# Fundamental rebound theories justify generalizing Newton's 340-year-old Third Law and its implications.

**Ajay Sharma**

Department of Physics. Career Point University. Hamirpur, HP 176041 India

Email ajoy.plus@gmail.com Mobile +91 94184-50899

**Abstract**

Consider compiling a set of one hundred observations of diverse rebounding bodies on a common surface. These bodies may have identical mass and composition but different shapes, such as spheres, cones, needles flat sheets, and irregular forms, and they rebound to different heights. These experiments are classified into realistic physical systems. In existing physics, motion of bodies is explained by coefficient of restitution method (CORM), the equations are based Newton's third law of motion (NTLM) and kinematical equations, which explains phenomena qualitatively. Hertz contact method also explains motion of bodies in very limited range. Under suitable conditions, a spherical ball may rebound nearly to its original height, retracing its path, and the rebound height may be treated as an indicator of reaction force. In contrast, a flat body rebounds to a minimal height whereas irregular bodies rebound along trajectories at varying angles. The typical motions of sphere, flat and irregular body may be explained with single equation quantitatively. Even recoil velocity of gun based on conservation of momentum, is not quantitatively confirmed at macroscopic level. Thus, NTLM may be used independently to explain the behavior of rebounding bodies, in realistic system NTLM is expressed as Reaction ($F_{BA}$) = − [$K_{shape} \times K_{composition} \times K_{target} \times K_{other}$] Action ($F_{AB}$). Accordingly for recoil of gun we get, $v_{gun} = - m_{bullet} . v_{bullet} / Z m_{gun}$. The proposed generalization may be examined through quantitative experiments involving diverse parameters, where *K* and *Z* represent experimentally determined coefficients. This law is specifically tailored for rebounding bodies in real-world scenarios, providing a phenomenological framework to describe experimental observations. Distinct theoretical treatments may be required for rebounding bodies in idealized versus real-world systems. The results motivate consideration of an extended interaction framework that incorporates geometric and surface-dependent factors within a Newtonian context.

**Key Words**. Third law, falling and rebounding bodies, shape, composition, and rocket.

## 1.0 Introduction

Newton originally formulated this law in *Principia Mathematica*, under the section Axioms, or Laws of Motion, presenting it in Latin and illustrating it with three simple examples typical of the era of natural philosophy [1]. Newton analyzed physical phenomena through the principles of proportions and proportionality [2], at a time when formal mathematical equations had not yet been developed. The law has been formulated during days of natural philosophy in Principia, which separated physics from natural philosophy.

When a basketball falls on the ground, and it rebounds, the interaction provides a simple, real-world, macroscopic demonstration of Newton's Third Law of Motion in a qualitative sense. Examples of this nature are extensively cited in the scientific literature. The quantitative confirmation of the law is required in such cases. What is exact height of rebound? What is the height predicted by equation? What are the factors influencing the results?

The Principia was translated into English by Andrew Motte in 1729, and Newton's third law of motion is quoted from this translation [3].

*"To every action there is an equal and opposite reaction, or the mutual actions of two bodies upon each other are always equal and directed to contrary parts."*

Reaction = −Action (1)

When one body strikes the other body, due to mutual interactions, simultaneously reaction arises, equal in magnitude of action but opposite in direction (mentioned by negative sign). The action and reaction occur in equal and opposite pairs.

In the second part of the law, Newton explained that the force exerted by the second body on the first body (reaction) equals the force exerted by the first body on the second body (action). The magnitudes of these forces are equal but directed oppositely along the same line of action.

In contemporary notation, action and reaction are not treated as physical quantities, since they are absent from the IUPAP-defined list of physical quantities; so, we need to assign a suitable parameter for action and reaction which may be quantitatively used to confirm Eq. (1). However, Newton himself expressed action and reaction in the first two applications of the law as push or pull i.e. force. Thus, law is expressed in terms of force. Eq. (1) may be written as

Force of Reaction of Body B on Body A ($F_{BA}$) =

- Force of Action of Body A on Body B ($F_{AB}$) (1)

In second part of the third law, it is defined in terms of the mutual interaction forces $F_{BA}$ and $F_{AB}$ , without reference to any additional parameters such as shape, composition of bodies and surface of interactions. Thus, the law appears independent of factors inherently associated with real-world observations and with the mutually interacting forces $F_{AB}$ and $F_{BA}$ , ; consequently, Eq. (1) may represent an idealized description of the interaction. The reaction force so generated in Eq. (1) will have its sustained impacts, like force.

Furthermore, the law has numerous applications in physics, mathematics, and engineering. Newton did not alter the definition and elaboration of the third law during the forty-year period from 1686 to 1726.

### 1.1 The Action ($F_{AB}$) and Reaction ($F_{BA}$) are simultaneous in nature.

In Newton's Third Law, *cause and effect are not implied* in the usual temporal sense; instead, the law describes a mutual, simultaneous interaction. $F_{Body \rightarrow Floor}(t) = -F_{Floor \rightarrow Body}(t)$ at the same instant t. The condition $\Delta t_{action-reaction}=0$, implies that action and reaction forces arise simultaneously, with no temporal separation between them, i.e., action force and reaction force. Both arise simultaneously during interaction and therefore cannot be interpreted as a simple cause–effect sequence.

Action and reaction always happen in pairs equal in magnitude but opposite direction [2,4]. The definition and equations of the third law of motion only take into account $F_{AB}$ and $F_{BA}$, but are independent of all other factors involved. According to the law, further characteristics of bodies are also insignificant.

### 1.2 The law is universal in nature as $F_{AB}$ and $F_{BA}$ are always equal and opposite.

The definition of NTLM and Eq. (1) imply that action (**$F_{AB}$**) and reaction (**$F_{BA}$**) are always equal in magnitude and opposite in direction in all cases without exception; therefore, the law is considered universal, or in simple terms, applicable to all interactions. There is no additional term in the definition or in Eq. (1) other than Action (**$F_{AB}$**) and reaction ( **$F_{BA}$**) that could modify their magnitudes or directions. Thus, the law is regarded as universal in nature, since its definition and Eq. (1) are considered valid without any stated constraints. All factors other than the action force $F_{AB}$ and the reaction force $F_{BA}$ are not included in the definition and mathematical formulation of the law. When bodies interact in realistic systems, physical factors such as the characteristics of the bodies (shape, size, composition, mass, asymmetry, etc.), the nature of the surface, the point or area of contact, orientation, angle of fall, spin, rotation, and similar parameters influence the observed results. Newton's Third Law applies equally to bodies with masses of a few milligrams or less and to those of several kilograms or more, consistent with the definition of 'body' given in Definition 1 of *Philosophiæ Naturalis Principia Mathematica*. These factors are neither taken into account in the definition of the law nor included in Eq. (1). When the law is critically examined under various realistic conditions, it may be concluded that the law holds strictly in idealized situations only. This discussion is restricted to macroscopic rebounding bodies (whether horizontal or vertical) and does not extend to other types of interactions in which the law is generally regarded as valid and well established.

### 1.3 Newton's definition of body or mass in Principia

Newton opens the *Principia* with Definition 1 for quantity of matter (mass or body), which lays the foundation for the subsequent formulations [1-3].

*The Quantity of matter is the measure of the same arising from its density and bulk conjunctly.*

Further, Newton stated that it is this quantity that I mean hereafter everywhere under the name of **body** or mass. Newton illustrated the concept of mass using examples such as snow, dust, and powdered substances. Thus, Newton formulated his laws based on macroscopic examples of mass and motion in Principia. In his first two illustrations of the third law, Newton depicted interactions involving a stone, a horse, and a finger. In the third example, Newton referred to interactions between bodies as defined in Definition I of the *Principia*. Thus, in the present perspective the law may apply to perfectly rigid bodies. Atoms, molecules, and subatomic particles were unknown or speculative in Newton's time (1642-1727). Dalton's Atomic Theory was developed from 1803 to 1808.

### 1.4 **Conditions For Complete Validity of the Third Law**

For Newton's Third Law to hold fully, two conditions must be satisfied

(i) The reaction force must equal the action force in magnitude.

(ii)The reaction must act in the exact opposite direction to the action.

In mathematical equations, the opposite direction of the reaction force is represented by a negative sign in Eq. (1) and in the subsequent related expressions, such as Eq. (2). Thus, for the law to be valid, both conditions must be satisfied simultaneously. If the magnitudes of action and reaction are equal but their directions differ, the law cannot be considered fully satisfied. Therefore, the fulfillment of both conditions is essential for the complete validity of the law. Directional observations are especially important in interactions involving bodies of varying shapes.

### 1.5 Newton's three original illustrative Examples in Principia (1686)

In Newton's usage, the term "action" denotes force. After defining the law in *Principia* [3], Newton illustrated it with three simple qualitative examples.

*Whatever draws or presses another is as much drawn or pressed by that other.*

When body B is drawn or pressed by an external force exerted by body A (action), body B simultaneously exerts an equal and opposite reaction force on body A. The mutual actions between two bodies are forces. Action and reaction forces act in pairs.

(i) Horse and Stone: If the horse pulls the stone tied to the rope, then the stone also pulls the horse equally backward.

(ii) Finger and Stone: If a finger presses a stone, the finger is also pressed by the stone.

In both examples, Newton considered small forces acting on a system that remained at rest.

The finger exerts force on the stone (action), while the stone exerts force on the finger. The action and reaction forces are equal and opposite, and the system therefore remains at rest. If the applied action force is below an optimum value, or the finger pushes the stone weakly, the stone nevertheless continues to remain at rest. If a sufficient force is applied by the finger, the stone begins moving. Newton did not consider this situation. Surface characteristics play a decisive role in the interactions which are not mentioned by Newton. Thus, in first two examples, Newton described idealistic situation or special case only.

(iii) **Movement of Projectile and Target after interaction:** In the third example of the third law p.20, Newton stated [3].

*"If a body impinges upon another, and by its force* ***change the motion of the other****; that body also (because of the quality of the mutual pressure)* ***will undergo an equal change****, in its own motion,* ***towards the contrary part".***

Therefore, throughout his illustrations, Newton assumed that the reaction (force exerted by the second body) always equals the action (force exerted by the first body) and acts in the opposite direction. In this case as well, Newton did not describe the physical characteristics of the interacting bodies or the nature of the surface involved.

Thus, Newton imposed strict conditions on interacting bodies, stating that the mutual forces they exert on each other are always equal in magnitude and opposite in direction, irrespective of the characteristics and physical properties of the interacting bodies and the surface of interaction. In the third illustration, he expanded the discussion to include interactions between macroscopic bodies as they move in response to these interactions. Practically, Newton considered the situation in which a force is applied to a body, causing it to move; that is, the body is displaced through a certain distance.

Here the motion is represented by the quantity of motion, namely, momentum. If a body A (projectile) of mass $M_p$ exerts a force on body B (target) of mass $M_t$; body A experiences an equal and opposite change in motion. $u_1$, $u_2$ are initial velocities, and $v_1$, $v_2$ final velocities for projectile and target. Then,

[Change in motion of the body A (projectile) when it impinges on body B due to mutual interactions] or [reaction]

= - [Change in motion of the body B (target) when the body A (projectile) impinges on it] or [action] (2)

$$[M_p v_1 - M_p u_1] = - [M_t v_2 - M_t u_2] \qquad (2)$$

$M_p v_1 + M_t v_2$ (Final momentum of system) = $M_t u_2 + M_p u_1$ (Initial momentum of system) (2)

So, the third example of Newton's third law of motion leads to the law of conservation of momentum. For simplicity, if $M_t$ and $M_p$ are equal, then

$$u_1 + u_2 = v_1 + v_2 \qquad (3)$$

The Eq.(3) implies if masses of projectile and target are same, then initial velocity of the system same as final velocity of the system i.e. it implies conservation of velocity. If the target is at rest, i.e., $u_2 = 0$, then Eq. (3) becomes

$$u_1 = v_1 + v_2 \qquad (4)$$

Thus various other equations are feasible depending upon depending different values of masses and velocities.

**Experimental status:** The law of conservation of momentum, Eq. (2), derived from the third application of Newton's third law of motion, requires experimental verification at the macroscopic level. Practical conditions regarding the bodies and the surface must be identified under which the equation remains valid. Realistically, at the macroscopic scale, interaction outcomes depend on the geometrical and material properties of the colliding bodies, as well as on the characteristics of the contact surface. These factors are not taken into account in the equations, as they involve only mass and velocity. The energy may be dissipated in various forms. Realistically, the definition and mathematical expression of the law must take into account all such relevant factors; however, it should be recognized that Newton formulated the law during the era of natural philosophy, when systematic theoretical and experimental methods had only just begun to develop. The mathematical equations were not prevalent in those days. Mathematical equations were not widely developed or systematically applied in physics during that period.

Therefore, multiple sets of experimental observations with different feasible parameter combinations are required to validate Eqs. (2–4). These equations, along with other related derived relations, need quantitative validation under all relevant conditions. Repeated theoretical interpretation alone is not sufficient to establish the complete validity of any equation. Eqs. (2–4), and other derived relations, remain to be experimentally confirmed through multiple experiments considering a wide range of parameters. Theoretical equations need to be confirmed by experimental confirmation for complete validity. These equations may be considered under ideal and realistic conditions (when bodies have different shapes, symmetries, compositions, and surfaces) for analysis. The results would vary as physical conditions change.

The law of conservation of momentum [4] is also explained with the help of the equation of the second law of motion.

$$F = ma = \frac{dp}{dt} \qquad (5)$$

If F=0 i.e. no force acts on the system or isolated system then,

p =constant or initial momentum (mu) = final momentum (mv) or u =v (6)

Thus, both Newton's third law and the equation F=ma, lead to the law of conservation of momentum.

Realistically, Eq. (6) is valid when a body is either at rest or moving with uniform velocity. This represents the second part of Newton's First Law of Motion; however, this deduction holds only under specific or ideal conditions, as a body travels a longer distance on a smooth floor than on rough ground. As surface smoothness increases, the distance traveled by the body correspondingly increases. The uniform motion of the body must be measured systematically. Suitable conditions should therefore be created as far as possible to determine how long a body can continue moving with uniform velocity. Clearly, the body must be symmetric and appropriately designed for this condition to be justified experimentally. In practical, realistic situations, an attempt may be made to approximate such a system in which the body moves with uniform velocity as predicted by Eq. (6), even within a limited region. Similarly, Newton's Third Law must be tested under various conditions. A practical demonstration of the transition or applicability of Newton's laws at an illustrative level would be both instructive and logically consistent. Thus, Newton's laws are fundamentally interrelated.

For precise validity of the equations, shapes, symmetries, compositions of bodies and surfaces must be appropriately chosen.

### 1.6 Universality of the Equations and Their Independence from Projectile–Target Characteristics

The definition and equation of Newton's Third Law are assumed to apply universally to all bodies, such as solids, liquids, gases, and semi-fluids, without regard to their mass, shape, material, or composition, etc.[5-6,7]. The law imposes no constraints on mathematical predictions because it considers only $F_{AB}$ and $F_{BA}$, excluding all additional influencing parameters explicitly therein. Newton further imposed the condition that, in every interaction, $F_{AB}$ and $F_{BA}$ are always equal in magnitude and opposite in direction.. Newton himself defined the body as he employed it in discussions, as stated in section (1.2) i.e. macroscopic level. In real-world systems, the characteristics of body A, body B, and the interaction surface are significant, their effects may be easily understood in experiments.

