Mathematical analysis of an Icosidodecahedron

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1. Introduction: An icosidodecahedron is a solid which has 20 congruent equilateral triangular & 12 congruent regular pentagonal faces each having equal edge length. It is made/generated either by cutting off all the vertices (corners) of a regular icosahedron (having 12 vertices & 20 congruent equilateral triangular faces) or by cutting off all the vertices of a regular dodecahedron (having 20 vertices & 12 congruent regular pentagonal faces) to generate 20 equilateral triangular & 12 regular pentagonal faces of equal edge length such that all the vertices lie on a sphere. In this paper, we will use the equations of right pyramid [1] & regular icosahedron [2] for calculating all the parameters of an icosidodecahedron. When a regular icosahedron is cut off from the vertex, a right pyramid, with base as a regular pentagon & certain normal height, is obtained. Since an icosahedron has 12 vertices hence we obtain 12 congruent cut off right pyramids each with a regular pentagonal base.

2. Cutting off a regular icosahedron

For ease of calculations, let there be a regular icosahedron with edge length $2a$ & its centre at the point C. Now all its 12 vertices are cut off to obtain an icosidodecahedron. Thus each of the congruent equilateral triangular faces with edge length $2a$ is changed into a new reduced equilateral triangular face with edge length a (see Figure 1) & we obtain 12 congruent cut off right pyramids with base as a regular pentagon corresponding to 12 vertices of the parent solid. (see Figure 1 which shows an equilateral triangular face & a right pyramid with regular pentagonal base & normal height h being cut off from the regular icosahedron).

$$No.of\ congruent\ equilateral\ triangular\ faces\ in\ the\ icosidodecahedron$$
$$= no.of\ faces\ in\ parent\ icosahedron = 20$$

$$No.of\ congruent\ regular\ pentagonal\ faces\ in\ icosidodecahedron$$
$$= no.of\ vertices\ in\ parent\ icosahedron = 12$$

$$No.of\ vertices\ in\ the\ icosidodecahedron = 5 \times 12 - 30 = 30$$

In order to generate a icosidodecahedron with edge length a then we have to cut off all 12 vertices of a regular icosahedron of edge length $2a$ (see Figure 1).

3. Analysis of icosidodecahedron by using equations of right pyramid & icosahedron

Now consider any of 12 congruent cut off right pyramids having base as a regular pentagon ABDEF with side length a, normal height h & an angle 60^o between any two consecutive lateral edges (see Figure 2 below).

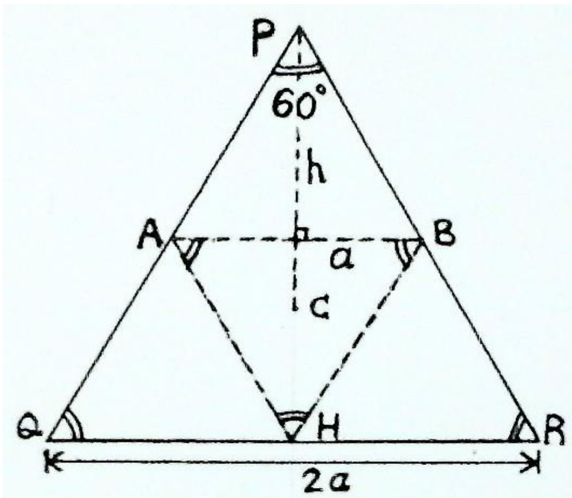

Figure 1: Each of 20 congruent equilateral triangular faces with edge length $2a$ of a regular icosahedron is changed into a new reduced equilateral triangular face with edge length a by cutting off all the vertices. A right pyramid with base as a regular pentagon with side length a & normal height h is being cut off from an icosahedron with edge length $2a$.

