On the analysis of trusses using complex numbers

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Abstract

In this letter we show how member forces in a truss can be computed using complex numbers. We give an example of a truss where equations with only one unknown member force are formulated by taking moments about a particular chosen point. A theorem provides an expression for a moment in terms of a complex valued force, a complex valued point of application and some basic operations on complex numbers. We consider the following two situations:

(i) The moments are taken about a joint of the truss.

(ii) The moments are taken about a point which is not a joint of the truss. A theorem provides an expression for the position of that point in terms of joints of the truss.

As an example we consider the truss of Fig. 1, which is made of nine members connected by six frictionless pins at joints located at the ends of each member. The rigid truss is supported by a pin at joint 1 and a roller in joint 2.

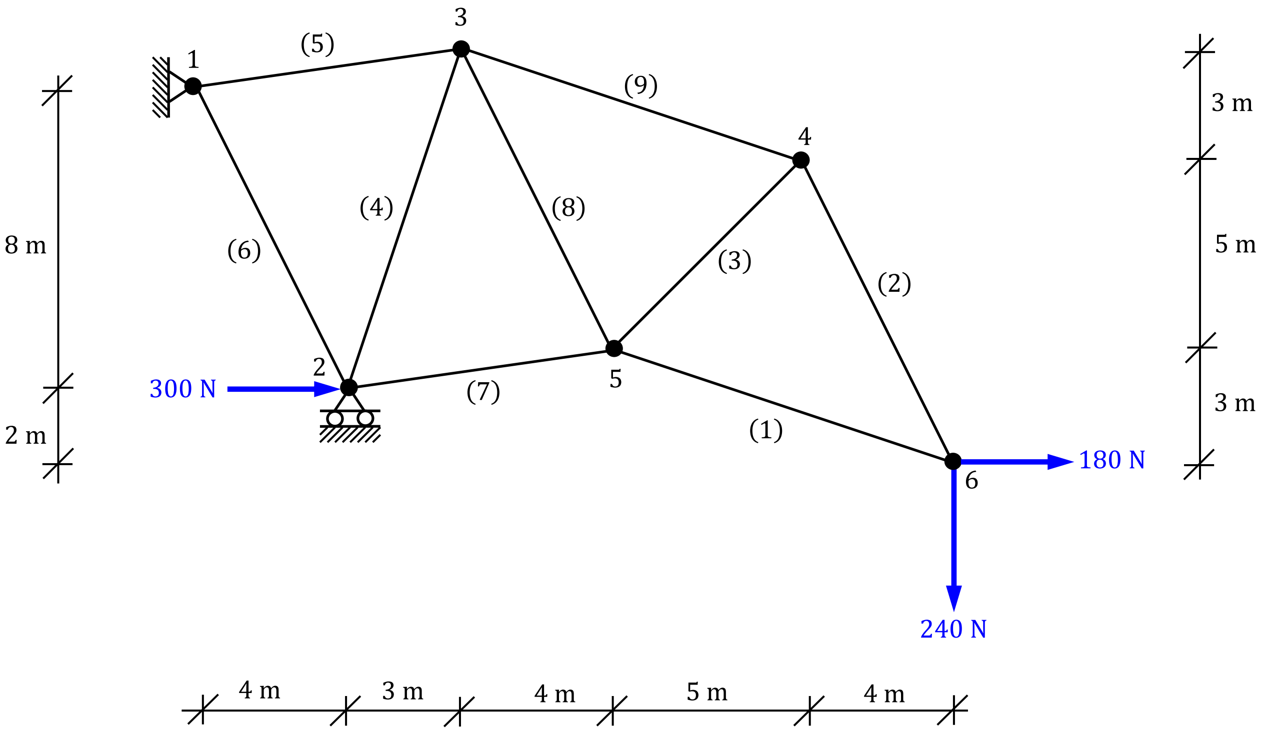

Figure 1

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A free-body diagram of the entire truss is given in Fig. 2; external forces consist of one applied load in joint 2, two applied loads in joint 6, two reactions in joint 1 and one reaction in joint 2.

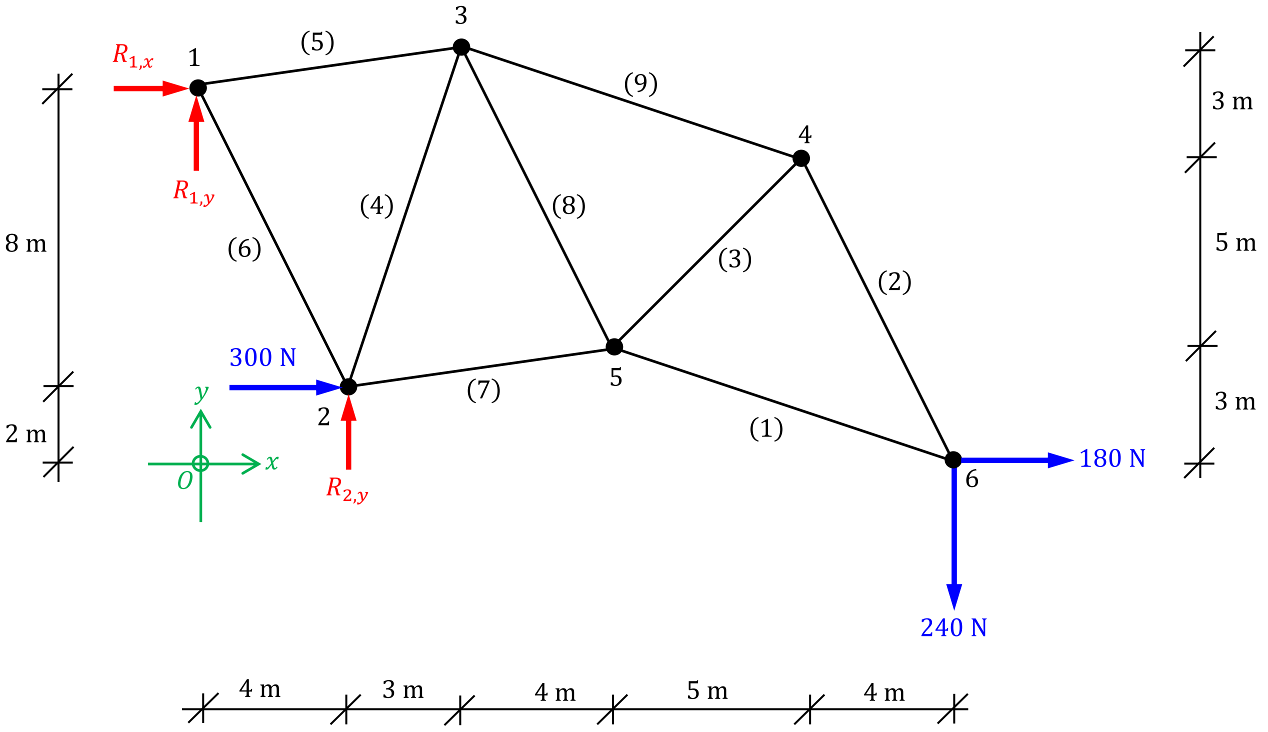

Figure 2

The necessary and sufficient conditions for the equilibrium of a rigid body in two dimensions may be expressed as

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_P = 0$$

The direction of the axes of coordinates and the location of point P may be chosen arbitrarily.
Using the complex-valued representation of a force $\underline{F} := F_x + i \cdot F_y$ the first two equations can be combined as

$$(0) + i \cdot (0) = (\sum F_x) + i \cdot (\sum F_y) = \sum(F_x + i \cdot F_y) = \sum \underline{F} = 0$$

For computation of the moments in the third equation we have the following theorem.

