Solid Angle Subtended by a Circular Plane at an Arbitrary Point in Space

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1. Introduction

When a circular plane of finite radius is observed from an arbitrary point in space, its projection generally appears as an ellipse, except in the straight-on configuration, namely, when the observation point lies on the axis normal to the plane and passing through its centre, in which case the circle appears unchanged. In this paper, a general approximate equation for the resulting elliptical plane is derived using the Approximation Formula. This formulation is then employed to estimate the solid angle subtended by a circular plane at an arbitrary point in space [1]. This approach contrasts with the closed-form solutions available for the solid angle subtended by polygonal planes at arbitrary points in space [2,3]. Figure 1 illustrates the straight-on view of a circular plane represented by line AOB and its off-axis projection as an elliptical plane A'MB in the corresponding front view.

2. Solid angle subtended by a circular plane at an arbitrary point

Let there be a circular plane with the centre 'O' & radius 'R' and any arbitrary point say 'P' at a distance 'd' from the centre 'O' such that the angle between normal through the centre 'O' of the plane & the line OP joining the given point 'P' to the centre 'O' is $'\theta'$ (as shown in the Figure 1 below).

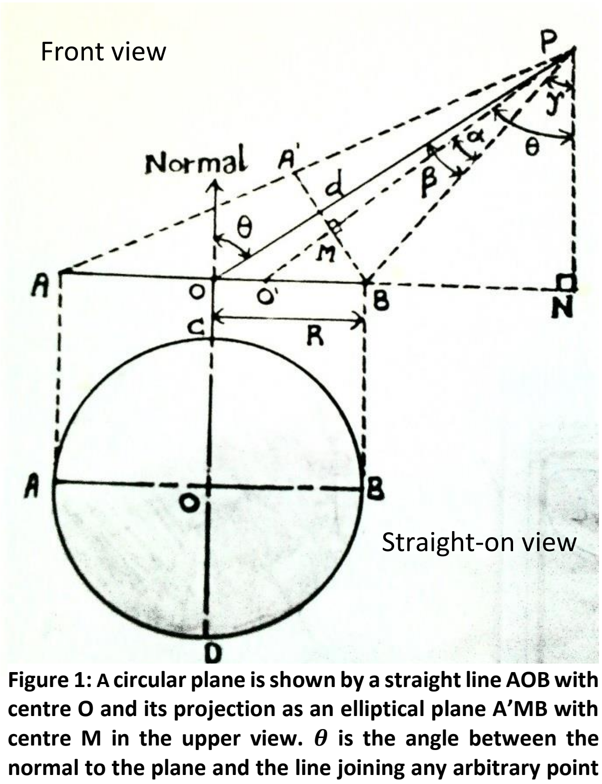

Figure 1: A circular plane is shown by a straight line AOB with centre O and its projection as an elliptical plane A'MB with centre M in the upper view. θ is the angle between the normal to the plane and the line joining any arbitrary point P to the centre O of the circular plane with a radius R.

In right ΔONP (Fig. 1),

$$\Rightarrow \sin\theta = \frac{ON}{OP} = \frac{ON}{d} \Rightarrow ON = d\sin\theta$$

$$\cos\theta = \frac{PN}{OP} = \frac{PN}{d} \Rightarrow PN = d\cos\theta$$

$$\Rightarrow BN = ON - OB = d\sin\theta - R$$

In right ΔPMB (Fig. 1),

$$\Rightarrow PB = \sqrt{PM^2 + MB^2} = \sqrt{h^2 + b^2} = k\ (let)$$

where, $\boldsymbol{PM} = \boldsymbol{h}$ and $\boldsymbol{MB} = \boldsymbol{A'M} = \boldsymbol{b}$ (Let's assume)

$$\Rightarrow \sin\angle BPM = \frac{BM}{BP} \Rightarrow \sin\alpha = \frac{b}{k}$$

