

# Gravitational Dynamics of Cantor Dust and Self-Interacting Dark Matter

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## Abstract

We develop a first-principles framework in which *self-interacting dark matter* (SIDM) emerges as an effective manifestation of primordial Cantor Dust. In this framework, Dark Matter resides on a multifractal support characterized by a scale-dependent Hausdorff dimension  $D < 3$ . We argue that the phenomenology of SIDM is a memory of primordial Cantor Dust, emerging *without* conventional particle scattering and cross sections. The mechanism described here reproduces the flat galactic rotation curves, baryonic Tully-Fisher relations, the SIDM transport cross section, explains the core-cusp problem, the evasion of Bullet Cluster bounds and the morphology of the cosmic web.

**Key words:** Self-interacting Dark Matter, Cantor Dust, anomalous diffusion, transport cross section, flat rotation curves, Tully-Fisher relations, core-cusp problem, Bullet Cluster bounds, cosmic web.

## **1. Introduction**

Explaining the nature of Dark Matter remains a long-standing challenge in cosmology and particle physics. While collisionless cold Dark Matter (CDM) successfully accounts for the behavior of many cosmological structures, persistent discrepancies on galactic scales—such as the core–cusp problem and the anomalous rotation curves—suggest that additional physics may be required. SIDM models attempt to address these issues by introducing an effective scattering cross section, but this approach faces challenges regarding the microscopic (“particle”-like) origin of SIDM phenomena, scale dependence, and compatibility with cluster constraints.

In this paper, we pursue an alternative route in which SIDM phenomenology arises from gravitational dynamics of Cantor Dust rather than from new particle interactions [22 – 23, 26 – 27]. We assume that Dark Matter resides on Cantor Dust support with arbitrary mass dimension  $D < 3$ . This single geometric ansatz fixes the gravitational field carried by Cantor Dust and implies a virial balance leading to the corrected velocity-dispersion

scaling. Fractal geometry also determines the density profile and the characteristic dynamical time. Because gravitational interactions in a Cantor Dust background are *collective* and *correlated*, the effective relaxation time is governed by a scale-dependent relationship. As a result, the system naturally separates into an *inner region* where diffusion (scattering) and thermalization dominate, and an *outer region* where relaxation is suppressed and a diffusion-free (collisionless) regime sets in.

The purpose of this introductory paper is to show that this framework provides a consistent and transparent explanation for SIDM-like behavior, core formation, and halo structure, while automatically satisfying constraints from galaxy clusters and large-scale structure.

For the sake of clarity and accessibility, the paper is conceived as a “toy model”, presented in bullet-style form, with technical content kept at a minimum and emphasis placed on building physical intuition. Independent reviews are needed to substantiate, develop or refute the tentative body of ideas described herein.

## 2. Cantor Dust as primordial gravitational structure

### 2.1 Mass as geometric measure

Consider a primordial spacetime region  $\mathcal{R}$ . Following [26 - 27], let  $\mu(\mathcal{R})$  denote a geometric measure defined on  $\mathcal{R}$  with the following properties:

1) Additivity:  $\mu(\mathcal{R}_1 \cup \mathcal{R}_2) = \mu(\mathcal{R}_1) + \mu(\mathcal{R}_2)$

2) Scale-covariance:  $\mu(\lambda\mathcal{R}) = \lambda^{d_H-1} \mu(\mathcal{R})$

where  $d_H$  is a generic Hausdorff dimension. In suitable mass units and for a measure domain  $C_i$  ( $i=1,2,\dots,n$ ), define a corresponding *mass measure* as

$$m_i = \int_{C_i} d\mu_i$$

This definition is suitable for describing the phenomenology of Cantor Dust, an early cosmological structure displaying fractal or multifractal properties.

We shall use it as substitute for the ordinary mass concept in all our subsequent derivations.

## 2.2 Galactic mass distribution

Let a baryonic mass distribution be supported on a random Cantor Dust background embedded in  $\mathbb{R}^3$ , with:

$$M(R) \equiv \int_{|x|<R} \rho(x) d^3x \sim M_0 \left(\frac{R}{L}\right)^D, \quad 1 < D < 3 \quad (1a)$$

where:

- $D$  is the *spatial* Hausdorff dimension,
- $L$  is the outer scale,
- $M_0 = M(L)$ .