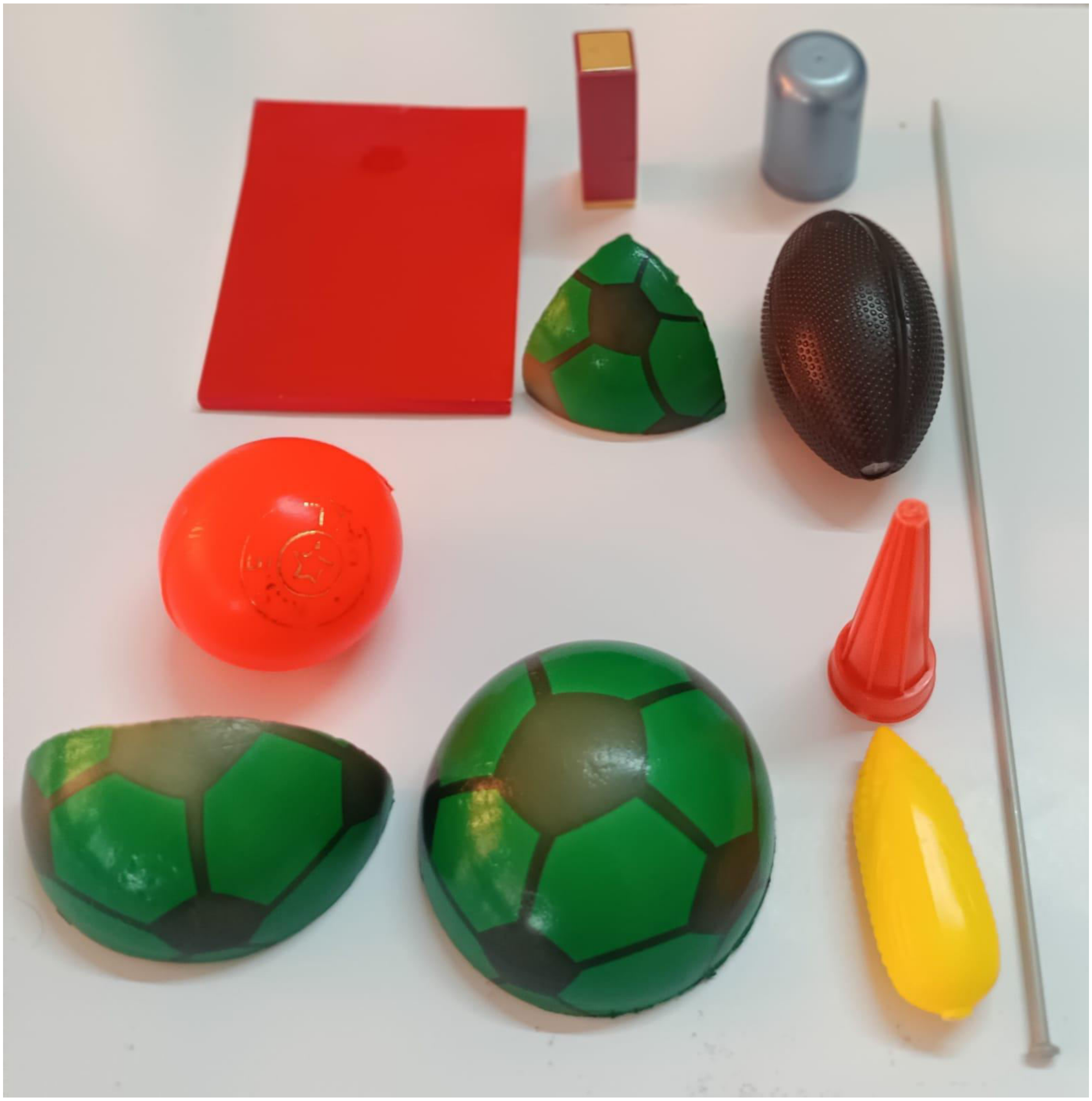
The shapes, compositions of bodies, and interaction surfaces are not mentioned in Eq.(1) as they are insignificant. For comparison, Eqs. (1) and (50) may be considered; the latter includes all factors affecting results.

**(i) First body A (projectile).**

This is the body (projectile) that exerts action or an action force on the second body during interaction. In the definition of the third law, further characteristics of the body are neither defined nor mentioned at all. The body may have many possible shapes, e.g., spheres, hemispheres, cylinders, triangles, polygons, cones, needle, long thin pipes, flat sheets, or arbitrary, irregular shapes or any other possible shapes, etc. The interacting body may differ in physical properties such as composition, material nature, elasticity, flexibility, mass, asymmetry, and size or related properties; the law applies equally in all cases. For bodies of differing shapes, angle of fall of body, the angles of contact with the floor or supporting surface play a significant role in the interaction. Newton's Third Law does not account for all these factors and is assumed to remain valid for all bodies. The angle of fall of body is not mentioned. For simplicity in preliminary free-fall experiments, the body is assumed not to spin or rotate during its motion.

However, modern highly elastic materials, e.g., Super Ball, Waboba Moon Ball, Super High Bounce Balls (laboratory-grade polybutadiene), may also be considered in experiments, as the law is assumed valid for all bodies. The synthesis of additional highly elastic materials beyond those previously described is presently underway. They will represent potential additional systems or bodies, once finalized, for experimental verification of Newton's third law.

**Figure 1 The bodies of different shapes but of same mass and composition falling on the same floor in realistic systems.**



**(ii) Second body B (target):**

The second body, or target, is the body on which the first body applies the action or force, and as a result of interaction, an equal and opposite reaction is generated by second body on first body simultaneously. Thus, action and reaction always occur as a pair, having equal magnitudes but opposite directions. Newton's formulation of the law is presented broadly and does not specify physical characteristics of the second interacting object (target), referring to it simply as a body.

The characteristics of the second body (target) are also extremely important, as they produce mutual interactions with the first body (projectile), generating an equal and opposite reaction force simultaneously. The equality of action and reaction, and their opposite directions, depend upon characteristics of both the projectile and the target.

In his definition, Newton refers only to "bodies," a general term encompassing both projectile and target. Newton defined the body in Definition I of the *Principia*. Newton explained the law using simple qualitative examples. The law implies that, without specifying characteristics of the first body (projectile) and the second body (target), the reaction equals the action but acts in the opposite direction.

When the law is examined critically within broader theoretical and experimental frameworks, the characteristics of the target (second body) should be described relative to those of the projectile (first body). Whether an object functions as a target depends on the projectile; for example, a stretched sheet of paper may serve as a target for a small body but not for a stone. Thus, the projectile and the target should be treated as relative entities in realistic analyses. Projectile–target pairs must therefore be chosen carefully and cannot be treated arbitrarily in quantitative analysis. This is further evident from the fact that Newton explained the third law through three qualitative examples, which were appropriate in the era of natural philosophy when Newton wrote Principia (1686), but now both experimentally and theoretically the predictions may be specifically confirmed with state -of -the -art equipment.

**(iii) Third Body C: Interaction Surface**

In interactions involving a horizontal surface, the physical characteristics of the surface play a crucial role in determining the observed response. This aspect needs to be critically discussed as it may cause a significant loss of energy comparatively. The physical characteristics of the surface influence the outcome of the interactions and must be taken into account.

If one marble A strikes the second marble B with the same force (action) on a different surface, e.g., on ground, floor or glass slab. Then, on the different surfaces, the velocities of marble A and marble B are different, after interactions. Thus, in horizontal interactions, the role of the surface is as important as that of the projectile and the target in understanding the interactions.

If interactions occur on a horizontal surface, then surface characteristics become significant and affect the results. This aspect must be critically discussed, as it may cause comparatively significant energy loss. The physical characteristics of the surface influence interaction outcomes and must be taken into account.

**Speculative conclusion in view of the second part of Newton's First Law of Motion:**

**(a)** If the system is completely free from resistive forces, and **marble A strikes marble B**, then the latter must move with uniform velocity, if the system is free from all other forces, as implied by the second part of Newton's First Law of Motion.
(b) In the case of freely falling and rebounding bodies, air resistance may be minimized or effectively eliminated by creating a partial or complete vacuum.

Thus, in some cases, the surface on which interactions of projectile and target occur may be termed as the third body, as it affects the results. If the bodies fall and rebound in a vacuum, the effect of the intervening medium is negligible. The purpose of discussing these factors is that Newton's law, in its original formulation, is silent on these aspects.

**2.0 Two categories of applications of Newton's third law are recognized.**

The understanding of physical phenomena and scientific laws is a continuous process. Newton illustrated the Third Law with three qualitative examples in the *Principia* (1686). Subsequent scientists extended its applications to a variety of physical phenomena.

**(i) Action reaction Motion Systems or Contact-based Action–Reaction Systems.** In the *Principia* (1686), Newton presented only three applications, largely qualitative in nature. Then the following scientists extended the application of the law in other examples, such as bouncing balls, swimming, rowing, walking, jumping from a boat, balloon deflation, swimming fish, flying birds, gun recoil, striking and rebounding bodies on the horizontal surface, etc. However, rigorous experimental verification of Newton's Third Law across diverse physical systems remains technically challenging and often requires high-precision instrumentation.

**Recoil Velocity of Gun:** By the sixteenth century, arquebuses and muskets were increasingly employed in English armies; however, Newton's writings do not make any reference to such firearms. Afterward Eq. (2) is applied to calculate recoil velocity of the gun. Initially both bullet and gun are rest, thus

$$0 = m_{bullet}\, v_{bullet} + m_{gun}\, v_{gum}$$

The recoil velocity of the gun (indicated by negative sign) is given by

$$v_{gun} = -\, m_{bullet} . v_{bullet} / m_{gun} \quad (7)$$

The Eq.(7) is theoretically discussed at numerous occasions, but it in needs to experimentally confirmed with quantitative observations. Here recoil velocity of gun ($v_{gun}$) depends only on $m_{bullet}$, $v_{bullet}$ and $m_{gun}$. This experiment illustrates the law of conservation of momentum at the macroscopic level; however, in the standard literature, quantitative data confirming Eq. (7) are not available. This implies that specific experiments are absolutely necessary, as without experiments theoretical predictions are not validated.

**One dimensional elastic collision:** The equation for conservation for kinetic energy is

$$½M_p v^2_1 + ½M_t v^2_2 \text{ (Final KE of system)} = ½M_t u^2_2 + ½M_p u^2_1 \text{ (Initial KE of system)} \quad (8)$$

and equation for conservation of momentum is as in Eq.(2)

In one dimensional elastic collision the conservation of momentum and kinetic energy holds good simultaneously , and equations for final velocities of projectile and target in standard literature are derivable as [2,7]

$$v_1 \text{ [ Final velocity of projectile]} = \frac{[M_p - M_t]u_1 + 2M_t u_2}{(M_p + M_t)} \quad (9)$$

$$v_2 \text{ [ Final velocity of target]} = \frac{[M_t - M_p]u_2 + 2M_p u_1}{[M_t + M_p]} \quad (10)$$

(a) If the target is at rest i.e. $u_2 = 0$ (say, a wall). Thus when a ball strikes the wall i.e. $M_t >> M_p$

$$v_1 \text{ (final velocity of ball)} = -u_1 \text{ (initial velocity of ball)} \quad (11)$$

Thus, the ball must rebound backward with the same velocity.

(b) Further, consider a ball or stone hanging in the path of train i.e. $M_t << M_p$.

$$v_1 \text{ (final velocity of train)} \approx u_1 \text{ (initial velocity of train)} \quad (12)$$

Thus, the train continues to move with its initial velocity even after colliding with the hanging ball.

$$v_2 \text{ (final velocity of ball)} \approx 2M_p\, u_1 / M_p = 2u_1 = 2 \text{ (initial velocity of train)} \quad (13)$$

Under the idealized assumption, when a train collides with a hanging ball, it is predicted to move with twice its initial velocity. This is a subtle and delicate prediction that requires careful experimental verification. It involves the simultaneous conservation of two fundamental laws of physics, namely momentum and kinetic energy. Experimental confirmation of Eq.(13) must be done and the conditions which equation is confirmed must be quantified.

Mathematical predictions are validated only through experiments, not by how long they have been taught. Therefore, experimental confirmation provides the true test of a law at any stage. Laws may be extended or modified at any time based on experimental observations, which constitutes the fundamental criterion that a law must satisfy.

The predictions given in Eqs. (9-13) are purely theoretical in nature; therefore, require experimental validation. These predictions have been discussed extensively in theoretical contexts on numerous occasions; however, they require rigorous experimental confirmation, and the corresponding parameters must be quantitatively determined. This would directly confirm that, experimentally, the shape, size, material properties of the bodies, the nature of the target surface, and other factors play significant roles. These factors are not accounted for in Eqs. (2-4 , 8–13) when interpreting the results they only involve m and v. The specific experimental conditions under which these predictions are satisfied must also be systematically verified.

**Note:** The predictions in Eqs. (9-13) are derived from the simultaneous conservation of kinetic energy and linear momentum, whereas Eq. (7) is based solely on the conservation of linear momentum. Even Eq. (7) is yet to be precisely confirmed for macroscopic bodies Both the laws of conservation of momentum and kinetic energy, as expressed in Eqs. (2) and (8), involve mass and velocity.

(ii) **Aerospace Propulsion Systems**. These systems constitute highly advanced and technically sophisticated applications of Newton's Third Law developed in the $20^{th}$ and $21^{st}$ centuries. These applications include rockets, fireworks, spacecraft, airplanes, missiles, helicopters, drones, and gliders, among others. Newton did not these applications in his works; therefore, they may be regarded as extended interpretations or applications of Newton's Third Law.

In aerospace and rocket propulsion systems, the exhaust gases expelled at high velocity produce the reaction force that propels the vehicle forward. This is a direct consequence of Newton's Third Law of Motion**.** Aeronautical and astronautical engineers and scientists apply Newton's third law in these systems. Thus, the law exhibits an exceptionally broad range of applications, a feature not commonly observed in other physical laws.

(a) Using Newton's third law, the ideal rocket equation [8] was derived by Russian teacher Konstantin Tsiolkovsky in 1897 and published in his paper *"Exploration of Outer Space by Means of Rocket Devices"* in 1903. The velocity of the rocket at any instant is given by

$$\Delta V = V_e \ln M_0 - V_e \ln M = V_e \ln M_0/M \quad \textbf{(14)}$$

The Eq.(14), which is based on law of conservation of momentum (based on Newton's Third Law) at macroscopic level, has undergone several refinements by aeronautical scientists over the years to improve its applicability and precision. This equation has been modified or extended by following scientists as Eqs. (15,17).

The applications of Newton's third law of motion extend to various realistic systems. The law is used with kinematic equations $v^2 - u^2 = 2gS$ and $S = ut + ½gt^2$ to calculate coefficient of restitution ( COR or e).

## 3.0 Historical Context before and after Principia.

Newton's third law of motion is extensively used in technology and academia, so its origin and related historical context must be noted for complete understanding of various aspects of the law. The phenomena of fireworks, rockets, and guns were well known centuries before the formulation of Newton's Third Law. However, Newton did not discuss the application of the Third Law to such devices in the *Principia*. A systematic description and analysis of these applications is therefore both scientifically important and historically interesting.

**(i) Descartes' Third Law of Nature [9]:** The third law of nature, as formulated by Descartes, appeared in 1644 in *Principles of Philosophy* (Chapter 2, Paragraph 40, p.34 ), predating Newton's version by about four decades.

The third law of nature: (a) if one body collides with another that is stronger than itself, it loses none of its motion; (b) if it collides with a weaker body, it loses the same amount of motion that it gives to the other body.

Both Descartes and Newton have discussed colliding bodies. The third application of Newton's Third Law of Motion quantitatively defines the relationship between interacting bodies in terms of action and reaction, i.e., forces that are equal in magnitude and opposite in direction, whereas Descartes' law offers only a qualitative description of the physical event.

(ii) During Newton's lifetime (1642–1727), mathematical equations were not prevalent in scientific inquiries, and natural phenomena were largely interpreted through qualitative reasoning and description. Newton interpreted phenomena geometrically by the method of proportion or proportionality [2]. Newton originally stated the law in Latin in *Philosophiae Naturalis Principia Mathematica,* and it was later translated into English by Andrew Motte, encouraged by his brother Benjamin Motte, a publisher who saw the book's commercial potential.

The English translation was published in 1729, two years after the death of Newton. Newton did not alter the definition of the third law for approximately 40 years (1686–1726), nor did he provide additional clarification beyond its original statement as illustrated in the 1686 Principia. The second and third editions of the *Principia* were published by Newton in 1713 and 1726, respectively. Newton stated the second law in proportionality form and the third law in equality form. In contrast, Newton's First Law provides a precise descriptive statement of the underlying physical phenomenon.

(iii) Swiss Leonhard Euler started relating force with mass and acceleration in 1736 in the book Mechanica [10, 11,12]. Euler [13,14] published the equation F=ma in 1776 when he was working at the Imperial Academy of Sciences in St. Petersburg, Russia**.** F = ma was published in the treatise *Nova methodus motuum corporum rigidorum determinandi* (A New Method for Determining the Motions of Rigid Bodies) in the journal Academia Scientiarum Imperialis Petropolitana (Imperial Academy of Sciences in Saint Petersburg) at pages 222-224 (E479).