3.1. Normal height (h) of cut off right pyramid: We know that the normal height of any right pyramid with regular polygonal base**,** having n no. of sides each of length a & an angle α between any two consecutive lateral edges, is given as

$$H=\frac{a}{2}\sqrt{cot^2\frac{\alpha}{2}-cot^2\frac{\pi}{n}}$$

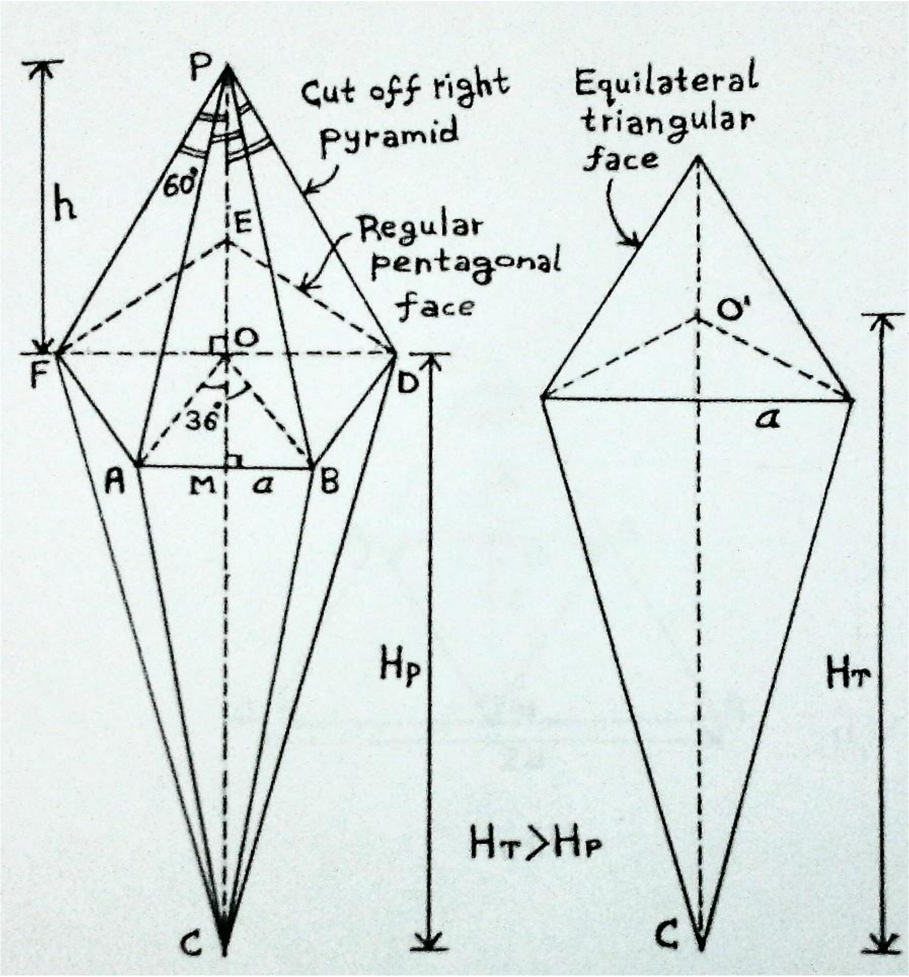

Figure 2: Normal distance (H_T) of equilateral triangular faces is always greater than the normal distance (H_p) of regular pentagonal faces measured from the centre C of any icosidodecahedron.

$$\therefore h=\frac{a}{2}\sqrt{cot^2\frac{60^o}{2}-cot^2\frac{\pi}{5}}=\frac{a}{2}\sqrt{3-\left(\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}\right)^2}\quad (for\ regular\ pentagonal\ base,\ \ n=5)$$

$$\Rightarrow h=\frac{a}{2}\sqrt{3-\frac{6+2\sqrt{5}}{10-2\sqrt{5}}}=\frac{a}{2}\sqrt{\frac{30-6\sqrt{5}-6-2\sqrt{5}}{10-2\sqrt{5}}}=\frac{a}{2}\sqrt{\frac{4(6-2\sqrt{5})}{10-2\sqrt{5}}}=\frac{2a}{2}\sqrt{\frac{(\sqrt{5}-1)^2}{10-2\sqrt{5}}}=\frac{(\sqrt{5}-1)a}{\sqrt{10-2\sqrt{5}}}$$

$$\therefore cut\ off\ normal\ height,\ \ h=\frac{(\sqrt{5}-1)a}{\sqrt{10-2\sqrt{5}}}\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(I)$$