Here,  $\rho(x)$  is *not a density field* in the classical sense; instead, it's the "coarse-grained" parameter

$$\rho(x) = \sum_i m_i \delta(x - x_i), \quad (1b)$$

where  $\{x_i\}$  forms a fractal-like distribution,  $\delta(\dots)$  is the delta function and  $m_i$  are the mass measures defined above.

Starting from (1a), the following kinematic relations hold:

Mass scaling

$$M(r) \sim r^D, \quad r = R/L$$

Gravitational acceleration

$$g(r) = \frac{GM(r)}{r^2} \sim r^{D-2}$$

Gravitational potential

$$\Phi(r) \sim \int g \, dr \sim r^{D-1}$$

Orbital velocity

The dynamic equilibrium/virial condition

$$g(r) = v^2(r)/r$$

leads to the velocity dispersion relationship,

$$\boxed{v^2(r) \sim r^{D-1}} \quad (2)$$

The above formulas are used in [26] to derive the flat galactic rotation curves and the baryonic Tully-Fisher relation.

### **2.3 Gravitational field generated by Cantor Dust**

The gravitational acceleration at point  $x$  is given by

$$\mathbf{a}(x) = -G \sum_i m_i \frac{x - x_i}{|x - x_i|^3} \quad (3)$$

Consistent with the attributes of Cantor Dust, gravitational acceleration (3) is *non-smooth*, has *scale-dependent properties*, and represents a *random field with long-range correlations*. We next proceed to characterize its statistics.

## **3. Force variance and scaling**

### **3.1 Contribution from a single shell**

Consider a thin spherical shell  $[r, r + dr]$ . The number of points in this shell scales as

$$dN(r) \sim r^{D-1} dr \quad (4)$$

Each point contributes an acceleration magnitude on the order of  $\sim Gm/r^2$ .

Because directions are random, one has

$$d\langle a^2 \rangle \sim dN(r) \left( \frac{Gm}{r^2} \right)^2 \sim r^{D-1} \frac{dr}{r^4} = r^{D-5} dr \quad (5)$$

### **3.2 Infrared dominance**

Integrating (5) from the inner cutoff  $\ell$  to the outer scale  $L$  gives

$$\langle a^2 \rangle \sim \int_{\ell}^L r^{D-5} dr \quad (6)$$

For  $D < 4$ , the integral is *IR dominated*, leading to

$$\langle a^2 \rangle \sim L^{D-4} \quad (7)$$

Equations (5) - (7) may be viewed as model of non-local gravitational fluctuations.

### **3. Force correlations along particle trajectories**

For a test particle moving along trajectory  $x(t)$ , define the force correlation function as

$$C(\tau) \equiv \langle \mathbf{a}(x(t)) \cdot \mathbf{a}(x(t + \tau)) \rangle \quad (8)$$

In addition to (8), let's assume:

- ergodicity,
- isotropy,
- ballistic motion at short times, that is,

$$|x(t + \tau) - x(t)| \approx v\tau \quad (9)$$

#### **3.1 Spatial to temporal mapping**

Using (8), it can be shown that spatial correlations scale as (the formal proof is lengthy and not included here)

$$\langle \mathbf{a}(x) \cdot \mathbf{a}(x + r) \rangle \sim r^{D-4} \quad (10)$$

Mapping to  $r \sim v\tau$  gives,

$$C(\tau) \sim (v\tau)^{D-4} \equiv \tau^{-\beta} \quad (11)$$

where

$$\boxed{\beta = 4 - D} \quad (1 < \beta < 3) \quad (12)$$

(11) and (12) are a description of *long-memory noise*, strongly dependent on the Hausdorff dimension. The space–time mapping of the type (11) is typical in:

- plasma kinetic theory,
- Chandrasekhar stochastic gravity,
- fractional Langevin dynamics.

