left(l_{sa2}+\delta l\left[\frac{1}{2}\,\Delta t[l_{sa2}]+t_{sb2}\right]\right)^2}+m0^2 c^4} +$$

$$\sum_{k8=1}^{sam}\left(\left(\frac{1}{2}\left(\frac{\pi}{\text{ArcTan}\left[\frac{l_{sa2}+\delta l\left[\frac{1}{2}\,\Delta t[l_{sa2}]+t_{sb2}\right]}{2\sqrt{(ram_{k8}[l_{sa2},t_{sb2}])^2-\frac{1}{4}\left(l_{sa2}+\delta l\left[\frac{1}{2}\,\Delta t[l_{sa2}]+t_{sb2}\right]\right)^2}}\right]}\right)^2 - \frac{\pi}{\text{ArcTan}\left[\frac{l_{sa2}+\delta l\left[\frac{1}{2}\,\Delta t[l_{sa2}]+t_{sb2}\right]}{2\sqrt{(ram_{k8}[l_{sa2},t_{sb2}])^2-\frac{1}{4}\left(l_{sa2}+\delta l\left[\frac{1}{2}\,\Delta t[l_{sa2}]+t_{sb2}\right]\right)^2}}\right]} + 2\right)\right.$$

$$\left.\sqrt{\frac{\hbar^2 c^2}{16}\times\frac{3}{\left(l_{sa2}+\delta l\left[\frac{1}{2}\,\Delta t[l_{sa2}]+t_{sb2}\right]\right)^2}+m0^2 c^4}\right);$$

```
   t_sb2 =., {sb2, 0, MMAX}];
  l_sa2 =., {sa2, 0, NMAX}]
```

sa2=0 l_{sa2}=1.61626×10^{-15}

sb2=0 t_{sb2}=0.

sb2=1 t_{sb2}=1.54803×10^{-24}

sb2=104 t_{sb2}=1.60995×10^{-22}

sa2=1 l_{sa2}=4.01584×10^{-13}

sb2=0 t_{sb2}=0.

sb2=1 t_{sb2}=1.54803×10^{-24}

sb2=104 t_{sb2}=1.60995×10^{-22}

sa2=10 l_{sa2}=4.00129×10^{-12}

sb2=0 t_{sb2}=0.

sb2=1 t_{sb2}=1.54803×10^{-24}

sb2=104 t_{sb2}=1.60995×10^{-22}

sa2=20 l_{sa2}=8.00097×10^{-12}

sb2=0 t_{sb2}=0.

sb2=1 t_{sb2}=1.54803×10^{-24}

sb2=104 t_{sb2}=1.60995×10^{-22}

sa2=30 l_{sa2}=1.20006×10^{-11}

sb2=0 t_{sb2}=0.

sb2=1 t_{sb2}=1.54803×10^{-24}

sb2=104 t_{sb2}=1.60995×10^{-22}

sa2=40 l_{sa2}=1.60003×10^{-11}

sb2=0 t_{sb2}=0.

sb2=1 t_{sb2}=1.54803×10^{-24}

sb2=104 t_{sb2}=1.60995×10^{-22}

sa2=50 l_{sa2}=$2.\times10^{-11}$

sb2=0 t_{sb2}=0.

sb2=1 t_{sb2}=1.54803×10^{-24}

sb2=104 t_{sb2}=1.60995×10^{-22}

In[ ]:=
```
IdJ22a = Interpolation[Flatten[Table[{{lmin + sa22 d4, sb22 δ4}, IdJ2a[[sa22,sb22]]},
         интерполировать  уплостить  таблица значений
    {sa22, 0, NMAX}, {sb22, 0, MMAX}], 1], InterpolationOrder → 5];
                                          порядок интерполяции

IdJ22a[l, t]
```

In[ ]:=
```
Do[l_aa2 = lmin + aa2 d4;
оператор цикла
 If[aa2 == 0 || aa2 == 1, Print["aa2=", aa2, " ", "l_aa2=", l_aa2], If[aa2 == f_1 || aa2 == f_2 ||
 условный оператор      печатать                              условный оператор
    aa2 == f_3 || aa2 == f_4 || aa2 == f_5, Print["aa2=", aa2, " ", "l_aa2=", l_aa2], Null]];
                                           печатать                                 пустой
 Do[τ_bb2 = bb2 δ4;
 оператор цикла
  If[(aa2 == 0 || aa2 == 1) && (bb2 == 0 || bb2 == 1 || bb2 == MMAX), Print["bb2=", bb2, " ",
  условный оператор                                                  печатать
    "τ_bb2=", τ_bb2], If[(aa2 == f_1 || aa2 == f_2 || aa2 == f_3 || aa2 == f_4 || aa2 == f_5) &&
                      условный оператор
     (bb2 == 0 || bb2 == 1 || bb2 == MMAX), Print["bb2=", bb2, " ", "τ_bb2=", τ_bb2], Null]];
                                            печатать                                 пустой
  J2a_aa2,bb2 = 1/(T[l_aa2] - τ_bb2) 1/T[l_aa2] NIntegrate[IdJ22a[l_aa2, t],
                                                квадратурное интегрирование
     {t, T[l_aa2] + τ_bb2, 2 T[l_aa2]}];
  τ_bb2 =., {bb2, 0, MMAX}];
 l_aa2 =., {aa2, 0, NMAX}]
```

```
aa2=0 l_aa2=1.61626×10^-15
bb2=0 τ_bb2=0.
bb2=1 τ_bb2=1.54803×10^-24
bb2=104 τ_bb2=1.60995×10^-22
aa2=1 l_aa2=4.01584×10^-13
bb2=0 τ_bb2=0.
bb2=1 τ_bb2=1.54803×10^-24
bb2=104 τ_bb2=1.60995×10^-22
aa2=10 l_aa2=4.00129×10^-12
bb2=0 τ_bb2=0.
bb2=1 τ_bb2=1.54803×10^-24
bb2=104 τ_bb2=1.60995×10^-22
aa2=20 l_aa2=8.00097×10^-12
bb2=0 τ_bb2=0.
bb2=1 τ_bb2=1.54803×10^-24
bb2=104 τ_bb2=1.60995×10^-22
aa2=30 l_aa2=1.20006×10^-11
bb2=0 τ_bb2=0.
bb2=1 τ_bb2=1.54803×10^-24
bb2=104 τ_bb2=1.60995×10^-22
aa2=40 l_aa2=1.60003×10^-11
bb2=0 τ_bb2=0.
bb2=1 τ_bb2=1.54803×10^-24
bb2=104 τ_bb2=1.60995×10^-22
aa2=50 l_aa2=2.×10^-11
bb2=0 τ_bb2=0.
bb2=1 τ_bb2=1.54803×10^-24
bb2=104 τ_bb2=1.60995×10^-22
```

```
In[]:= J22a = Interpolation[Flatten[Table[{{lmin + aa22 d4, bb22 δ4}, J2a_aa22,bb22},
          интерполировать  уплостить  таблица значений
        {aa22, 0, NMAX}, {bb22, 0, MMAX}], 1], InterpolationOrder → 5];
                                                порядок интерполяции

J22a[l, τ]
```

```
In[]:= Do[If[(aam == f_1 || aam == f_2 || aam == f_3 || aam == f_4 || aam == f_5), Print["aam=", aam], Null];
        условный оператор                                                        печатать            пустой
  l_aam = lmin + aam d4;
  Int2a_aam = NIntegrate[J22a[l_aam, τ], {τ, 0, T[l_aam] - ϵ4}];
               квадратурное интегрирование
  l_aam =., {aam, 0, NMAX}]
```

```
aam=10

aam=20

aam=30

aam=40

aam=50
```

In[◦]:=
```
F2a =
  Interpolation[Table[{lmin + a9 d4, Int2a_a9}, {a9, 0, NMAX}], InterpolationOrder → 5];
  интерполировать  таблица значений                        порядок интерполяции
```

In[◦]:=
```
I2a = NIntegrate[F2a[l], {l, lmin, lmax4}]
      квадратурное интегрирование
```

Out[◦]=

8.64432×10^{-12}

In[◦]:= $N\left[\frac{1}{lP} \text{Na I2a}\right]$

численное приближение

Out[◦]=

3.95244×10^{103}

In[◦]:=
```
I0a = I1a + I2a
```

Out[◦]=

1.72891×10^{-11}

In[◦]:= $N\left[\frac{1}{lP} \Delta_1 \text{Na I0a}\right]$

численное приближение

Out[◦]=

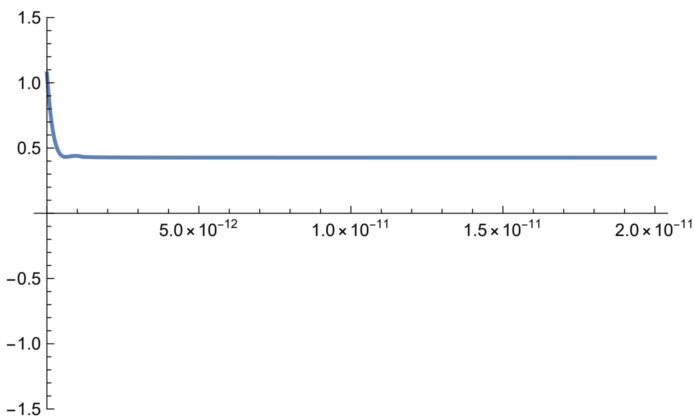
5.84186×10^{103}

$N\left[\frac{1}{lP} \Delta_2 \text{Na I0a}\right]$

численное приближение

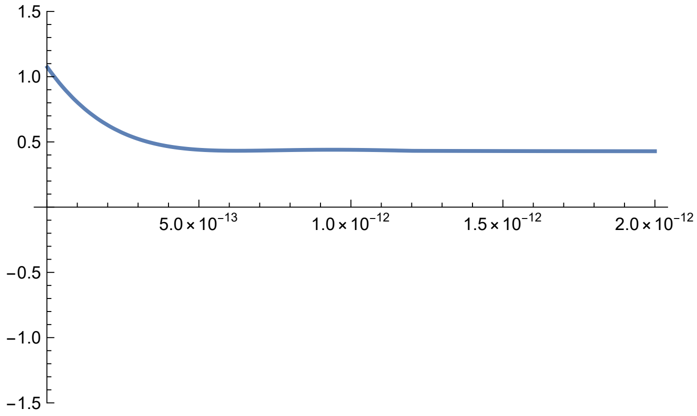
In[◦]:=
```
Plot[F2a[l], {l, lmin, lmax4}, PlotRange → {-1.5, 1.5}]
график функции                  отображаемый диапазон графика
```

Out[◦]=



In[ ]:=
```
Plot[F2a[l], {l, lmin, 2×10^-12}, PlotRange → {-1.5, 1.5}]
```
график функции — отображаемый диапазон графика