3.2. Volume (V') of cut off right pyramid: We know that the volume of a right pyramid is given as

$$=\frac{1}{3}(area\ of\ base\ (regular\ pentagon))\times(normal\ height)$$

$$\therefore V'=\frac{1}{3}\left(\frac{1}{4}na^2cot\frac{\pi}{n}\right)\times h=\frac{1}{3}\left(\frac{1}{4}\times 5\times a^2cot\frac{\pi}{5}\right)\times\frac{(\sqrt{5}-1)a}{\sqrt{10-2\sqrt{5}}}$$

$$=\frac{5a^3}{12}\times\frac{(\sqrt{5}+1)}{\sqrt{10-2\sqrt{5}}}\times\frac{(\sqrt{5}-1)a}{\sqrt{10-2\sqrt{5}}}=\frac{5a^3}{12}\times\frac{4}{10-2\sqrt{5}}=\frac{5a^3(10+2\sqrt{5})}{3(10-2\sqrt{5})(10+2\sqrt{5})}=\frac{5a^3(10+2\sqrt{5})}{3(100-20)}$$

$$=\frac{a^3(10+2\sqrt{5})}{48}=\frac{(5+\sqrt{5})a^3}{24}$$

$$\boldsymbol{V'=\frac{(5+\sqrt{5})a^3}{24}}\qquad \ldots\ldots\ldots\ldots\ldots\ldots(II)$$

3.3. Normal distance (H_T) of equilateral triangular faces from the centre of icosidodecahedron: The normal distance (H_T) of each of 20 equilateral triangular faces from the centre C of icosidodecahedron is given as

$$H_T=O'C\quad (from\ the\ figure\ 2\ above)$$

$$\Rightarrow\ H_T=(inner\ (inscribed)radius\ of\ parent\ icosahedron\ with\ edge\ length\ 2a)$$

$$\Rightarrow\ H_T=\frac{(3+\sqrt{5})(2a)}{4\sqrt{3}}=\frac{(3+\sqrt{5})a}{2\sqrt{3}}\qquad \text{(inner radius is given from HCR's Formula [2])}$$

$$\Rightarrow\ \boldsymbol{H_T=\frac{(3+\sqrt{5})a}{2\sqrt{3}}\approx 1.511522628a}\qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(III)$$

It's clear that all 20 congruent equilateral triangular faces are at an equal normal distance H_T from the centre of any icosidodecahedron.

3.4. Solid angle (ω_T) subtended by each of the equilateral triangular faces at the centre icosidodecahedron: we know that the solid angle (ω) subtended by any regular polygon with each side of length a at any point lying at a distance H on the vertical axis passing through the centre of plane is given by HCR's Theory of Polygon [3,4] as follows

$$\omega=2\pi-2n\sin^{-1}\left(\frac{2Hsin\frac{\pi}{n}}{\sqrt{4H^2+a^2cot^2\frac{\pi}{n}}}\right)$$

Hence, by substituting the corresponding values in the above expression, we get the solid angle subtended by each equilateral triangular face at the centre of icosidodecahedron as follows

$$\omega_T=2\pi-2\times 3\sin^{-1}\left(\frac{2\left(\frac{(3+\sqrt{5})a}{2\sqrt{3}}\right)sin\frac{\pi}{3}}{\sqrt{4\left(\frac{(3+\sqrt{5})a}{2\sqrt{3}}\right)^2+a^2cot^2\frac{\pi}{3}}}\right)$$

$$= 2\pi - 6\sin^{-1}\left(\frac{\frac{(3+\sqrt{5})}{\sqrt{3}}\times\frac{\sqrt{3}}{2}}{\sqrt{\frac{9+5+6\sqrt{5}}{3}+\frac{1}{3}}}\right) = 2\pi - 6\sin^{-1}\left(\frac{3+\sqrt{5}}{2\sqrt{\frac{15+6\sqrt{5}}{3}}}\right) = 2\pi - 6\sin^{-1}\left(\frac{3+\sqrt{5}}{2\sqrt{5+2\sqrt{5}}}\right)$$

$$\boldsymbol{\omega_T = 2\pi - 6\sin^{-1}\left(\frac{3+\sqrt{5}}{2\sqrt{5+2\sqrt{5}}}\right) \approx 0.179853499\ sr} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(IV)$$