(iv) The gravitational acceleration ($g = 9.80665\ m/s^2$) was measured accurately in 1888 by the French Service géographique de l'Armée [15]. In 1901**,** at the 3rd General Conference on Weights and Measures (CGPM), weight was formally defined as mg.

In Euler's equation F = ma, when the acceleration a replaces the acceleration due to gravity g, the resulting expression W = mg represents the weight of the body near the Earth's surface. Realistically, this became the greatest breakthrough in classical mechanics related to the quantitative aspects of Newton's third law.

Thus, expressing Newton's Third Law in precise mathematical form (for falling and rebounding bodies) became practically feasible only about 215 years after the publication of the *Principia*. In the case of a body freely falling towards the surface, the action can be regarded as the gravitational pull of the Earth on the body, quantitatively given by its weight, mg.

(v) The equations describing freely falling and rebounding bodies became feasible about 125 years ago, around 1901. During this period, Newton's third law was established as a fundamental principle primarily at a qualitative level. Historically, the law has been supported largely by qualitative illustrations, adequate during the early development of mechanics but insufficient for modern quantitative verification standards. However, systematic and fundamental quantitative experiments on falling and rebounding bodies have not yet been conducted to determine the precise conditions governing rebound heights or distances. This constitutes a significant aspect of the present discussion.

**(vi) Fireworks and Rockets:** Gunpowder-propelled devices, primarily used for recreational purposes such as fireworks, were developed in China around the 9th century A.D., nearly 700 years before Newton's *Principia*. Rockets were employed in warfare for the first time during the Mongol-Chinese conflict in 1232, using gunpowder (solid fuel); their motion was completely uncontrolled after ignition, similar to fireworks. The earliest recorded fireworks display in England occurred at the wedding of King Henry VII and Elizabeth of York in 1486. Despite these developments, Newton (1642–1727) did not discuss rockets in the *Principia* or elsewhere, nor did he mention his Third Law to explain their motion. In Philosophiæ Naturalis Principia Mathematica, Newton elaborated the Third Law using several qualitative examples.

**(vii)** After the publication of Newton's *Principia* (1687), his Third Law of Motion quickly became a cornerstone of European physics. Universities such as Cambridge, Oxford, Paris, Naples, and Göttingen incorporated Newtonian mechanics into their curricula, educating scholars and engineers who applied these principles in research, technology, and practical sciences. Beginning in the early 1800s, the British East India Company's schools, military academies, and surveying programs systematically incorporated European scientific knowledge, including the principles of Newtonian mechanics, into colonial India's education system. This foundation paved the way for the formal inclusion of Newton's laws in the curricula of the first modern Indian universities e.g., Calcutta, Bombay, and Madras, established in 1857.

It is pertinent to note that the acceleration due to gravity, $g=9.8005\ \mathrm{m\ s^{-2}}$, was experimentally determined in France in 1888, and the concept of weight was formally defined in 1901. By this time, Newton's laws had already been incorporated into academic curricula worldwide, about one and a half century before.

**(viii)** In 19$^{th}$ century, the gliders were considerably well developed. Otto Lilienthal (1848–1896) was a German aviation pioneer known as the "Father of Gliding".

Wright Flyer I, invented by the Wright brothers in 1903, fundamentally relied on Newton's Third Law of Motion for its flight. Here, wings produce lift and the flyer is controlled by elevators, wing wrapping and rudder etc. A glider flies without an engine, while the Wright Flyer was the first aircraft to fly using an engine and propellers.

In 1907, Paul Cornu (France) achieved one of the first manned vertical lifts using a twin-rotor craft i.e., practical perception of a helicopter.

(ix) Goddard refined the ideal rocket equation in 1919 in a monograph [16], introducing a generalized form as follows.

$$\Delta V = V_e \ln\frac{M0}{M} - \int_0^t g\,dt - \int_0^t \frac{D(v)}{m}\,dt \qquad (15)$$

where $V_e$ is the effective velocity, $M_0$ and M are the initial and final masses, g is the acceleration due to gravity, D is the drag force, t is the burn time, etc.

In 1926, Goddard became the first astronautical pioneer to experimentally launch a liquid-fueled rocket (often called Nell), achieving a flight lasting about 2.5 seconds. The fuel consisted of gasoline (fuel) and liquid oxygen (oxidizer). By further improving the design, Goddard extended the rocket's flight time to approximately 22.3 seconds in 1937. Although technology and knowledge have since advanced in astronautical engineering and computational modeling, space missions may succeed or fail, yet Newton's third law, formulated about 340 years ago in the *Principia*, remains valid or unquestioned. In such experiments, technological advancement is equally crucial.

Thus, Newton's Third Law of Motion finds diverse applications across science and engineering.

Now the exhaust velocity ( $v_e$) is written in terms of specific impulse ($I_{sp}$) and acceleration due to gravity $g_0$

$$v_e = I_{sp}.g_0 \qquad (16)$$

In terms of specific impulse, Eqs. (14) are expressed as follows:

$$\Delta v = I_{sp}\, g_0 \ln (m_0/m_f) \qquad (17)$$

where $I_{sp}$ specific impulse and $g_0$ is standard gravity.

### 4.0 Perception of some observations about rebounding bodies, and their quantitative explanation using existing or novel methods.

Scientific laws are formulated on the basis of consistent collective observations, not on the explanation of isolated or discrete observations. Consider a compiled dataset of 100 macroscopic observations of freely falling bodies from varying heights and different angles onto the same surface, rebounding to different heights and trajectories, involving different shapes but identical mass and composition. The shapes of various bodies are mentioned in section (1.6) e.g. spheres, hemispheres, cylinders, triangles, polygons, cones, needle, long thin pipes, flat sheets, or arbitrary, irregular shapes or any other possible shapes, etc. These experimental observations can be conducted easily and reproducibly under controlled conditions.

A theoretical law is established only when its predictions are confirmed by experiments. Therefore, the rebound height of each body must be measured individually and precisely, and quantitatively explained using existing theories. Since the coefficient of restitution depends on multiple parameters, the rebound height is typically determined for bodies of regular shape with sizes between 10 mm and 20 mm. These results can then be compared with independent formulations (Eqs. 50, 53), as determining the coefficient of restitution and thus the rebound height for bodies of varying shapes is an exceptionally tedious process when a wide range of parameters is considered. This analysis is restricted to rebounding bodies within realistic macroscopic systems exclusively; it excludes other contexts where Newton's third law is already empirically validated.

When observations span a wide range of parameters, existing theories are employed to explain the results comprehensively. In such cases the existing theories (like coefficient of restitution methods, CORM) explain the observations qualitatively. Such situations necessitate the development of an alternative or new theories for accurate interpretation. The ultimate aim of scientific pursuits is quantitative explanation of phenomena with suitable theory.

**Primary requirement:** Thus, the primary requirement is that such observations as cited above, be performed at the macroscopic level with utmost care, utilizing different parameters; then existing theories must be applied for quantitative explanation. The existing theory appears applicable in only within a limited domain when carefully analyzed, their applications may be extended within domain of their applicability

**Simplicity to complexity:** For simplicity, the bodies are released in such a way that they undergo pure translational motion, without any spin or rotational motion during free fall. Initial observations with bodies of different shapes, but identical mass and composition on the same surface, are considered for simplicity; afterward, more realistic parameters may be incorporated in the observations for explanation. These observations may include vertical or horizontal motion of bodies on different surfaces. Then complex observations involving various other cases may be considered for comprehensiveness. Subsequently, alternative theoretical approaches to explain observations are considered.

We recommend conducting experiments progressively, from simpler to more complex setups, with quantitative explanations of the results.

**4.1 Why above rebounding observations are not explained with help of Newton's law only.**

Existing methods for explaining rebound height invoke Newton's third law along with kinematic equations i.e. Eqs. (18,23). But these results are not quantitative. Newton's third law is not used alone, the various reasons may be deliberated.

**First**, Newton's Third Law was originally formulated without a mathematical equation in *the Principia* and Newton interpreted it with three qualitative examples. So, Newton himself could not justify his third law of motion in such cases due to theoretical and experimental limitations.

**Second**, a spherical body appears to rebound toward its nearly original position, in the opposite direction after striking a surface under certain conditions, and this behavior seems consistent with the law, but only qualitatively. Thus, scientists did not quantitatively confirm the law due to the experimental complexity associated with the interactions. Hence, the law was only established qualitatively in many situations.

**Third,** existing literature employs Newton's third law alongside kinematic equations to estimate rebound heights. Kinematic equations i.e., Eqs. (18,23) are derived for uniformly accelerated motion. However, the coefficient of restitution method (CORM) that utilized Newton's third law as basis along with kinematic equations remains largely qualitative and limited in applicability.

**Fourth,** the precise acceleration due to gravity, g, was determined in 19$^{th}$ century enabling a mathematical treatment only after the standardized definition of weight (mg) became established in 1901. By this time, the law was widely regarded as qualitatively verified in many cases. In the case of freely falling and rebounding bodies, the equations involving weight are significant when formulating the expressions for action and reaction forces.

**Fifth,** in the 20$^{th}$ and 21$^{st}$ centuries, the law enabled significant development in rockets, aircraft, and propulsion systems, gliders, drones etc. which redirected focus toward astronautical engineering and computer-based design, rather than fundamental experimental assessments in the rebounding bodies.

**Sixth,** in established physics, force is associated with acceleration, i.e., F = ma. NTLM can be directly related to rebound height, $H_r$ (horizontal and vertical) here the effective term is the reaction force. Thus, rebound height (distance) must be related to reaction force. Here rebound height or distance is regarded as indicator of reaction force or reaction force manifests in terms of $H_r$ . The rebound height is related to reaction force in equations (50,53). In the existing methods the rebound heights have been expressed by Eqs. (29–30, 34), taking into account kinematic equations and original height, velocity, and time. Furthermore, velocity, time, and distance traveled are inherently interrelated.

For comparison, rebound heights from the new method i.e. Eqs. (50,53) and existing methods are presented alongside existing methods in Table II.

**4.2 Analysis of rebound heights using Newton's third law with kinematic equations and F=ma.**

These basic pedagogical observations regarding rebound height in physics, engineering and applied mathematics, demand quantitative explanation. In the existing physics to explain the rebound height, coefficient of restitution method (CORM) , Impact Force Method ( Herz's contact Theory, 1882) and equation ) $H_r = v^2_{after}/2g$ are used. In the existing physics, various qualitative methods have been used to explain rebounding bodies, but only in a descriptive manner.

**First Method**: Coefficient of Restitution Method (CORM)

The coefficient of restitution is obtained using Newton's Third Law of Motion as basis and applying kinematic equations $v^2 - u^2 = 2gS$ and $S = ut+½gt^2$. These kinematic equations are derivable and hence applicable under conditions when acceleration is uniform. Thus, it puts constraints on application of coefficient of Restitution Methods.

**(i) The simplest way to measure rebound height by Coefficient of Restitution Method**

We have kinematic Eq. (18), which is derivable, hence applicable to uniformly accelerated motion.

$$v^2 - u^2 = 2gS \qquad (18)$$

Consider a body of mass 1 kg falling from rest (u=0) from a height of 1 m (H). Neglecting air resistance, the velocity of the body immediately before interaction ($v^2_{before}$) with the floor is given by Eq. (18) as

$$v^2_{before} - 0 = 2gH \qquad (19)$$

During the mutual interaction between the body and the floor or surface, in accordance with Newton's Third Law, an impulsive reaction force is generated simultaneously, resulting in the rebound of the body. The action and reaction occur in pairs and are equal in magnitude and opposite in direction. After separation from the floor, the velocity of the body is termed $v_{after}$; the body then rises to a maximum rebound height $H_r$, momentarily comes to rest, and subsequently falls again. Now Eq. (18) can be written as:

$$0 - v^2_{after} = -2gH_r$$

$$v^2_{after} = 2gH_r \qquad (20)$$

Now dividing Eq.(20) with Eq.(19)

$$\frac{v_{after}}{v_{before}} = \sqrt{\frac{H_r}{H}} \qquad (21)$$

**Calculation of rebound height.**

First, when a body is dropped from a height H, it interacts with the floor and rebounds to a height $H_r$ ; the ratio $\frac{v_{after}}{v_{before}}$ defines the coefficient of restitution (COR or e), as given in Eq. (21). Hence, the ratio $\sqrt{\frac{H_r}{H}}$ also represents the coefficient of restitution. This provides the simplest method to estimate the rebound height by relating the original height to the value of e. However, the ratio $\sqrt{\frac{H_r}{H}}$ must be precisely equal to $\frac{v_{after}}{v_{before}}$ for the relation to remain valid.

The coefficient of restitution (COR or e) is defined as

$$e = \frac{v_{after}}{v_{before}} = \sqrt{\frac{H_r}{H}} \qquad (22)$$

Equation (21) must be satisfied with precision; otherwise, the determination of rebound height remains only qualitative. The applicability of Equation (21) may be considered theoretically limited because it depends on Eq. (18), which is valid only for uniformly accelerated motion. In horizontal motion, the magnitudes of velocities and corresponding deflections must be measured carefully, along with the relevant influencing factors. Equation (21) must be quantitatively verified before applying this method to calculate the rebound height ($H_r = e^2 H$)

**(ii) Coefficient of Restitution in terms of time of fall ($T_{fall}$) and time of rebound ($T_{reboud}$) or extended form of Coefficient of Restitution**

Consider another kinematical equation that relates the distance traveled to the time elapsed, expressed as follows:

$$S = ut + \tfrac{1}{2}gt^2 \qquad (23)$$

This equation is again derivable and applicable to uniformly accelerated motion. It provides an alternative interpretation of the COR through temporal symmetry, that is, by comparing the time of fall of the body to the surface ($T_{fall}$) with the rebound time ($T_{rebound}$); however, this approach may complicate the measurement of COR, as it relates the same to temporal symmetry. Now Eq. (23) may be written for the falling and rebounding phases.

For the falling motion:

$$H = \tfrac{1}{2}gT^2_{fall} \qquad (24)$$

After interaction with the surface (floor), the body rebounds upward due to the reaction force generated in accordance with Newton's Third Law of Motion; at the maximum height, the velocity becomes zero. The time of travel from the surface to the highest point is termed the time of rebound ($T_{rebound}$). At this point, the body momentarily comes to rest and then falls again due to gravity.

$$H_r \text{ (upward)} = \tfrac{1}{2} gT^2_{rebound} \qquad (25)$$

Now dividing Eq.(25) with Eq.(24).

$$\text{or} \quad \sqrt{\frac{H_r}{H}} = \frac{T_{rebound}}{T_{fall}} \qquad (26)$$

Comparing Eq.(26) with Eq.(22)

$$\sqrt{\frac{H_r}{H}} = \frac{T_{rebound}}{T_{fall}} = \frac{v_{after}}{v_{before}} \qquad (27)$$

Thus, we get another extended equation for coefficient of restitution as

$$e = \frac{v_{after}}{v_{before}} = \sqrt{\frac{H_r}{H}} = \frac{T_{rebound}}{T_{fall}} \qquad (28)$$

Now the ratio of $T_{rebound}$ and $T_{before}$ i.e. $\frac{T_{rebound}}{T_{fall}}$ must be equal to $\frac{v_{after}}{v_{before}}$ further equal to $\sqrt{\frac{H_r}{H}}$. This places additional constraints on the applicability of the coefficient of restitution. Equation (27) must be satisfied for all bodies, irrespective of their physical characteristics, even when they are externally dropped from different heights. Thus COR may also be measured in terms of ratio of $T_{rebound}$ and $T_{before}$ .

The variables i.e. $v_{after}$ , $v_{before}$ , $T_{rebound}$, $T_{fall}$ , $H_r$ and H may be experimentally determined and tabulated for a particular body at a given values of H, and then similarly tabulated for another body under different conditions.

Equation (28) must be satisfied for all bodies under systematic and controlled observations. Reliable conclusions can then be drawn; as stated in Section (4.0), a sufficiently large number of observations is necessary to obtain confirmatory results.