Out[ ]=



In[ ]:=
```
Tm2 = AbsoluteTime[];
```
абсолютное значение

In[ ]:=
```
ComputationTime = Tm2 - Tm1
```

Out[ ]=

17 558.4553638

```
In[1]:= me = 9.11 × 10^-31;
        mμ = 1.88356 × 10^-28;
        mτ = 3.16751 × 10^-27;
In[2]:= uplmνe = 3.6 × 10^-36;
In[3]:= uplmνμ = 3.39 × 10^-31;
In[4]:= uplmντ = 3.2445 × 10^-29;
        mu = 4.2785 × 10^-30;
        md = 8.4675 × 10^-30;
        ms = 1.78266 × 10^-28;
        mc = 2.22833 × 10^-27;
        mb = 8.46765 × 10^-27;
        mt = 3.052 × 10^-25;
In[5]:= VU = 3.6 × 10^80;
        Na = 7.39 × 10^79;
        VA = 4/3 π (r_max^3 - r_min^3);
        m = me;
        c = 299 792 458;
        h = 6.6260755 × 10^-34;
        ħ = h/(2 π);
In[12]:= r_min := 10^-11
In[13]:= lmin := 10^20 lP
In[14]:= lP := 1.616255 × 10^-35
In[15]:= r_max := 2.5 × 10^-11
In[93]:= r_max := 3.1 × 10^-11
In[16]:= lmax := 2.00008 × 10^-11
In[17]:= VAp_1 = 6.12611 × 10^-32;
         VAp_2 = 1.20599 × 10^-31;
         VAp_3 = 9.0059 × 10^-31;
         VAp_4 = 1.43257 × 10^-30;
         VAp_5 = 2.25659 × 10^-31;
         VAp_6 = 1.14899 × 10^-29;
         VAp_7 = 1.14616 × 10^-30;
         VAp_8 = 5.57109 × 10^-30;
         VAp_9 = 1.4133 × 10^-29;
         VAp_10 = 4.1846 × 10^-30;
```

In[27]:=
$$\Delta_1 = 0.739;$$
$$\Delta_2 = 0.24;$$
$$\Delta_3 = 0.0104;$$
$$\Delta_4 = 0.0046;$$
$$\Delta_5 = 0.0013;$$
$$\Delta_6 = 0.0011;$$
$$\Delta_7 = 0.00096;$$
$$\Delta_8 = 0.00065;$$
$$\Delta_9 = 0.00058;$$
$$\Delta_{10} = 0.00044;$$

In[37]:= `l = lmin;`

In[38]:=
$$R11 = \Big(16\sqrt{3}\,c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3\sqrt{3}\,l^2 \hbar^4 + 1024\,c^4 l^5 m^4 r_{min} + 384\,c^2 l^3 m^2 \hbar^2 r_{min} + 36\,l\,\hbar^4 r_{min} + 1024\,c^4 l^4 m^4 r_{min}^2 + 384\,c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36\,\hbar^4 r_{min}^2 + \sqrt{\Big(8\,l\,\hbar^2 \left(16 c^2 l^2 m^2 + 3\hbar^2\right) r_{min} \left(l\left(16\sqrt{3}\,c^2 l^2 m^2 + 3\left(1+\sqrt{3}\right)\hbar^2\right) + 2\sqrt{3}\left(16 c^2 l^2 m^2 + 3\hbar^2\right) r_{min}\right)\left(\sqrt{3}\,l\,\hbar^2 + \left(32 c^2 l^2 m^2 + 6\hbar^2\right) r_{min}\right) + \left(l^2\hbar^2\left(16\sqrt{3}\,c^2 l^2 m^2 + 3\left(-1+\sqrt{3}\right)\hbar^2\right) + 4\,l\left(16 c^2 l^2 m^2 + 3\hbar^2\right)^2 r_{min} + 4\left(16 c^2 l^2 m^2 + 3\hbar^2\right)^2 r_{min}^2\right)^2\Big)}\Big) \Big/ \left(4\left(16 c^2 l^2 m^2 + 3\hbar^2\right)\left(\sqrt{3}\,l\,\hbar^2 + \left(32 c^2 l^2 m^2 + 6\hbar^2\right) r_{min}\right)\right) \;//\; N$$

численное приближение

Out[38]= 1.00016×10^{-11}

In[39]:= `l =.`

In[40]:= `l = lmax;`

In[41]:=
$$R12 = \Big(16\sqrt{3}\,c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3\sqrt{3}\,l^2 \hbar^4 + 1024\,c^4 l^5 m^4 r_{min} + 384\,c^2 l^3 m^2 \hbar^2 r_{min} + 36\,l\,\hbar^4 r_{min} + 1024\,c^4 l^4 m^4 r_{min}^2 + 384\,c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36\,\hbar^4 r_{min}^2 + \sqrt{\Big(8\,l\,\hbar^2 \left(16 c^2 l^2 m^2 + 3\hbar^2\right) r_{min} \left(l\left(16\sqrt{3}\,c^2 l^2 m^2 + 3\left(1+\sqrt{3}\right)\hbar^2\right) + 2\sqrt{3}\left(16 c^2 l^2 m^2 + 3\hbar^2\right) r_{min}\right)\left(\sqrt{3}\,l\,\hbar^2 + \left(32 c^2 l^2 m^2 + 6\hbar^2\right) r_{min}\right) + \left(l^2\hbar^2\left(16\sqrt{3}\,c^2 l^2 m^2 + 3\left(-1+\sqrt{3}\right)\hbar^2\right) + 4\,l\left(16 c^2 l^2 m^2 + 3\hbar^2\right)^2 r_{min} + 4\left(16 c^2 l^2 m^2 + 3\hbar^2\right)^2 r_{min}^2\right)^2\Big)}\Big) \Big/ \left(4\left(16 c^2 l^2 m^2 + 3\hbar^2\right)\left(\sqrt{3}\,l\,\hbar^2 + \left(32 c^2 l^2 m^2 + 6\hbar^2\right) r_{min}\right)\right) \;//\; N$$

численное приближение

Out[41]= 3.00003×10^{-11}

In[42]:= `l =.`

In[43]:= `l = lmin;`

In[44]:=
$$R21 = \Bigg(\left(\sqrt{3}\,l\,\hbar^2 + \left(32 c^2 l^2 m^2 + 6\hbar^2\right) r_{min}\right)\Big(16\sqrt{3}\,c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3\sqrt{3}\,l^2 \hbar^4 + 1 \Big/ \left(\left(16 c^2 l^2 m^2 + 3\hbar^2\right)\left(\sqrt{3}\,l\,\hbar^2 + \left(32 c^2 l^2 m^2 + 6\hbar^2\right) r_{min}\right)\right)\; 256\,c^4 l^5 m^4 \Big(16\sqrt{3}\,c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3\sqrt{3}\,l^2 \hbar^4 + 1024\,c^4 l^5 m^4 r_{min} + 384\,c^2 l^3 m^2 \hbar^2 r_{min} + 36\,l\,\hbar^4 r_{min} + 1024\,c^4 l^4 m^4 r_{min}^2 + 384\,c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36\,\hbar^4 r_{min}^2 + \sqrt{\Big(8\,l\,\hbar^2\left(16 c^2 l^2 m^2 + 3\hbar^2\right) r_{min}\left(l\left(16\sqrt{3}\,c^2 l^2 m^2 + 3\left(1+\sqrt{3}\right)\hbar^2\right) + \right.}$$

$$2\sqrt{3}\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)\ r_{min})\ (\sqrt{3}\ l\,\hbar^2+(32\,c^2\,l^2\,m^2+6\,\hbar^2)\ r_{min})\ +$$
$$(l^2\,\hbar^2\ (16\sqrt{3}\ c^2\,l^2\,m^2+3\ (-1+\sqrt{3})\ \hbar^2)+4\,l\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)^2\ r_{min}\ +$$
$$4\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)^2\ r_{min}^2)^2))\ +$$
$$1\Big/\ ((16\,c^2\,l^2\,m^2+3\,\hbar^2)\ (\sqrt{3}\ l\,\hbar^2+(32\,c^2\,l^2\,m^2+6\,\hbar^2)\ r_{min}))\quad 96\,c^2\,l^3\,m^2$$
$$\hbar^2\ (16\sqrt{3}\ c^2\,l^4\,m^2\,\hbar^2-3\,l^2\,\hbar^4+3\sqrt{3}\ l^2\,\hbar^4+1024\,c^4\,l^5\,m^4\,r_{min}\ +$$
$$384\,c^2\,l^3\,m^2\,\hbar^2\,r_{min}+36\,l\,\hbar^4\,r_{min}+1024\,c^4\,l^4\,m^4\,r_{min}^2\ +$$
$$384\,c^2\,l^2\,m^2\,\hbar^2\,r_{min}^2+36\,\hbar^4\,r_{min}^2+\surd(8\,l\,\hbar^2\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)\ r_{min}$$
$$(l\ (16\sqrt{3}\ c^2\,l^2\,m^2+3\ (1+\sqrt{3})\ \hbar^2)+2\sqrt{3}\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)\ r_{min})$$
$$(\sqrt{3}\ l\,\hbar^2+(32\,c^2\,l^2\,m^2+6\,\hbar^2)\ r_{min})+(l^2\,\hbar^2\ (16\sqrt{3}\ c^2\,l^2\,m^2+3\ (-1+\sqrt{3})\ \hbar^2)\ +$$
$$4\,l\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)^2\ r_{min}+4\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)^2\ r_{min}^2)^2))\ +$$
$$1\Big/\ ((16\,c^2\,l^2\,m^2+3\,\hbar^2)\ (\sqrt{3}\ l\,\hbar^2+(32\,c^2\,l^2\,m^2+6\,\hbar^2)\ r_{min}))\quad 9\,l\,\hbar^4$$
$$(16\sqrt{3}\ c^2\,l^4\,m^2\,\hbar^2-3\,l^2\,\hbar^4+3\sqrt{3}\ l^2\,\hbar^4+1024\,c^4\,l^5\,m^4\,r_{min}\ +$$
$$384\,c^2\,l^3\,m^2\,\hbar^2\,r_{min}+36\,l\,\hbar^4\,r_{min}+1024\,c^4\,l^4\,m^4\,r_{min}^2\ +$$
$$384\,c^2\,l^2\,m^2\,\hbar^2\,r_{min}^2+36\,\hbar^4\,r_{min}^2+\surd(8\,l\,\hbar^2\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)\ r_{min}$$
$$(l\ (16\sqrt{3}\ c^2\,l^2\,m^2+3\ (1+\sqrt{3})\ \hbar^2)+2\sqrt{3}\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)\ r_{min})$$
$$(\sqrt{3}\ l\,\hbar^2+(32\,c^2\,l^2\,m^2+6\,\hbar^2)\ r_{min})+(l^2\,\hbar^2\ (16\sqrt{3}\ c^2\,l^2\,m^2+3\ (-1+\sqrt{3})\ \hbar^2)\ +$$
$$4\,l\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)^2\ r_{min}+4\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)^2\ r_{min}^2)^2))\ +$$
$$(64\,c^4\,l^4\,m^4\ (16\sqrt{3}\ c^2\,l^4\,m^2\,\hbar^2-3\,l^2\,\hbar^4+3\sqrt{3}\ l^2\,\hbar^4+1024\,c^4\,l^5\,m^4\,r_{min}+384$$
$$c^2\,l^3\,m^2\,\hbar^2\,r_{min}+36\,l\,\hbar^4\,r_{min}+1024\,c^4\,l^4\,m^4\,r_{min}^2+384\,c^2\,l^2\,m^2\,\hbar^2\,r_{min}^2+36\,\hbar^4\,r_{min}^2\ +$$
$$\surd(8\,l\,\hbar^2\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)\ r_{min}\ (l\ (16\sqrt{3}\ c^2\,l^2\,m^2+3\ (1+\sqrt{3})\ \hbar^2)\ +$$
$$2\sqrt{3}\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)\ r_{min})\ (\sqrt{3}\ l\,\hbar^2+(32\,c^2\,l^2\,m^2+6\,\hbar^2)\ r_{min})\ +$$
$$(l^2\,\hbar^2\ (16\sqrt{3}\ c^2\,l^2\,m^2+3\ (-1+\sqrt{3})\ \hbar^2)+4\,l\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)^2$$
$$r_{min}+4\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)^2\ r_{min}^2)^2))^2)\Big/$$
$$((16\,c^2\,l^2\,m^2+3\,\hbar^2)^2\ (\sqrt{3}\ l\,\hbar^2+(32\,c^2\,l^2\,m^2+6\,\hbar^2)\ r_{min})^2)\ +$$
$$(24\,c^2\,l^2\,m^2\,\hbar^2\ (16\sqrt{3}\ c^2\,l^4\,m^2\,\hbar^2-3\,l^2\,\hbar^4+3\sqrt{3}\ l^2\,\hbar^4+1024\,c^4\,l^5\,m^4\,r_{min}+384\,c^2$$
$$l^3\,m^2\,\hbar^2\,r_{min}+36\,l\,\hbar^4\,r_{min}+1024\,c^4\,l^4\,m^4\,r_{min}^2+384\,c^2\,l^2\,m^2\,\hbar^2\,r_{min}^2+36\,\hbar^4\,r_{min}^2\ +$$
$$\surd(8\,l\,\hbar^2\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)\ r_{min}\ (l\ (16\sqrt{3}\ c^2\,l^2\,m^2+3\ (1+\sqrt{3})\ \hbar^2)\ +$$
$$2\sqrt{3}\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)\ r_{min})\ (\sqrt{3}\ l\,\hbar^2+(32\,c^2\,l^2\,m^2+6\,\hbar^2)\ r_{min})\ +$$
$$(l^2\,\hbar^2\ (16\sqrt{3}\ c^2\,l^2\,m^2+3\ (-1+\sqrt{3})\ \hbar^2)+4\,l\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)^2$$
$$r_{min}+4\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)^2\ r_{min}^2)^2))^2)\Big/$$
$$((16\,c^2\,l^2\,m^2+3\,\hbar^2)^2\ (\sqrt{3}\ l\,\hbar^2+(32\,c^2\,l^2\,m^2+6\,\hbar^2)\ r_{min})^2)\ +$$
$$(9\,\hbar^4\ (16\sqrt{3}\ c^2\,l^4\,m^2\,\hbar^2-3\,l^2\,\hbar^4+3\sqrt{3}\ l^2\,\hbar^4+1024\,c^4\,l^5\,m^4\,r_{min}+384\,c^2\,l^3\,m^2\,\hbar^2\,r_{min}\ +$$
$$36\,l\,\hbar^4\,r_{min}+1024\,c^4\,l^4\,m^4\,r_{min}^2+384\,c^2\,l^2\,m^2\,\hbar^2\,r_{min}^2+36\,\hbar^4\,r_{min}^2\ +$$
$$\surd(8\,l\,\hbar^2\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)\ r_{min}\ (l\ (16\sqrt{3}\ c^2\,l^2\,m^2+3\ (1+\sqrt{3})\ \hbar^2)\ +$$
$$2\sqrt{3}\ (16\,c^2\,l^2\,m^2+3\,\hbar^2)\ r_{min})\ (\sqrt{3}\ l\,\hbar^2+(32\,c^2\,l^2\,m^2+6\,\hbar^2)\ r_{min})\ +$$