3.5. Normal distance (H_P) of regular pentagonal faces from the centre of icosidodecahedron: The normal distance (H_P) of each of 12 regular pentagonal faces from the centre C of icosidodecahedron is given as

$$H_P = OC = CP - OP \quad (from\ the\ figure\ 2\ above)$$

$$\Rightarrow\ H_P = (outer\ (circumscribed)\ radius\ of\ parent\ icosahedron) - h$$

$$\Rightarrow\ H_P = \frac{(2a)\sqrt{10+2\sqrt{5}}}{4} - \frac{(\sqrt{5}-1)a}{\sqrt{10-2\sqrt{5}}} \qquad \text{(outer radius is given from HCR's Formula [2])}$$

$$= \frac{\left(\sqrt{(100-20)} - 2\sqrt{5} + 2\right)a}{2\sqrt{10-2\sqrt{5}}} = \frac{(4\sqrt{5}-2\sqrt{5}+2)a}{2\sqrt{10-2\sqrt{5}}} = \frac{(\sqrt{5}+1)a}{\sqrt{10-2\sqrt{5}}}$$

$$\Rightarrow\ \boldsymbol{H_P = \frac{(\sqrt{5}+1)a}{\sqrt{10-2\sqrt{5}}} = a\cot 36^o \approx 1.37638192a} \quad \ldots\ldots\ldots\ldots\ldots.(V)$$

It's clear that all 12 congruent regular pentagonal faces are at an equal normal distance H_P from the centre of any icosidodecahedron.

It's also clear from eq(III) & (V) $H_T > H_P$ i.e. the normal distance (H_T) of equilateral triangular faces is greater than the normal distance (H_P) of regular pentagonal faces from the centre of icosidodecahedron i.e. pentagonal faces are much closer to the centre as compared to the equilateral triangular faces in any icosidodecahedron.

3.6. Solid angle (ω_P) subtended by each of the regular pentagonal faces at the centre of icosidodecahedron: we know that the solid angle (ω) subtended by any regular polygon is given by HCR's Theory of Polygon [3,4] as follows

$$\omega = 2\pi - 2n\sin^{-1}\left(\frac{2H\sin\frac{\pi}{n}}{\sqrt{4H^2 + a^2\cot^2\frac{\pi}{n}}}\right)$$

Hence, by substituting the corresponding value of normal distance $H = H_P$ in the above expression, we get the solid angle subtended by each regular pentagonal face at the centre of icosidodecahedron as follows

$$\omega_P = 2\pi - 2\times 5\sin^{-1}\left(\frac{2\left(\frac{(\sqrt{5}+1)a}{\sqrt{10-2\sqrt{5}}}\right)\sin\frac{\pi}{5}}{\sqrt{4\left(\frac{(\sqrt{5}+1)a}{\sqrt{10-2\sqrt{5}}}\right)^2 + a^2\cot^2\frac{\pi}{5}}}\right) \quad (for\ regular\ pentagon,\ \ n = 5)$$

$$= 2\pi - 10\sin^{-1}\left(\frac{\frac{2(\sqrt{5}+1)}{\sqrt{10-2\sqrt{5}}} \times \frac{\sqrt{10-2\sqrt{5}}}{4}}{\sqrt{4\left(\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}\right)^2 + \left(\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}\right)^2}}\right) = 2\pi - 10\sin^{-1}\left(\frac{(\sqrt{5}+1)}{2\left(\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}\right)\sqrt{5}}\right)$$

$$= 2\pi - 10\sin^{-1}\left(\frac{\sqrt{10-2\sqrt{5}}}{2\sqrt{5}}\right) = 2\pi - 10\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{10-2\sqrt{5}}{5}}\right)$$

$$\boldsymbol{\omega_P = 2\pi - 10\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{10-2\sqrt{5}}{5}}\right) \approx 0.747441718\ sr} \quad \ldots\ldots\ldots\ldots\ldots..(VI)$$

It's clear that the solid angle subtended by each of the regular pentagonal faces is greater than the solid angle subtended by each of the equilateral triangular faces at the centre of any icosidodecahedron.