Theorem

Let $\underline{F} := F_x + i \cdot F_y$ and $\underline{Z} := Z_x + i \cdot Z_y$ be the complex-valued representation of a force and its point of application respectively, see Fig. 3. Then the moment M_P of the force about the point P can be obtained as

$$M_P = \operatorname{Im}(\underline{F} \cdot \underline{Z}^*)$$

Proof

$$\begin{aligned}\underline{F} \cdot \underline{Z}^* &= (F_x + i \cdot F_y) \cdot (Z_x + i \cdot Z_y)^* = (F_x + i \cdot F_y) \cdot (Z_x - i \cdot Z_y) \\ &= (F_x \cdot Z_x + F_y \cdot Z_y) + i \cdot (F_y \cdot Z_x - F_x \cdot Z_y)\end{aligned}$$

$$\Rightarrow \quad \operatorname{Im}(\underline{F} \cdot \underline{Z}^*) = \operatorname{Im}\left((F_x \cdot Z_x + F_y \cdot Z_y) + i \cdot (F_y \cdot Z_x - F_x \cdot Z_y)\right) = F_y \cdot Z_x - F_x \cdot Z_y$$

□

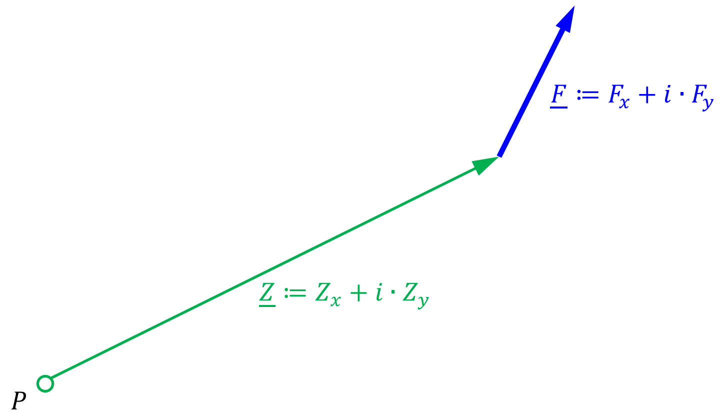

Figure 3: Vector diagram consisting of the force $\underline{F}$ and its point of application $\underline{Z}$.

The three unknown reactions $R_{1,x}$, $R_{1,y}$ and $R_{2,y}$ in the free-body diagram of Fig. 2 can be determined from the following two equations of equilibrium

$$\sum \underline{F} = 0, \quad \sum M_1 = 0$$

Equilibrium of the entire truss.
Determination of the reaction $\underline{R}_2 = R_{2,y} \cdot i$.
Taking moments about joint 1 to eliminate $R_{1,x}$ and $R_{1,y}$ from the computation.
$\sum M_1 = 0$:

$$\mathrm{Im}\left(\left(300 + R_{2,y} \cdot i\right) \cdot (4 - 8i)^* + (180 - 240i) \cdot (20 - 10i)^*\right) = 0$$

$$\mathrm{Im}\left(\left(R_{2,y} \cdot i + 300\right) \cdot (4 + 8i) + (180 - 240i) \cdot (20 + 10i)\right) = 0$$

$$\mathrm{Im}\left(\left(R_{2,y}\right) \cdot (i) \cdot (4 + 8i) + (300) \cdot (4 + 8i) + (180 - 240i) \cdot (20 + 10i)\right) = 0$$

$$\mathrm{Im}\left(R_{2,y} \cdot (-8 + 4i) + (1200 + 2400i) + (6000 - 3000i)\right) = 0$$

$$\mathrm{Im}\left(R_{2,y} \cdot (-8 + 4i) + (7200 - 600i)\right) = 0$$

$$4 \cdot R_{2,y} - 600 = 0$$

$$R_{2,y} = \frac{600}{4} = 150 \text{ N}$$

$$\underline{R}_2 = R_{2,y} \cdot i = 150i \text{ N}$$

The two moments in this equation follow from the two vector diagrams shown in Fig. 4.

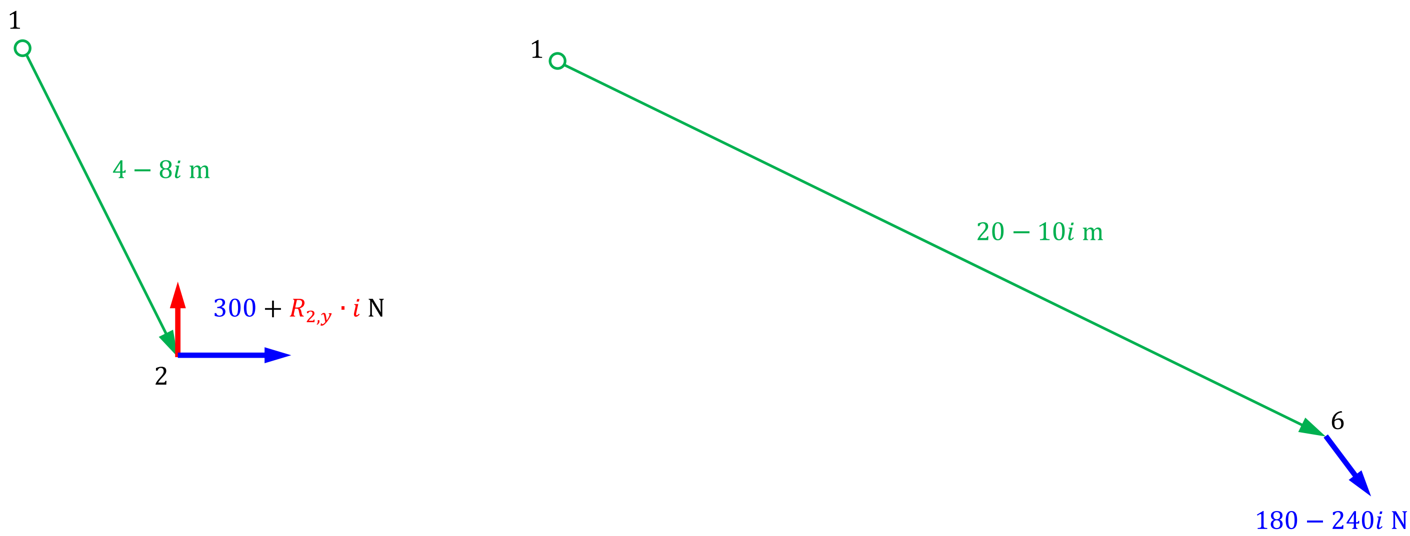

Figure 4

Determination of the reaction $\underline{R_1} = R_{1,x} + R_{1,y} \cdot i$.
Substituting the previous result for $\underline{R_2}$ into
$\sum \underline{F} = 0$:

$$\underline{R_1} + \underline{R_2} + (300) + (180 - 240i) = 0$$

$$\underline{R_1} + 150i + 300 + 180 - 240i = 0$$

$$\underline{R_1} + 480 - 90i = 0$$

$$\underline{R_1} = -480 + 90i = R_{1,x} + R_{1,y} \cdot i \text{ N}$$

$$R_{1,x} = -480 \text{ N}$$

$$R_{1,y} = 90 \text{ N}$$

To visualize the internal forces cuts are made in member (r) the vicinity of its joints p and q, see Fig. 5. Each member is acted up on by two forces, one at each end; the forces have the same magnitude, same line of action, and opposite sense, see Fig. 6.