$$\left(l^2 \hbar^2 \left(16 \sqrt{3}\ c^2 l^2 m^2 + 3 \left(-1 + \sqrt{3}\right) \hbar^2\right) + 4\, l \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2\right.$$

$$\left.\left.\left. r_{min} + 4 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2 r_{min}^2\right)^2\right)\right)^2\right) \Big/$$

$$\left(4 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2 \left(\sqrt{3}\ l \hbar^2 + \left(32 c^2 l^2 m^2 + 6 \hbar^2\right) r_{min}\right)^2\right) +$$

$$\sqrt{\left(1 \Big/ \left(2 \left(\sqrt{3}\ l \hbar^2 + \left(32 c^2 l^2 m^2 + 6 \hbar^2\right) r_{min}\right)^3\right) l \hbar^2 \left(16 \sqrt{3}\ c^2 l^4 m^2 \hbar^2 -\right.\right.}$$

$$3 l^2 \hbar^4 + 3 \sqrt{3}\ l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3 m^2 \hbar^2 r_{min} + 36\, l \hbar^4 r_{min} +$$

$$1024 c^4 l^4 m^4 r_{min}^2 + 384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 + \sqrt{\left(8\, l \hbar^2 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)\right.}$$

$$r_{min} \left(l \left(16 \sqrt{3}\ c^2 l^2 m^2 + 3 \left(1 + \sqrt{3}\right) \hbar^2\right) + 2 \sqrt{3} \left(16 c^2 l^2 m^2 + 3 \hbar^2\right) r_{min}\right)$$

$$\left(\sqrt{3}\ l \hbar^2 + \left(32 c^2 l^2 m^2 + 6 \hbar^2\right) r_{min}\right) + \left(l^2 \hbar^2 \left(16 \sqrt{3}\ c^2 l^2 m^2 + 3 \left(-1 + \sqrt{3}\right)\right.\right.$$

$$\left.\left.\left.\hbar^2\right) + 4\, l \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2 r_{min} + 4 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2 r_{min}^2\right)^2\right)\Big)$$

$$\Big(16 \sqrt{3}\ c^2 l^4 m^2 \hbar^2 + 3 l^2 \hbar^4 + 3 \sqrt{3}\ l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3 m^2$$

$$\hbar^2 r_{min} + 64 \sqrt{3}\ c^2 l^3 m^2 \hbar^2 r_{min} + 36\, l \hbar^4 r_{min} + 12 \sqrt{3}\ l \hbar^4 r_{min} + 1024 c^4 l^4 m^4 r_{min}^2 +$$

$$384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 + \sqrt{\left(8\, l \hbar^2 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right) r_{min}\right.}$$

$$\left(l \left(16 \sqrt{3}\ c^2 l^2 m^2 + 3 \left(1 + \sqrt{3}\right) \hbar^2\right) + 2 \sqrt{3} \left(16 c^2 l^2 m^2 + 3 \hbar^2\right) r_{min}\right)$$

$$\left(\sqrt{3}\ l \hbar^2 + \left(32 c^2 l^2 m^2 + 6 \hbar^2\right) r_{min}\right) + \left(l^2 \hbar^2 \left(16 \sqrt{3}\ c^2 l^2 m^2 + 3 \left(-1 + \sqrt{3}\right)\right.\right.$$

$$\left.\left.\left.\hbar^2\right) + 4\, l \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2 r_{min} + 4 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2 r_{min}^2\right)^2\right)\Big)$$

$$\Big(144 c^2 l^4 m^2 \hbar^2 + 27 l^2 \hbar^4 + 3 \sqrt{3}\ l^2 \hbar^4 + 2048 \sqrt{3}\ c^4 l^5 m^4 r_{min} +$$

$$192 c^2 l^3 m^2 \hbar^2 r_{min} + 768 \sqrt{3}\ c^2 l^3 m^2 \hbar^2 r_{min} + 36\, l \hbar^4 r_{min} + 72 \sqrt{3}\ l \hbar^4 r_{min} +$$

$$1024 \sqrt{3}\ c^4 l^4 m^4 r_{min}^2 + 384 \sqrt{3}\ c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \sqrt{3}\ \hbar^4 r_{min}^2 +$$

$$\sqrt{3} \sqrt{\left(8\, l \hbar^2 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right) r_{min} \left(l \left(16 \sqrt{3}\ c^2 l^2 m^2 + 3 \left(1 + \sqrt{3}\right) \hbar^2\right) +\right.\right.}$$

$$\left.2 \sqrt{3} \left(16 c^2 l^2 m^2 + 3 \hbar^2\right) r_{min}\right) \left(\sqrt{3}\ l \hbar^2 + \left(32 c^2 l^2 m^2 + 6 \hbar^2\right) r_{min}\right) +$$

$$\left(l^2 \hbar^2 \left(16 \sqrt{3}\ c^2 l^2 m^2 + 3 \left(-1 + \sqrt{3}\right) \hbar^2\right) + 4\, l \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2\right.$$

$$\left.\left. r_{min} + 4 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2 r_{min}^2\right)^2\right)\Big) +$$

$$\Big(16 \sqrt{3}\ c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3 \sqrt{3}\ l^2 \hbar^4 + \Big(256 c^4 l^5 m^4 \Big(16 \sqrt{3}\ c^2 l^4 m^2 \hbar^2 -$$

$$3 l^2 \hbar^4 + 3 \sqrt{3}\ l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3 m^2 \hbar^2 r_{min} +$$

$$36\, l \hbar^4 r_{min} + 1024 c^4 l^4 m^4 r_{min}^2 + 384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 +$$

$$\sqrt{\left(8\, l \hbar^2 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right) r_{min} \left(l \left(16 \sqrt{3}\ c^2 l^2 m^2 + 3 \left(1 + \sqrt{3}\right) \hbar^2\right) +\right.\right.}$$

$$\left.2 \sqrt{3} \left(16 c^2 l^2 m^2 + 3 \hbar^2\right) r_{min}\right) \left(\sqrt{3}\ l \hbar^2 + \left(32 c^2 l^2 m^2 + 6 \hbar^2\right) r_{min}\right) +$$

$$\left(l^2 \hbar^2 \left(16 \sqrt{3}\ c^2 l^2 m^2 + 3 \left(-1 + \sqrt{3}\right) \hbar^2\right) + 4\, l\right.$$

$$\left.\left.\left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2 r_{min} + 4 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2 r_{min}^2\right)^2\right)\Big)\Big) \Big/$$

$$\left(\left(\left(16 c^2 l^2 m^2 + 3 \hbar^2\right) \left(\sqrt{3}\ l \hbar^2 + \left(32 c^2 l^2 m^2 + 6 \hbar^2\right) r_{min}\right)\right) + \Big(96 c^2\right.$$

$$l^3 m^2 \hbar^2 \Big(16 \sqrt{3}\ c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3 \sqrt{3}\ l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3$$

$$m^2 \hbar^2 r_{min} + 36\, l \hbar^4 r_{min} + 1024 c^4 l^4 m^4 r_{min}^2 + 384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 +$$

$$\sqrt{\left(8\, l \hbar^2 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right) r_{min} \left(l \left(16 \sqrt{3}\ c^2 l^2 m^2 + 3 \left(1 + \sqrt{3}\right) \hbar^2\right) +\right.\right.}$$

$$\left.2 \sqrt{3} \left(16 c^2 l^2 m^2 + 3 \hbar^2\right) r_{min}\right) \left(\sqrt{3}\ l \hbar^2 + \left(32 c^2 l^2 m^2 + 6 \hbar^2\right) r_{min}\right) +$$

$$\left(l^2 \hbar^2 \left(16 \sqrt{3}\ c^2 l^2 m^2 + 3 \left(-1 + \sqrt{3}\right) \hbar^2\right) + 4\, l\right.$$