4. Important parameters of an icosidodecahedron

1. **Inner (inscribed) radius$(\boldsymbol{R_i})$:** It is the radius of the largest sphere inscribed (trapped inside) by the icosidodecahedron. The largest inscribed sphere always touches all 12 congruent regular pentagonal faces but does not touch any of 20 congruent equilateral triangular faces at all since all 12 pentagonal faces are closer to the centre as compared to all 20 triangular faces. Thus, inner radius is always equal to the normal distance (H_P) of regular pentagonal faces from the centre of an icosidodecahedron & is given from the eq(V) as follows

$$\boldsymbol{R_i = H_P = \frac{(\sqrt{5}+1)a}{\sqrt{10-2\sqrt{5}}} \approx 1.37638192a}$$

Hence, the **volume of inscribed sphere** is given as

$$V_{inscribed} = \frac{4}{3}\pi(R_i)^3 = \boldsymbol{\frac{4}{3}\pi\left(\frac{(\sqrt{5}+1)a}{\sqrt{10-2\sqrt{5}}}\right)^3 \approx 10.92208337a^3}$$

2. **Outer (circumscribed) radius$(\boldsymbol{R_o})$:** It is the radius of the smallest sphere circumscribing a given icosidodecahedron or it's the radius of a spherical surface passing through all 30 vertices of a given icosidodecahedron. It is calculated as follows (see Figure 2 above).

$$R_o = distance\ of\ any\ of\ the\ vertices\ from\ the\ centre\ C = CA = CB = CD = CE = CF$$

In right ΔOMA

$$sin\angle AOM = \frac{AM}{OA} \Rightarrow sin36^o = \frac{\left(\frac{a}{2}\right)}{OA} \quad or\ \boldsymbol{OA = \frac{a}{2sin36^o}} \qquad \left(since,\ \ \angle AOB = \frac{2\pi}{5} = 72^o\right)$$

In right ΔAOC

$$\Rightarrow CA = \sqrt{(OA)^2 + (OC)^2} = \sqrt{\left(\frac{a}{2sin36^o}\right)^2 + \left(\frac{(\sqrt{5}+1)a}{\sqrt{10-2\sqrt{5}}}\right)^2} \qquad (since,\ \ OC = H_p)$$

$$= a\sqrt{\left(\frac{2}{\sqrt{10-2\sqrt{5}}}\right)^2 + \left(\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}\right)^2} = a\sqrt{\frac{4+6+2\sqrt{5}}{10-2\sqrt{5}}} = a\sqrt{\frac{(10+2\sqrt{5})(10+2\sqrt{5})}{(10-2\sqrt{5})(10+2\sqrt{5})}}$$

$$= \frac{(10+2\sqrt{5})a}{4\sqrt{5}} = \frac{(\sqrt{5}+1)a}{2} = outer\ (circumscribed)\ radius$$

Hence, outer radius of icosidodecahedron is given as

$$\boldsymbol{R_o = \frac{(\sqrt{5}+1)a}{2} \approx 1.618033989a}$$

Hence, the **volume of circumscribed sphere** is given as

$$V_{circumscribed} = \frac{4}{3}\pi(R_o)^3 = \boldsymbol{\frac{4}{3}\pi\left(\frac{(\sqrt{5}+1)a}{2}\right)^3 = 17.74400005a^3}$$

3. **Surface area**$(\boldsymbol{A_s})$**:** We know that an icosidodecahedron has 20 congruent equilateral triangular & 12 congruent regular pentagonal faces each of edge length a. Hence, its surface area is given as follows

$$A_s = 20 \times (area\ of\ equilateral\ triangle) + 12 \times (area\ of\ regular\ pentagon)$$