With only a limited number of observations, the theory cannot be considered universally valid for all bodies under varying physical conditions.

However, comparing the COR and the initial height H to calculate the rebound height $H_r$ remains the simplest qualitative method. In contrast, Eq. (28) provides quantitative verification, as the ratios of velocities, times, and heights are compared numerically and systematically. Each term in Eq. (28) may be experimentally confirmed.

**Calculation of Rebound Height**

The rebound height may be precisely calculated if the Eq. (28) is satisfied, it is first and foremost condition is that Eq.(28) is obeyed for various bodies. To get the rebound height in terms of time squaring Eq. (27) we get

$$H_r = H\left[\frac{T_{rebonud}^2}{T_{fall}^2}\right] \quad (29)$$

$$H_r = H\left[\frac{v_{after}^2}{v_{before}^2}\right] \quad (30)$$

The coefficient of restitution (COR) for a hard rubber ball (spherical) typically lies in range 0.7–0.8. Here it added the value of COR or e, must be determined if the various terms i.e. $\frac{v_{after}}{v_{before}}$ , $\sqrt{\frac{H_r}{H}}$ and $\frac{T_{rebound}}{T_{fall}}$ are equal.

Let the body is dropped from certain heigh. Hence by measuring velocities, $v_{after}$ , $v_{before}$ , $T_{fall}$ and $T_{rebound}$ may be easily estimated. If the effective COR for the ball–surface system is taken as e=0.7, then the corresponding post-impact velocity $v_{after}$ , time of rebound $T_{rebound}$ and the maximum rebound height $H_r$ are theoretically given by

$$H_r = 0.49 \text{ m} \quad (31)$$

$$v_{after} = 0.7\, v_{before} \quad (32)$$

$$T_{rebound} = 0.7\, T_{before} \quad (33)$$

**Estimation of $H_r$ in existing literature** : The experimental measurement of rebound height using the coefficient of restitution is accurate only within approximately 5–6 % for rigid spherical bodies under ideal conditions, whereas for soft, non-spherical, or realistic bodies the uncertainty is considerably higher. The rebound height directly depends on the coefficient of restitutions which is variable dependent on impact velocity, material properties, ram drop height , consequently rebound height also varies [17-19]. For example, the rebound heights of cone and sphere of the same mass and composition are different.

Thus, determining the coefficient of restitution and using it to find rebound height for various bodies is an exceptionally tedious process

Thus, this method cannot be regarded as strictly quantitative. Moreover, the rebound height should also be verified independently in terms of the time of fall and the time of rebound. In various observations, bodies of different masses, shapes, and compositions on diverse surfaces must be used that is, the domain of observations must extend beyond standard laboratory conditions or standard bodies.

Further values of $v_{before}$ and $v_{after}$ be precisely measured.

This method generally involves dropping symmetrical bodies from relatively small heights (e.g., 1 m); however, the observations may be repeated by releasing the bodies from greater heights, such as 4–5 m or other values. Consequently, both approaches for measuring rebound height are predominantly qualitative and require systematic experimental confirmation repeatedly for various bodies. Therefore, Eqs. (26–33) must be validated experimentally, each term must be specifically measured quantitatively.

**Typical comments on Eqs.(27-28).**

Currently rebound heights are calculated by the equation $H_r=e^2H$, and this method is qualitative in nature, as numerous experiments are feasible depending upon characteristics of bodies and surface. This discussion also associates the coefficient of restitution with time symmetry. Thus, it puts some constraints on applicability of Eq. (27); firstly Eq. (27) must be quantitatively confirmed for various values of velocities, times, and heights. Only then Eq. (28) may be applied to calculate rebound heights.

As an additional ratio of time of rebound and time of fall appears in the equation, it therefore imposes additional but strict constraints on applicability of rebound height equation at quantitative level. Thus some other method for calculating rebound heights may be formulated or proposed.

**(iii) A simpler method to estimate rebound height ($H_r$) using Eq. (18) i.e.** $v^2 - u^2 = 2gS$

The rebound height may also be calculated directly from Eq. (18,34), without involving the coefficient of restitution (COR or e). When the body rebounds from the surface or floor after interactions with initial velocity (initial velocity, $v_{after}$), it rises to a maximum height $H_r$, where the velocity becomes zero (v=0). Thus,

$$0 - v^2_{after} = -2gH_r$$

$$H_r = v^2_{after}/2g \qquad (34)$$

The qualitative explanation is straightforward: if a body is dropped from 1 m, 2 m, or a greater height, and its velocity after interaction ($v^2_{after}$) is larger, then the rebound height will be correspondingly higher. For quantitative analysis, Eqs. (29-33) must be verified for all bodies under various conditions. In addition, $H_r$ from Eq. (34) must be quantitatively confirmed.

However, Eqs. (18) and (23) are derived for uniformly accelerated motion, whereas rebounding bodies may exhibit arbitrarily variable motion during impact. These equations may also be applied to horizontal motion when a body strikes a target; in such cases, resistive forces and dissipative effects of the system must be properly evaluated. Moreover, bodies of irregular shape may rebound at varying angles under such conditions. All methods of measurement $H_r$ must coincide. Consequently, such experiments in basic physics would lead to novel outcomes. The Eqs.(21,27,34) may be repeated by dropping bodies from different heights ( H1, H2 , H3 ………) to experimentally confirm the various equations relating to rebound height. Likewise rebound height ($H_r$) may be calculated by measuring Eq.(23) as

$$S = H_r \text{ (upward)} = ½\, gT^2_{rebound} \qquad (25)$$

Thus all values of $H_r$ given in terms of various parameters may be determined . All values of $H_r$ must coincide for suitable parameters.

### 4.3 Hertz Contact Theory (1882) or Impact Force Method to calculate Rebound Height ($H_r$)

Hertz Contact Theory (HCT) is applicable only under idealized elastic and geometric conditions (spherical bodies), and hence its applicability to complex real-world interactions is limited the equation for contact time [20] is given by

$$T_c = K \sqrt[5]{\left(\frac{m^2}{RE^2 v}\right)} \qquad (35)$$

where K constant nearly equal to 2.94, m is mass of ball, R is radius of curvature of sphere, E is combined stiffness ( elastic modulus) and v is velocity. This theory assumes purely elastic, frictionless, non-adhesive contacts under quasi-static loading between smooth, non-conforming half-spaces. Its application is exceptionally complex in such phenomena.

**Applicability in rebounding bodies with Newton's second and third laws along with kinematic equations.**

This perception may be mentioned in application of estimation for rebound height for the completeness only as it is not applicable in general cases. Thus the Eqs.(36-45) are speculative applications of HCT (1882)

Let body of mass 1kg, falls from height 1m. Its change in momentum can be determined as

$$v^2 = 2gS = 19.6\ m^2/s^{-2} \quad \text{or } v = 4.43\ ms^{-1} \quad \text{or} \quad dp = 4.43\ kgms^{-1} \qquad \mathbf{(36)}$$

When a body freely falls on the floor, due to mutual interactions, reaction force arises in opposite direction, and body rebounds upward. According to Newton's third law, action and reaction forces act simultaneously in equal–opposite pairs. According the above theory, the duration of contact between the rubber ball and the surface is of the order of $10^{-4}$ s-$10^{-3}$ s as reported in the literature for several impact scenarios in engineering mechanics. Accordingly, the magnitude of the impact force is given by equation of second law of motion i.e. Eq. (5) as

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = 4.43/10^{-4} = 44300 \text{ newtons} \qquad (37)$$

When body rebounds upward, then weight of body mg ( 9.8 newton's) acts downward.

The net upward force = 44300 -9.8 (mg) = 44290 newtons. (38)

Consequently, exceptionally high force 44290 newtons act on the body. Thus, for body of mass 1kg, effective acceleration (**a**$_{\mathbf{eff}}$ ) produced will be

$$\mathbf{a_{eff}} = 4.43\text{x}10^4\ m/s^2. \qquad (39)$$

**Calculation of rebound height in terms of-acceleration, S = ut + ½ at$^2$**

Consider a thought experiment, if this acceleration were considered sustained for $10^{-4}$s or $10^{-3}$s, the predicted rebound heights would be on the order of 0.215mm or 0.0215cm. Eq. (23) relates distance travelled with time. Thus,

$$H_r = ½ (a_{eff})\ t^2 \qquad (40)$$

or $H_r = ½ (4.43\text{x}10^4\ m/s^2)\ t^2$ (41)

If the body is assumed to be sustain high value of acceleration for $10^{-4}$ s , then

S or $H_r = ½ (4.3\text{x}10^4\ \text{x}10^{-8}) = 0.215$mm (42)

If the body is assumed to be sustain high value of acceleration for $10^{-3}$ s , then

S or $H_r = ½ (4.3\text{x}10^4\ \text{x}10^{-6}) = 0.0215$cm (43)

If this peak acceleration $4.3\text{x}10^4$ is assumed to be sustained by body for a duration of 0.1 s or 0.01 s,

S or $H_r = ½ (4.3\text{x}10^4\ \text{x}10^{-2}) =$ 221 m (44)

S or $H_r = ½ (4.3\text{x}10^4\ \text{x}10^{-4}) =$ 2.21 m (45)

Thus, various cases are theoretically considered here for the sake of completeness. However, these values of heights appear very inconsistent, scientists may speculate experiments so that contact durations increase.

Thus, in existing physics, scientists have to make sufficiently systematic and practical attempts to quantitatively determine the rebound heights or distances of bodies striking a target and rebounding at the macroscopic level. Theoretically, many equations have been derived for analytical discussion and interpretation of the phenomenon.

**Table 1 General Comparison on Existing perceptions and Generalized form of Third law about rebound height.**

| Sr No. | Attribute | $v^2 - u^2 = 2gs$ (Kinematic equation) | $S = ut + ½gt^2$ (Kinematic equation) | Newton's Original Law | Newton's Generalized or Extended Law |
|---|---|---|---|---|---|
| 1 | Applicability | Uniformly accelerated | Uniformly accelerated | Uniform or non-uniform | All types with additional factors. |
| 2 | Basis for Rebound Height | Newton's Third Law | Newton's Third Law | Third Law (Qualitative) | Third Law (Quantitative) |
| 3 | Parameters | H, u, v, g, and S | H, u, v, g, and t | Action ($F_{AB}$), Reaction ($F_{BA}$) $F_{BA} = - F_{AB}$ | $F_{BA}$, $F_{AB}$, shape, composition, target, other factors Reaction ($F_{BA}$) = − [$K_{shape} \times K_{composition} \times K_{target} \times K_{other}$] Action ($F_{AB}$) |
| 4 | $H_r$ using coefficient of restitution | $H_r = H\left[\frac{v^2_{after}}{v^2_{before}}\right]$ | $H_r = H\left[\frac{T^2_{rebound}}{T^2_{fall}}\right]$ | $F_{BA} = - F_{AB}$ Applied in experiments | Action ($F_{AB}$) = - K Action ($F_{BA}$) Rebound height = $D_f$ Reaction force = $D_f$ [$K_{shape}$ x $K_{composition}$ x $K_{target}$ x $K_{other}$ ] Action ($F_{AB}$) |
| 5 | $H_r$ using single equation | $H = \frac{v^2}{2g}$ | --- | as above | as above |
| 6 | $H_r$ using impact force method | ---- | $H_r = ½ (a_{eff})t^2$ $a_{eff} = 4.3x10^4 m/s^2$ | as above | as above |
| 7 | Nature of Experiments | Qualitative | Qualitative | Limited parameters | All possible parameters |
| 8 | Conservation of momentum | ----- | ----- | $v_{gun} = - m_{bullet} . v_{bullet} / m_{gun}$ (idealized equation ) | $v_{gun} = - m_{bullet} . v_{bullet} / Zm_{gun}$ (realistic equation) |

| | | | | | |
|---|---|---|---|---|---|
| 9 | Comments | Force: not explicitly related | Force: not explicitly related | $F_{BA}$, $F_{AB}$ are explicitly related | $F_{BA}$, $F_{AB}$ are explicitly related along with other factors. (Ki's) |

**5.0 Explanation of rebound experiments with help of Newton's Third Law of Motion.**

In existing physics, several mathematical equations involving the coefficient of restitution (COR or e) are used to calculate rebound height. These equations, namely Eqs. (21, 25, 27, 34), are primarily analytical in form and require systematic verification for quantitative validity. Our objective is to examine each equation individually and compare the predicted rebound heights with those obtained from alternative formulations. In the available literature, no comprehensive attempts have been made to test these equations over a wide range of parameters. Moreover, the impact-force method is applicable only within a limited regime of conditions.

Since the existing methods for calculating rebound height have not been rigorously validated across broad experimental ranges, their quantitative reliability remains restricted.

In science, more than one method may exist to explain a particular observation. Therefore, Newton's Third Law of Motion, which forms the conceptual basis of existing approaches, may be employed independently and exclusively to analyze and explain the behavior of rebounding bodies.

(a) In existing approaches used to explain rebound heights, Newton's Third Law of Motion (NTLM) is taken as the foundational basis, together with the kinematic equations $v^2 - u^2 = 2gS$ and $S = ut + ½gt^2$**.** These equations are derived under the assumption of uniformly accelerated motion and are therefore applicable only within that restricted framework. As discussed in Section (1.2), in such treatments NTLM, as the underlying principle, does not explicitly incorporate several significant physical factors. Consequently, the coefficient of restitution method (CORM) remains essentially qualitative in nature. Hence, certain limitations exist in the conventional explanation of rebounding bodies.

Therefore, as a trial approach, Newton's Third Law of Motion (NTLM) may be applied independently and exclusively to explain rebound heights, since the coefficient of restitution method (CORM) is conceptually based on it. This approach can incorporate all relevant factors influencing the results, provided that NTLM is appropriately generalized for the analysis of falling and rebounding bodies

(b) A freely falling basketball moves downward under the action of gravity. When it strikes the ground, according to Newton's Third Law of Motion, the interaction between the ball and the floor generates a reaction force simultaneously in the opposite direction; consequently, the ball rebounds upward. The action and reaction forces act in pairs. If the qualitative behavior of the ball is attributed to explain this reaction force, then the quantitative aspects of the motion may also be analyzed in terms of the same interaction force. Thus, if NTLM is used to explain the qualitative motion, it may likewise be employed to analyze the quantitative motion. There is no fundamental reason in physics why Newton's Third Law should be restricted to explaining only the qualitative motion of a ball and not its quantitative behavior.

(c) Newton originally illustrated the Third Law in the *Principia* through qualitative examples, such as a horse pulling a stone, a finger pushing a stone, and one body striking another. In this manner, he justified the action–reaction principle by expressing forces as mutual interactions between bodies.

Subsequently, scientists extended the applications of Newton's Third Law into broader domains, including interaction-based (action–reaction) systems and aerospace propulsion systems. These diverse phenomena are interpreted on the basis of Newton's Third Law of Motion. If NTLM is applied to explain such complex systems, then it may also be applied to rebounding bodies. The definition and mathematical expression of the law are regarded as universally applicable, as discussed in Sections (1.1–1.2).

### 5.1 Quantitative Explanation Using Newton's third law in rebounding bodies.

One may compile a dataset of approximately 100 macroscopic experimental observations as mentioned in section (4.0) using bodies of different shapes but identical mass and composition falling on the same floor. After analyzing simple examples, more complex cases may be systematically investigated and their results interpreted accordingly. As mentioned in Section (5.0), Newton's law may be applied in such cases also. Thus, this discussion applies to realistic or real-world conditions when motion of bodies with different shapes are considered and large numbers of observations are taken.

Here the reason is simple that Newton's third law of motion is over 340 years old and continuously used; hence to critically study the law we need sufficient experimental data, rather than few observations. Only then confirmatory conclusions may be drawn.