$$\cos\angle BPM = \frac{PM}{BP} \Rightarrow \cos\alpha = \frac{h}{k}$$

In right ΔPNB (Fig. 1),

$$\Rightarrow BN^2 + PN^2 = PB^2$$

$$\Rightarrow (d\sin\theta - R)^2 + (d\cos\theta)^2 = d^2\sin^2\theta + R^2 - 2dR\sin\theta + d^2\cos^2\theta$$

$$\Rightarrow\ d^2+R^2-2dR\sin\theta=k^2=h^2+b^2 \qquad \ldots\ldots(1)$$

$$\Rightarrow \sin\angle BPN=\frac{BN}{BP}\Rightarrow \sin\gamma=\frac{d\sin\theta-R}{k}$$

$$\cos\angle BPM=\frac{PN}{BP}\Rightarrow \cos\gamma=\frac{d\cos\theta}{k}$$

In ΔAOP applying Sine-Rule (Fig. 1) as follows

$$\frac{\sin\angle OPA}{AO}=\frac{\sin\angle AOP}{AP}$$

$$\Rightarrow \frac{\sin\left(\alpha-\left(\theta-(\alpha+\gamma)\right)\right)}{R}=\frac{\sin\left(\frac{\pi}{2}+\theta\right)}{\sqrt{PN^2+AN^2}}=\frac{\cos\theta}{\sqrt{(d\cos\theta)^2+(R+d\sin\theta)^2}}$$

$$\Rightarrow \sin\left((2\alpha+\gamma)-\theta\right)=\frac{R\cos\theta}{\sqrt{d^2\cos^2\theta+R^2+d^2\sin^2\theta+2dR\sin\theta}}=\frac{R\cos\theta}{\sqrt{d^2+R^2+2dR\sin\theta}}$$

$$\sin\left((2\alpha+\gamma)-\theta\right)=l\ \text{(let)}$$

$$\text{where,}\quad l=\frac{R\cos\theta}{\sqrt{d^2+R^2+2dR\sin\theta}} \qquad \ldots\ldots..(2)$$

$$\Rightarrow \sin(2\alpha+\gamma)\cos\theta-\cos(2\alpha+\gamma)\sin\theta=l$$

$$\Rightarrow (\sin2\alpha\cos\gamma+\cos2\alpha\sin\gamma)\cos\theta-(\cos2\alpha\cos\gamma-\sin2\alpha\sin\gamma)\sin\theta=l$$

$$\Rightarrow \left((2\sin\alpha\cos\alpha)\cos\gamma+(2\cos^2\alpha-1)\sin\gamma\right)\cos\theta-\left((2\cos^2\alpha-1)\cos\gamma-(2\sin\alpha\cos\alpha)\sin\gamma\right)\sin\theta=l$$

$$\Rightarrow (2\sin\alpha\cos\alpha\cos\gamma+2\cos^2\alpha\sin\gamma-\sin\gamma)\cos\theta-(2\cos^2\alpha\cos\gamma-\cos\gamma-2\sin\alpha\cos\alpha\sin\gamma)\sin\theta=l$$

Now on setting the corresponding values, we obtain

$$\Rightarrow \left\{2\left(\frac{b}{k}\right)\left(\frac{h}{k}\right)\left(\frac{d\cos\theta}{k}\right)+2\left(\frac{h}{k}\right)^2\left(\frac{d\sin\theta-R}{k}\right)-\left(\frac{d\sin\theta-R}{k}\right)\right\}\cos\theta$$
$$-\left\{2\left(\frac{h}{k}\right)^2\left(\frac{d\cos\theta}{k}\right)-\left(\frac{d\cos\theta}{k}\right)-2\left(\frac{b}{k}\right)\left(\frac{h}{k}\right)\left(\frac{d\sin\theta-R}{k}\right)\right\}\sin\theta=l$$

$$\Rightarrow \left(\frac{2(d\sin\theta-R)\cos\theta}{k^3}-\frac{2d\sin\theta\cos\theta}{k^3}\right)h^2+b\left(\frac{2d\cos^2\theta}{k^3}+\frac{2(d\sin\theta-R)\sin\theta}{k^3}\right)h$$
$$+\left(\frac{d\sin\theta\cos\theta+R\cos\theta-d\sin\theta\cos\theta}{k^3}\right)=l$$

$$\Rightarrow -\left(\frac{2R\cos\theta}{k^3}\right)h^2+2b\left(\frac{d-R\sin\theta}{k^3}\right)h+\left(\frac{R\cos\theta}{k}-l\right)=0$$