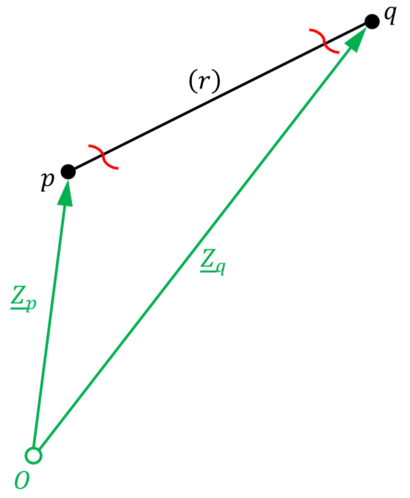

Figure 5: Cuts in the vicinity of joints p and q are denoted by red tildes.

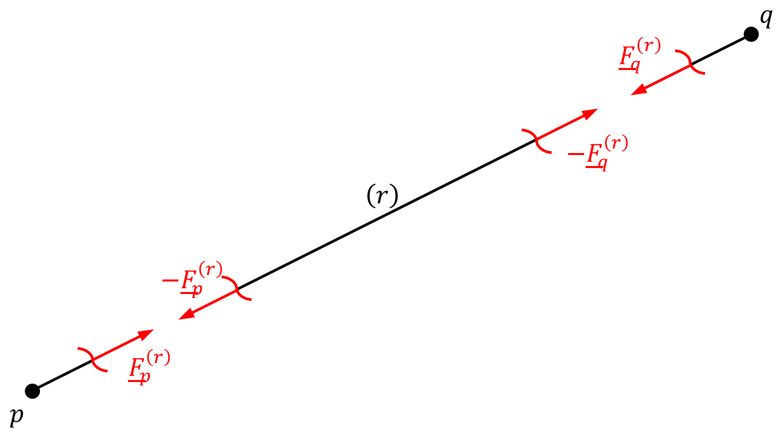

Figure 6: Internal forces drawn away from the cuts as if the member (r) is tension.

The common magnitude of the forces exerted by member (r) on the joints p and q allows to express the internal forces $\underline{F}_p^{(r)}$ and $\underline{F}_q^{(r)}$ through the scalar member force $F^{(r)}$as

$$\underline{F}_p^{(r)} := F^{(r)} \cdot \frac{\underline{Z}_q - \underline{Z}_p}{|\underline{Z}_q - \underline{Z}_p|}, \qquad \underline{F}_q^{(r)} := F^{(r)} \cdot \frac{\underline{Z}_p - \underline{Z}_q}{|\underline{Z}_p - \underline{Z}_q|}$$

Remark: note that

$$\underline{F}_p^{(r)} + \underline{F}_q^{(r)} = 0$$

We assume that all members are in tension. A positive value obtained for member force $F^{(r)}$indicates that member (r) is in tension. A negative value obtained for member force $F^{(r)}$ indicates that member (r) is in compression. If $F^{(r)} = 0$ then member (r) is said to be a zero-force member.

The nine unknown member forces $F^{(r)}, r \in \{1,2,\cdots,9\}$ can be determined by the method of sections.

In Fig. 7 a section has been passed through the members (1) and (2). The internal forces $\underline{F}_6^{(1)}$ and $\underline{F}_6^{(2)}$ acting on joint 6 then can be expressed through their member forces $F^{(1)}$and $F^{(2)}$, respectively, as

$$\underline{F}_6^{(1)} = F^{(1)} \cdot \frac{\underline{Z}_5 - \underline{Z}_6}{|\underline{Z}_5 - \underline{Z}_6|} = F^{(1)} \cdot \frac{(11+3i)-(20)}{|(11+3i)-(20)|} = F^{(1)} \cdot \frac{-9+3i}{|-9+3i|} = F^{(1)} \cdot \frac{-9+3i}{\sqrt{90}} = F^{(1)} \cdot \frac{-3+i}{\sqrt{10}}$$

$$\underline{F}_6^{(2)} = F^{(2)} \cdot \frac{\underline{Z}_4 - \underline{Z}_6}{|\underline{Z}_4 - \underline{Z}_6|} = F^{(2)} \cdot \frac{(16+8i)-(20)}{|(16+8i)-(20)|} = F^{(2)} \cdot \frac{-4+8i}{|-4+8i|} = F^{(2)} \cdot \frac{-4+8i}{\sqrt{80}} = F^{(2)} \cdot \frac{-1+2i}{\sqrt{5}}$$

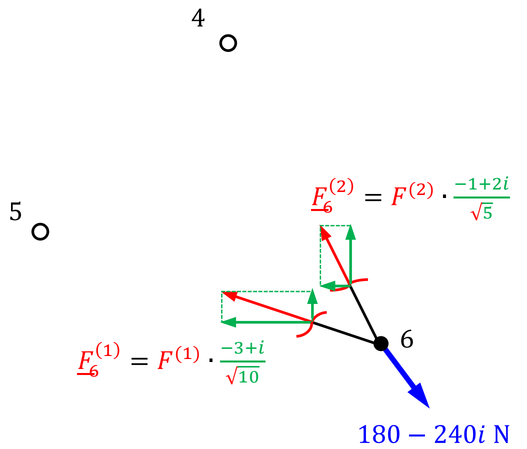

Figure 7

***Equilibrium of joint* 6.**

Determination of member force $F^{(1)}$.

Taking moments about joint 4 to eliminate $F^{(2)}$ from the computation.

$\sum M_4 = 0$:

$$\mathrm{Im}\left(\left(F^{(1)} \cdot \frac{-3+i}{\sqrt{10}}\right) \cdot (4-8i)^* + (180-240i) \cdot (4-8i)^*\right) = 0$$

$$\mathrm{Im}\left(\frac{F^{(1)}}{\sqrt{10}} \cdot (-3+i) \cdot (4+8i) + (180-240i) \cdot (4+8i)\right) = 0$$

$$\mathrm{Im}\left(\frac{F^{(1)}}{\sqrt{10}} \cdot (-20-20i) + (2640+480i)\right) = 0$$

$$\frac{-20}{\sqrt{10}} \cdot F^{(1)} + 480 = 0$$

$$480 = \frac{20}{\sqrt{10}} \cdot F^{(1)}$$

$$F^{(1)} = \frac{480}{20} \cdot \sqrt{10} = +24\sqrt{10}\ \mathrm{N}$$

The two moments in this equation follow from the vector diagram shown in Fig. 8.