**General Discussion**

Let the body of mass 1kg freely fall in vacuum from point $A_0$ at height H (1 m). Then it must reach at the bottom at time 0.45 s, i.e.

$$T_f = \sqrt{\frac{2s}{g}} = 0.45\text{s} \qquad (46)$$

(a) The body falls downward with acceleration due to gravity g (9.78–9.83 m $s^{-2}$ , in different regions) arising from the mutual gravitational attraction between the body and the Earth. The downward force acting on the body is the gravitational force, or its weight, which is expressed as mg. In this framework, the action force corresponds to the gravitational force or weight acting downward towards floor /surface. Consequently, for bodies of identical mass (1kg), the magnitude of the action remains the same (mg), independent of shape and other physical characteristics. The definition and Eq. (1) does not put any constraints on the universality of the law.

$$\text{Action } (F_{BF}) = \text{Force} = \text{weight (mg)} = 9.8 \text{ newtons} \qquad (47)$$

(b) The body strikes the surface ( floor, say), with action ($F_{BF}$) 9.8 newtons. According to Newton's third law, a reaction force ($F_{FB}$) arises simultaneously due to mutual interactions between a body and the floor; its magnitude equals that of the action (i.e., 9.8 N) and is directed upward, opposite to the downward action of the body. The action and reaction exist in equal and opposite pairs as given by Eq. (48).

$$\text{Reaction } (F_{FB}) = - \text{ Action } (F_{BF}) = -9.8 \text{ newtons} \qquad (48)$$

Due to this this reaction force body rebounds upward. Newton's third law assumes reaction force (9.8 newtons) is precisely equal to action force (9.8 newton) but opposite in direction. Thus, Newton considered ideal cases when both forces are equal before and after interaction.

**Eq.(1) and Eq.(50) :** Newton's Third Law applies equally to particles with masses of a few milligrams or less, as well as to bodies weighing several kilograms or more. Newton defined a body in Definition 1 of the *Principia*; therefore, such observations can logically be extended beyond laboratory conditions or idealized calculations involving rebounding bodies. Equation (1) represents an idealized form. The original law implies that equal and opposite reaction force arises spontaneously, neglecting the role of other factors and the surface on which the interaction occurs. The other factors also influence the results. This constitutes the core issue. In contrast, Equation (50) accounts for all relevant factors, making it a more realistic representation, here applied in case of freely falling and rebounding bodies. When a body falls, the acceleration due to gravity is +g, and when the body rebounds upward, the acceleration is −g.

**Quantitative nature of Discussion**

(i) **Spherical bodies:** Let the spherical body (symmetric), under suitable conditions, may rebound to nearly the original point $A_0$ at height H in the opposite direction. This observation describes the standard situation. Thus, in this case, the magnitude of the reaction force is equal to the action and opposite in direction. According to Newton's third law, when a body strikes the bottom surface (floor), the reaction is generated simultaneously; action and reaction occur in pairs, equal in magnitude and opposite in direction; reaction cannot be diminished in any way. Thus, in view of Newton's Third Law, we can write

Action ($F_{BF}$) = -Reaction ($F_{FB}$) = -9.8 newtons. (opposite direction) **(48)**

**Law is fully justified under ideal conditions:** If the action and reaction are equal and opposite, as in Eq. **(48),** the body rebounds to the original point $A_0$, retracing its original path. In case, under some conditions, the body rebounds to its original point ($H=H_r$ =1m) or retraces the same trajectory, then the 'rebound angle' (RA) may be taken as zero. Under standard or ideal conditions, Newton's Third Law may be regarded as fully satisfied for a spherical body, since both conditions (equality of action and reaction magnitudes, and opposite direction) are satisfied simultaneously in this limiting particular case.

Thus, the rebound height may be regarded as a measurable physical indicator of the reaction force; alternatively, $H_r$ may be interpreted as a macroscopic manifestation of that reaction force, or the reaction force may be quantitatively calibrated in terms of the observed rebound distance.

**Standard conditions of the system:** Consequently, ideal conditions of the body need to be determined when third law is completely obeyed in the observations. For simplicity in one of the ways, of the ideal body may be spherical in shape with suitable material or composition, and the target may consist a carpet of 0.5 m thickness with dimensions 3 m × 3 m of uniform suitable composition. The mass, composition, shape of body and nature of target may be regarded as ideal if the body rebounds to height H, i.e., $H = H_r$ retracing original path. At present it appears quite complex, but with suitable choice of body (highly elastic bodies are being discovered in laboratories) and target, it may be feasible in practice. The practical realization of ideal conditions is an extremely tedious process, as numerous combinations of different materials and interacting substances may need to be systematically considered.

Thus the rebound height is manifestation of reaction force. In the third illustrative example of the Third Law, Newton explained that when one body impinges upon a second body, the reaction force produces a change in the motion of the second body, causing it to move through a certain distance. The greater the applied force, the greater the resulting displacement of the body. Moreover, due to the reaction force, the first body also experiences a corresponding change in motion and moves backward.

(ii) **Bodies of different shapes:** In real-world conditions, the bodies of identical mass and composition but of different shapes (e.g., sphere, hemispheres, cylinders, triangular and polygonal bodies, cones, needles, elongated pipes, thin sheets, and other arbitrary or irregular forms) rebound to lower heights and varying angles, than spheres of the same mass and composition. Moreover, bodies of irregular shape rebound along different trajectories, exhibiting variations in both direction and spatial path after impact.

According to Newton's Third Law, the action and reaction are assumed to be same for bodies of different shapes (when the mass is the same), as expressed by Eqs. (48**)** analogous to the case of a spherical ball. However, the rebounding behaviors is different from theoretical predictions. Action and reaction occur as equal and opposite force pairs. According to Newton's third law body of different shape of mass 1kg , when freely fall , it has same action as that of spherical body , so Eq.(47) becomes

$$\text{Action } (F_{dsF}) = \text{Force} = \text{weight } (mg) = 9.8 \text{ newtons} \quad (49)$$

**Strikingly Divergent Observations:** However, a flat body (having same mass and composition as that of spherical body interacting with same surface) is observed to rebound to the minimum height, whereas irregular bodies exhibit variable and often different rebound heights and varying angles. Since the masses are identical (1kg), the action force (9.8 newtons) is the same as that of the spherical body. Should we therefore conclude that the experimental reaction force is greater for a spherical ball and smallest for a flat body? If so, such an inference must be logically and theoretically justified. Theoretically, according the action and reaction for such bodies are expressed by Eqs. (47-49**)** and their values are identical to those obtained for a spherical body. However, the experimental behavior is significantly different from that of a spherical body. In the existing framework, Eqs. (21, 25, 27,34) do not address such effects.

Here scientifically, along with $F_{AB}$ and $F_{BA}$, all other influencing factors are discussed in section (6.0). These observations may be repeated for bodies in free fall from varying heights ($H_1$, $H_2$, $H_3$, etc.) across multiple trials.

**(iii) Factors responsible for reduced rebound height ($H_r$) varying rebound angles.**

**(a) Loss of energy during interactions.**

These factors may be systematically evaluated within this context. The causes for bodies rebounding to a lower height $H_r$ , include energy losses, such as heat energy, sound energy, other associated forms, and additional significant effects depending on interacting bodies. An effort should be made to minimize the energy losses as much as possible. Non-deformable bodies must be selected to achieve conceptual simplicity.

These energies must be carefully quantified, and it should be examined whether they are solely responsible for bodies rebounding to a reduced height, or if other factors demand consideration in the quantitative analysis.

(b) Other inherent factors: Thus, additional factors beyond $F_{AB}$ and $F_{BA}$ are likely responsible for the observed anomalous behavior, yet remain unaccounted for within the existing law and scientific framework. These factors include the shape of body as composition, material properties of the interacting bodies and mass are the same. The other relevant experimental parameters involved either implicitly or explicitly [21-22] need to be discussed. These factors may be assessed using varied observations and systematic critical evaluation. This aspect is discussed in section (6.0).

As discussed in Sections (1.0–1.3), according to the definition and Eq. (1), only $F_{AB}$ and $F_{BA}$ are significant, while other influencing factors are not included. This may be attributed to the fact that Newton formulated the law during the era of natural philosophy, when systematic theoretical development and precise experimental verification were not as advanced as they are today.

(iv) **Time of fall and rebound to maximum height:** Under ideal conditions (in vacuum), if a spherical body of mass 1 kg falls from a height of 1 m and reaches the floor from point **$A_0$** in 0.45 s (action is 9.8 newtons) as in **Eq. (46),** it should rebound (reaction is 9.8 newtons but in the opposite direction) and return to point **$A_0$** in the same duration.

In the existing literature, Newton's Third Law is commonly interpreted using parameters as in Eqs. (18-23), F = ma, coefficient of restitution, and action–reaction forces. Alternatively, it may be examined in terms of temporal symmetry by comparing the time of fall of a body to the surface ($T_{fall}$) with the time taken after rebound ($T_{rebound}$) to return to its initial position $A_0$ from surface. Under ideal conditions, these two times are expected to be equal. Because the motion of falling and rebounding bodies has not been analyzed quantitatively in the existing literature, the practical relevance of the time of descent to the point of impact ($T_{fall}$) and the time of rebound to the maximum height ($T_{rebound}$) or original point has remained largely unrecognized.

Had the law been experimentally verified quantitatively for falling and rebounding bodies, fundamental aspects such as the difference between the time of fall to the surface ($T_{fall}$) and the time of rebound ($T_{reboud}$) to the maximum height would have been recognized much earlier. Further, the bodies may be made to fall from different heights ($H_1$, $H_2$, $H_3$ etc.)

(v) **Angle of rebound.** Newton's third law states that the magnitude of the reaction equals that of the action, and its direction is exactly opposite to the action. In Eq. (1), the negative sign is introduced externally to denote the opposite direction of the reaction force. Here, the body is assumed to fall freely without spin or rotation, thereby simplifying the analysis and calculations. However, separate sets of observations may be conducted to examine the effects of spin, rotation, and other relevant factors influencing falling and rebounding bodies.

**Different angles:** For freely falling bodies, a spherical ball rebounds along the line of fall, while bodies of other shapes, such as semi-spheres, cylinders, triangles, polygons, cones, long thin pipes, flat sheets, or arbitrary forms, rebound at angles deviating from the line of fall. For these bodies, the rebound angles vary depending on the specific conditions of impact. The angle of fall also plays a significant role in determining the nature of the interaction and the subsequent rebound when a body strikes a surface. These variations must be fully harnessed to realize their maximum potential, and this behavior must be analyzed and discussed in detail following experimental measurements. Thus, $v_{before}$, $v_{after}$ , $T_{rebound}$ , $T_{fall}$, $H_r$ , H and angle of rebound must be noted in all observations for completeness.

(vi) **Horizontal impact of the body with the target:** In this case body has to be externally pushed with some known force. When a ball or body travels horizontally across a surface, strikes a target, and rebounds, frictional forces and energy dissipation (as heat and sound) become significant factors that must be quantified along with other relevant parameters. Thus, the properties of the surface on which the interaction occurs play a significant role. In this case, the action force must be precisely measured using external instruments. The role of the target's characteristics becomes increasingly significant in this context.

The use of asymmetrical bodies often results in irregular or unpredictable forward and backward motion, making the accurate measurement of associated quantities both challenging and extremely difficult. Similarly, experiments that involve varying the composition of bodies and targets are highly cumbersome to conduct with quantitative precision.

Therefore, falling and rebounding experiments involving bodies of identical mass and composition but different shapes are discussed first, owing to their simplicity and their fundamental importance in examining the validity of the Third Law. For the obvious reasons simple translation motion of falling bodies is considered without spin or rotation. In addition, we come across far more complicated experiments to confirm Newton's third law. Such typical and complex experiments for the confirmation of the third law may be conducted subsequently. If such experiments are conducted quantitatively and their results are rigorously interpreted, they could have far-reaching implications for classical mechanics.

### 5.2 Futuristic Experiments

In the future, such experiments may be performed in low-gravity environments, such as on the Moon, where the acceleration due to gravity is approximately one-sixth of Earth's value. The lower value of g increases the duration of free fall and modifies the time of ascent during rebound. At the International Space Station, or in other microgravity environments, such experiments may also be explored under conditions of weightlessness.

Furthermore, researchers are exploring elastic materials more advanced than Super Balls and Sky Balls to achieve even greater bounce performance. Thus, the law needs to undergo quantitative testing in such experimental situations. Thus, the concise formulation of Newton's third law underpins a wide range of experimental studies, highlighting both its applications and possible departures. These must be exploited or performed to their fullest extent.

**Speculated experiments in view of the simultaneous application of Newton's First and Third laws:** Speculated experiments may be conceived on a completely frictionless surface where no external force acts on the system. When body A pushes body B, the interaction produces equal and opposite forces on both bodies simultaneously, in accordance with Newton's third law.

If no net external force acts, body B should then continue to move with uniform velocity after interaction, consistent with Newton's first law. Similarly, body A should also move backward as a consequence of the reaction force. Since a perfectly frictionless system or a condition of zero external force cannot be realized in practice, experiments should be designed to progressively approximate such idealized conditions, and the results should be interpreted with this limitation in mind.

### 5.3 Experimental Approach Using Low-Cost Instrumentation

Experiments on falling and rebounding bodies may be conducted using low-cost instrumentation while retaining adequate quantitative precision. A high-frame-rate USB camera enables recording of the impact event, and frame-by-frame analysis provides measurement of contact duration and post-impact kinematics. Photogate timers allow independent determination of fall and rebound times for cross-validation of results.

Rebound height may be estimated from calibrated slow-motion video using a reference scale within the field of view. The rebound angle relative to the initial line of fall can be obtained either through digital angle sensors or from motion analysis of recorded video. Where higher spatial resolution is required, laser displacement or optical range sensors may be incorporated, although such devices are not essential. This integrated video-based methodology permits systematic investigation of impact dynamics, transient deformation, and geometry-dependent rebound behavior under controlled experimental conditions.

### 5.4 State-of-the-art experiments regarding Newton's third law of motion.

Further, Newton's Third Law is increasingly investigated in diverse and complex systems, where reciprocity is being broken. Newton's third law predicts equal and opposite reactions, implying that sperm whipping their tails in viscous mucus should face symmetric drag that slows motion and dissipates energy. However, Kyoto University research published in *PRX Life* reveals [23] "odd elasticity," where asymmetric flagellar bending creates non-reciprocal forces, enabling efficient swimming and inspiring future medical micro-robots.

In a 2026 NYU experiment [24] levitating Styrofoam beads trapped in acoustic waves exhibit nonreciprocal interactions:

larger beads scatter strong sound waves and exert significant forces on smaller beads, while the smaller beads scatter weakly and fail to produce an equal opposing force, breaking action–reaction symmetry.

### 6.0 Generalized or Extended or Realistic Form of Newton's Third Law for Rebounding Bodies

Systematic shape-dependent variations in rebound behavior suggest that macroscopic impact interactions exhibit effective asymmetries not captured by idealized Newton's third-law formulations. Eq. (1) is justified under ideal conditions for perfectly rigid bodies of suitable composition on appropriate surfaces. The action and reaction symmetry is exhibited only under ideal conditions. A general explanation is required for realistic systems that represent everyday phenomena. Thus, the original form of NTLM may be theoretically extended and applied to such phenomena of rebounding bodies. Realistically there is a smooth transition in applications of NTLM from idealized systems to real-world systems. Here examples of rebounding bodies under different circumstances or conditions may be considered. Thus, various parameters such as shape, composition, surface, and other effective factors need to be taken into account. The fundamental and practical reasons for generalizing Newton's Third Law of Motion are outlined below.

(i) In existing physics, Eqs. (21, 25, 27, 34) are used to explain the rebound height of bodies. However, these equations describe the rebound heights qualitatively only . They are not specifically confirmed or cited in standard literature for this purpose. These equations involve velocity, time, gravitational acceleration g, and the original height H from which the body is dropped. They implicitly rely on Newton's Third Law of Motion as the basis: when a body interacts with the floor, an equal and opposite reaction force is simultaneously generated, causing the body to rebound.

(ii) In science, even if one method explains a phenomenon, an alternative or new method may be developed and applied. In scientific inquiry, the approach that explains phenomena in a simpler, clearer, and more comprehensive manner is generally preferred.

(iii) Newton's Third Law is widely extended in application to action–reaction phenomena such as bouncing balls, swimming, rowing, walking, jumping from a boat, balloon deflation, swimming fish, flying birds, gun recoil, and bodies striking and rebounding from horizontal surfaces. It is also applied in aerospace propulsion systems including rockets, fireworks, spacecraft, airplanes, missiles, helicopters, and drones.