Now, for the convenience, let's assume

$$\boldsymbol{A=2\left(\frac{R\cos\theta}{k^3}\right),\quad B=2\left(\frac{d-R\sin\theta}{k^3}\right),\quad C=\left(\frac{R\cos\theta}{k}-l\right)}$$

Thus, we get a biquadratic equation for calculating the value of h given as follows

$$\Rightarrow -Ah^2 + Bbh + C = 0 \quad \Rightarrow -Ah^2 + C = -Bbh \qquad \text{(Squaring both the sides)}$$

$$\Rightarrow (Ah^2 - C)^2 = (Bbh)^2 \Rightarrow A^2h^4 + C^2 - 2ACh^2 = B^2b^2h^2$$

$$\Rightarrow A^2h^4 + C^2 - 2ACh^2 = B^2h^2(k^2 - h^2) = B^2k^2h^2 - B^2h^4 \quad \text{(from eq(1))}$$

$$\Rightarrow \quad (A^2+B^2)h^4 - (2AC + B^2k^2)h^2 + C^2 = 0 \qquad \ldots\ldots\ldots\ldots(3)$$

where, $\boldsymbol{k = \sqrt{d^2 + R^2 - 2dR\sin\theta}}$ & $\boldsymbol{l = \frac{R\cos\theta}{\sqrt{d^2 + R^2 + 2dR\sin\theta}}}$ (from eq(1) & (2))

The above equation (3) is biquadratic in terms of arbitrary variable 'h'. It is called Auxiliary Equation.

In this case, the projection of the circular plane, in the direction of line OP at an angle $'\theta'$ with the normal through the centre 'O' of the circular plane, will be an elliptical plane having centre 'M', major axis CD = diameter $= 2R$ & minor axis A'B$= 2b$ (let) when viewed from any arbitrary point P in the space (as shown in Figure 2 below).

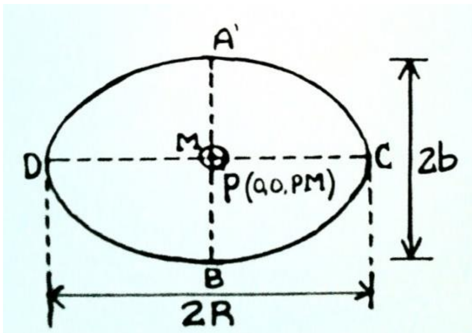

Figure 2: A circular plane with a radius R, always appearing as an elliptical plane with an imaginary centre except the straight-on-position, is projected as an elliptical plane with centre M, major axis *2R* and minor axis *2b*.

In this case, the solid angle subtended by the given circular plane at the arbitrary point 'P' in the space = solid angle subtended by the elliptical plane of projection of the circular plane at the same point 'P'

Now, using HCR's Approximation Formula for the elliptical plane [1], the approximate value of solid angle subtended by the projected elliptical plane, with major axis $2a$ and minor axis $2b$ at a point lying at a distance h on the vertical axis passing through the centre is given as follows

$$\boldsymbol{\omega \cong 2\pi\left[1 - \frac{1}{\sqrt{1 + \left(\frac{ab}{h^2}\right)}}\right]}$$

On setting the values of $a = DM = R,\ b = \sqrt{k^2 - h^2}$ & $h = PM$ (from eq(1))

Hence, we get the approximate value of solid angle subtended by the circular plane at the given point in the space, as follows

$$\omega \cong 2\pi\left[1 - \frac{1}{\sqrt{1 + \left(\frac{R\sqrt{k^2 - h^2}}{h^2}\right)}}\right] \qquad \text{(Substituting the value of } k^2 \text{ from eq(1))}$$

$$\Rightarrow \omega \cong 2\pi\left[1 - \frac{1}{\sqrt{1 + \left(\frac{R\sqrt{d^2 + R^2 - 2dR\sin\theta - h^2}}{h^2}\right)}}\right] \qquad \ldots\ldots\ldots\ldots(4)$$