We know that **area of any regular n-polygon** with each side of length a is given as

$$\boldsymbol{A = \frac{1}{4}na^2cot\frac{\pi}{n}}$$

Hence, by substituting all the corresponding values in the above expression, we get

$$A_s = 20 \times \left(\frac{1}{4} \times 3a^2cot\frac{\pi}{3}\right) + 12 \times \left(\frac{1}{4} \times 5a^2cot\frac{\pi}{5}\right) = 15a^2 \times \frac{1}{\sqrt{3}} + 15a^2cot36^o = \left(5\sqrt{3} + 15cot36^o\right)a^2$$

$$\boldsymbol{A_s = \left(5\sqrt{3} + 15cot36^o\right)a^2 \approx 29.30598285a^2}$$

4. **Volume**$(\boldsymbol{V})$**:** We know that an icosidodecahedron with edge length a is obtained by cutting a regular icosahedron with edge length $2a$ at all its 12 vertices. Thus, 12 congruent right pyramids with regular pentagonal base are cut off from the parent icosahedron. Hence, the volume (V) of the icosidodecahedron is given as follows

$$V = (volume\ of\ parent\ regular\ icosahedron) - 12(volume\ of\ cut\ off\ right\ pyramid)$$

$$= \frac{5}{12}(3+\sqrt{5})(2a)^3 - 12 \times (V') \quad (volume\ of\ regular\ icosahedron\ is\ given\ from\ HCR's\ formula)$$

$$= \frac{10}{3}(3+\sqrt{5})a^3 - 12 \times \frac{(5+\sqrt{5})a^3}{24} \quad (substituting\ the\ value\ of\ V'\ from\ eq\ (II))$$

$$= \frac{10}{3}(3+\sqrt{5})a^3 - \frac{(5+\sqrt{5})a^3}{2} = \frac{(60+20\sqrt{5}-15-3\sqrt{5})a^3}{6} = \frac{(45+17\sqrt{5})a^3}{6}$$

$$\boldsymbol{V = \frac{(45+17\sqrt{5})a^3}{6} \approx 13.83552594a^3}$$

5. **Mean radius**$(\boldsymbol{R_m})$**:** It is the radius of the sphere having a volume equal to that of a given icosidodecahedron. It is calculated as follows

$$volume\ of\ sphere\ with\ mean\ radius\ R_m = volume\ of\ given\ icosidodecahedron$$

$$\frac{4}{3}\pi(R_m)^3 = \frac{(45+17\sqrt{5})a^3}{6} \Rightarrow (R_m)^3 = \frac{(45+17\sqrt{5})a^3}{8\pi} \quad or \quad R_m = a\left(\frac{45+17\sqrt{5}}{8\pi}\right)^{\frac{1}{3}}$$

$$\boldsymbol{R_m = a\left(\frac{45+17\sqrt{5}}{8\pi}\right)^{\frac{1}{3}} \approx 1.489254843a}$$

It's clear from above results that $\boldsymbol{R_i < R_m < R_o}$

5. Construction of a solid icosidodecahedron: In order to construct a solid icosidodecahedron with edge length a there are two methods

5.1. Construction from elementary right pyramids: In this method, first we construct all elementary right pyramids as follows

Step 1. Construct 20 congruent right pyramids with equilateral triangular base of side length a & normal height (H_T)

$$H_T = \frac{(3+\sqrt{5})a}{2\sqrt{3}} \approx 1.511522628a$$

Step 2. Construct 12 congruent right pyramids with regular pentagonal base of side length a & normal height (H_P)

$$H_P = \frac{(\sqrt{5}+1)a}{\sqrt{10-2\sqrt{5}}} = a\cot 36^o \approx 1.37638192a$$

Step 3. Now, paste/bond by joining all these elementary right pyramids by overlapping their lateral surfaces & keeping their apex points coincident with each other such that all the edges of each equilateral triangular base (face) coincide with the edges of three regular pentagonal bases (faces). Thus, a solid icosidodecahedron, with 20 congruent equilateral triangular & 12 congruent regular pentagonal faces each of edge length a, is obtained.