Since Newton's Third Law is applied to numerous phenomena and forms the conceptual basis of Eqs. (21, 25, 27, 34), it may also be independently and exclusively used to determine the rebound height of bodies. If generalized appropriately, it can incorporate additional significant factors that influence the results.

(iv) Furthermore, in certain state-of-the-art experiments [23-24 ], action and reaction forces have not been strictly confirmed as equal in magnitude and opposite in direction, as discussed in Section (5.4); that is, non-reciprocal interactions have been reported. In practical engineering designs, gliders and drones possess specific shapes, and aquatic animals adjust their body shapes while swimming, indicating the importance of geometry in such phenomena. Accordingly, the effect of shape is incorporated in Eq. (50) through Eq. (51) as $K_{shape}$ .

(v) Quantitative macroscopic experiments: The recoil velocity of a gun, derived from the law of conservation of momentum as given by Eq. (7), has not been comprehensively verified quantitatively at the macroscopic level. Additionally, equations for one-dimensional elastic collisions are based on the simultaneous application of conservation of momentum and kinetic energy; similar to Eq. (7), Eqs. (2, 8–13) require specific macroscopic quantitative confirmation. Purely theoretical discussions are insufficient for complete understanding; therefore, such predictions must be explicitly validated. Standard textbooks do not provide extensive macroscopic experimental verification of these results.

**The Logical and Consistent Generalization of NTLM**

Thus, an alternative or extended theoretical framework can be developed by generalizing Newton's Third Law of Motion, as this formulation accounts for all relevant factors influencing the interaction, and may therefore be regarded as a more realistic representation of the third law. Realistically these factors include shape, composition, symmetry of bodies, characteristics of surfaces, etc. These factors are not taken into account by the original form of NSLM and Eq. (1). The definition of the third law and Eq. (1) takes into account $F_{BA}$ and $F_{AB}$ only; other factors never appear in the equation

. This discussion is confined to rebounding bodies in realistic macroscopic systems; in all other cases where Newton's Third Law has been experimentally validated, the classical formulation remains applicable.

As mentioned in Section (1.2), the definition of NTLM and Eq. (1) consider only $F_{AB}$ and $F_{BA}$ while other significant influencing factors are neglected. In experimental determination of rebound heights (vertical motion) or distances (horizontal motion), the shape and composition of bodies, the nature of the surface, and additional factors play important roles. These factors may be incorporated into NTLM through an appropriate generalization or extension [5–6, 21-22, 25-30], remaining within the conceptual scope of the original law. Such an approach enhances the descriptive range of the law.

Therefore, the generalization of Newton's Third Law is logically consistent and may also be regarded as a hypothesis to address possible conceptual limitations. The hypothesis or postulate will be validated if supported by experimental evidence. This scientific tradition also applies to this case and Eqs. (50, 53). Bohr's quantized orbit hypothesis and Einstein's explanation of the photoelectric effect are notable examples of physical hypotheses confirmed by experiments. The present generalization is intended only to explain rebound heights of bodies and does not affect the numerous other situations where the law has been experimentally verified. Accordingly, the proposed formulation seeks to provide a broader quantitative framework for describing rebound behavior and may be expressed as:

$$\text{Reaction} \propto \text{Action} \quad \text{or} \quad \text{Reaction } (F_{BA}) = -\,K \text{ Action } (F_{AB}) \qquad (50)$$

where K is a proportionality coefficient or effective phenomenological parameter. If =1, Eq. (50) reduces to Eq. (1); otherwise, the reaction force may differ in magnitude from the action force. Unlike the simplified approach of Equation (1), which only considers $F_{AB}$ and $F_{BA}$ Equation (50) integrates all relevant parameters. This makes Equation (50), the more comprehensive choice for analyzing complex, real-world scenarios. If the value of coefficient K is unity, then Eq. (50) reduces to Eq. (1) and hence explains all those effects explained by Eq. (1). Thus, there is a smooth transition from Eq. (50) to Eq. (1). There would have been a clear inconsistency if Eq. (1) was not recovered from Eq. (50).

The value of K may be determined experimentally under different conditions. Equation (50) is further elaborated through Eq. (51) when all relevant experimental factors are included. Since the primary objective is to determine rebound heights (vertical motion) and rebound distances (horizontal motion), Eq. (53) is formulated explicitly in conjunction with Eq. (50).
Eq. (1) applies to ideal systems, whereas Eq. (50) suits realistic systems where various external factors arise in experiments. Newton himself defined the body in Definition I of the *Principia*; the law must be interpreted in the spirit of this definition. While this generalized equation accurately models rebounding bodies in real-world conditions, it is not intended for cases that rely on specific experimental derivations.

### 6.1 K as Experimental Coefficient or a Phenomenological Parameter.

In Eq.(50) K is a dimensionless parameter, and its magnitude may vary from one observation to another, depending on the associated experimental variables.

The coefficient K incorporates several subtle factors not included in the original law, such as shape, size, asymmetry, composition, material properties, physical characteristics of the bodies, properties of the target, nature of surface , energy conversion during interaction, and all other influential variables that implicitly or explicitly affect the outcomes. Since bodies rebound at different angles due to shape, size, geometry, and related factors, K also indirectly governs variations in rebound angles. It also accounts for possible variations in results in case bodies are dropped from different heights.

Consequently, the law exhibits applicability across a broad range of physical phenomena. Hence, the value of K depends on experimental conditions and therefore varies from one experiment to another. The value of K may be determined through standardization, calibration, or comparison, depending on the practicality and feasibility of the experimental procedure. Thus, a substantial body of experimental data must be systematically collected to enable rigorous analysis and reliable interpretation. Thus, the value of K may be expressed in one of the following ways:

$$K \text{ (coefficient or additional factor)} = K_{shape} \times K_{composition} \times K_{target} \times K_{other} \qquad \textbf{(51)}$$

($K_{shape} \times K_{composition} \times K_{target} \times K_{other}$ ) is an additional expression as in Eq. (51) which implies the reaction force may deviate from the action depending on various influencing factors and effects. The original formulation of Newton's third law, as well as other existing approaches used to explain rebounding bodies, do not explicitly account for such factors. K may be better called a phenomenological parameter as it varies from one set of observation to other. The reaction force may or may not be equal to the action force, and the direction of the reaction force depends on the asymmetry of the interacting system. Realistically, Eq. (50) becomes in comprehensive form as

$$\textbf{Reaction } (\mathbf{F_{BA}}) = \mathbf{-}\ [K_{shape} \times K_{composition} \times K_{target} \times K_{other}]\ \textbf{Action } (\mathbf{F_{AB}}) \qquad (50)$$

$K_{shape}$ accounts for the influence of shape, size, geometry, asymmetry, and related factors; $K_{composition}$ represents the effects of material composition and characteristics such as elasticity and flexibility; and $K_{target}$ describes all measurable effects associated with the properties of the target. Newton treated the target simply as a second body without specifying its characteristics, although these factors are important during interaction. $K_{other}$ represents contributions from the interaction surface and other residual effects, including energy transformation and redistribution between interacting bodies, as well as all additional elusive influencing factors; deviations from the normal rebound angle may also be incorporated within it. The measurement of $K_{other}$ is more challenging than the determination of other $K_i$ parameters**.** If bodies are dropped from different heights in various observations, the corresponding $K_i$ values may vary. Such observations are required for completeness. So Eq.(50) is practically a **phenomenological generalization or extension as it takes in account all** experimental factors. If Eqs. (50, 53) are justified by experiments, then this extension or generalization will be experimentally confirmed. As these equations are newly proposed, experiments should therefore be planned and executed meticulously.

The value of K may be determined in a manner analogous to coefficients such as thermal conductivity, viscosity, friction, drag, etc. Several other parameters of this type are discussed in the literature. Analogous to the coefficients in the Bethe–Weizsäcker mass formula, it encapsulates complex interactions. Thus, $K_{other}$ is equally important for the completeness of this model, since some interactions are inherently complex in measurements.

Thus, the coefficient K is a key factor in extending idealized formulations of Newton's Third Law to realistic macroscopic interactions. K approaches unity in idealized limits, thereby recovering the conventional form of Newton's third law. If K=1, then Eq. (50) reduces to Eq. (1) i.e. generalized equation gives same results as original law.

Experimental measurements of the original height (H), rebound height ($H_r$), time required to reach the floor ($T_{fall}$), the time required to rebound the maximum height ($T_{rebound}$), velocity $v_{reboud}$ , $v_{before}$ and rebound angle, may be measured and tabulated for integrated interpretation. The observations be taken for different values of H ($H_1$, $H_2$, $H_3$….). Thus, pre-requite to develop a theory is that to conduct experiments over wide range of parameters, and draw conclusions. The conclusions not may be drawn from limited theoretical analysis and observations.

### 6.2 Generalized or Extended Form of Newton's Third Law as a Natural Extension of the Original Principle

The generalized or extended form aims to unify the effects of shape, composition, target properties, surface characteristics, and other relevant factors within a single framework, rather than treating them as separate domains. It does not replace Newton's third law but extends it, embedding macroscopic realities into a unified operator equation. This formulation illustrates how the generalized law integrates separate factors into a structured equation, instead of leaving them fragmented across physics, mathematics, and engineering subfields. It thereby serves as a bridge between classical mechanics (some theoretical interpretations are valid only for idealized systems.) and real-world impact phenomena.

The proposed extension does not replace Newton's third law but identifies the conditions under which its classical form holds exactly and those under which a generalized or extended interpretation is necessary. The original law is recovered as a limiting case, ensuring full conceptual consistency with Newtonian mechanics. Eq. (50) provides a theoretical extension of Newton's third law and calls for experimental verification for conclusive analysis over wider range of parameters.

### 6.3 Generalized Formulation of Newton's Third Law of Motion

Therefore, Newton's third law of motion, in its generalized or extended form, may be conceptualized in the following way:

*"Every action has a proportional reaction; the magnitude and direction of the reaction would be precisely equal and opposite depending upon experimental factors such as the shape, size, characteristics, and other involved factors, etc., of the interacting bodies."*

Under idealized conditions, the proportionality reduces to equality, and the generalized formulation converges to the original law. Under some conditions the net effect of $K_i$'s may be unity or each $K_i$ is individually unity. Thus, the above definition of the law applies to realistic systems, where all factors affect the results and must therefore be taken into account. Consequently, all factors must be included for the law's comprehensiveness. This generalization or extension is inconsequential for cases where the third law is experimentally justified; it targets rebounding bodies only.

### 6.4 Difference between generalized Eq. (53) and Original Eq. (1) of the third law.

Equation (1) contains only two terms: action ($F_{AB}$) and reaction ($F_{BA}$) which are defined as equal in magnitude and opposite in direction. No additional term, variable, or parameter appears in Eq. (1), implying that action and reaction are equal and opposite under all specified conditions. In this sense, Newton's Third Law of Motion (NTLM) is regarded as universal in nature. There is no explicit term in Eq. (1) that modifies the magnitudes or directions of the action ($F_{AB}$) and reaction ($F_{BA}$), forces.

. In the existing literature coefficient of restitution method (CORM), NTLM is used as the foundational principle to calculate rebound height together with the kinematic equations (18, 23), namely $v^2 - u^2 = 2gS$ and $S=ut+½gt^2$. Even in this formulation, the results remain essentially qualitative in character.

In contrast, Equation (50) represents an extended form of Eq. (1) within the conceptual domain of Newton's Third Law; under certain limiting conditions, reduces to Eq. (1). Equation (50) includes action ($F_{AB}$) and reaction ($F_{BA}$), together with additional parameters, such as $K_{shape}$ , $K_{composition}$ , $K_{target}$ and $K_{other}$. Thus, Eq. (50) is more realistic in formulation, as it incorporates factors that influence the observed results in falling and rebounding bodies.

In idealized situations, Eq. (1) may remain applicable, as discussed in Sections (1.1–1.2); however, Eq. (50), which accounts for shape, composition, target characteristics, and other relevant factors, is intended to apply under realistic physical conditions. It is applied to describe the motion of falling and rebounding bodies, and rebounding height is expressed in Eq. (53). Experimental investigations may help to assess the validity and applicability of Eqs. (50) and (53).

**Table 2 Comparison of Newton's Original form of Third law and Generalized form of Third Law**.

| Sr. No. | Characteristics | Newton's Third Law | Generalized form of Third Law |
|---|---|---|---|
| 1 | System | Ideal system | Realistic System |
| 2 | Definition | Action and Reaction are equal and opposite | Action and Reaction are proportional, to each other |
| 3 | Direction and magnitude | Precisely equal and opposite | May or may not be equal and opposite |
| 4 | Equation | $F_{BA} =- F_{AB}$ | Reaction ($F_{BA}$) =-K Action ($F_{AB}$); $F_{BA} = -[K_{shape} \times K_{composition} \times K_{target} \times K_{other}] F_{AB}$ |

| | | | |
|---|---|---|---|
| 5 | Special case | It is itself a special case | If $K_{shape} \times K_{composition} \times K_{target} \times K_{other} = 1$ then $F_{BA} = -F_{AB}$ An extended or generalized case |
| 6 | Application | Applied in various situations | Initially meant for Rebounding Bodies |
| 7 | Status | Established | To be established |
| 8 | Vision | For 340 years part of literature | Will be generalized form of literature |
| 9 | Necessity | It is special case | Experiments will establish it as a general case |
| 10 | Law of Conservation of Momentum | Based on it. | Explained if K=1 |
| 11 | Rebound Height | Used in Coefficient of Restitution, $H_r = e^2H$ | $H_r$ = H Action $[K_{shape} \times K_{composition} \times K_{target} \times K_{other}]$ |
| 12 | State-of-the-art experiments | Some deviations in peer review journals | Experiments may be conducted |

### 6.5 Area of touch or contact (AOT).

The area of touch (AOT) or contact can be defined as the actual area of the body (projectile, which may have various shapes) that directly touches, contacts, or strikes the floor. The areas of touch/contact may be identical for different bodies, e.g., sphere, long thin pipe, cone, needle-like shape, flat, irregular body, triangle, etc. The effect of AOT depends on the shape, size, symmetry, and other geometrical features of the body. The area of touch may be considered as a parameter associated with the body's shape in the following manner.

$$K_{shape} \propto \frac{\mathbf{1}}{\boldsymbol{area\ of\ touch\ or\ contact}} \quad \text{or} \quad K_{shape} = \frac{\boldsymbol{k}}{\boldsymbol{area\ of\ touch}} \qquad (52)$$

k depends on the properties of the bodies and the related experimental conditions.

Consider a flat body and a sphere (with the same mass and composition) both dropped identically onto the same surface. In the case of the flat body, the effective area of touch is larger than that of the sphere, so the value of $K_{shape}$ is smaller, as in Eq. (52). Consequently, the reaction is reduced for the flat body in Eq. (50), and thus the flat body rebounds to a lower height ($H_r$<1m). The current value of K should be regarded as a qualitative estimate; rigorous and repeated experimentation is necessary to establish its accurate quantitative measurement. This aspect is also highlighted in Section (6

### 6.6 Experimental Simplicity in Confirming the Generalized or Extended Formulation

The closest approach of NASA's Parker Solar Probe to the Sun is about 6.1 million kilometers, and scientists have developed extremely complex and sophisticated methods for producing antiparticles at CERN, Fermilab, SLAC National Accelerator Laboratory, and other facilities.

Therefore, such basic experiments involving falling and rebounding bodies are extremely simple compared to those scientific achievements and are essential for developing a fundamental understanding of the basic laws of physics. The discussion is equally applicable to the horizontal motion of bodies; however, in this case, the characteristics of the surface become significantly more influential.

**6.7 A Phenomenological Relationship Linking Rebound Height and Reaction Force**

Conceptually, *Principia* relates displacement or distance to impressed force. In Definition IV, Newton associated impressed force with a change in uniform motion; in the First Law, uniform motion is altered by an impressed force; in the Second Law, the change in motion is proportional to the impressed force; and in the Third Law, when one body exerts a force on another, it produces a change in the motion of the second body.

The motion of a body changes when it moves from one point to another or travels a certain distance. Thus, a change in the motion of a body implies a change in its position; therefore, any change in motion inherently corresponds to a change in position or displacement. Accordingly, in Newton's Third Law, the reaction force may be associated with rebound height (in vertical motion) or distance traveled (in horizontal motion). This interpretation provides a unified explanation of Newton's laws and may be applied to observable physical situations.