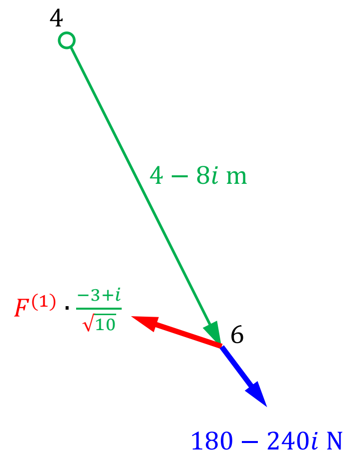

Figure 8

Determination of member force $F^{(2)}$.
Taking moments about joint 5 to eliminate $F^{(1)}$ from the computation.
$\sum M_5 = 0$:

$$\mathrm{Im}\left(\left(F^{(2)} \cdot \frac{-1+2i}{\sqrt{5}}\right) \cdot (9-3i)^* + (180-240i) \cdot (9-3i)^*\right) = 0$$

$$\mathrm{Im}\left(\frac{F^{(2)}}{\sqrt{5}} \cdot (-1+2i) \cdot (9+3i) + (180-240i) \cdot (9+3i)\right) = 0$$

$$\mathrm{Im}\left(\frac{F^{(2)}}{\sqrt{5}} \cdot (-15+15i) + (-2340+1620i)\right) = 0$$

$$\frac{15}{\sqrt{5}} \cdot F^{(2)} + 1620 = 0$$

$$1620 = \frac{15}{\sqrt{5}} \cdot F^{(2)}$$

$$F^{(2)} = \frac{1620}{15} \cdot \sqrt{5} = +108\sqrt{5}\ \mathrm{N}$$

The two moments in this equation follow from the vector diagram shown in Fig. 9.

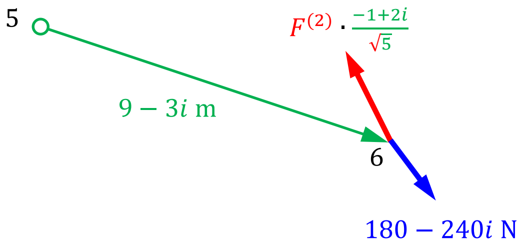

Figure 9

Alternatively, the member force $F^{(2)}$ can be determined by substituting the previous result for $F^{(1)}$ into $\sum \underline{F} = 0$, see Fig. 7.

$$F^{(1)} \cdot \frac{-3+i}{\sqrt{10}} + F^{(2)} \cdot \frac{-1+2i}{\sqrt{5}} + 180 - 240i = 0$$

$$24 \cdot \sqrt{10} \cdot \frac{-3+i}{\sqrt{10}} + F^{(2)} \cdot \frac{-1+2i}{\sqrt{5}} + 180 - 240i = 0$$

$$F^{(2)} \cdot \frac{-1+2i}{\sqrt{5}} - 72 + 24i + 180 - 240i = 0$$

$$F^{(2)} \cdot \frac{-1+2i}{\sqrt{5}} + 108 - 216i = 0$$

$$F^{(2)} \cdot \frac{-1+2i}{\sqrt{5}} = -108 + 216i$$

$$F^{(2)} = \frac{\sqrt{5}}{-1+2i} \cdot (-108+216i) = \frac{-108+216i}{-1+2i} \cdot \sqrt{5} = +108\sqrt{5}\ \mathrm{N}$$

In Fig. 10, a section has been passed through the members (7), (8) and (9). The section divides the truss into completely separate parts but does not intersect more than three members. These parts are denoted by the joints they contain.

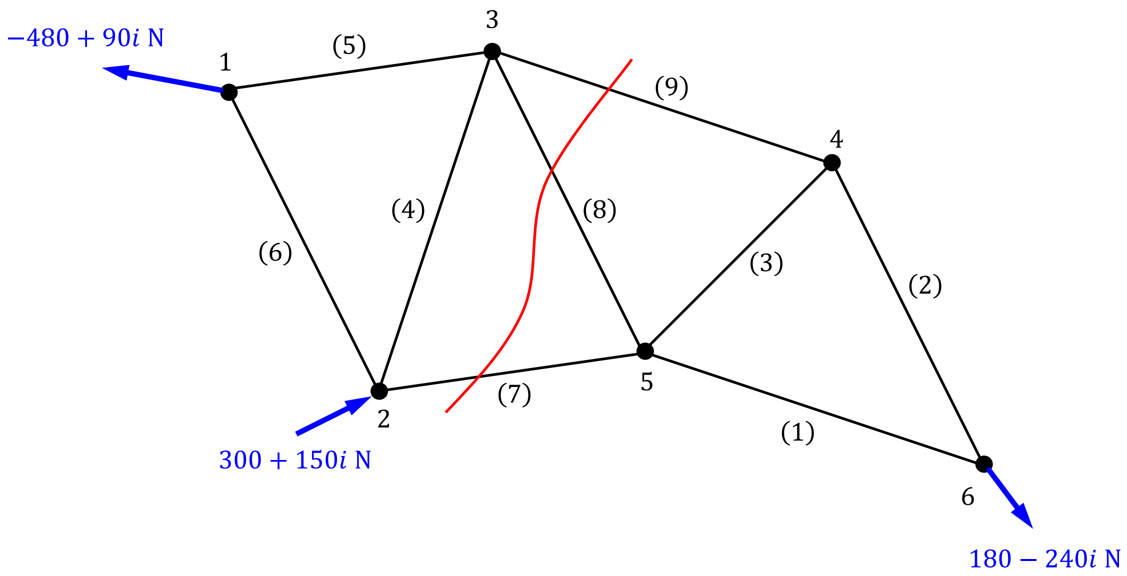

Figure 10

The member forces $F^{(7)}$, $F^{(8)}$ and $F^{(9)}$ act on part 1,2,3 and part 4,5,6 of the truss, see Fig. 11.

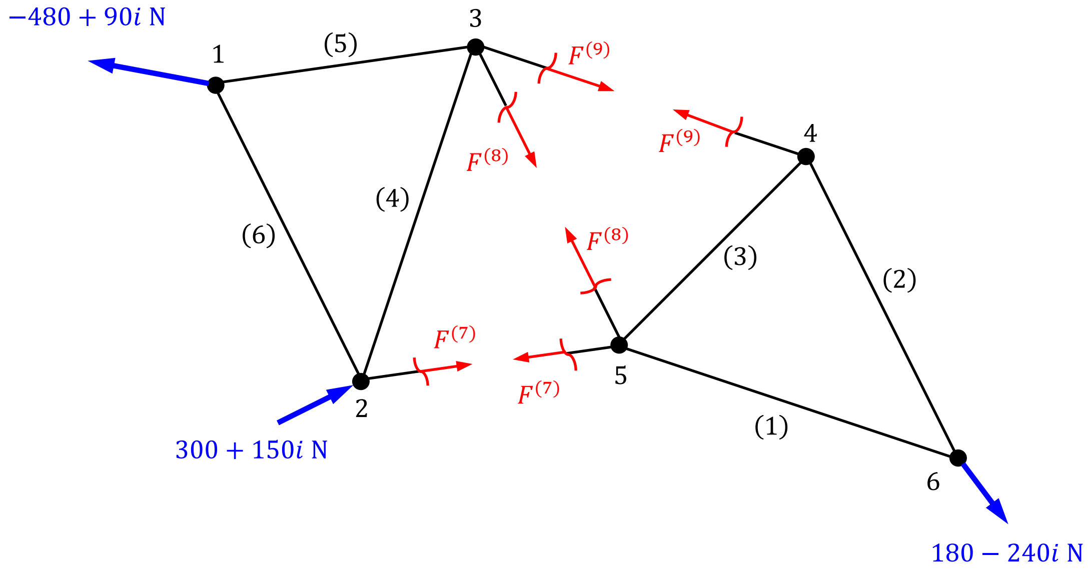

Figure 11

The internal forces $\underline{F_5}^{(7)}$, $\underline{F_5}^{(8)}$and $\underline{F_4}^{(9)}$ acting on part 4,5,6 in Fig. 12 can be expressed as through their member forces $F^{(7)}$, $F^{(8)}$ and $F^{(9)}$, respectively, as