It relies only on:

1. Stationarity of the underlying field,
2. Short-time ballistic motion,

### 3. Scale-free correlations

## 4. Velocity diffusion from correlated gravitational fluctuations

### 4.1 Mean-square velocity increment

Velocity increments can be computed from

$$\Delta v(t) = \int_0^t a(t') dt' \quad (13a)$$

and the mean-square velocity from

$$\langle \Delta v^2(t) \rangle = 2 \int_0^t (t - \tau) C(\tau) d\tau \quad (13b)$$

Inserting (11) in (13b) yields:

$$\langle \Delta v^2(t) \rangle \sim \int_0^t (t - \tau) \tau^{-\beta} d\tau \sim t^{2-\beta} \quad (14)$$

Using (12) and (14) leads to an *anomalous diffusion* relationship

$$\boxed{\langle \Delta v^2(t) \rangle \sim t^{D-2}} \quad (15)$$

underlying three basic scenarios,

- $D = 2$ : logarithmic diffusion
- $D > 2$ : superdiffusion
- $D < 2$ : subdiffusion

The key observation here is that (15) *does not follow* from ordinary scattering/collision processes involving quantum particles, but from velocity diffusion induced by gravitational fluctuations.

## **5. Emergence of an effective diffusion rate**

### **5.1 Definition of relaxation time**

Define  $t_{\text{rel}}$  by the relationship,

$$\langle \Delta v^2(t_{\text{rel}}) \rangle \sim v^2 \quad (16)$$

By (15), one gets

$$t_{\text{rel}} \sim v^{\frac{2}{D-2}} \quad (17)$$

## **5.2 Effective diffusion frequency**

Define next the effective diffusion frequency as

$$\boxed{\Gamma_{\text{eff}} \equiv t_{\text{rel}}^{-1}} \quad (18)$$

Although (18) is of purely gravitational nature, it plays the role of a collision rate.

## **6. Mapping to SIDM transport theory**

### **6.1 SIDM relaxation time**

The scattering frequency in SIDM models is given by

$$t_{\text{SIDM}}^{-1} = n_{\text{DM}} \sigma_T v \quad (19)$$

where  $\sigma_T$  is the transport cross section and  $n_{\text{DM}}$  the number of Dark Matter particles involved in the transport process.

## 6.2 Identification

Equating (18) and (19) gives

$$\sigma_T^{\text{eff}} \sim \frac{1}{n_{\text{DM}} v} v^{-\frac{2}{D-2}} \quad (20a)$$

or

$$\boxed{\sigma_T^{\text{eff}} \propto v^{-\alpha}, \quad \alpha = \frac{2}{D-2} + 1} \quad (20b)$$

## 6.3 Key result

For a fractal structure whose spatial dimension falls close to the ordinary space dimension, namely  $D \approx 2.66 - 3$  we obtain

$$\sigma_T \propto v^{-4} \text{ to } v^{-3}$$

which matches the SIDM velocity dependence of the transport cross-section, *without* invoking particle self-interactions. Note that SIDM velocity-dependent models are essential for reconciling the large cross sections

required to explain dwarf galaxy observations with the much tighter constraints from galaxy clusters.

In summary, the line of arguments developed so far shows that,

1. Cantor Dust  $\Rightarrow M(r) \sim r^D$
2. Cantor Dust  $\Rightarrow$  long-range temporal correlations
3. Correlations  $\Rightarrow$  anomalous velocity diffusion
4. Diffusion  $\Rightarrow$  finite relaxation time
5. Relaxation time  $\Rightarrow$  effective diffusion rate
6. Diffusion rate  $\Rightarrow$  SIDM-like transport cross section

Next sections elaborate on how this framework can explain several features of Dark Matter phenomena, such as core-cusp transition, the evasion of the Bullet Cluster bounds and halo formation.

## 7. Derivation of the core–cusp transition

### 7.1 What must be shown

Astrophysical observations distinguish between the formation of *cusps* from the formation of *cores* in the halo distribution of Dark Matter. Our goal is to show that Cantor Dust–induced gravitational diffusion produces a finite-density core below a radius  $r_c$ , while leaving the outer halo diffusion-free (collisionless). For more on the theoretical basis of cores and cusps formation, refer to Appendix B.

### 7.2 Jeans equation of galactic dynamics

The Jeans momentum equation with an effective collisional pressure is [16]

$$\frac{d}{dr}(\rho\sigma_v^2) = -\rho\frac{d\Phi}{dr} \quad (27)$$

where  $\sigma_v^2 = \langle v^2 \rangle$  and  $\Phi(\mathbf{x}, t)$  is the Newtonian gravitational potential, defined by  $\mathbf{g} = -\nabla\Phi$ . Inside the region where relaxation is efficient,

$$t_{\text{rel}}(r) \ll t_{\text{dyn}}(r) \quad (28)$$

Here, the dynamical time  $t_{\text{dyn}} \sim (G\rho)^{-\frac{1}{2}}$  represents the characteristic timescale for a self-gravitating system to settle down. Condition (28) leads to the onset of *local thermal equilibrium*.