Above is the required expression for calculating the approximate value of solid angle subtended by the circular plane at the given point in the space. The value of h^2 is found out from the auxiliary equation (3) given as follows

$$\Rightarrow [(A^2+B^2)h^4 - (2AC + B^2k^2)h^2 + C^2 = 0] \qquad \forall\ h \geq \sqrt{d^2 - 2dR\sin\theta}$$

where,

$$\boldsymbol{A = 2\left(\frac{R\cos\theta}{(d^2 + R^2 - 2dR\sin\theta)^{\frac{3}{2}}}\right), \qquad B = 2\left(\frac{d - R\sin\theta}{(d^2 + R^2 - 2dR\sin\theta)^{\frac{3}{2}}}\right),}$$

$$\boldsymbol{C = R\cos\theta\left(\frac{1}{\sqrt{d^2 + R^2 - 2dR\sin\theta}} - \frac{1}{\sqrt{d^2 + R^2 + 2dR\sin\theta}}\right)}, \text{ and } \boldsymbol{k = \sqrt{d^2 + R^2 - 2dR\sin\theta}}$$

2.1. Condition for the appropriate value of arbitrary variable

Since, the auxiliary equation is a biquadratic equation in terms of arbitrary variable h or a quadratic equation in terms of h^2 hence, it will give two values of h^2 out of which only one value (appropriate) will be accepted while other value will be discarded by applying the following condition,

$$\because \text{minor axis} \leq \text{major axis} \Rightarrow 2b \leq 2a \ \ or \ \ b \leq a \Rightarrow \sqrt{k^2 - h^2} \leq R \qquad (\because \text{major axis}, 2a = 2R)$$

$$\Rightarrow k^2 - h^2 \leq R^2 \Rightarrow d^2 + R^2 - 2dR\sin\theta - h^2 \leq R^2 \qquad (\text{setting the value of } k)$$

$$h^2 \geq d^2 + R^2 - 2dR\sin\theta - R^2 \Rightarrow \boldsymbol{h \geq \sqrt{d^2 - 2dR\sin\theta}}$$

Above is the required condition to decide the appropriate value of arbitrary variable h.

2.2. Important deductions

If the given point is lying on the normal axis passing through the centre of the circular plane then the solid angle subtended by the circular plane at the same point is obtained by setting $\theta = 0$ in the above Eq(4) as follows

$$\omega \cong 2\pi\left[1 - \frac{1}{\sqrt{1 + \left(\frac{R\sqrt{d^2 + R^2 - 2dR\sin0 - h^2}}{h^2}\right)}}\right] = 2\pi\left[1 - \frac{1}{\sqrt{1 + \left(\frac{R\sqrt{d^2 + R^2 - h^2}}{h^2}\right)}}\right] \qquad \ldots\ldots(5)$$

The value of h^2 is determined as follows

$$A = 2\left(\frac{R\cos0}{(d^2 + R^2 - 2dR\sin0)^{\frac{3}{2}}}\right) = 2\left(\frac{R}{(d^2 + R^2)^{\frac{3}{2}}}\right)$$

$$B = 2\left(\frac{d - R\sin0}{(d^2 + R^2 - 2dR\sin0)^{\frac{3}{2}}}\right) = 2\left(\frac{d}{(d^2 + R^2)^{\frac{3}{2}}}\right)$$

$$C = R\cos(0)\left(\frac{1}{\sqrt{d^2 + R^2 - 2dR\sin0}} - \frac{1}{\sqrt{d^2 + R^2 + 2dR\sin0}}\right) = R\left(\frac{1}{\sqrt{d^2 + R^2}} - \frac{1}{\sqrt{d^2 + R^2}}\right) = 0$$