$$\underline{F_5}^{(7)} = F^{(7)} \cdot \frac{\underline{Z_2} - \underline{Z_5}}{|\underline{Z_2} - \underline{Z_5}|} = F^{(7)} \cdot \frac{(4+2i)-(11+3i)}{|(11+3i)-(20)|} = F^{(7)} \cdot \frac{-7-i}{|-7-i|} = F^{(7)} \cdot \frac{-7-i}{\sqrt{50}} = F^{(7)} \cdot \frac{-7-i}{5\sqrt{2}}$$

$$\underline{F_5}^{(8)} = F^{(8)} \cdot \frac{\underline{Z_3} - \underline{Z_5}}{|\underline{Z_3} - \underline{Z_5}|} = F^{(8)} \cdot \frac{(16+8i)-(20)}{|(16+8i)-(20)|} = F^{(8)} \cdot \frac{-4+8i}{|-4+8i|} = F^{(8)} \cdot \frac{-4+8i}{\sqrt{80}} = F^{(8)} \cdot \frac{-1+2i}{\sqrt{5}}$$

$$\underline{F_4}^{(9)} = F^{(9)} \cdot \frac{\underline{Z_3} - \underline{Z_4}}{|\underline{Z_3} - \underline{Z_4}|} = F^{(9)} \cdot \frac{(7+11i)-(16+8i)}{|(7+11i)-(16+8i)|} = F^{(9)} \cdot \frac{-9+3i}{|-9+3i|} = F^{(9)} \cdot \frac{-4+8i}{\sqrt{90}} = F^{(9)} \cdot \frac{-3+i}{\sqrt{10}}$$

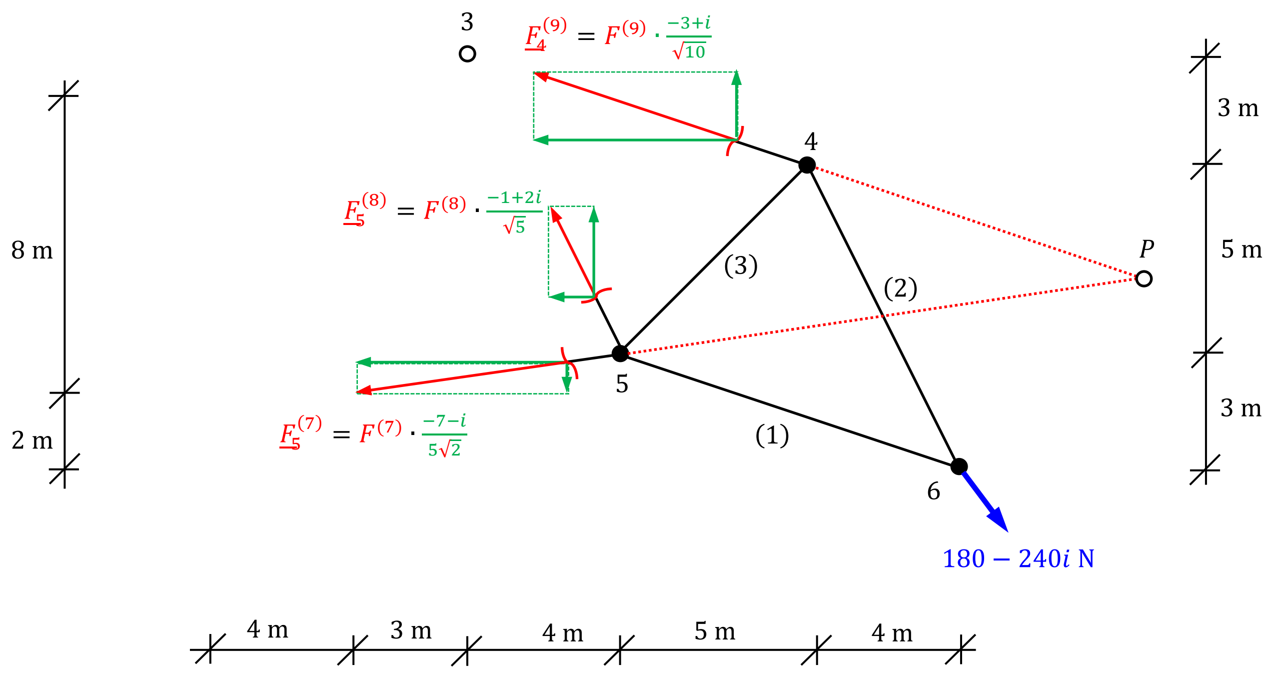

Figure 12

***Equilibrium of part* $4, 5, 6$.**

Determination of member force $F^{(7)}$.

Taking moments about joint 3 to eliminate $F^{(8)}$ and $F^{(9)}$ from the computation.

$\sum M_3 = 0$:

$$\mathrm{Im}\left(\left(F^{(7)} \cdot \frac{-7-i}{5\sqrt{2}}\right) \cdot (4-8i)^* + (180-240i) \cdot (13-11i)^*\right) = 0$$

$$\mathrm{Im}\left(\frac{F^{(7)}}{5\sqrt{2}} \cdot (-7-i) \cdot (4+8i) + (180-240i) \cdot (13+11i)\right) = 0$$

$$\mathrm{Im}\left(\frac{F^{(7)}}{5\sqrt{2}} \cdot (-20-60i) + (4980-1140i)\right) = 0$$

$$\frac{-60}{5\sqrt{2}} \cdot F^{(7)} - 1140 = 0$$

$$-1140 = \frac{60}{5\sqrt{2}} \cdot F^{(7)}$$

$$F^{(7)} = \frac{-1140}{60} \cdot 5\sqrt{2} = -95\sqrt{2}\ \mathrm{N}$$

The two moments in this equation follow from the vector diagram shown in Fig. 13.

