Deduction of the Electron Charge

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Abstract

In this work the charge of the electron will be deduced from Gauss's flow law.

Keywords: electron, Gauss's law, speed of light, electron mass, Bohr radius

1. Introduction

The charge of electron is considered a fundamental constan of nature, which means its value must be determined experimentally. The current value is known with very high precision due to a series of rigorous experiments. The most famous is Millikan's oil drop experiment. [1]

However, in this work it will be deduced from other more fundamental constants such as the electron radius, the speed of light and electron mass.

2. Gauss's Flux Law and Electron Charge

Gauss's flux law: [2,3]

$$\Phi = \iiint \nabla \cdot \vec{\boldsymbol{F}} dV \qquad (1)$$

Since $\vec{\boldsymbol{F}} = \omega^2 \vec{\boldsymbol{r}} \mu_e = \frac{v^2}{r^2} \vec{\boldsymbol{r}} \mu_e$ [4]is the centripetal force of the electron, where μ_e is the reduced mass of the electron in the hydrogen atom, v is the velocity of the electron in the hydrogen atom, and r is the Bohr radius.

Then, from (1), the following is obtained:

$$\Phi = 4\pi v_e^2 r_b \mu_e \qquad v_e = \frac{\hbar}{\mu_e r_b}$$

From here, we get:

$$\frac{\Phi}{4\pi} = \frac{\hbar^2}{r_b \mu_e} \qquad (2)$$

3. Electron Charge in ues

From (2):

$$\sqrt{\frac{\Phi}{4\pi}} = e = \pm \hbar \sqrt{\frac{1}{r_b \mu_e}} \qquad (3)$$

Given:

$$\hbar = 1.055 \times 10^{-27} erg \cdot s$$

$$r_b = 5.3 \times 10^{-9}\ cm$$

$$\mu_e = 9.11 \times 10^{-28}\ gr$$

Substituting these values into (3):

$$e = \pm 4.8 \times 10^{-10}\ ues\ [5]$$

4. Origen of the Electyric Permitivity ε_0 and Magnetic Permeability μ_0 Constants

As these constants are related to the deduction of electrical charge in Coulombs, it is incorrect to use them without knowing their true origin.

Speed of light squared is $c^2 = 9 \times 10^{16}\ \frac{m^2}{s^2}$. The trick is to multiply and divide this quantity by 4π and by the units $\frac{N}{A^2}$, and then distribute them in a special way as shown below: [6]

$$c^2 = \frac{4\pi}{4\pi} \times 9 \times 10^{16} \frac{1}{\frac{s^2}{m^2}} = \frac{1}{\left(\frac{4\pi}{10^7}\right)\left(\frac{1}{4\pi \times 9 \times 10^9}\right)\left(\frac{N}{A^2}\right)\frac{(A^2)}{N}}$$

Ampere $(A) \times$ second (s)=Coulomb (C)

$$c^2 = \frac{1}{\left(\frac{4\pi}{10^7}\right)\left(\frac{N}{A^2}\right)\left(\frac{1}{4\pi \times 9 \times 10^9}\right)\frac{C^2}{N \cdot m^2}}$$

$$\mu_0 = \frac{4\pi}{10^7}\ \frac{N}{A^2}$$

$$\varepsilon_0 = \left(\frac{1}{4\pi \times 9 \times 10^9}\right) \frac{C^2}{N \cdot m^2} = 8.84 \times 10^{-12}\ \frac{C^2}{N \cdot m^2}$$

Therefore:

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

5. Electron Charge in Coulombs

Also fron formula (2), multiplying both sides by ε_0 but in MKS units, we obtain the following:

$$\Phi\varepsilon_0 = \frac{4\pi\hbar^2\varepsilon_0}{r_b\mu_e}$$

$$\sqrt{\Phi\varepsilon_0} = e = \pm\hbar\sqrt{\frac{4\pi\varepsilon_0}{r_b\mu_e}} \quad (4)$$

Using:

$$\varepsilon_0 = 8.84 \times 10^{-12}\ \frac{C^2}{N\cdot m^2}$$

$$\mu_e = 9.11 \times 10^{-31} kg$$

$$r_b = 5.3 \times 10^{-11}\ m$$

$$\hbar = 1.055 \times 10^{-34} J \cdot s$$

Substituting these values into (4):

$$e\ = \pm 1.6 \times 10^{-19}\ C$$

6. Conclusions

The charge of electron is not a fundamental constant it is the result of the combination of other fundamental constants, such as the Bohr radius, the mass of the electron, and the reduced Planck constant.

The sign of the charge arises from the mathematical nature of the square root, naturally allowing for the existence of electrons, positrons.

Likewise, Magnetic Permeability constantsμ_0 and Electric Permittivity constants ε_0 are not fundamental constants but rather mathematical arrangements.

7. References

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