### 7.3 Condition for the core radius

The core radius  $r_c$  is defined by the condition

$$t_{\text{rel}}(r_c) = t_{\text{dyn}}(r_c) \quad (29)$$

with,

$$t_{\text{dyn}}(r) \sim \frac{r}{v(r)}, \quad v^2(r) \sim \frac{GM(r)}{r} \quad (30)$$

By (2), (30) leads to

$$\boxed{t_{\text{dyn}}(r) \sim r^{1-(D-1)/2} = r^{(3-D)/2}} \quad (31)$$

This scaling is *monotonically increasing* with  $r$  for  $D < 3$ .

## 7.4 Relaxation time in Cantor Dust

Following Appendix A, in the Cantor Dust picture, we write

$$t_{\text{rel}}(r) \sim \frac{t_{\text{dyn}}(r)}{\epsilon_{\text{eff}}(r)}$$

where the effective coupling scales as

$$\epsilon_{\text{eff}}(r) \sim r^{D-2}$$

Therefore,

$$t_{\text{rel}}(r) \sim r^{(3-D)/2} r^{2-D}$$

or

$$\boxed{t_{\text{rel}}(r) \sim r^{\frac{7-3D}{2}}} \quad (32)$$

For more on the effective coupling  $\epsilon_{\text{eff}}$  and its dynamic implications, refer to Appendix A.

## 7.5 Core-cusp transition

Core forms where relaxation is fast relative to Hubble time,

$$t_{\text{rel}}(r) \lesssim t_H$$

From (32),

$$r^{(7-3D)/2} \lesssim t_H$$

This defines a finite core radius

$$\boxed{r_c \sim t_H^{2/(7-3D)}} \quad (33)$$

(33) implies the following scenarios:

- For  $D < 7/3 \approx 2.33$ ,: the finite core radius grows with the Hubble time, relaxation and diffusion become increasingly efficient, thermalization is enabled and large cores develop.
- For  $D \rightarrow 3$ : the finite core radius drops down with the Hubble time, relaxation is suppressed, and cusps start to develop.

## 8. Halo density profile

Density (defined by (1b)), follows from straightforward geometric considerations, that is,

$$\rho(r) = \frac{dM}{dr} \frac{1}{4\pi r^2} \sim r^{D-3}$$

or

$$\boxed{\rho(r) \sim r^{-(3-D)}} \quad (34)$$

Thus:

- $D \approx 2 \rightarrow$  density goes up as  $r$  drops down, favoring the formation of cores in *inner* Dark Matter haloes.
- $D \approx 2.6 \rightarrow$  marginal behavior.
- $D \rightarrow 3 \rightarrow$  density drops or stays nearly constant as  $r$  goes up, enabling the formation of cusps in *outer* Dark Matter haloes.

## 9. Evasion of the Bullet Cluster bound

Let's begin by recalling representative scaling relationships derived from the fractal dimension  $D$ , namely,

$$v^2(r) \propto r^{D-1} \quad (35)$$

$$\rho(r) \propto r^{D-3} \quad (36)$$

$$t_{\text{dyn}}(r) \sim \frac{r}{v} \propto r^{\frac{3-D}{2}} \quad (37)$$

Gravitational relaxation time of galactic dynamics is given by [16]

$$t_{\text{rel}}(r) \sim \frac{v^3}{G^2 m \rho \ln \Lambda} \propto r^{\frac{7-3D}{2}} \quad (38)$$

where  $\Lambda$  measures how efficiently uncorrelated scattering events accumulate to cause relaxation.

The transport efficiency scales as (Appendix A):

$$\epsilon_{\text{eff}}(r) \equiv \frac{t_{\text{dyn}}}{t_{\text{rel}}} \propto r^{D-2} \quad (39)$$

Bullet Clusters constrain momentum exchange during a fast cluster collision.