Thus, Newton associated force with change in motion, which consequently produces changes in the body's position. This implies that Newton connected force with both motion and displacement. In simple terms, when a reaction force acts on a body, the body travels a corresponding distance.

Even in daily observations, displacement is related to the impressed force: if a body is pushed with a smaller force, it moves a shorter distance; if pushed with a greater force, it moves a larger distance. When a body strikes a target and rebounds, the rebound distance is produced by the reaction force. Accordingly, the reaction force may be related to rebound height (vertical motion) or backward distance (horizontal motion), as expressed in Eq. (53).

$$\text{Reaction Force} \propto \text{displacement} \quad \text{or} \quad \text{Rebound height } (H_r) \propto \text{Reaction force } (F_{BA})$$

The Reaction force ($F_{BA}$) is given by Eq. (50), thus the rebound height of body may be written as

$$\text{Rebound height } (H_r) = D_f \text{ Reaction force } (F_{BA}) = D_f\,[K_{shape} \times K_{composition} \times K_{target} \times K_{other}]\text{ Action } (F_{AB}) \qquad (53)$$

where $D_f$ is a dimensional constant with dimensions $M^{-1} L^0 T^2$ and magnitude unity. It is introduced solely to ensure dimensional homogeneity, since in classical mechanics force is related to 'ma', which has dimensional formula $MLT^{-2}$.

The existing methods for measuring rebound heights, namely Eqs. (21, 25, 27, 34), do not explicitly incorporate relevant factors such as shape, composition of bodies, and properties of the target surface. They lack quantitative verification for this purpose and yield primarily indirect or qualitative results. In those equations, rebound height is associated with velocity, time, displacement, and gravitational acceleration. In contrast, Eq. (53) directly relates rebound height or distance to reaction force and the factors $K_{shape}$, $K_{composition}$, $K_{target}$ and $K_{other}$. This constitutes the essential distinction of the proposed alternative framework.

Equation (53) may be examined theoretically to test the internal consistency of the proposed formulation. However, systematic experiments are required for quantitative verification over a wide range of parameters.

**Ideal situation.** If a body is dropped from height H and rebounds to the same height $H_r$, such that $H = H_r = 1\text{m}$. It represents the ideal condition where K equals unity, implying each constituent of K may be equal unity or their combined effect equals unity. The rebound height is given by

Rebound height ($H_r$) = $D_f$ [$K_{shape}$ x $K_{composition}$ x $K_{target}$ x $K_{other}$ ] Action ($F_{AB}$)

or $D_f$ [$K_{shape}$ x $K_{composition}$ x $K_{target}$ x $K_{other}$ ] Action ($F_{AB}$) = 9.8 m (54)

K =1 (54)

Rebound height ($H_r$) = $D_f$ Action ($F_{AB}$)

Such ideal conditions must be clearly identified and experimentally verified before their applicability can be established.

**General situation:** In real-world situations, bodies of different shapes, while having the same mass, composition, target, and other properties, may rebound to a height of 4.9 m; accordingly, Eq. (53) becomes

Rebound height($H_r$) = $D_f$ Reaction force = $D_f$ $K_{shape}$ Action ($F_{AB}$) = 4.9 m (55)

Here $K_{composition}$, $K_{target}$ and $K_{other}$ are regarded as unity, as body's composition, characteristics of target etc. are regarded as standard. The value of $K_{shape}$ deviates from unity because the shapes considered are different from the spherical form, the composition and target are in standard form. The magnitude of dimensional constant is unity.

$K_{shape}$ 9.8 = 4.9 m or $K_{shape}$ = ½ (56)

If the body rebounds to progressively lower heights, then the value of K,( $K_{shape}$ , as $K_{composition}$, $K_{target}$, $K_{other}$ are unity ) as given by Eq. (53) will differ from unity or will be lesser than unity. Furthermore, the angle at which the body rebounds is also taken into account through $K_{other}$ , along with additional factors not explicitly represented by the remaining $K_i$ terms.

**Flat Body :** Consider the same spherical body is changed to flat body (of same mass and composition) is dropped from height of 1m , on the same floor , and it rebounds to only 0.1m , then Eq.(53) may be written as

Rebound height ($H_r$) = $D_f$ [$K_{shape}$ ] Action ($F_{AB}$) = 0.1 m or $K_{shape}$ 9.8 = 0.1m

Hence, $K_{shape}$ = 0.1/9.8 = 0.01 (57)

Thus here body rebounds to least height and value of $K_{shape}$ is minimum accordingly. In this perception reaction force is lesser.

**Body does not rebound at all**: Consider the situation when a flat body (exceptionally thin comparatively, having same mass and composition as that of sphere) falls on the same floor, but does not rebound all (say) i.e. $H_r$ =0,

Rebound height ($H_r$) = $D_f$ $K_{shape}$ Action ($F_{AB}$) =0

$D_f$ =1, Action ($F_{AB}$) = 9.8 newtons

So, $K_{shape}$ = 0 (58)

Thus this model of rebound of bodies based on applications of generalized or extended form of third law, explains the rebound of bodies in different ways. Therefore, these experiments need to be systematically performed to measure the corresponding values of $K_i$'s. The existing equations i.e. Eqs. (21,25,27,34) describe the rebound heights under different measurement parameters; however, these interpretations remain primarily qualitative in nature.

**Horizontal Motion:**

Furthermore, Eqs. (50,53) can also be used to calculate the rebound distance of a body when it strikes a target and moves backward in horizontal path, which constitutes an additional advantage of the proposed formulation. The effects arising from surface properties are incorporated through the parameters $K_i$'s. As these factors are not incorporated in Eq.(21,25,27,28 ), but Eq. (53) provides a more general, comprehensive and advantageous formulation. When body strikes a target then it rebounds due to reaction force, thus in all such phenomena reaction force is significant.

The results from existing theories and generalized form of Newton's third law of motion are shown in Table 1. It is evident that the generalized form the third law takes all possible factors in account regarding rebounding bodies, hence needs to be experimentally confirmed.

Thus, from the above discussion, Eq. (1) applies to ideal systems, whereas Eq. (50) specifically meant for realistic systems. Eq. (1) accounts only for $F_{AB}$ and $F_{BA}$ but neglects all other factors, whereas Eq. (50) includes all factors (such as shape, composition of bodies, surface and target characteristics) along with $F_{AB}$ and $F_{BA.}$ In practice, all factors must be considered; only then is the law applicable to realistic situations. This discussion is only confined to rebounding bodies in realistic systems.

**6.8 In Newton's second law Force as Change in Motion and Euler's Force as Acceleration**

In *Definition IV* of the *Principia*, Newton described force as an agency that changes a body's state of rest or uniform motion. In the First Law of Motion, he stated that such changes occur only under the action of an external force. In the Second Law of Motion, Newton established a proportional relationship between the impressed force and the change in motion.

*The alteration of motion is ever proportional to the motive force impressed, and is made in the direction of the right line in which that force is impressed.*

Newton did not provide a mathematical expression for force in the *Principia*, in the era of natural philosophy. Further, a change in motion implies a change in position (distance traveled). Thus, Newton connected force with alteration of motion, and alteration of motion is inherently related to displacement. Also, Newton did not mention acceleration as (a = $\frac{dv}{dt}$ ) in the Principia or elsewhere.

**Euler defined acceleration, and related force with force in mathematical forms.**

Whereas the explicit concept of acceleration as the time rate of change of velocity (a = $\frac{dv}{dt}$ ) was introduced later by Euler [10] in *Mechanica* (1736) about a decade after death of Newton.

Euler [10-13] began relating force with acceleration in 1736 in different forms (F =ma/n in 1736, F =2ma in 1749, F =ma/2g in 1765, n and g are constants), and finally derived F=ma in 1776. Subsequently, F =ma was associated with the Second Law of Motion by later scientists, naturally after 1776 or about 50 years after Newton's death. However as discussed above Newton related force with change in motion (change in distance travelled).

The concept of physical dimensions was systematically formalized by Fourier [31-32] in 1822, nearly 135 years after the publication of Newton's *Principia*. Thus Leonhard Euler (1703-1783) mathematically worked without such constraints in his mathematical treatises.

**Conceptual development:** It is mentioned in the first paragraph of Section (6.7) that in the *Principia* Newton articulated the concept of force in Definition IV, in the First Law of Motion, in the Second Law of Motion, and in the application of the Third Law, consistently relating force to a change in the state of rest or of uniform motion or motion.

Euler [10-13] began expressing force in terms of acceleration in 1736 and finally formulated the equation F=ma in 1776, thereby relating force to acceleration (the rate of change of motion with respect to time). Nearly fifty years after Newton's death, later scientists associated Newton's Second Law with Euler's equation (F=ma). Thus, 'the rate of change of motion with time' became formally identified with force.

However, there is also a second conceptual aspect of force. Newton related force to a change in a body's motion, and such a change in motion necessarily involves a change in position, distance, or displacement; consequently, force is indirectly connected with the corresponding distance or displacement. In simple terms, if force produces a change in motion, the body must traverse some distance.

This second aspect of Newton's laws, namely, that a change in motion implies a change in displacement and hence a connection between force and distance—is also incorporated in Eq. (53). Thus, Eq. (53) may be regarded as reflecting a broader or extended interpretation of Newton's laws, in which force is related both to the change in motion and to the associated displacement. Thus, it may be regarded as the fullest interpretation of Newton's law that force is related with change in motion and change in distance or displacement.

**Priority Issue:** Historically, Euler's pivotal role in explicitly formulating F = ma has received little attention in standard textbooks, despite its foundational significance in classical mechanics. This represents a relatively rare instance in the history of science where the contributor of a foundational formalism is not consistently acknowledged in introductory physics literature. It is worth considering whether Euler's name should be explicitly associated with the equation F=ma, one of the most fundamental relations in science, in standard introductory textbooks of physics and mathematics. Consequently, grasping the conceptual and historical evolution of Newton's laws is an ongoing process.

### 7.0 Law of conservation of momentum and Newton's Third Law of Motion

In the Definition I of the Principia, Newton has defined mass or body, giving examples at macroscopic level.

If the equation is meant for ideal conditions, then its applications may lead to different results in realistic conditions. For examples, equations $v^2 - u^2 = 2gS$ and $S = ut + \frac{1}{2}gt^2$, are derived when acceleration is constant, and applicable under that condition only.

**Ideal Systems: The** mathematical equation for the third law of motion is given by

$$F_{BA} = - F_{AB} \qquad (1)$$

or

$$F_{BA} + F_{AB} = 0 \qquad (1)$$

Thus, the net force on the system is zero. In Eq. (1), no factors other than $F_{BA}$ and $F_{AB}$ , appear during collision; therefore, the characteristics of the bodies and the nature of the surface do not appear in equation. These factors are not mentioned in Eq. (1) because the third law is defined in an idealized form. Under ideal conditions, the law expressed by Eq. (1) leads to the law of conservation of momentum, which may hold strictly for ideal systems. For a more comprehensive understanding, we consider Eq. (50), which intentionally associates reaction with action along with additional parameters as experimentally these factors affect the results. The third law of motion has many extended applications.

$$\text{Reaction } (F_{BA}) = -\,[K_{shape} \times K_{composition} \times K_{target} \times K_{other}] \text{ Action } (F_{AB}). \qquad (50)$$

Newton also presented a third example to illustrate the third law in Section (1.5), which leads to Eq. (2), i.e., the law of conservation of momentum.

$$M_p u_1 + M_t u_2 \text{ (Initial momentum of system)} = M_p v_1 + M_t v_2 \text{ (Final momentum of system)} \qquad (2)$$

If masses of projectile ($M_p$) and target ($M_t$) are equal ($M_t = M_p$), then Eq.(2) becomes

$u_1$(initial velocity of projectile) + $u_2$ (initial velocity of target)

= $v_1$ (final velocity of projectile) + $v_2$ (final velocity of target) (3)

$$u_1 + u_2 = v_1 + v_2 \quad (3)$$

If the target is much heavier than the projectile, i.e., $M_t >> M_p$, for example when a ball moving with initial velocity ($u_1$) strikes a stationary boulder ($u_2 = 0$). Then the target (boulder) remains approximately at rest after collision, i.e., $v_2 \approx 0$.

Hence,

$u_1$ (initial velocity of ball)= $v_1$ (final velocity of ball) (59)

**Prediction :** Equation (59) implies that the final velocity of the ball ($v_1$) should be equal to its initial velocity ($u_1$), in both magnitude and direction. Since no negative sign appears in Eq. (59), reversal of direction is not predicted. Thus, the theoretical prediction suggests that the ball (projectile), after collision, should continue moving forward with its initial velocity, similar to motion in air or vacuum without interaction.

**Actual Observations:** However, experimental observations show that the ball rebounds in the backward direction. The rebound velocity ($v_1$) differs from the initial velocity ($u_1$); therefore, the observed behavior is not consistent with Eq. (59). The rebound velocity of the ball depends on the characteristics of the ball (shape, composition, etc.), the characteristics of the boulder, and the nature of the surface on which the interaction occurs. The ball does not displace or eliminate the boulder and continue with the same velocity in straight line as suggested by Eq. (59).

Thus, in a realistic system, the application of Newton's third law in ideal form does not adequately describe the interaction in either magnitude or direction. Under realistic conditions, the experimental outcome differs from idealized predictions. In addition, Eq. (2) has not been systematically verified for different bodies under varying conditions; therefore, theoretical interpretation alone is insufficient for scientifically establishing an equation.

Moreover, Newton's third law i.e. Eq. (1) by itself does not quantitatively explain the motion of rebounding bodies, even when used together with Eqs. (18, 23). This interpretation results in Coefficient of Restitution Method (CORM) for measurement of rebounding heights, which is qualitative. Consequently, Eq. (1) has been intentionally generalized as Eq. (50) to taken in account the effective factors through K . Further, Eq. (50), in conjunction with Eq. (53), is employed to explain the motion of rebounding bodies by relating force to rebound height in vertical motion and to backward displacement in horizontal motion.

**Simple Comparison with Eq.( 11)** :

Equation (11) models a ball colliding with a massive wall, where the target's mass greatly exceeds the projectile's.

Here

$v_1$ (final velocity of the ball), = $-u_1$ (initial velocity of the ball), (11)

Equation (11) shows rebound upon wall collision: the negative sign indicates backward motion, with final speed equaling initial speed. In contrast, Equation (59) has a positive sign, indicating no rebound; the ball continues forward with its original velocity. This marks the key difference between the equations. Thus, all theoretical predictions require clear illustration, whether experimentally verified or not.

**Recoil of gun: Realistic System**

When an equation is defined under ideal conditions, it may not necessarily remain applicable under realistic conditions. In realistic situations, certain modifications may be required. Equations derived from Eq. (1) must therefore be critically examined through experiments under realistic conditions. Accordingly, the equation for recoil velocity of the gun, i.e., Eq. (7), should be subjected to systematic experimental verification.

$v_{gun} = - m_{bullet} . v_{bullet} / m_{gun}$

$$v_{gun} = -\frac{m_{bullet}.v_{bullet}}{m_{gun}} \qquad (7)$$

This equation has been extensively treated within theoretical frameworks at the macroscopic level and widely taught; however, rigorous experimental confirmation is still desirable for completeness. There is no clear evidence in the literature that this equation has been directly and systematically verified experimentally. Only consistent and repeatable experimental observations establish validity in realistic situations.