$$(16 c^2 l^2 m^2 + 3 \hbar^2)^2 r_{min} + 4 (16 c^2 l^2 m^2 + 3 \hbar^2)^2 r_{min}^2)^2))) /$$

$$((16 c^2 l^2 m^2 + 3 \hbar^2) (\sqrt{3}\, l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2) r_{min})) +$$

$$(9 l \hbar^4 (16 \sqrt{3}\, c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3 \sqrt{3}\, l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3$$

$$m^2 \hbar^2 r_{min} + 36 l \hbar^4 r_{min} + 1024 c^4 l^4 m^4 r_{min}^2 + 384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 +$$

$$\sqrt{} (8 l \hbar^2 (16 c^2 l^2 m^2 + 3 \hbar^2) r_{min} (l (16 \sqrt{3}\, c^2 l^2 m^2 + 3 (1 + \sqrt{3}) \hbar^2) +$$

$$2 \sqrt{3} (16 c^2 l^2 m^2 + 3 \hbar^2) r_{min}) (\sqrt{3}\, l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2) r_{min}) +$$

$$(l^2 \hbar^2 (16 \sqrt{3}\, c^2 l^2 m^2 + 3 (-1 + \sqrt{3}) \hbar^2) + 4 l$$

$$(16 c^2 l^2 m^2 + 3 \hbar^2)^2 r_{min} + 4 (16 c^2 l^2 m^2 + 3 \hbar^2)^2 r_{min}^2)^2))) /$$

$$((16 c^2 l^2 m^2 + 3 \hbar^2) (\sqrt{3}\, l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2) r_{min})) + (64 c^4$$

$$l^4 m^4 (16 \sqrt{3}\, c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3 \sqrt{3}\, l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3 m^2$$

$$\hbar^2 r_{min} + 36 l \hbar^4 r_{min} + 1024 c^4 l^4 m^4 r_{min}^2 + 384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 +$$

$$\sqrt{} (8 l \hbar^2 (16 c^2 l^2 m^2 + 3 \hbar^2) r_{min} (l (16 \sqrt{3}\, c^2 l^2 m^2 + 3 (1 + \sqrt{3}) \hbar^2) +$$

$$2 \sqrt{3} (16 c^2 l^2 m^2 + 3 \hbar^2) r_{min}) (\sqrt{3}\, l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)$$

$$r_{min}) + (l^2 \hbar^2 (16 \sqrt{3}\, c^2 l^2 m^2 + 3 (-1 + \sqrt{3}) \hbar^2) +$$

$$4 l (16 c^2 l^2 m^2 + 3 \hbar^2)^2 r_{min} + 4 (16 c^2 l^2 m^2 + 3 \hbar^2)^2 r_{min}^2)^2))^2) /$$

$$((16 c^2 l^2 m^2 + 3 \hbar^2)^2 (\sqrt{3}\, l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2) r_{min})^2) + (24 c^2 l^2$$

$$m^2 \hbar^2 (16 \sqrt{3}\, c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3 \sqrt{3}\, l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3 m^2$$

$$\hbar^2 r_{min} + 36 l \hbar^4 r_{min} + 1024 c^4 l^4 m^4 r_{min}^2 + 384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 +$$

$$\sqrt{} (8 l \hbar^2 (16 c^2 l^2 m^2 + 3 \hbar^2) r_{min} (l (16 \sqrt{3}\, c^2 l^2 m^2 + 3 (1 + \sqrt{3}) \hbar^2) +$$

$$2 \sqrt{3} (16 c^2 l^2 m^2 + 3 \hbar^2) r_{min}) (\sqrt{3}\, l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)$$

$$r_{min}) + (l^2 \hbar^2 (16 \sqrt{3}\, c^2 l^2 m^2 + 3 (-1 + \sqrt{3}) \hbar^2) +$$

$$4 l (16 c^2 l^2 m^2 + 3 \hbar^2)^2 r_{min} + 4 (16 c^2 l^2 m^2 + 3 \hbar^2)^2 r_{min}^2)^2))^2) /$$

$$((16 c^2 l^2 m^2 + 3 \hbar^2)^2 (\sqrt{3}\, l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2) r_{min})^2) +$$

$$(9 \hbar^4 (16 \sqrt{3}\, c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3 \sqrt{3}\, l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3 m^2$$

$$\hbar^2 r_{min} + 36 l \hbar^4 r_{min} + 1024 c^4 l^4 m^4 r_{min}^2 + 384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 +$$

$$\sqrt{} (8 l \hbar^2 (16 c^2 l^2 m^2 + 3 \hbar^2) r_{min} (l (16 \sqrt{3}\, c^2 l^2 m^2 + 3 (1 + \sqrt{3}) \hbar^2) +$$

$$2 \sqrt{3} (16 c^2 l^2 m^2 + 3 \hbar^2) r_{min}) (\sqrt{3}\, l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)$$

$$r_{min}) + (l^2 \hbar^2 (16 \sqrt{3}\, c^2 l^2 m^2 + 3 (-1 + \sqrt{3}) \hbar^2) +$$

$$4 l (16 c^2 l^2 m^2 + 3 \hbar^2)^2 r_{min} + 4 (16 c^2 l^2 m^2 + 3 \hbar^2)^2 r_{min}^2)^2))^2) /$$

$$(4 (16 c^2 l^2 m^2 + 3 \hbar^2)^2 (\sqrt{3}\, l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2) r_{min})^2))^2)))) /$$

$$(2 (16 c^2 l^2 m^2 + 3 \hbar^2) (16 \sqrt{3}\, c^2 l^4 m^2 \hbar^2 + 3 l^2 \hbar^4 +$$

$$3$$

$$\sqrt{3}$$

$$l^2$$

$$\hbar^4 + 1024$$

c^4
l^5
m^4
$r_{min} + 384$
c^2
l^3
m^2
$\hbar^2$
$r_{min} + 64$
$\sqrt{3}$
c^2
l^3
m^2
$\hbar^2$
$r_{min} + 36$
l
$\hbar^4$
$r_{min} + 12$
$\sqrt{3}$
l
$\hbar^4$
$r_{min} + 1024$
c^4
l^4
m^4
$r_{min}^2 + 384$
c^2
l^2
m^2
$\hbar^2$
$r_{min}^2 + 36$
$\hbar^4$
$r_{min}^2 +$

$$\sqrt{\left(8\, l\, \hbar^2 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right) r_{min} \left(l \left(16 \sqrt{3}\, c^2 l^2 m^2 + 3 \left(1 + \sqrt{3}\right) \hbar^2\right) + 2 \sqrt{3} \left(16 c^2 l^2 m^2 + 3 \hbar^2\right) r_{min}\right) \left(\sqrt{3}\, l\, \hbar^2 + \left(32 c^2 l^2 m^2 + 6 \hbar^2\right) r_{min}\right) + \left(l^2 \hbar^2 \left(16 \sqrt{3}\, c^2 l^2 m^2 + 3 \left(-1 + \sqrt{3}\right) \hbar^2\right) + 4\, l \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2 r_{min} + 4 \left(16 c^2 l^2 m^2 + 3 \hbar^2\right)^2 r_{min}^2\right)^2\right)} \;//\; N$$

численное приближение

Out[44]= 1.00032×10^{-11}

In[45]:= `l =.`

In[46]:= `l = lmax;`

In[47]:= R22 = $\Big(\Big(\left(\sqrt{3}\, l\, \hbar^2 + \left(32 c^2 l^2 m^2 + 6 \hbar^2\right) r_{min}\right) \Big(16 \sqrt{3}\, c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3 \sqrt{3}\, l^2 \hbar^4 +$

$1 \Big/ \left(\left(16 c^2 l^2 m^2 + 3 \hbar^2\right) \left(\sqrt{3}\, l\, \hbar^2 + \left(32 c^2 l^2 m^2 + 6 \hbar^2\right) r_{min}\right)\right) \;\; 256 c^4 l^5 m^4$

$\Big(16 \sqrt{3}\, c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3 \sqrt{3}\, l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3 m^2 \hbar^2 r_{min} +$

$36\, l\, \hbar^4 r_{min} + 1024 c^4 l^4 m^4 r_{min}^2 + 384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 +$

$$\sqrt{} \Big(8\, l\, \hbar^2 \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) r_{min} \Big(l \left(16 \sqrt{3}\, c^2\, l^2\, m^2 + 3 \left(1 + \sqrt{3}\right) \hbar^2\right) +$$

$$2 \sqrt{3} \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) r_{min}\Big) \left(\sqrt{3}\, l\, \hbar^2 + \left(32\, c^2\, l^2\, m^2 + 6\, \hbar^2\right) r_{min}\right) +$$

$$\Big(l^2\, \hbar^2 \left(16 \sqrt{3}\, c^2\, l^2\, m^2 + 3 \left(-1 + \sqrt{3}\right) \hbar^2\right) + 4\, l \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2 r_{min} +$$

$$4 \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2 r_{min}^2\Big)^2\Big)\Big) +$$

$$1 \Big/ \left(\left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) \left(\sqrt{3}\, l\, \hbar^2 + \left(32\, c^2\, l^2\, m^2 + 6\, \hbar^2\right) r_{min}\right)\right) \quad 96\, c^2\, l^3\, m^2$$

$$\hbar^2 \Big(16 \sqrt{3}\, c^2\, l^4\, m^2\, \hbar^2 - 3\, l^2\, \hbar^4 + 3 \sqrt{3}\, l^2\, \hbar^4 + 1024\, c^4\, l^5\, m^4\, r_{min} +$$

$$384\, c^2\, l^3\, m^2\, \hbar^2\, r_{min} + 36\, l\, \hbar^4\, r_{min} + 1024\, c^4\, l^4\, m^4\, r_{min}^2 +$$

$$384\, c^2\, l^2\, m^2\, \hbar^2\, r_{min}^2 + 36\, \hbar^4\, r_{min}^2 + \sqrt{} \Big(8\, l\, \hbar^2 \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) r_{min}$$

$$\left(l \left(16 \sqrt{3}\, c^2\, l^2\, m^2 + 3 \left(1 + \sqrt{3}\right) \hbar^2\right) + 2 \sqrt{3} \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) r_{min}\right)$$

$$\left(\sqrt{3}\, l\, \hbar^2 + \left(32\, c^2\, l^2\, m^2 + 6\, \hbar^2\right) r_{min}\right) + \Big(l^2\, \hbar^2 \left(16 \sqrt{3}\, c^2\, l^2\, m^2 + 3 \left(-1 + \sqrt{3}\right) \hbar^2\right) +$$

$$4\, l \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2 r_{min} + 4 \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2 r_{min}^2\Big)^2\Big)\Big) +$$

$$1 \Big/ \left(\left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) \left(\sqrt{3}\, l\, \hbar^2 + \left(32\, c^2\, l^2\, m^2 + 6\, \hbar^2\right) r_{min}\right)\right) \quad 9\, l\, \hbar^4$$

$$\Big(16 \sqrt{3}\, c^2\, l^4\, m^2\, \hbar^2 - 3\, l^2\, \hbar^4 + 3 \sqrt{3}\, l^2\, \hbar^4 + 1024\, c^4\, l^5\, m^4\, r_{min} +$$

$$384\, c^2\, l^3\, m^2\, \hbar^2\, r_{min} + 36\, l\, \hbar^4\, r_{min} + 1024\, c^4\, l^4\, m^4\, r_{min}^2 +$$