$$k = \sqrt{d^2 + R^2 - 2dR\sin0} \Rightarrow k = \sqrt{d^2 + R^2}$$

Now, setting the above values in the auxiliary equation (3), we have

$$(A^2+B^2)h^4+(2AC-B^2k^2)h^2+C^2=0$$

$$\Rightarrow\left(4\left(\frac{R}{(d^2+R^2)^{\frac{3}{2}}}\right)^2+4\left(\frac{d}{(d^2+R^2)^{\frac{3}{2}}}\right)^2\right)h^4$$

$$-\left(4\left(\frac{R}{(d^2+R^2)^{\frac{3}{2}}}\right)(0)+4\left(\frac{d}{(d^2+R^2)^{\frac{3}{2}}}\right)^2\left(\sqrt{d^2+R^2}\right)^2\right)h^2+(0)^2=0$$

$$\Rightarrow\quad \frac{4(d^2+R^2)}{(d^2+R^2)^3}h^4-\frac{4d^2(d^2+R^2)}{(d^2+R^2)^3}h^2=\frac{4}{(d^2+R^2)^2}(h^4-d^2h^2)=0$$

$$\Rightarrow\quad (h^2-d^2)h^2=0\quad\Rightarrow\quad \boldsymbol{h^2=d^2},\qquad \boldsymbol{h^2=0}$$

Thus, there arise two cases depending on the values of arbitrary variable h as follows

Case 1: When the given point is lying on the normal axis through the centre of circular plane at a positive distance from its centre (i.e. $d>0\ \Rightarrow h^2=d^2$)

Now, on setting $h^2=d^2$ in the above expression (5), we get the approximate solid angle, subtended by the circular plane at any point lying on the normal axis passing through its centre as follows

$$\Rightarrow\omega\cong 2\pi\left[1-\frac{1}{\sqrt{1+\left(\frac{R\sqrt{d^2+R^2-d^2}}{d^2}\right)}}\right]=2\pi\left[1-\frac{1}{\sqrt{1+\left(\frac{R^2}{h^2}\right)}}\right]=2\pi\left[1-\frac{h}{\sqrt{h^2+R^2}}\right]$$

$$\Rightarrow\omega\cong 2\pi\left[1-\frac{h}{\sqrt{h^2+R^2}}\right]\ \Rightarrow\quad \boldsymbol{\omega=2\pi\left[1-\frac{h}{\sqrt{h^2+R^2}}\right]}$$

The above result denotes the exact value of the solid angle subtended by a circular plane at any point lying on the vertical axis passing through its centre. Hence the result is correct.

Case 2: When the given point is lying on the circular plane completely inside the boundary (i.e. $d=0\ \Rightarrow h^2=0$)

Now, on setting $h^2=0$ in the above expression (5), we get the approximate solid angle, subtended by the circular plane at any point lying on the normal axis passing through its centre as follows

$$\Rightarrow\omega\cong 2\pi\left[1-\frac{1}{\sqrt{1+\left(\frac{R\sqrt{d^2+R^2-(0)^2}}{(0)^2}\right)}}\right]=2\pi\left[1-\frac{1}{\sqrt{1+\infty}}\right]=2\pi[1-0]$$

$$\omega\cong 2\pi\ \Rightarrow\quad \boldsymbol{\omega=2\pi\ \mathrm{sr}}$$

The above result denotes that solid angle subtended by any plane at any point lying on it within the boundary is always 2π sr i.e. any point lying on a plane within in the boundary, sees that plane all around it.

3. Dimensions of the elliptical plane of projection for a circular plane when viewed from any arbitrary point (off-centre position) in the space

The projection of the circular plane, in the direction of line OP at an angle $'\theta'$ with the normal through the centre 'O' of the circular plane, will be an elliptical plane having centre 'M', major axis CD = diameter $= 2R$ and minor axis A'B $= 2b$ when viewed from any arbitrary point P in the space (as shown in the Figure 2 above). All the important dimensions of the elliptical plane of projection are calculated as follows

$$\text{Major axis,}\quad 2a = \text{Diameter of circular plane} \;= 2R,\quad \text{Minor axis,}\, 2b = 2\sqrt{k^2 - h^2}$$

$$\text{Eccentricity,}\, e = \sqrt{1 - \frac{b^2}{a^2}} \qquad (0 \le e < 1 \;\;\forall\;\; b \le a)$$