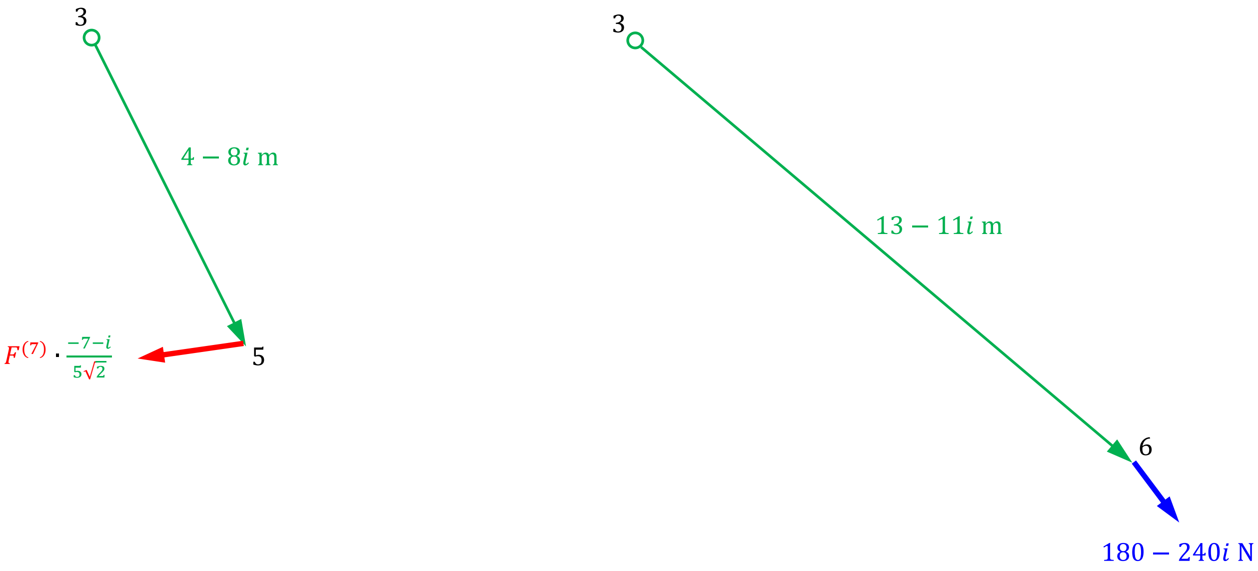

Figure 13

Determination of member force $F^{(8)}$.

The lines of action of the internal forces $\underline{F}_5^{(7)}$ and $\underline{F}_4^{(9)}$ intersect at point P, see Fig. 12.

To determine the location of point P of the moments we have the following theorem.

Theorem

Given the line with position vector $\underline{Z}_p$ and direction vector $\underline{D}_p$. Given another line with position vector $\underline{Z}_q$ and direction vector $\underline{D}_q$, see Fig. 14. If these lines are non-parallel then the position vector $\underline{Z}$ of the point of intersection can be obtained as

$$\underline{Z} = \frac{\underline{D}_p \cdot \mathrm{Im}\left(\underline{Z}_q \cdot \underline{D}_q^*\right) - \underline{D}_q \cdot \mathrm{Im}\left(\underline{Z}_p \cdot \underline{D}_p^*\right)}{\mathrm{Im}\left(\underline{D}_p \cdot \underline{D}_q^*\right)}$$

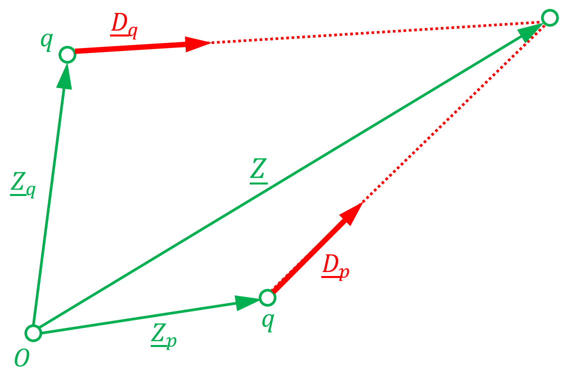

Figure 14

Proof

$\underline{Z} = \underline{Z}_p + \lambda_p \cdot \underline{D}_p, \qquad \lambda_p \in \mathbb{R}$

$\underline{Z} = \underline{Z}_q + \lambda_q \cdot \underline{D}_q, \qquad \lambda_q \in \mathbb{R}$

$$\underline{Z}_p + \lambda_p \cdot \underline{D}_p = \underline{Z}_q + \lambda_q \cdot \underline{D}_q \quad \Leftrightarrow \quad \underline{Z}_p - \underline{Z}_q + \lambda_p \cdot \underline{D}_p = \lambda_q \cdot \underline{D}_q$$

$$\Rightarrow \quad \mathrm{Im}\left(\left(\underline{Z}_p - \underline{Z}_q + \lambda_p \cdot \underline{D}_p\right) \cdot \underline{D}_q^*\right) = \mathrm{Im}\left(\left(\lambda_q \cdot \underline{D}_q\right) \cdot \underline{D}_q^*\right) = 0 \quad \Leftrightarrow \quad \lambda_p = \frac{\mathrm{Im}\left((\underline{Z}_q - \underline{Z}_p) \cdot \underline{D}_q^*\right)}{\mathrm{Im}\left(\underline{D}_p \cdot \underline{D}_q^*\right)}$$

$$\underline{Z} = \underline{Z}_p + \lambda_p \cdot \underline{D}_p = \underline{Z}_p + \frac{\mathrm{Im}\left((\underline{Z}_q - \underline{Z}_p) \cdot \underline{D}_q^*\right)}{\mathrm{Im}\left(\underline{D}_p \cdot \underline{D}_q^*\right)} \cdot \underline{D}_p = \frac{\underline{Z}_p \cdot \mathrm{Im}\left(\underline{D}_p \cdot \underline{D}_q^*\right) + \underline{D}_p \cdot \mathrm{Im}\left((\underline{Z}_q - \underline{Z}_p) \cdot \underline{D}_q^*\right)}{\mathrm{Im}\left(\underline{D}_p \cdot \underline{D}_q^*\right)}$$

$$= \frac{\underline{Z}_p \cdot \left(\underline{D}_p \cdot \underline{D}_q^* - \underline{D}_p^* \cdot \underline{D}_q\right) + \underline{D}_p \cdot \left((\underline{Z}_q - \underline{Z}_p) \cdot \underline{D}_q^* - (\underline{Z}_q^* - \underline{Z}_p^*) \cdot \underline{D}_q\right)}{2i \cdot \mathrm{Im}\left(\underline{D}_p \cdot \underline{D}_q^*\right)} = \frac{\underline{Z}_p \cdot \underline{D}_p \cdot \underline{D}_q^* - \underline{Z}_p \cdot \underline{D}_p^* \cdot \underline{D}_q + \underline{Z}_q \cdot \underline{D}_p \cdot \underline{D}_q^* - \underline{Z}_p \cdot \underline{D}_p \cdot \underline{D}_q^* - \underline{Z}_q^* \cdot \underline{D}_p \cdot \underline{D}_q + \underline{Z}_p^* \cdot \underline{D}_p \cdot \underline{D}_q}{2i \cdot \mathrm{Im}\left(\underline{D}_p \cdot \underline{D}_q^*\right)}$$

$$= \frac{-\underline{Z}_p \cdot \underline{D}_p^* \cdot \underline{D}_q + \underline{Z}_q \cdot \underline{D}_p \cdot \underline{D}_q^* - \underline{Z}_q^* \cdot \underline{D}_p \cdot \underline{D}_q + \underline{Z}_p^* \cdot \underline{D}_p \cdot \underline{D}_q}{2i \cdot \mathrm{Im}\left(\underline{D}_p \cdot \underline{D}_q^*\right)} = \frac{\underline{Z}_q \cdot \underline{D}_p \cdot \underline{D}_q^* - \underline{Z}_q^* \cdot \underline{D}_p \cdot \underline{D}_q - \underline{Z}_p \cdot \underline{D}_p^* \cdot \underline{D}_q + \underline{Z}_p^* \cdot \underline{D}_p \cdot \underline{D}_q}{2i \cdot \mathrm{Im}\left(\underline{D}_p \cdot \underline{D}_q^*\right)}$$

$$= \frac{\underline{D}_p \cdot \left(\underline{Z}_q \cdot \underline{D}_q^* - \underline{Z}_q^* \cdot \underline{D}_q\right) - \underline{D}_q \cdot \left(\underline{Z}_p \cdot \underline{D}_p^* - \underline{Z}_p^* \cdot \underline{D}_p\right)}{2i \cdot \mathrm{Im}\left(\underline{D}_p \cdot \underline{D}_q^*\right)} = \frac{\underline{D}_p \cdot \mathrm{Im}\left(\underline{Z}_q \cdot \underline{D}_q^*\right) - \underline{D}_q \cdot \mathrm{Im}\left(\underline{Z}_p \cdot \underline{D}_p^*\right)}{\mathrm{Im}\left(\underline{D}_p \cdot \underline{D}_q^*\right)}$$

□

Applying the previous theorem to determine the point of intersection of the lines of action of the internal forces $\underline{F}_5^{(7)}$ and $\underline{F}_4^{(9)}$.