$$384\, c^2\, l^2\, m^2\, \hbar^2\, r_{min}^2 + 36\, \hbar^4\, r_{min}^2 + \sqrt{} \Big(8\, l\, \hbar^2 \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) r_{min}$$

$$\left(l \left(16 \sqrt{3}\, c^2\, l^2\, m^2 + 3 \left(1 + \sqrt{3}\right) \hbar^2\right) + 2 \sqrt{3} \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) r_{min}\right)$$

$$\left(\sqrt{3}\, l\, \hbar^2 + \left(32\, c^2\, l^2\, m^2 + 6\, \hbar^2\right) r_{min}\right) + \Big(l^2\, \hbar^2 \left(16 \sqrt{3}\, c^2\, l^2\, m^2 + 3 \left(-1 + \sqrt{3}\right) \hbar^2\right) +$$

$$4\, l \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2 r_{min} + 4 \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2 r_{min}^2\Big)^2\Big)\Big) +$$

$$\Big(64\, c^4\, l^4\, m^4 \Big(16 \sqrt{3}\, c^2\, l^4\, m^2\, \hbar^2 - 3\, l^2\, \hbar^4 + 3 \sqrt{3}\, l^2\, \hbar^4 + 1024\, c^4\, l^5\, m^4\, r_{min} + 384$$

$$c^2\, l^3\, m^2\, \hbar^2\, r_{min} + 36\, l\, \hbar^4\, r_{min} + 1024\, c^4\, l^4\, m^4\, r_{min}^2 + 384\, c^2\, l^2\, m^2\, \hbar^2\, r_{min}^2 + 36\, \hbar^4\, r_{min}^2 +$$

$$\sqrt{} \Big(8\, l\, \hbar^2 \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) r_{min} \Big(l \left(16 \sqrt{3}\, c^2\, l^2\, m^2 + 3 \left(1 + \sqrt{3}\right) \hbar^2\right) +$$

$$2 \sqrt{3} \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) r_{min}\Big) \left(\sqrt{3}\, l\, \hbar^2 + \left(32\, c^2\, l^2\, m^2 + 6\, \hbar^2\right) r_{min}\right) +$$

$$\Big(l^2\, \hbar^2 \left(16 \sqrt{3}\, c^2\, l^2\, m^2 + 3 \left(-1 + \sqrt{3}\right) \hbar^2\right) + 4\, l \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2$$

$$r_{min} + 4 \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2 r_{min}^2\Big)^2\Big)\Big)^2\Big) \Big/$$

$$\left(\left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2 \left(\sqrt{3}\, l\, \hbar^2 + \left(32\, c^2\, l^2\, m^2 + 6\, \hbar^2\right) r_{min}\right)^2\right) +$$

$$\Big(24\, c^2\, l^2\, m^2\, \hbar^2 \Big(16 \sqrt{3}\, c^2\, l^4\, m^2\, \hbar^2 - 3\, l^2\, \hbar^4 + 3 \sqrt{3}\, l^2\, \hbar^4 + 1024\, c^4\, l^5\, m^4\, r_{min} + 384\, c^2$$

$$l^3\, m^2\, \hbar^2\, r_{min} + 36\, l\, \hbar^4\, r_{min} + 1024\, c^4\, l^4\, m^4\, r_{min}^2 + 384\, c^2\, l^2\, m^2\, \hbar^2\, r_{min}^2 + 36\, \hbar^4\, r_{min}^2 +$$

$$\sqrt{} \Big(8\, l\, \hbar^2 \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) r_{min} \Big(l \left(16 \sqrt{3}\, c^2\, l^2\, m^2 + 3 \left(1 + \sqrt{3}\right) \hbar^2\right) +$$

$$2 \sqrt{3} \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) r_{min}\Big) \left(\sqrt{3}\, l\, \hbar^2 + \left(32\, c^2\, l^2\, m^2 + 6\, \hbar^2\right) r_{min}\right) +$$

$$\Big(l^2\, \hbar^2 \left(16 \sqrt{3}\, c^2\, l^2\, m^2 + 3 \left(-1 + \sqrt{3}\right) \hbar^2\right) + 4\, l \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2$$

$$r_{min} + 4 \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2 r_{min}^2\Big)^2\Big)\Big)^2\Big) \Big/$$

$$\left(\left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2 \left(\sqrt{3}\, l\, \hbar^2 + \left(32\, c^2\, l^2\, m^2 + 6\, \hbar^2\right) r_{min}\right)^2\right) +$$

$$\Big(9\, \hbar^4 \Big(16 \sqrt{3}\, c^2\, l^4\, m^2\, \hbar^2 - 3\, l^2\, \hbar^4 + 3 \sqrt{3}\, l^2\, \hbar^4 + 1024\, c^4\, l^5\, m^4\, r_{min} + 384\, c^2\, l^3\, m^2\, \hbar^2\, r_{min} +$$

$$36\, l\, \hbar^4\, r_{min} + 1024\, c^4\, l^4\, m^4\, r_{min}^2 + 384\, c^2\, l^2\, m^2\, \hbar^2\, r_{min}^2 + 36\, \hbar^4\, r_{min}^2 +$$

$$\sqrt{\;}\Big(8\, l\, \hbar^2\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)\, r_{min}\, \big(l\, (16\, \sqrt{3}\, c^2\, l^2\, m^2 + 3\, (1 + \sqrt{3})\, \hbar^2) +$$

$$2\, \sqrt{3}\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)\, r_{min}\big)\, (\sqrt{3}\, l\, \hbar^2 + (32\, c^2\, l^2\, m^2 + 6\, \hbar^2)\, r_{min}) +$$

$$\big(l^2\, \hbar^2\, (16\, \sqrt{3}\, c^2\, l^2\, m^2 + 3\, (-1 + \sqrt{3})\, \hbar^2) + 4\, l\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)^2$$

$$r_{min} + 4\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)^2\, r_{min}^2\big)^2\Big)\Big)^2\Big) \Big/$$

$$\Big(4\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)^2\, (\sqrt{3}\, l\, \hbar^2 + (32\, c^2\, l^2\, m^2 + 6\, \hbar^2)\, r_{min})^2\Big) +$$

$$\sqrt{\;}\Big(1 \Big/ \big(2\, (\sqrt{3}\, l\, \hbar^2 + (32\, c^2\, l^2\, m^2 + 6\, \hbar^2)\, r_{min})^3\big)\, l\, \hbar^2\, \Big(16\, \sqrt{3}\, c^2\, l^4\, m^2\, \hbar^2 -$$

$$3\, l^2\, \hbar^4 + 3\, \sqrt{3}\, l^2\, \hbar^4 + 1024\, c^4\, l^5\, m^4\, r_{min} + 384\, c^2\, l^3\, m^2\, \hbar^2\, r_{min} + 36\, l\, \hbar^4\, r_{min} +$$

$$1024\, c^4\, l^4\, m^4\, r_{min}^2 + 384\, c^2\, l^2\, m^2\, \hbar^2\, r_{min}^2 + 36\, \hbar^4\, r_{min}^2 + \sqrt{\;}\Big(8\, l\, \hbar^2\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)$$

$$r_{min}\, \big(l\, (16\, \sqrt{3}\, c^2\, l^2\, m^2 + 3\, (1 + \sqrt{3})\, \hbar^2) + 2\, \sqrt{3}\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)\, r_{min}\big)$$

$$(\sqrt{3}\, l\, \hbar^2 + (32\, c^2\, l^2\, m^2 + 6\, \hbar^2)\, r_{min}) + \big(l^2\, \hbar^2\, (16\, \sqrt{3}\, c^2\, l^2\, m^2 + 3\, (-1 + \sqrt{3})$$

$$\hbar^2) + 4\, l\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)^2\, r_{min} + 4\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)^2\, r_{min}^2\big)^2\Big)\Big)$$

$$\Big(16\, \sqrt{3}\, c^2\, l^4\, m^2\, \hbar^2 + 3\, l^2\, \hbar^4 + 3\, \sqrt{3}\, l^2\, \hbar^4 + 1024\, c^4\, l^5\, m^4\, r_{min} + 384\, c^2\, l^3\, m^2$$

$$\hbar^2\, r_{min} + 64\, \sqrt{3}\, c^2\, l^3\, m^2\, \hbar^2\, r_{min} + 36\, l\, \hbar^4\, r_{min} + 12\, \sqrt{3}\, l\, \hbar^4\, r_{min} + 1024\, c^4\, l^4\, m^4\, r_{min}^2 +$$

$$384\, c^2\, l^2\, m^2\, \hbar^2\, r_{min}^2 + 36\, \hbar^4\, r_{min}^2 + \sqrt{\;}\Big(8\, l\, \hbar^2\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)\, r_{min}$$

$$\big(l\, (16\, \sqrt{3}\, c^2\, l^2\, m^2 + 3\, (1 + \sqrt{3})\, \hbar^2) + 2\, \sqrt{3}\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)\, r_{min}\big)$$

$$(\sqrt{3}\, l\, \hbar^2 + (32\, c^2\, l^2\, m^2 + 6\, \hbar^2)\, r_{min}) + \big(l^2\, \hbar^2\, (16\, \sqrt{3}\, c^2\, l^2\, m^2 + 3\, (-1 + \sqrt{3})$$

$$\hbar^2) + 4\, l\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)^2\, r_{min} + 4\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)^2\, r_{min}^2\big)^2\Big)\Big)$$

$$\Big(144\, c^2\, l^4\, m^2\, \hbar^2 + 27\, l^2\, \hbar^4 + 3\, \sqrt{3}\, l^2\, \hbar^4 + 2048\, \sqrt{3}\, c^4\, l^5\, m^4\, r_{min} +$$

$$192\, c^2\, l^3\, m^2\, \hbar^2\, r_{min} + 768\, \sqrt{3}\, c^2\, l^3\, m^2\, \hbar^2\, r_{min} + 36\, l\, \hbar^4\, r_{min} + 72\, \sqrt{3}\, l\, \hbar^4\, r_{min} +$$

$$1024\, \sqrt{3}\, c^4\, l^4\, m^4\, r_{min}^2 + 384\, \sqrt{3}\, c^2\, l^2\, m^2\, \hbar^2\, r_{min}^2 + 36\, \sqrt{3}\, \hbar^4\, r_{min}^2 +$$

$$\sqrt{3}\, \sqrt{\;}\Big(8\, l\, \hbar^2\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)\, r_{min}\, \big(l\, (16\, \sqrt{3}\, c^2\, l^2\, m^2 + 3\, (1 + \sqrt{3})\, \hbar^2) +$$

$$2\, \sqrt{3}\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)\, r_{min}\big)\, (\sqrt{3}\, l\, \hbar^2 + (32\, c^2\, l^2\, m^2 + 6\, \hbar^2)\, r_{min}) +$$

$$\big(l^2\, \hbar^2\, (16\, \sqrt{3}\, c^2\, l^2\, m^2 + 3\, (-1 + \sqrt{3})\, \hbar^2) + 4\, l\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)^2$$

$$r_{min} + 4\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)^2\, r_{min}^2\big)^2\Big)\Big) +$$