The value of h^2 is calculated from the auxiliary (biquadratic) equation (3) given as follows

$$\Rightarrow [(A^2 + B^2)h^4 - (2AC + B^2k^2)h^2 + C^2 = 0] \qquad \forall\;\; h \ge \sqrt{d^2 - 2dR\sin\theta}$$

Where

$$A = 2\left(\frac{R\cos\theta}{(d^2 + R^2 - 2dR\sin\theta)^{\frac{3}{2}}}\right), \qquad B = 2\left(\frac{d - R\sin\theta}{(d^2 + R^2 - 2dR\sin\theta)^{\frac{3}{2}}}\right)$$

$$C = R\cos\theta\left(\frac{1}{\sqrt{d^2 + R^2 - 2dR\sin\theta}} - \frac{1}{\sqrt{d^2 + R^2 + 2dR\sin\theta}}\right) \;\&\;\; k = \sqrt{d^2 + R^2 - 2dR\sin\theta}$$

Above expressions are useful to calculate all the parameters such as major axis, minor axis and eccentricity of the elliptical plane of projection of a circular plane when viewed from any arbitrary point in the space.

4. Illustrative Numerical Example

Example 1: Calculate major axis, minor axis & eccentricity of the elliptical plane of projection of a circular plane, with a radius 25 units, when viewed from a point at a distance 70 units from the centre such that the angle between normal to the centre & the line joining the given point to the centre of circular plane is 30^o. Also calculate the approximate value of the solid angle subtended by the circular plane at the same point in the space.

Solution. Given that

$$R = \text{Radius of the circular plane} = 25 \text{ units}, \qquad d = \text{distance from the centre} = 70 \text{ units}, \;\&$$

$$\theta = \text{angle with the normal} = 30^o$$

First of all, let's calculate all the arbitrary constants *A, B, C* & *k* with the help of given values as follows

$$\boldsymbol{A} = 2\left(\frac{R\cos\theta}{(d^2 + R^2 - 2dR\sin\theta)^{\frac{3}{2}}}\right) = 2\left(\frac{(25)\cos 30^o}{((70)^2 + (25)^2 - 2(70)(25)\sin 30^o)^{\frac{3}{2}}}\right) \approx 1.866917824 \times 10^{-4}$$

$$\boldsymbol{B} = 2\left(\frac{d - R\sin\theta}{(d^2 + R^2 - 2dR\sin\theta)^{\frac{3}{2}}}\right) = 2\left(\frac{70 - 25\sin 30^o}{((70)^2 + (25)^2 - 2(70)(25)\sin 30^o)^{\frac{3}{2}}}\right) \approx 4.958181338 \times 10^{-4}$$

$$\boldsymbol{C} = R\cos\theta\left(\frac{1}{\sqrt{d^2+R^2-2dR\sin\theta}} - \frac{1}{\sqrt{d^2+R^2+2dR\sin\theta}}\right)$$

$$= 25\cos30^o\left(\frac{1}{\sqrt{(70)^2+(25)^2-2(70)(25)\sin30^o}} - \frac{1}{\sqrt{(70)^2+(25)^2+2(70)(25)\sin30^o}}\right) \approx 0.098544198$$

$$\boldsymbol{k} = \sqrt{d^2+R^2-2dR\sin\theta} = \sqrt{(70)^2+(25)^2-2(70)(25)\sin30^o} \approx 61.44102864$$

The value of h^2 is calculated from the auxiliary (biquadratic) equation given as follows

$$\Rightarrow (A^2+B^2)h^4 - (2AC+B^2k^2)h^2 + C^2 = 0$$

Substituting the corresponding values in the above equation, we obtain

$$((1.866917824\times10^{-4})^2 + (4.958181338\times10^{-4})^2)h^4 - (2(1.866917824\times10^{-4})(0.098544198) + (4.958181338\times10^{-4})^2(61.44102864)^2)h^2 + (0.098544198)^2 = 0$$

$$\Rightarrow\ \boldsymbol{2.806894434\times10^{-7}h^4 - 9.648242563\times10^{-4}h^2 + 9.710958959\times10^{-3} = 0}$$