This is equivalent to requiring:

$$\tau \equiv \int \rho \sigma_{\text{eff}} dl \ll 1 \Leftrightarrow t_{\text{rel}} \gg t_{\text{coll}} \quad (40)$$

where the collision time is:

$$t_{\text{coll}} \sim \frac{L}{V_{\text{rel}}} \sim 10^8 \text{ yr} \quad (41)$$

The constraint (40) is evaluated at cluster scales on the order of:

$$r_{\text{BC}} \sim 10^2\text{--}10^3 \text{ kpc} \quad (42)$$

Using (38) gives:

$$t_{\text{rel}}(r_{\text{BC}}) \propto r_{\text{BC}}^{\frac{7-3D}{2}} \quad (43)$$

For the physically relevant range  $D > 2$ , if  $(7 - 3D)/2 \leq 1/2$ , (43) is very large and

$$t_{\text{rel}}(r_{\text{BC}}) \gg t_{\text{coll}} \quad (44)$$

The relaxation suppression implied by (44) arises because, at large  $r$ :

- density drops as  $\rho \propto r^{D-3}$
- velocity dispersion rises only mildly:  $v^2 \propto r^{D-1}$

Thus, the diffusion rate:

$$\Gamma \sim t_{\text{rel}}^{-1} \propto r^{-\frac{7-3D}{2}} \quad (45)$$

is strongly suppressed on cluster scales, galactic mergers exhibit collisionless behavior, and Bullet Cluster bounds are naturally evaded.

## **10. Formation of the cosmic web**

The cosmic web forms if:

1. Dynamics is collisionless on large scales,
2. Phase-space sheets can fold without thermalization,
3. Gravity is non-harmonic (exhibiting anisotropic collapse)

Referring to (38)

$$t_{\text{rel}}(r) \propto r^{\frac{7-3D}{2}}$$

for any  $D > 2$ :

$$t_{\text{rel}}(r) \xrightarrow{r \rightarrow \text{Mpc}} \infty \quad (46)$$

Thus:

$$t_{\text{rel}}(r) \gg t_H \quad (47)$$

(47) shows that  $D > 2$  implies that relaxation is strongly suppressed, and both diffusion and transport “freeze”.

Cantor Dust implies:

$$M(r) \propto r^D \Rightarrow \Phi(r) \propto r^{D-1} \quad (48)$$

For  $D < 2$ , collapse rates depend on direction and perturbations grow anisotropically. This naturally produces:

- sheets (1 collapsing axis),
- filaments (2 collapsing axes),

- nodes (3 collapsing axes  $\rightarrow$  halos).

Using (39):

$$\epsilon_{\text{eff}}(r) \propto r^{D-2}$$

implies that, for  $D < 2$  and at cosmic-web scales  $r \gg r_c$ :

$$\epsilon_{\text{eff}} \ll 1 \quad (49)$$

transport is suppressed, filaments do not thermalize and stay long-lived.

In summary,

a) Gravitational relaxation diverges on large scales, ensuring collisionless dynamics during cluster mergers and cosmic-web formation, while remaining effective only at sufficiently small radii,

b) Cantor Dust gravity yields a consistent multiscale picture: collisionless dynamics on cosmic-web scales, emergence of haloes below a finite radius, and core formation for  $2 < D < 7/3$ . The separation of regimes is controlled

entirely by the fractal dimension  $D$ , with no need for tuning cross sections or postulating exotic particle physics.

## **Conclusions**

We have shown that the SIDM phenomenology can emerge from a relic form of fractal gravity without introducing new Dark Matter interactions. Beginning with the Cantor Dust mass scaling, Eq. (1), Newtonian gravity uniquely fixes the gravitational field and the velocity dispersion. This corrected scaling leads to a relaxation time, Eqs. (17) and (32), that grows rapidly with radius, producing thermalized cores where the condition Eq. (28) is satisfied and collisionless behavior elsewhere.

The resulting halo density profile, Eqs. (33) and (34), interpolate smoothly between “cored” and “cuspy” limits as a function of the fractal dimension  $D$ , while the outer halo obeys the collisionless Jeans equation, Eq. (27). Because momentum exchange is collective and nonlocal, merging clusters exhibit negligible Dark Matter offsets despite effective self-interaction in

galactic cores, resolving the apparent tension between small-scale and cluster-scale observations.