$$\Big(16\, \sqrt{3}\, c^2\, l^4\, m^2\, \hbar^2 - 3\, l^2\, \hbar^4 + 3\, \sqrt{3}\, l^2\, \hbar^4 + \Big(256\, c^4\, l^5\, m^4\, \Big(16\, \sqrt{3}\, c^2\, l^4\, m^2\, \hbar^2 -$$

$$3\, l^2\, \hbar^4 + 3\, \sqrt{3}\, l^2\, \hbar^4 + 1024\, c^4\, l^5\, m^4\, r_{min} + 384\, c^2\, l^3\, m^2\, \hbar^2\, r_{min} +$$

$$36\, l\, \hbar^4\, r_{min} + 1024\, c^4\, l^4\, m^4\, r_{min}^2 + 384\, c^2\, l^2\, m^2\, \hbar^2\, r_{min}^2 + 36\, \hbar^4\, r_{min}^2 +$$

$$\sqrt{\;}\Big(8\, l\, \hbar^2\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)\, r_{min}\, \big(l\, (16\, \sqrt{3}\, c^2\, l^2\, m^2 + 3\, (1 + \sqrt{3})\, \hbar^2) +$$

$$2\, \sqrt{3}\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)\, r_{min}\big)\, (\sqrt{3}\, l\, \hbar^2 + (32\, c^2\, l^2\, m^2 + 6\, \hbar^2)\, r_{min}) +$$

$$\big(l^2\, \hbar^2\, (16\, \sqrt{3}\, c^2\, l^2\, m^2 + 3\, (-1 + \sqrt{3})\, \hbar^2) + 4\, l$$

$$(16\, c^2\, l^2\, m^2 + 3\, \hbar^2)^2\, r_{min} + 4\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)^2\, r_{min}^2\big)^2\Big)\Big)\Big) \Big/$$

$$\Big((16\, c^2\, l^2\, m^2 + 3\, \hbar^2)\, (\sqrt{3}\, l\, \hbar^2 + (32\, c^2\, l^2\, m^2 + 6\, \hbar^2)\, r_{min})\Big) + \Big(96\, c^2$$

$$l^3\, m^2\, \hbar^2\, \Big(16\, \sqrt{3}\, c^2\, l^4\, m^2\, \hbar^2 - 3\, l^2\, \hbar^4 + 3\, \sqrt{3}\, l^2\, \hbar^4 + 1024\, c^4\, l^5\, m^4\, r_{min} + 384\, c^2\, l^3$$

$$m^2\, \hbar^2\, r_{min} + 36\, l\, \hbar^4\, r_{min} + 1024\, c^4\, l^4\, m^4\, r_{min}^2 + 384\, c^2\, l^2\, m^2\, \hbar^2\, r_{min}^2 + 36\, \hbar^4\, r_{min}^2 +$$

$$\sqrt{\;}\Big(8\, l\, \hbar^2\, (16\, c^2\, l^2\, m^2 + 3\, \hbar^2)\, r_{min}\, \big(l\, (16\, \sqrt{3}\, c^2\, l^2\, m^2 + 3\, (1 + \sqrt{3})\, \hbar^2) +$$

$$2\sqrt{3}\ (16 c^2 l^2 m^2 + 3 \hbar^2)\ r_{min})\ (\sqrt{3}\ l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)\ r_{min}) +$$
$$(l^2 \hbar^2\ (16 \sqrt{3}\ c^2 l^2 m^2 + 3\ (-1 + \sqrt{3})\ \hbar^2) + 4 l$$
$$(16 c^2 l^2 m^2 + 3 \hbar^2)^2\ r_{min} + 4\ (16 c^2 l^2 m^2 + 3 \hbar^2)^2\ r_{min}^2)^2)))\Big/$$
$$((16 c^2 l^2 m^2 + 3 \hbar^2)\ (\sqrt{3}\ l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)\ r_{min})) +$$
$$(9 l \hbar^4\ (16 \sqrt{3}\ c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3 \sqrt{3}\ l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3$$
$$m^2 \hbar^2 r_{min} + 36 l \hbar^4 r_{min} + 1024 c^4 l^4 m^4 r_{min}^2 + 384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 +$$
$$\sqrt{}(8 l \hbar^2\ (16 c^2 l^2 m^2 + 3 \hbar^2)\ r_{min}\ (l\ (16 \sqrt{3}\ c^2 l^2 m^2 + 3\ (1 + \sqrt{3})\ \hbar^2) +$$
$$2\sqrt{3}\ (16 c^2 l^2 m^2 + 3 \hbar^2)\ r_{min})\ (\sqrt{3}\ l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)\ r_{min}) +$$
$$(l^2 \hbar^2\ (16 \sqrt{3}\ c^2 l^2 m^2 + 3\ (-1 + \sqrt{3})\ \hbar^2) + 4 l$$
$$(16 c^2 l^2 m^2 + 3 \hbar^2)^2\ r_{min} + 4\ (16 c^2 l^2 m^2 + 3 \hbar^2)^2\ r_{min}^2)^2)))\Big/$$
$$((16 c^2 l^2 m^2 + 3 \hbar^2)\ (\sqrt{3}\ l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)\ r_{min})) + (64 c^4$$
$$l^4 m^4\ (16 \sqrt{3}\ c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3 \sqrt{3}\ l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3 m^2$$
$$\hbar^2 r_{min} + 36 l \hbar^4 r_{min} + 1024 c^4 l^4 m^4 r_{min}^2 + 384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 +$$
$$\sqrt{}(8 l \hbar^2\ (16 c^2 l^2 m^2 + 3 \hbar^2)\ r_{min}\ (l\ (16 \sqrt{3}\ c^2 l^2 m^2 + 3\ (1 + \sqrt{3})\ \hbar^2) +$$
$$2\sqrt{3}\ (16 c^2 l^2 m^2 + 3 \hbar^2)\ r_{min})\ (\sqrt{3}\ l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)$$
$$r_{min}) + (l^2 \hbar^2\ (16 \sqrt{3}\ c^2 l^2 m^2 + 3\ (-1 + \sqrt{3})\ \hbar^2) +$$
$$4 l\ (16 c^2 l^2 m^2 + 3 \hbar^2)^2\ r_{min} + 4\ (16 c^2 l^2 m^2 + 3 \hbar^2)^2\ r_{min}^2)^2))^2)\Big/$$
$$((16 c^2 l^2 m^2 + 3 \hbar^2)^2\ (\sqrt{3}\ l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)\ r_{min})^2) + (24 c^2 l^2$$
$$m^2 \hbar^2\ (16 \sqrt{3}\ c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3 \sqrt{3}\ l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3 m^2$$
$$\hbar^2 r_{min} + 36 l \hbar^4 r_{min} + 1024 c^4 l^4 m^4 r_{min}^2 + 384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 +$$
$$\sqrt{}(8 l \hbar^2\ (16 c^2 l^2 m^2 + 3 \hbar^2)\ r_{min}\ (l\ (16 \sqrt{3}\ c^2 l^2 m^2 + 3\ (1 + \sqrt{3})\ \hbar^2) +$$
$$2\sqrt{3}\ (16 c^2 l^2 m^2 + 3 \hbar^2)\ r_{min})\ (\sqrt{3}\ l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)$$
$$r_{min}) + (l^2 \hbar^2\ (16 \sqrt{3}\ c^2 l^2 m^2 + 3\ (-1 + \sqrt{3})\ \hbar^2) +$$
$$4 l\ (16 c^2 l^2 m^2 + 3 \hbar^2)^2\ r_{min} + 4\ (16 c^2 l^2 m^2 + 3 \hbar^2)^2\ r_{min}^2)^2))^2)\Big/$$
$$((16 c^2 l^2 m^2 + 3 \hbar^2)^2\ (\sqrt{3}\ l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)\ r_{min})^2) +$$
$$(9 \hbar^4\ (16 \sqrt{3}\ c^2 l^4 m^2 \hbar^2 - 3 l^2 \hbar^4 + 3 \sqrt{3}\ l^2 \hbar^4 + 1024 c^4 l^5 m^4 r_{min} + 384 c^2 l^3 m^2$$
$$\hbar^2 r_{min} + 36 l \hbar^4 r_{min} + 1024 c^4 l^4 m^4 r_{min}^2 + 384 c^2 l^2 m^2 \hbar^2 r_{min}^2 + 36 \hbar^4 r_{min}^2 +$$
$$\sqrt{}(8 l \hbar^2\ (16 c^2 l^2 m^2 + 3 \hbar^2)\ r_{min}\ (l\ (16 \sqrt{3}\ c^2 l^2 m^2 + 3\ (1 + \sqrt{3})\ \hbar^2) +$$
$$2\sqrt{3}\ (16 c^2 l^2 m^2 + 3 \hbar^2)\ r_{min})\ (\sqrt{3}\ l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)$$
$$r_{min}) + (l^2 \hbar^2\ (16 \sqrt{3}\ c^2 l^2 m^2 + 3\ (-1 + \sqrt{3})\ \hbar^2) +$$
$$4 l\ (16 c^2 l^2 m^2 + 3 \hbar^2)^2\ r_{min} + 4\ (16 c^2 l^2 m^2 + 3 \hbar^2)^2\ r_{min}^2)^2))^2)\Big/$$
$$(4\ (16 c^2 l^2 m^2 + 3 \hbar^2)^2\ (\sqrt{3}\ l \hbar^2 + (32 c^2 l^2 m^2 + 6 \hbar^2)\ r_{min})^2))^2)))\Big/$$
$$(2\ (16 c^2 l^2 m^2 + 3 \hbar^2)\ (16 \sqrt{3}\ c^2 l^4 m^2 \hbar^2 + 3 l^2 \hbar^4 +$$
$$3$$

$\sqrt{3}$
l^2
$\hbar^4 + 1024$
c^4
l^5
m^4
$r_{min} + 384$
c^2
l^3
m^2
$\hbar^2$
$r_{min} + 64$
$\sqrt{3}$
c^2
l^3
m^2
$\hbar^2$
$r_{min} + 36$
l
$\hbar^4$
$r_{min} + 12$
$\sqrt{3}$
l
$\hbar^4$
$r_{min} + 1024$
c^4
l^4
m^4
$r_{min}^2 + 384$
c^2
l^2
m^2
$\hbar^2$
$r_{min}^2 + 36$
$\hbar^4$
$r_{min}^2 +$

$$\sqrt{\left(8\, l\, \hbar^2 \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) r_{min} \left(l \left(16 \sqrt{3}\, c^2\, l^2\, m^2 + 3\left(1 + \sqrt{3}\right) \hbar^2\right) + 2\sqrt{3}\, \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right) r_{min}\right) \left(\sqrt{3}\, l\, \hbar^2 + \left(32\, c^2\, l^2\, m^2 + 6\, \hbar^2\right) r_{min}\right) + \left(l^2\, \hbar^2 \left(16\sqrt{3}\, c^2\, l^2\, m^2 + 3\left(-1 + \sqrt{3}\right) \hbar^2\right) + 4\, l \left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2 r_{min} + 4\left(16\, c^2\, l^2\, m^2 + 3\, \hbar^2\right)^2 r_{min}^2\right)^2\right)}\ \text{// N}$$