Thus, Eq. (7), being derived from Eqs. (1,2), has not been experimentally verified in a systematic manner and therefore represents an idealized formulation, whereas Eq. (50) is proposed as an extension, intended to incorporate various factors that influence the results. In view of this phenomenological generalized form of Newton's Third Law of motion, Eq. (2) may be written as:

$$[M_p v_1 - M_p u_1] = - Z [ M_t v_2 - M_t u_2 ] \qquad (60)$$

Now applying law of conservation of momentum , the equation for the recoil velocity of the gun is given by:

$$0 = M_p v_1 + Z M_t v_2 \quad \text{or} \quad 0 = m_{bullet} . v_{bullet} + Z\, m_{gun} . v_{gun}$$

$$v_{gun} = -\frac{m_{bullet}.v_{bullet}}{Z\ m_{gun}} \qquad (61)$$

where Z is a coefficient that depends on experimental conditions, i.e., the characteristics of the gun (shape, size, symmetry, composition, etc.), the bullet, and the nature of the surface (state, roughness, etc.) . This factor is missing in in Eq.(7).
For definitive assessment of the influence of these parameters, the recoil velocity of the gun must be measured precisely along with the associated variables in systematic experiments. Such experiments are essential for a proper understanding and validation of Eqs. (7, 60) and their applications.
The classical recoil equation corresponds to an ideal isolated system. In realistic macroscopic systems, additional momentum carriers and interaction effects introduce an effective correction factor Z, leading to the modified recoil expression.

**Is Eq.(50) obeys law of conservation of momentum**

Here practical difference between Eq.(1) and Eq.(50) is that Eq.(1) is meant for ideal system, as it taken in account only $F_{AB}$ and $F_{BA}$ and Eq.(50) is meant for realistic or real world systems. It takes in account $F_{AB}$ , $F_{BA}$ , $K_{shape}$ , $K_{composition}$ , $K_{target}$ and $K_{other}$ , thus it is not only important but a necessity in physics. For simplicity we can understand the Newton's original law and generalized law in the following way.

(i) The law of conservation of momentum is meant for ideal system or isolated system i.e. no external force acts on the body. In idealized Newtonian mechanics, interacting bodies are treated as rigid or point-like objects, leading to the symmetric relation $F_{BA}=-F_{AB}$ or $F_{BA} + F_{AB} = 0$. Interacting bodies are treated as point like objects, leading to symmetric relation.

(ii) In addition to idealized systems , we too have realistic systems. Thus extended equation

Reaction ($F_{BA}$) + [$K_{shape}$ x $K_{composition}$ x $K_{target}$ x $K_{other}$ ] Action ($F_{AB}$) **=0**

is applicable to these systems and it takes the additional factors in account which are elusive to Newton's idealized equations.

In case additional factor is, [$K_{shape}$ x $K_{composition}$ x $K_{target}$ x $K_{other}$ ] is unity then extended equation reduces to Eq. (1), thus ideal conditions. In this case extended equation obeyes law of conservation of momentum as original equations does. Thus there is consistency, in case equation $F_{BA}=-K\ F_{AB}$ have not given equation, $F_{BA}=-F_{AB}$ there would have been inconsistency .Hence like Newton's equation, generalized equation, also obeys law conservation of momentum under the conditions Newton's original law obeys. There is smooth transition of generalized equation to original equation.

Scientifically one can not confine thinking to Newton's law only when there are other available phenomena , not accounted for by Newton's equation. Hence for the sake of comprehensiveness law has been logically extended may be called extended form of Newton's Third Law.

We conclude that Newton established the idealized symmetry ($F_{AB}=-F_{BA}$), whereas $K$, in equation ( $F_{AB}=-KF_{BA}$), provides the **empirical parameters** defining realistic, dissipative physical interactions as dictated by **geometry, material composition, and interfacial dynamics**. It may be justified by two physical examples.

(a) **Horizontal and vertical jumping from a boat**

An analogous situation occurs when a person jumps from a boat: if only the person is observed, momentum appears to be generated, but when the boat is included which move backward, total momentum is conserved. Water friction acts as external force afterward, slowing boat, but during instantaneous jump, conservation holds for isolated person-boat system.
Conducting precise aquatic experiment by measuring the mass and velocity of a jumper (80kg, 20 km/s) against the boat's mass (200 kg) and recoil velocity (v) and quantitative analysis of resistive forces. This data facilitates estimating the transfer coefficient (K) in Reaction = -K action, and exact validity ideal law, Reaction = -Action.

One may assume suitable values in theoretical estimates number of times. But a specific experiment and calculation of K would end to such speculations which is scientific. Here K may be called the 'momentum transfer coefficient,' the coefficient of dynamic recoil.

Slight variation in experiment may be visualized if a person jumps vertical upward (rather than horizontal as in first case), then momentum of the boat needs to be determined. The specific quantitative experiments would give realistic values of various parameters.

**(b) Falling and rebounding bodies**

Specifically in the case of falling and rebounding bodies, the interaction cannot be regarded as an idealized two-particle collision. The **shape of the falling body**, its **material composition**, the **nature of the target surface**, and the **surface properties** significantly influence deformation and stress distribution during impact. So now system is no longer ideal but realistic.

Under such realistic conditions, the effective force relation during contact may be represented as $F_{BA}=-KF_{AB}$ where K depends on geometric, material and surface characteristics. This apparent **force asymmetry** does not imply a breakdown of law of conservation of momentum; rather, it reflects that the falling body and the target surface alone do not form a perfectly isolated system.

The reason being that that the two-body subsystem is not perfectly isolated, since deformation, surface interaction, and target response redistribute momentum during impact.

In rebounding experiments, during impact, momentum is redistributed through different shapes and composition of bodies internal deformation, stress-wave propagation, and coupling with the supporting structure and the Earth. When the complete interacting system is considered, total momentum remains conserved, and the coefficient **K characterizes effective macroscopic interaction dynamics due to various involved factors** rather than a violation of fundamental conservation laws.

**8.0 Unphysical results in theoretical discussion.**

Now Eq. (9) may be written as

$$u_1 \text{ [Initial Velocity of projectile]} = \frac{[v1\,(M_p+M_t)-2M_t u_2}{[M_p-M_t]} \qquad (62)$$

The determinacy property of an equation implies that if values $M_p$ $M_t$, $v_1$ and $u_2$ are given, then the value of $u_1$ may be determined from Equation 62. Depending upon projectile and target masses, the following possibilities arise:

(i) Here denominator depends on the $M_p$ and $M_t$ only, these can be chosen in experiments. In masses $M_p$ and $M_t$ may be approach to each other or become equal, then predictions of initial velocity $u_1$ is not consistent from Eq. (62). Depending upon values of $M_p$ and $M_t$ , the velocity $u_1$ may approach to speed of light or becomes equals speed of light. It is not consistent with Special Theory of Relativity. If speed of body becomes equal to that of light then mass becomes infinite or mass of body of 1kg or less, becomes more than mass of universe or multiverses, which is inconsistent and unphysical result.

Here, the denominator depends on $M_p$ and $M_t$ exclusively; these can be selected in experiments. If masses $M_p$ and $M_t$ converge toward one another or become equal, then predictions for initial velocity $u_1$ are inconsistent with Equation (62). Depending upon values for $M_p$ and $M_t$ the velocity $u_1$ might approach or equal the speed of light. This is inconsistent with the Special Theory of Relativity. If a body's speed equals that of light, then mass becomes infinite; the mass of a body of 1kg or less, exceeds the mass of the universe or multiverses, which is an inconsistent and unphysical result.

(ii) Further from Eq.(10) we get

$$u_2 \text{ [initial velocity of target]} = \frac{v_2[M_t+M_p]-2M_p u_1}{[M_t-M_p]} \qquad (63)$$

In this case, the denominator remains ($M_t$-$M_p$) representing a mathematical singularity. Thus, under similar conditions for values of $M_p$ and $M_t$ , again similar inconsistent or unphysical results are obtained for velocity $u_2$ as in Equation ( 62). It must be noted that Equations (9 and 10), hence Equations (62 and 63), are based on the simultaneous conservation of momentum and kinetic energy; these are two foundational laws not only of mechanics but of physics and science. Thus, here theoretical deductions based on or related to Newton's third law of motion are objectively and logically discussed, but inconsistent and unphysical results are obtained.

**Simple conclusion of this discussed.**

It is a logical deduction that when theoretical predictions are inconsistent, quantitative experiments must be performed to reach definitive conclusions. This will resolve diverse speculations, and the original law must be rigorously evaluated in light of these empirical observations.

Consequently, specifically in Section 4.0, diverse experiments are proposed so that empirical data for rebound heights can be determined. Initially, experimental results might be interpreted using various traditional methods which are derived from Newton's third law of motion. These approaches are analyzed in Sections 4.0–4.3. It should be noted that such existing models are not quantitatively validated for every potential value of parameters involved in trials. The justification is that such experimental tests have not been performed and data is currently unavailable. However, Equation 50 is introduced as an extended version of Newton's third law, in

theoretical form; it could assist in interpreting these newly acquired and previously unconfirmed experimental results. Therefore, Equation (50) may interpret the rebound height defined by Equation ( 53), or alternative outcomes can be clarified using these expressions. Consequently, there might be extensive speculative or theoretical discourse regarding the application of Newton's third law as utilized over the past 340 years.

However, several straightforward, easily reproducible experiments would ultimately determine the status of this original law in realistic scenarios. The coefficient K will be either unity or deviate from unity. If a value for K other than unity is eventually confirmed consistently across specific cases, then the expression, , Reaction ($F_{BA}$) = - [$K_{shape}$ x $K_{composition}$ x $K_{target}$ x $K_{other}$ ] Action ($F_{AB}$) would be established as the Generalized form of Newton's third law.

## 9.0 Extension and development of laws is an established process.

Refinements, extensions, or replacements of established laws are integral to scientific and physical progress.

(i) Newton presented three qualitative examples in the *Principia* (1687) to illustrate the third law of motion. Subsequent scientists applied the law qualitatively to explain phenomena such as bouncing balls, swimming, the recoil of a gun, etc. In the 20$^{th}$ and 21$^{st}$ centuries, the law was also widely used in the explanation of aerospace and propulsion systems. Attempts have been made to experimentally and quantitatively verify the law for falling and rebounding bodies at the macroscopic level.

(ii) Newton's seventeenth-century corpuscular theory of light was eventually supplanted by the wave theory advanced by Young and Fresnel. Maxwell later in 1865 unified optical phenomena with electromagnetism, identifying light as an electromagnetic wave. During the twentieth century, quantum theory introduced photons, distinct from Newton's corpuscles. Contemporary physics describes light using quantum electrodynamics, a fundamentally different framework today. While the seeds were planted in the 1920s–1930s, the complete quantum electrodynamical theory emerged in the 1940s.

(iii) Dalton's atomic theory (1803-1808) constituted a foundational classical description of matter. Subsequent experimental and theoretical contributions by J. J. Thomson, Rutherford, Bohr, Moseley, and Chadwick progressively revealed subatomic structure, nuclear organization, quantized energy levels, atomic number, and isotopic variation. These advances led to a refined and generalized understanding of atomic behavior beyond Dalton's original formulation.

(iv) Newton expressed the speed of sound in a medium as

$$v = \frac{P}{D_m} \tag{64}$$

**Pierre-Simon Laplace** in 1816 corrected this relation, yielding Equation (65) , because Newton's expression did not produce the experimentally correct value for the speed of sound in air. Hence,

$$v = \gamma \frac{P}{D_m} \tag{65}$$

where P denotes the pressure, $D_m$ the density of the medium, and γ the ratio of specific heats (specific heat at constant pressure to that at constant volume). Equation (65) provides an accurate value for the speed of sound in air. Now Eq.(65) is called the Newton–Laplace equation for the velocity of sound in gases.

Realistically, Newton assumed that the propagation of sound waves is isothermal in nature; however, Laplace later demonstrated that the process is actually adiabatic.

(v) Tsiolkovsky published Eq. (8), the ideal rocket equation, in 1903 based on Newton's third law of motion or law of conservation of momentum. Goddard later extended this formulation in 1919 as Eq. (9) by incorporating the effects of gravity and aerodynamic drag. Consequently, Goddard's equation accounts for additional influences such as gravitational force and aerodynamic resistance. The equation has been further improved by other scientists.

(vi) Furthermore, Newton's third law implies that $F_{AB}$ and $F_{BA}$ are the only significant factors governing the interaction of bodies in Eq. (1). In contrast, Eq. (50) shows that for falling and rebounding bodies, in addition to $F_{AB}$ and $F_{BA}$ factors such as shape, composition, asymmetry, the nature and properties of the target, nature of surface, and other associated parameters also become significant.

(vii) This discussion aligns with the various historical developments of scientific laws and with Mach's proposition in the 1880s and 1890s that the metaphysical nature of mechanical laws must be empirically tested. Thus, interpretation of basic law of physics, mathematics, engineering etc. has been properly investigated.

A generalized form of a theory is considered valid only when its predictions are consistently supported by repeated experimental evidence. Newton's laws govern most everyday phenomena, but deviations appear at quantum scales, relativistic speeds, and in deformed or asymmetric interactions, requiring generalized formulations. The historical development of classical physics demonstrates that foundational laws are frequently refined through experimentally motivated generalizations, as exemplified by the Newton–Laplace correction to the velocity of sound and the generalized forms of Hooke's law and Ohm's law. In this case, the original formulation remains valid as a special or limiting case, while the generalized relation incorporates additional physical effects.

Likewise, a logical generalization of Newton's Third Law of Motion is proposed in the present work; however, its ultimate acceptance necessarily depends on rigorous experimental validation.

**10.0 The rebound experiments are different than those in Aerospace and Propulsion Systems**.

The quantitative verification of Aerospace and Propulsion Systems (fireworks, rockets, spacecraft, missiles, gliders and drones) is fundamentally different from Action–Reaction Motion Systems or contact-based action–reaction interactions (bouncing balls, swimming, rowing, walking, jumping, balloon deflation, and gun recoil). The former system was mainly developed in the $20^{th}$ and $21^{st}$ centuries due to the efforts of astronautical engineers in various subfields, and later was the earliest system of applications of Newton's third law of motion developed qualitatively.

The rockets are guided and stabilized through computer-controlled systems, whereas fireworks travel along uncontrolled and purely ballistic trajectories.

The cruise missiles function under continuous powered flight and utilize advanced guidance technologies, such as GPS, radar, and inertial navigation systems.

The motion of drones is regulated by onboard flight-control electronics, integrated sensors, and externally commanded inputs.

Airplanes are directed through pilot inputs or automatic control systems and operate along predetermined and navigationally defined flight paths.

A glider moves in the atmosphere using aerodynamic lift and gravity**,** not by engines or propellers. The Airbus Perlan 2 has achieved the highest altitude ever, 76,124 feet. Gliders are typically constructed from lightweight materials and feature cambered (curved upper surface and flatter lower surface), high-aspect-ratio wings**,** a streamlined fuselage**,** and a conventional tail**,** which together optimize lift, minimize drag, and ensure stable flight.

Such designs are consistent with the generalized coefficient K, as it explicitly accounts for the body's shape **(**$K_{shape}$**)**, material composition **(**$K_{composition}$**)**, $K_{other,}$ etc. The combination of aerodynamic geometry and low-mass construction directly influences the glider's ascent in rising air and descent in still air**,** reflecting how variations in shape and material modify the effective action–reaction forces as expressed in Eq. (18).

Thus, diverse experiments dealing with action-reaction systems may be individually confirmed as their nature is different from Aerospace and Propulsion Systems. The different values of $K_i$'s are feasible for different experiments.

## 11.0 Conclusions

Newton's Third Law is a fundamental principle of physics and underlies many applications across physics, various branches of engineering, mathematics, and related fields. Realistically the law is meant for ideal systems, as factors like shape, composition, and nature of surfaces do not appear in the mathematical equation; only FBA and FAB occur. Existing method (coefficient of restitution) for estimating rebound height involve Newton's third law and kinematic equations, but explains motion of bodies qualitatively. The rebound heights of bodies need to be quantitatively linked to mathematical expressions. Preliminary observations suggest that rebound behavior depends systematically on geometric shape, material composition, and impact conditions. The spherical bodies largely follow classical action–reaction behavior, whereas asymmetric or flat bodies show measurable deviations. Motivated by these trends, a generalized formulation of Newton's third law for realistic systems is proposed within its classical domain, introducing dimensionless coefficients that represent structural and material asymmetries, target properties, and other interaction parameters. The analysis identifies quantitatively unexplored experimental tests of Newton's third law, particularly for rebounding bodies of identical mass and composition but different shapes at the macroscopic level. The reaction is force is also proposed to be associated with rebound distance. The conclusions, therefore, emphasize the need for dedicated experimental investigations controlling all variables. This law is specifically tailored for rebounding bodies in real-world scenarios, providing a phenomenological framework to describe experimental observations.

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