Now, solving the above biquadratic equation in terms of h^2 as follows,

$$\boldsymbol{h^2} = \frac{9.648242563\times10^{-4} \pm \sqrt{(9.648242563\times10^{-4})^2 - 4(2.806894434\times10^{-7})(9.710958959\times10^{-3})}}{2(2.806894434\times10^{-7})}$$

$$= \frac{9.648242563\times10^{-4} \pm \sqrt{9.308858455\times10^{-7} - 1.090305466\times10^{-8}}}{5.613788868\times10^{-7}}$$

$$= \frac{9.648242563\times10^{-4} \pm 9.591573337\times10^{-4}}{5.613788868\times10^{-7}}$$

But, we know that $h \geq \sqrt{d^2-2dR\sin\theta}$ or $h \geq \sqrt{(70)^2-2(70)(25)\sin30^o}$ or $\boldsymbol{h \geq 56.1248608}$

Case 1: Taking positive sign

$$\boldsymbol{h^2} = \frac{9.648242563\times10^{-4} + 9.591573337\times10^{-4}}{5.613788868\times10^{-7}} \approx \boldsymbol{3427.242519}\ \Rightarrow\ \boldsymbol{h \approx 58.54265555}$$

$\Rightarrow\ h \approx 58.54265555 \geq 56.1248608$

Hence this value of arbitrary variable h is accepted.

Case 2: Taking negative sign

$$\boldsymbol{h^2} = \frac{9.648242563\times10^{-4} - 9.591573337\times10^{-4}}{5.613788868\times10^{-7}} \approx \boldsymbol{10.09464861}\ \Rightarrow\ \boldsymbol{h \approx 3.177207675}$$

But, $h \geq 56.1248608$ hence this value of arbitrary variable h is discarded.

Now, all the important dimensions of the elliptical plane of projection are calculated as follows

Major axis, $\boldsymbol{2a}$ = diameter of circular plane $= 2R = \boldsymbol{2\times25 = 50}$ **units**

Minor axis, $\boldsymbol{2b} = 2\sqrt{k^2-h^2} = 2\sqrt{(61.44102864)^2 - 3427.242519} \approx \boldsymbol{37.29651358}$ **units**

$$\textbf{Eccentricity}, e = \sqrt{1 - \frac{(18.64825679)^2}{(25)^2}} \approx \mathbf{0.666024046} \quad (\forall\, 0 \le e < 1 \ \ \forall \ \ b \le a)$$

Hence, the approximate value of the solid angle subtended by the circular plane at the given point in the space is calculated by using approximate formula as follows

$$\boldsymbol{\omega} \cong 2\pi\left[1 - \frac{1}{\sqrt{1 + \left(\frac{R\sqrt{d^2 + R^2 - 2dR\sin\theta - h^2}}{h^2}\right)}}\right]$$

$$\cong 2\pi\left[1 - \frac{1}{\sqrt{1 + \left(\frac{(25)\sqrt{(70)^2 + (25)^2 - 2(70)(25)\sin 30^o - 3427.242519}}{3427.242519}\right)}}\right] \approx \mathbf{0.388168533\ sr}$$

Thus, the given circular plane with a radius 25 units subtends solid angle approximately $0.388168533\ sr$ at the given point from which it appears as an elliptical plane with major axis $2a = 50$ units, minor axis $2b \approx 37.29651358$ units, and an eccentricity $e \approx 0.666024046$.

Conclusion: The formulae derived in this work provide a practical method for estimating the solid angle subtended by a circular plane at an arbitrary point in space. These relations are also applicable to determining the geometric parameters of the elliptical plane obtained as the projection of a circular plane when viewed from an off-centre position, including the major axis, minor axis, and eccentricity. All results have been obtained using the approximation formula developed by the author, as presented in his book Advanced Geometry, and offer a simple and effective approach for solving related geometric problems.

Note: Above articles had been derived *& illustrated by Mr H.C. Rajpoot (B Tech, Mechanical Engineering)*

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