The line of action of the internal force $\underline{F}_5^{(7)}$ is defined by the position vector $\underline{Z}_5 = 11 + 3i$ m and the direction vector $\underline{D}_5 = 3 - i$.

The line of action of the internal force $\underline{F}_4^{(9)}$ is defined by the position vector $\underline{Z}_4 = 16 + 8i$ m and the direction vector $\underline{D}_4 = 7 + i$.

$$\underline{Z} = \frac{\underline{D}_5 \cdot \mathrm{Im}\left(\underline{Z}_4 \cdot \underline{D}_4^*\right) - \underline{D}_4 \cdot \mathrm{Im}\left(\underline{Z}_5 \cdot \underline{D}_5^*\right)}{\mathrm{Im}\left(\underline{D}_5 \cdot \underline{D}_4^*\right)} = \frac{(7+i) \cdot \mathrm{Im}\left((16+8i) \cdot (3-i)^*\right) - (3-i) \cdot \mathrm{Im}\left((11+3i) \cdot (7+i)^*\right)}{\mathrm{Im}\left((7+i) \cdot (3-i)^*\right)} = \frac{(7+i) \cdot \mathrm{Im}(40+40i) - (3-i) \cdot \mathrm{Im}(80+10i)}{\mathrm{Im}(20+10i)}$$

$$= \frac{(7+i) \cdot (40) - (3-i) \cdot (10)}{(10)} = \frac{(280+40i) - (30-10i)}{(10)} = \frac{250+50i}{10} = 25 + 5i \text{ m}$$

Taking moments about point P to eliminate $F^{(7)}$ and $F^{(9)}$ from the computation.

$\sum M_P = 0$:

$$\mathrm{Im}\left(\left(F^{(8)} \cdot \frac{-1+2i}{\sqrt{5}}\right) \cdot (-14 - 2i)^* + (180 - 240i) \cdot (-5 - 5i)^*\right) = 0$$

$$\mathrm{Im}\left(\frac{F^{(8)}}{\sqrt{5}} \cdot (-1 + 2i) \cdot (-14 + 2i) + (180 - 240i) \cdot (-5 + 5i)\right) = 0$$

$$\mathrm{Im}\left(\frac{F^{(8)}}{\sqrt{5}} \cdot (10 - 30i) + (300 + 2100i)\right) = 0$$

$$\frac{-30}{\sqrt{5}} \cdot F^{(8)} + 2100 = 0$$

$$2100 = \frac{30}{\sqrt{5}} \cdot F^{(8)}$$

$$F^{(8)} = \frac{2100}{30} \cdot \sqrt{5} = +70\sqrt{5} \text{ N}$$

The two moments in this equation follow from the vector diagram shown in Fig. 15.

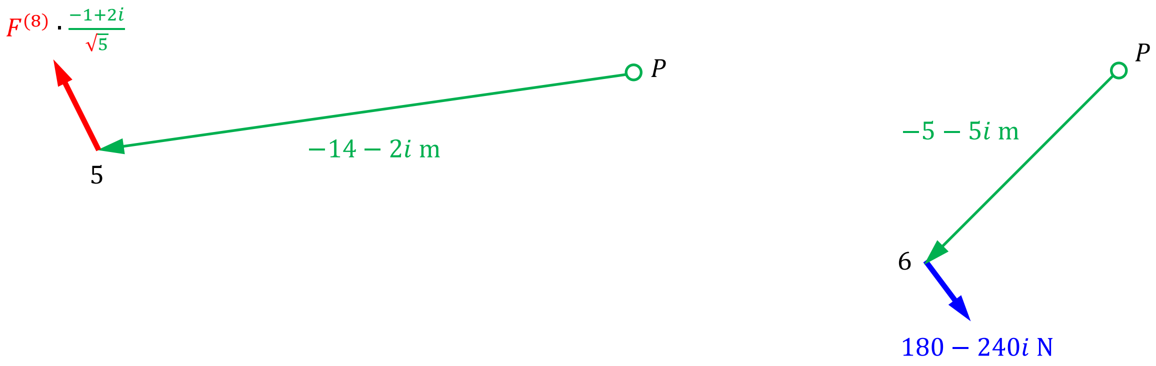

Figure 15

In Fig. 16, a section has been passed through the members (4), (5) and (7).

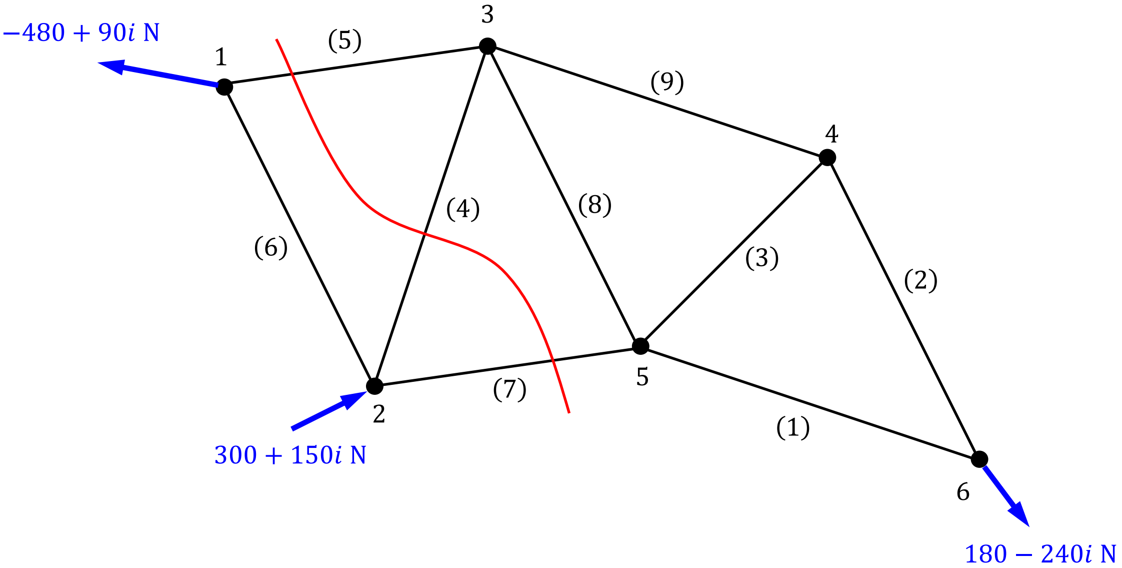

Figure 16

The internal forces $\underline{F}_3^{(4)}$, $\underline{F}_3^{(5)}$ and $\underline{F}_5^{(7)}$ acting on part 3,4,5,6 in Fig. 17 can be expressed as through their member forces $F^{(4)}$, $F^{(5)}$ and $F^{(7)}$, respectively, as