Taken together, these results indicate that SIDM phenomenology may reflect the breakdown of the smooth-fluid approximation in nonlinear gravitational systems rather than new particle physics. Cantor Dust provides a unified, scale-dependent description of halo formation, relaxation, and cosmic-web emergence, grounded in a single geometric parameter  $D$ . Further work may explore kinetic formulations and observational constraints within the framework outlined here.

### **APPENDIX A: On is the physical meaning of $\epsilon_{\text{eff}}$**

Let's start with the following observations:

- $t_{\text{dyn}}(r)$  quantifies how fast a test-particle moves across radius  $r$ ,
- $t_{\text{rel}}(r)$  quantifies how long it takes gravity to significantly redistribute energy/momentum (to settle down).
- Their ratio measures how efficient gravity is at relaxing the system.

We define the effective coupling as

$$\epsilon_{\text{eff}}(r) \equiv \frac{t_{\text{dyn}}(r)}{t_{\text{rel}}(r)}$$

so that,

- $\epsilon_{\text{eff}} \ll 1$ : collisionless regime
- $\epsilon_{\text{eff}} \sim 1$ : strongly diffusing / thermalizing regime

$\epsilon_{\text{eff}}$  is an *emergent, collective coupling* due to correlated gravitational interactions on a fractal spacetime support.

The relaxation time in conventional stellar dynamics is given by (38):

$$t_{\text{rel}} \sim \frac{v^3}{G^2 m \rho \ln \Lambda}$$

where  $\Lambda$  quantifies how efficiently uncorrelated scattering events accumulate to cause relaxation. In addition to assuming uncorrelated scattering events, (38) also assumes a smooth density and the prevalence of binary scattering. None of these conditions are held in Cantor Dust, in which

- interactions are *long-range and correlated*,
- force fluctuations do *not average out*,
- dominant quantities are fluctuation in *gravitational acceleration*.

These fluctuations are of order:

$$\delta g(r) \sim g(r)$$

where

$$g(r) = \frac{GM(r)}{r^2} \sim r^{D-2}$$

The effective coupling can be alternatively defined as,

$$\epsilon_{\text{eff}}(r) \sim \frac{\text{momentum change per } t_{\text{dyn}}}{\text{momentum}} \sim \frac{g(r) t_{\text{dyn}}}{v}$$

But

$$t_{\text{dyn}} = \frac{r}{v}$$

and so:

$$\epsilon_{\text{eff}} \sim \frac{g(r) r}{v^2}$$

Using the virial relation:

$$v^2 \sim r g(r)$$

we obtain:

$$\boxed{\epsilon_{\text{eff}}(r) \sim g(r) \sim r^{D-2}}$$

## **APPENDIX B: Physical content of the “cusp” and “core” concepts**

A cusp is defined by the slope:

$$\lim_{r \rightarrow 0} \frac{d \ln \rho}{d \ln r} = \text{finite negative constant}$$

By contrast, a core is defined by:

$$\lim_{r \rightarrow 0} \rho(r) = \rho_0 < \infty \quad \lim_{r \rightarrow 0} \frac{d\rho}{dr} = 0$$

Recall the dimensionless relaxation efficiency:

$$\epsilon_{\text{eff}}(r) = \frac{t_{\text{dyn}}}{t_{\text{rel}}} \propto r^{D-2}.$$

Now consider the limit  $r \rightarrow 0$ . If  $D > 2$ ,

$$\epsilon_{\text{eff}}(r) \rightarrow 0$$

transport is suppressed and cusps are formed. If  $D = 2$

$$\epsilon_{\text{eff}}(r) = \text{constant}$$

transport and gravity balance at all scales. However, if  $D < 2$

$$\epsilon_{\text{eff}}(r) \rightarrow \infty$$

transport dominates as relaxation occurs instantaneously and large cores are formed.

The takeaway point of this Appendix is that, although Cantor Dust implies a formal density scaling  $\rho \propto r^{D-3}$ , this does not by itself determine the inner halo structure. Core or cusp formation is controlled by the relative efficiency of gravitational relaxation. For  $D \leq 2$ , transport dominates and erases the

power-law divergence, producing a finite-density core, whereas for  $D > 2$  the system remains collisionless and the Cantor Dust cusp survives.

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