численное приближение

Out[47]= 5.00008×10^{-11}

In[48]:= `l =.`

In[49]:= $a11 := R11 - lmin \dfrac{R12 - R11}{lmax - lmin}$

In[50]:= $k11 := \dfrac{R12 - R11}{lmax - lmin}$

In[51]:= $a22 := R21 - lmin \frac{R22 - R21}{lmax - lmin}$

In[52]:= $k22 := \frac{R22 - R21}{lmax - lmin}$

In[53]:= a1 := a11

In[54]:= k1 := k11

In[55]:= a2 := a22

In[56]:= k2 := k22

In[57]:= $r_1 := a1 + k1\, l$

In[58]:= $r_2 := a2 + k2\, l$

In[59]:= $P_2 := R21 + (R22 - R21) \frac{l - lmin}{lmax - lmin}$

In[60]:= $u := \frac{lmax - lmin}{10}$

In[61]:= l1 := lmin + u

In[62]:= l2 := lmin + 2 u

In[63]:= l3 := lmin + 3 u

In[64]:= l4 := lmin + 4 u

In[65]:= l5 := lmin + 5 u

In[66]:= l6 := lmin + 6 u

In[67]:= l7 := lmin + 7 u

In[68]:= l8 := lmin + 8 u

In[69]:= l9 := lmin + 9 u

Inoculating values:

In[70]:= $a_2 := a2$

In[71]:= $k_2 := k2$

In[72]:= $\Delta l_{min} := \frac{\sqrt{3}\, l^2}{\sqrt{3}\, l + 6\, r_{min} + 32\, r_{min}\, l^2 \times \frac{m^2 c^2}{\hbar^2}}$

In[73]:= $\Delta l_1 := \Delta l_{min} \,/.\, r_{min} \to r_1$

In[74]:= $\Delta l_2 := \Delta l_1 \,/.\, r_1 \to r_2$

In[75]:= m0 = m;

In[76]:=
```
If[(m != uplmνe || m != uplmνμ || m != uplmντ), For[n = 2;
условный оператор                                цикл ДЛЯ
   l = lmin, Pn ≤ rmax, n++, l =.;
   m =.;
   c =.;
   ħ =.;
```

```
a2 =.;
k2 =.;
a_n =.;
k_n =.;
Δl_n =.;
```

$$\Delta l_{n+1} = \frac{\sqrt{3}\ l^2}{\sqrt{3}\ l + 6\, R_{n+1} + 32\, R_{n+1}\, l^2 \times \frac{m^2 c^2}{\hbar^2}};$$

$$\Delta l_n = \frac{\sqrt{3}\ l^2}{\sqrt{3}\ l + 6\, r_n + 32\, r_n\, l^2 \times \frac{m^2 c^2}{\hbar^2}};$$

$$P_{n+1} = R_{n+1} \text{ /. Flatten}\left[\text{Solve}\left[R_{n+1} - \frac{1}{2}\left(l - \Delta l_{n+1}\right) == r_n + \frac{1}{2}\left(l - \Delta l_n\right),\ R_{n+1}\right]\right][[1]];$$

уплостить решить уравнения

```
l = lmin;
m = m0;
c = 299 792 458;
ℏ = 1.05361 × 10^-34;
a2 = a22;
k2 = k22;
```

$$a_n = y_{1,n} - (y_{2,n} - y_{1,n}) \times \frac{lmin}{l1 - lmin};$$

$$k_n = \frac{y_{2,n} - y_{1,n}}{l1 - lmin};$$

$If[P_{n+1} > 0,\ l =.;$

условный оператор

```
m =.;
c =.;
ℏ =.;
a2 =.;
k2 =.;
a_n =.;
k_n =.;
```

$$r_{n+1} = R_{n+1} \text{ /. Flatten}\left[\text{Solve}\left[R_{n+1} - \frac{1}{2}\left(l - \Delta l_{n+1}\right) == r_n + \frac{1}{2}\left(l - \Delta l_n\right),\ R_{n+1}\right]\right][[1]];$$

уплостить решить уравнения

```
l = lmin;
m = m0;
c = 299 792 458;
ℏ = 1.05361 × 10^-34;
a2 = a22;
k2 = k22;
```

$$a_n = y_{1,n} - (y_{2,n} - y_{1,n}) \times \frac{lmin}{l1 - lmin};$$

$$k_n = \frac{y_{2,n} - y_{1,n}}{l1 - lmin};$$

$y_{1,n+1} = r_{n+1};$

```
l =.;
l = l1;
```

$y_{2,n+1} = r_{n+1};$

$$d_{n+1} = y_{1,n+1} + (y_{2,n+1} - y_{1,n+1}) \times \frac{x - lmin}{l1 - lmin};$$

```
x = l2;
```

```
l =.;
l = 12;
If[Abs[r_{n+1} - d_{n+1}] < 10^-3, l =.;
  абсолютное значение
 x =.;
 l = 13;
 x = 13;
 If[Abs[r_{n+1} - d_{n+1}] < 10^-2.8, l =.;
   абсолютное значение
  x =.;
  l = 14;
  x = 14;
  If[Abs[r_{n+1} - d_{n+1}] < 10^-2.8, l =.;
  ⋯ абсолютное значение
   x =.;
   l = 15;
   x = 15;
   If[Abs[r_{n+1} - d_{n+1}] < 10^-2.8, l =.;
     абсолютное значение
    x =.;
    l = 16;
    x = 16;
    If[Abs[r_{n+1} - d_{n+1}] < 10^-2.7, l =.;
    ⋯ абсолютное значение
     x =.;
     l = 17;
     x = 17;
     If[Abs[r_{n+1} - d_{n+1}] < 10^-2.7, l =.;
     ⋯ абсолютное значение
      x =.;
      l = 18;
      x = 18;
      If[Abs[r_{n+1} - d_{n+1}] < 10^-2.7, l =.;
      ⋯ абсолютное значение
       x =.;
       l = 19;
       x = 19;
       If[Abs[r_{n+1} - d_{n+1}] < 10^-2.6, l =.;
       ⋯ абсолютное значение
        x =.;
        l = lmax;
        x = lmax;
        If[Abs[r_{n+1} - d_{n+1}] < 10^-2.6 , l =.;
        ⋯ абсолютное значение
         x =.;
         r_{n+1} =.;
         a_n =.;
         k_n =.;
         r_{n+1} = a_{n+1} + k_{n+1} l;
         P_{n+1} =.;
         P_{n+1} = a_{n+1} + k_{n+1} l;
         a_{n+1} = y_{1,n+1} - (y_{2,n+1} - y_{1,n+1}) × lmin/(l1 - lmin);
```

```
            k_{n+1} = (y_{2,n+1} - y_{1,n+1}) / (l1 - lmin);
            l = lmin, Null];, Null];, Null];, Null];, Null];, Null];, Null];, Null];,
                      пустой   пустой   пустой   пустой   пустой   пустой   пустой   пустой
   Null];, P_{n+1} =.;
   пустой
  l =.;
  m =.;
  c =.;
  ħ =.;
  a2 =.;
  k2 =.;
  a_n =.;
  k_n =.;
  P_{n+1} = R_{n+1} /. Flatten[Solve[R_{n+1} - 1/2 (l - Δl_{n+1}) == r_n + 1/2 (l - Δl_n), R_{n+1}]][[2]];
                       уплостить решить уравнения

  r_{n+1} = R_{n+1} /. Flatten[Solve[R_{n+1} - 1/2 (l - Δl_{n+1}) == r_n + 1/2 (l - Δl_n), R_{n+1}]][[2]];
                       уплостить решить уравнения

  l = lmin;
  m = m0;
  c = 299 792 458;
  ħ = 1.05361 × 10^-34;
  a2 = a22;
  k2 = k22;
  a_n = y_{1,n} - (y_{2,n} - y_{1,n}) × lmin / (l1 - lmin);
  k_n = (y_{2,n} - y_{1,n}) / (l1 - lmin);
  y_{1,n+1} = r_{n+1};
  l =.;
  l = l1;
  y_{2,n+1} = r_{n+1};
  d_{n+1} = y_{1,n+1} + (y_{2,n+1} - y_{1,n+1}) × (x - lmin) / (l1 - lmin);
  x = l2;
  l =.;
  l = l2;
  If[Abs[r_{n+1} - d_{n+1}] < 10^-3, l =.;
     абсолютное значение
   x =.;
   l = l3;
   x = l3;
   If[Abs[r_{n+1} - d_{n+1}] < 10^-2.8, l =.;
      абсолютное значение
    x =.;
    l = l4;
    x = l4;
    If[Abs[r_{n+1} - d_{n+1}] < 10^-2.8, l =.;
       абсолютное значение
     x =.;
     l = l5;
     x = l5;
```

```
If[Abs[r_{n+1} - d_{n+1}] < 10^-2.8, l =.;
   абсолютное значение
 x =.;
 l = 16;
 x = 16;
 If[Abs[r_{n+1} - d_{n+1}] < 10^-2.7, l =.;
    абсолютное значение
  x =.;
  l = 17;
  x = 17;
  If[Abs[r_{n+1} - d_{n+1}] < 10^-2.7, l =.;
     абсолютное значение
   x =.;
   l = 18;
   x = 18;
   If[Abs[r_{n+1} - d_{n+1}] < 10^-2.7, l =.;
      абсолютное значение
    x =.;
    l = 19;
    x = 19;
    If[Abs[r_{n+1} - d_{n+1}] < 10^-2.6, l =.;
       абсолютное значение
     x =.;
     l = lmax;
     x = lmax;
     If[Abs[r_{n+1} - d_{n+1}] < 10^-2.6 , l =.;
        абсолютное значение
      x =.;
      r_{n+1} =.;
      a_n =.;
      k_n =.;
      r_{n+1} = a_{n+1} + k_{n+1} l;
      P_{n+1} =.;
      P_{n+1} = a_{n+1} + k_{n+1} l;
      a_{n+1} = y_{1,n+1} - (y_{2,n+1} - y_{1,n+1}) × lmin/(l1 - lmin);
      k_{n+1} = (y_{2,n+1} - y_{1,n+1})/(l1 - lmin);
      l = lmin, Null];, Null];, Null];, Null];, Null];, Null];, Null];, Null];,
                пустой  пустой  пустой  пустой  пустой  пустой  пустой  пустой
Null];];], Null] // Timing
           пустой  затраченное время
```

Out[76]= {2674., Null}

In[77]:= n

Out[77]= 9281

In[78]:= l =.