$$\underline{F}_3^{(4)} = F^{(4)} \cdot \frac{\underline{Z}_2 - \underline{Z}_3}{|\underline{Z}_2 - \underline{Z}_3|} = F^{(4)} \cdot \frac{(4+2i)-(7+11i)}{|(4+2i)-(7+11i)|} = F^{(4)} \cdot \frac{-3-9i}{|-3-9i|} = F^{(4)} \cdot \frac{-3-9i}{\sqrt{90}} = F^{(4)} \cdot \frac{-1-3i}{\sqrt{10}}$$

$$\underline{F}_3^{(5)} = F^{(5)} \cdot \frac{\underline{Z}_1 - \underline{Z}_3}{|\underline{Z}_1 - \underline{Z}_3|} = F^{(5)} \cdot \frac{(10i)-(7+11i)}{|(10i)-(7+11i)|} = F^{(5)} \cdot \frac{-7-i}{|-7-i|} = F^{(5)} \cdot \frac{-7-i}{\sqrt{50}} = F^{(5)} \cdot \frac{-7-i}{5\sqrt{2}}$$

$$\underline{F}_5^{(7)} = F^{(7)} \cdot \frac{\underline{Z}_2 - \underline{Z}_5}{|\underline{Z}_2 - \underline{Z}_5|} = F^{(7)} \cdot \frac{(4+2i)-(11+3i)}{|(4+2i)-(11+3i)|} = F^{(7)} \cdot \frac{-7-i}{|-7-i|} = F^{(7)} \cdot \frac{-7-i}{\sqrt{50}} = F^{(7)} \cdot \frac{-7-i}{5\sqrt{2}}$$

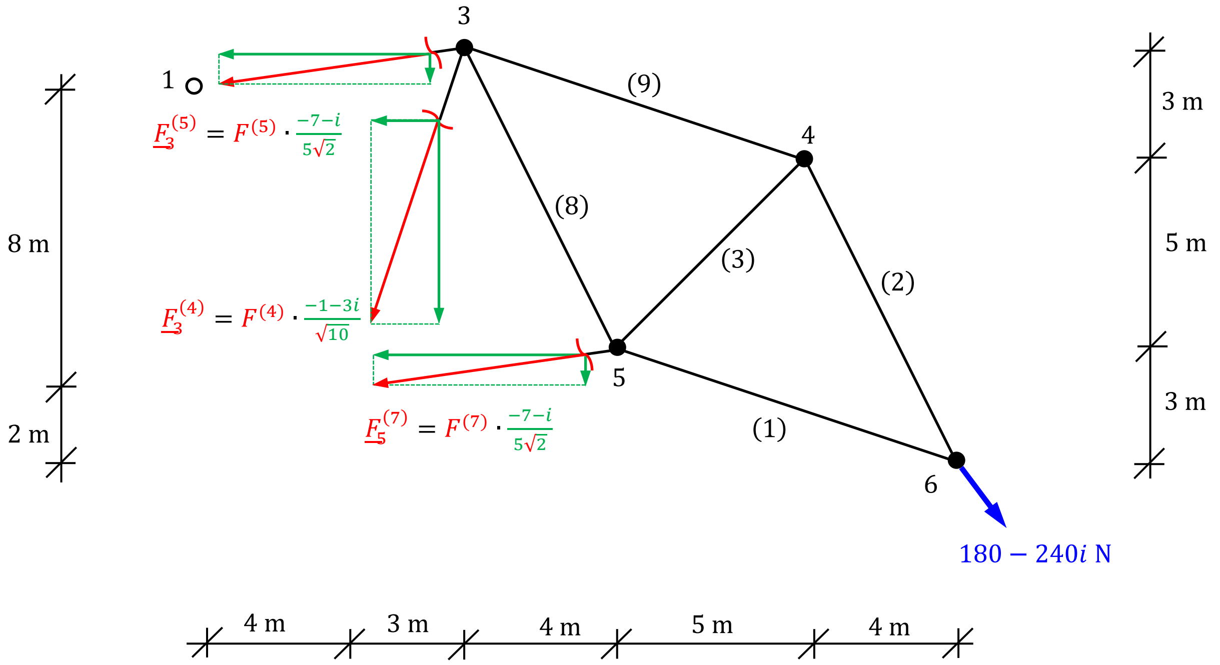

Figure 17

The equation of equilibrium $\sum \underline{F} = 0$ for part 3,4,5,6 in Fig. 17 can be visualized by the vector diagram shown in Fig. 18. All the forces then can be considered to act on an arbitrary chosen point, for example joint 3.

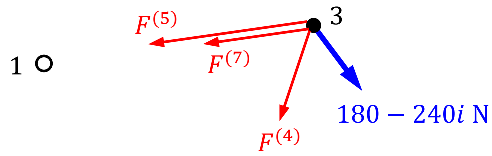

Figure 18

Remark: in Fig. 17 the lines of action of the member forces $F^{(5)}$ and $F^{(7)}$ are parallel, whereas in Fig. 18 the lines of action of these member forces coincide.

Taking moments about joint 1 to eliminate $F^{(5)}$ and $F^{(7)}$ from the computation.

$\sum M_1 = 0$:

$$\operatorname{Im}\left(\left(F^{(4)} \cdot \frac{-1-3i}{\sqrt{10}}\right) \cdot (7+i)^* + (180-240i) \cdot (7+i)^*\right) = 0$$

$$\operatorname{Im}\left(\frac{F^{(4)}}{\sqrt{10}} \cdot (-1-3i) \cdot (7-i) + (180-240i) \cdot (7-i)\right) = 0$$

$$\operatorname{Im}\left(\frac{F^{(4)}}{\sqrt{10}} \cdot (-10-20i) + (1020-1860i)\right) = 0$$

$$\frac{-20}{\sqrt{10}} \cdot F^{(4)} - 1860 = 0$$

$$-1860 = \frac{20}{\sqrt{10}} \cdot F^{(4)}$$

$$F^{(4)} = \frac{-1860}{20} \cdot \sqrt{10} = -93\sqrt{10}\ \mathrm{N}$$

The two moments in this equation follow from the vector diagram shown in Fig. 19.

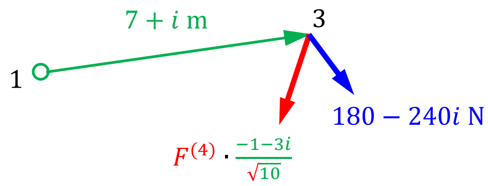

Figure 19

The member forces $F^{(5)}$ and $F^{(6)}$ can be determined in a similar fashion as the member forces $F^{(1)}$ and $F^{(2)}$. The member forces $F^{(9)}$ can be determined in a similar fashion as the member forces $F^{(7)}$. The member forces $F^{(3)}$ can be determined in a similar fashion as the member forces $F^{(4)}$.

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