In[79]:= Table[$a_k = y_{1,k} - (y_{2,k} - y_{1,k}) \times \frac{lmin}{l1 - lmin}$, {k, 3, n}];
таблица значений

In[80]:= $\text{Table}\left[k_i = \frac{y_{2,i} - y_{1,i}}{l1 - lmin}, \{i, 3, n\}\right];$
таблица значений

In[81]:= $\text{Table}[r_j = a_j + k_j\, l, \{j, 3, n\}];$
таблица значений

In[82]:= $l = lmin;$

In[83]:= r_n

Out[83]= 2.5×10^{-11}

In[84]:= r_{n-1}

Out[84]= 2.49984×10^{-11}

In[85]:= $l =.$

In[86]:= $\text{Table}\left[\Delta l_k = \frac{\sqrt{3}\, l^2}{\sqrt{3}\, l + 6\, r_k + 32\, r_k\, l^2 \times \frac{m^2\, c^2}{\hbar^2}}, \{k, 3, n\}\right];$
таблица значений

In[87]:= $N1_{min} := \frac{\pi}{\text{ArcTan}\left[\frac{l - \Delta l_{min}}{2\sqrt{r_{min}^2 - \frac{1}{4}(l - \Delta l_{min})^2}}\right]}$

In[88]:= $\text{Table}\left[N1_i = \frac{\pi}{\text{ArcTan}\left[\frac{l - \Delta l_i}{2\sqrt{r_i^2 - \frac{1}{4}(l - \Delta l_i)^2}}\right]}, \{i, 1, n\}\right];$
таблица значений

In[89]:= $\text{Reduce}\left[r_n \geq \frac{1}{2}\left(l - \frac{\sqrt{3}\, l^2}{\sqrt{3}\, l + 6\, r_n + 32\, r_n\, l^2 \times \frac{m^2\, c^2}{\hbar^2}}\right), l\right]$
привести

... Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[89]= $-1.07745 \times 10^{-15} \leq l \leq -1.07743 \times 10^{-15} \,||\, l > -1.07739 \times 10^{-15}$

In[90]:= $\text{Reduce}\left[r_{min} \geq \frac{1}{2}\left(l - \frac{\sqrt{3}\, l^2}{\sqrt{3}\, l + 6\, r_{min} + 32\, r_{min}\, l^2 \times \frac{m^2\, c^2}{\hbar^2}}\right), l\right]$
привести

... Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Out[90]= $l \leq 2.00008 \times 10^{-11}$

In[91]:= $l = lmax;$

In[92]:= $N1_{min}$

Out[92]= 2.00094

In[93]:= $l =.$

In[94]:= $l = lmin;$

In[95]:= $N1_{min}$

Out[95]= 38 876.8

In[96]:= l =.

In[97]:= $Int1 = NIntegrate\left[\left(\frac{N1_{min}^2}{2} - N1_{min} + 2\right)\sqrt{\frac{\hbar^2 c^2}{16} \times \frac{3}{\left(l - \Delta l_{min}\right)^2} + m^2 c^4} + \sum_{j=1}^{n-1}\left(\left(\frac{N1_j^2}{2} - N1_j + 2\right)\sqrt{\frac{\hbar^2 c^2}{16} \times \frac{3}{\left(l - \Delta l_j\right)^2} + m^2 c^4}\right), \{l, lmin, lmax\}\right]$

квадратурное интегрирование

Out[97]= 9.1214×10^{-12}

In[98]:= $N\left[\frac{1}{lP} Na\, \Delta_1\, Int1\right]$

численное приближение

Out[98]= 3.08206×10^{103}

$N\left[\frac{1}{lP} Na\, \Delta_2\, Int1\right]$

численное приближение

In[99]:= $NIntegrate\left[\left(\frac{N1_{min}^2}{2} - N1_{min} + 2\right)\sqrt{\frac{\hbar^2 c^2}{16} \times \frac{3}{\left(l - \Delta l_{min}\right)^2} + m^2 c^4}, \{l, lmin, lmax\}\right]$

квадратурное интегрирование

Out[99]= 5.1712×10^{-18}

More precise value

In[100]:= $N\left[\frac{4.17191 \times 10^{103}}{\frac{1}{lP} Na\, VA \int_{lmin}^{lmax} \frac{1}{l^3}\sqrt{\frac{\hbar^2 c^2}{16} \times \frac{3}{l^2} + m^2 c^4}\, dl}\right]$

численное приближение

... Power: Infinite expression $\frac{1}{0.}$ encountered.

Out[100]= ComplexInfinity

In[101]:= VA =.

In[102]:= $N\left[\frac{4}{3} \pi\, r_{max}^3\right]$

численное приближение

Out[102]= 6.54498×10^{-32}

In[103]:= $VA := \frac{4}{3} \pi\, r_{max}^3$

In[104]:= $N\left[\frac{4.17191\times 10^{103}}{\frac{1}{lP}\,Na\,VA\int_{lmin}^{lmax}\frac{1}{l^3}\sqrt{\frac{\hbar^2 c^2}{16}\times\frac{3}{l^2}+m^2 c^4}\,dl}\right]$
численное приближение

Power: Infinite expression $\frac{1}{0.}$ encountered.

Out[104]= ComplexInfinity

In[105]:= r_1 =.

In[106]:= n =.

In[107]:= r_1 := l + r_{min}

In[108]:= Table[r_n =., {n, 2, 9281}];
таблица значений

In[109]:= For[n = 1; l = lmin, $r_n \le r_{max}$, n++, r_{n+1} = (n + 1) l + r_{min}] // Timing
цикл ДЛЯ затраченное время

Out[109]= {0.03125, Null}

In[110]:= n

Out[110]= 9281

In[111]:= r_n

Out[111]= 2.50005×10^{-11}

In[112]:= r_{n-1}

Out[112]= 2.49988×10^{-11}

In[113]:= l =.

In[114]:= $N2_{min} := \frac{\pi}{ArcTan\left[\frac{l}{2\sqrt{r_{min}^2-\frac{1}{4}l^2}}\right]}$

In[115]:= $Table\left[N2_i = \frac{\pi}{ArcTan\left[\frac{l}{2\sqrt{r_i^2-\frac{1}{4}l^2}}\right]}, \{i, 1, n\}\right];$
таблица значений

In[116]:= $Int2 = NIntegrate\left[\left(\frac{N2_{min}^2}{2} - N2_{min} + 2\right)\sqrt{\frac{\hbar^2 c^2}{16}\times\frac{3}{l^2}+m^2 c^4} + \sum_{j=1}^{n-1}\left(\left(\frac{N2_j^2}{2} - N2_j + 2\right)\sqrt{\frac{\hbar^2 c^2}{16}\times\frac{3}{l^2}+m^2 c^4}\right), \{l, lmin, lmax\}\right]$
квадратурное интегрирование

Out[116]= 1.55936×10^{-13}

In[117]:= $N\left[\frac{1}{lP} Na\, \Delta_1\, Int2\right]$
численное приближение

Out[117]= 5.26895×10^{101}

$N\left[\frac{1}{lP} Na\, \Delta_2\, Int2\right]$
численное приближение

In[118]:= $NIntegrate\left[\left(\frac{N2_{min}^2}{2} - N2_{min} + 2\right) \sqrt{\frac{\hbar^2 c^2}{16} \times \frac{3}{l^2} + m^2 c^4}, \{l, lmin, lmax\}\right]$
квадратурное интегрирование

Out[118]= 5.16977×10^{-18}

In[126]:= $\frac{3.08206 \times 10^{103}}{5.26895 \times 10^{101}}$

Out[126]= 58.4948

In[120]:= VA =.

In[121]:= $VA := \frac{4}{3} \pi \left(r_{max}^3 - r_{min}^3\right)$

In[122]:= $VM := \sum_{i=1}^{10} (Na\, \Delta_i\, VAp_i)$

In[123]:= Vs := VU - VM

In[124]:= $r_{max} = \left(\frac{3}{4\pi} Vs\right)^{\frac{1}{3}}$

Out[124]= 4.41304×10^{26}

In[125]:= $Limit\left[ArcTan\left[\frac{l}{x}\right], x \to 0\right]$
предел арктангенс

Out[125]= $\frac{\sqrt{l^2}\, \pi}{2\, l}$

Conclusion

In the paper an approach to estimation of vacuum energy in empty space and in the limitation vacuum effect has been developed and improved. Two most common and widespread elements – hydrogen and helium in the Universe were considered and for them the vacuum energy at account the two fermions of SM – the electron and the *u* quark has been calculated with the aid of the unique computer program, especially developed for this article. Also for accurately the same case of the matter and the vacuum the calculation has been carried out on the program which was used in the previous paper, and these two results were compared to find out progress of the entire work.

The *u* quark free vacuum in the new approach with the linear dispersion has the large enough lower limit of the energy in comparison with the free electron vacuum, and in the previous approach without dispersion they have approximately the equal lower limits of the energies with small enough difference. The difference between the two approaches of the calculation of the free vacuum energy lower boundary for the *u* quark is therefore large enough and it is not essential for the electron. In the previous approach the lower boundary of the *u* quark free vacuum energy is a little great than the electron free vacuum energy. The lower limit *u* quark vacuum energy in the presence of the heavy nuclear matter, i.e. the effect in the new approach sufficiently severe differs from the value of the same kind of the energy lower boundary for the electron vacuum, due to the *u* quark is enough heavy in comparison with the electron. As the new computation and the computation on the previous scheme show, the hydrogen atom severer wraps the vacuum, despite its smaller sizes than the helium atom has and the half of the charge of the helium's nucleus. The difference between the lower limits of the vacuum energies near the matter for the same particles in the new approach, therefore, is greater for the hydrogen and is smaller for the helium. The differences between the new approach and the previous approach for the *u* quark are greater, than for the electron, due to the greater mass of the quark.

The electron neutrino contribute greatly in comparison with the muon and the taon neutrinos in the lower limit of the free vacuum energy in the new approach, and in this approach the all neutrino vacuums have much more great the lower boundaries of the vacuum energies in contrast those the same values in the previous approach. That is why the difference of the lower limits between these two approaches for the neutrinos is enough great. In the previous calculation the all values of the lower limits of the neutrinos vacuum energies differ a little, and the highest energy has the heaviest neutrino – the taon neutrino. This is explainable. The lower boundaries of the vacuum energies in presence of the effect in the previous approach have the same order of the quantity that is in the new approach. That says the two approaches do not contradict each other and the new one complements the old one. The difference between the *u* quark vacuum energy lower boundary and the electron vacuum energy lower boundary near the matter in the previous approach is less than the same one in the new approach. That is correct for the both atoms. The difference between the lower boundaries of the vacuum energies without the limitation vacuum effect, the difference of them between these two approaches behave the same way, like the difference between the lower limits of the vacuum energies and the difference of them between the two approaches with the effect.

The wrapping vacuum coefficients show that the heavy nuclear matter of the atoms really wraps the vacuums, in the new approach mostly for the hydrogen, that we should wait exactly on the numbers, and these numbers one can explain, probably, by the most abundance of hydrogen, but for the electron, and not for the *u* quark, like for the lower limits of the vacuum energies (the numbers) with the effect and without it. It is needed to say that this fact is not obvious; it obviously seems correct that the helium's nucleus must mostly wrap the vacuums, due to the doubled charge of it and the more spatial sizes in comparison with the electron. For the helium the logical dependence recovers in the both cases: the new one and the previous one. In the previous approach the logical dependence restores, i.e. the most wrapping of the vacuum takes place in the helium atom for the *u* quark and the least wrapping has the hydrogen atom for the electron-positron vacuum. The $u\bar{u}$ vacuum in the hydrogen atom wraps smaller, than the corresponding one in the helium atom in the previous approach. The non-integer value of the electric charge of the *u* quark does not impact on non-prevailing of any values for this vacuum. The exploring effect is very small for the new approach and it is much great for the previous approach.

Appendix

Any approximate expression has its exact analog (from the right to the left)

$$\sum_{n=0}^{n_{\max}} f\left(l_{\min} + nl_P\right) \approx \frac{1}{l_P} \int_{l_{\min}}^{l_{\max}} f\left(l\right) dl ,$$

where

$$n_{\max} = \text{Floor}\left(\frac{l_{\max} - l_{\min}}{l_P}\right),$$

but due to enormous number of summands cannot be calculated and, therefore, must not consider. (Here the function 'Floor' means rounding down)

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