5.2. Facing a solid sphere: It is a method of facing, first we select a **blank as a solid sphere** of certain material (i.e. metal, alloy, composite material etc.) & with suitable diameter in order to obtain the maximum desired edge length of icosidodecahedron. Then, we perform facing operations on the solid sphere to generate 20 congruent equilateral triangular & 12 congruent regular pentagonal faces each of equal edge length.

5.2.1. Edge length: Let there be a blank as a solid sphere with a diameter D. Then the edge length a, of an icosidodecahedron of the maximum volume to be produced, can be co-related with the diameter D by **relation of outer radius (R_o) with edge length $(a)$of a icosidodecahedron** as follows

$$R_o = \frac{(\sqrt{5}+1)a}{2}$$

Now, substituting $R_o = {D}/{2}$ in the above expression, we have

$$\frac{D}{2} = \frac{(\sqrt{5}+1)a}{2} \quad or \quad a = \frac{D}{(\sqrt{5}+1)}$$

$$\boldsymbol{a = \frac{D}{(\sqrt{5}+1)} \approx 0.309016994D}$$

Above relation is very useful for determining the edge length a of an icosidodecahedron to be produced from a solid sphere with known diameter D for manufacturing purpose.

5.2.2. Maximum volume of icosidodecahedron produced from the solid sphere is given as follows

$$V_{max} = \frac{(45+17\sqrt{5})a^3}{6} = \frac{(45+17\sqrt{5})}{6}\left(\frac{D}{(\sqrt{5}+1)}\right)^3 = \frac{(45+17\sqrt{5})D^3}{6(5\sqrt{5}+1+15+3\sqrt{5})}$$

$$= \frac{(45+17\sqrt{5})D^3}{48(\sqrt{5}+2)} = \frac{(45+17\sqrt{5})(\sqrt{5}-2)D^3}{48(\sqrt{5}+2)(\sqrt{5}-2)} = \frac{(45\sqrt{5}+85-90-34\sqrt{5})D^3}{48(5-4)} = \frac{(11\sqrt{5}-5)D^3}{48}$$

$$\boldsymbol{V_{max} = \frac{(11\sqrt{5}-5)D^3}{48} \approx 0.408265578D^3}$$

5.2.3. M**inimum volume of material removed** is given as

$$(V_{removed})_{min} = (volume\ of\ parent\ sphere\ with\ diameter\ D) - (volume\ of\ icosidodecahedron)$$

$$= \frac{\pi}{6}D^3 - \frac{(11\sqrt{5}-5)D^3}{48} = \left(\frac{\pi}{6} - \frac{11\sqrt{5}-5}{48}\right)D^3$$

$$\boldsymbol{(V_{removed})_{min} = \left(\frac{\pi}{6} - \frac{11\sqrt{5}-5}{48}\right)D^3 \approx 0.115333197D^3}$$

5.2.4. Percentage $(\%)$ **of minimum volume of material removed**

$$\%\ \boldsymbol{of}\ \boldsymbol{V_{removed}} = \frac{minimum\ volume\ removed}{total\ volume\ of\ sphere} \times 100$$

$$= \frac{\left(\frac{\pi}{6} - \frac{11\sqrt{5}-5}{48}\right)D^3}{\frac{\pi}{6}D^3} \times 100 = \boldsymbol{\left(1 - \frac{11\sqrt{5}-5}{8\pi}\right) \times 100 \approx 22.03\%}$$

It's obvious that when an icosidodecahedron of the maximum volume is produced from a solid sphere then about 22.03% of material is removed as scraps. Thus, we can select optimum diameter of blank as a solid sphere to produce a solid icosidodecahedron of the maximum volume (or with maximum desired edge length).

Conclusions: Let there be any icosidodecahedron having 20 congruent equilateral triangular & 12 congruent regular pentagonal faces each with edge length a then all its important parameters are calculated/determined as tabulated below.

Congruent polygonal faces	No. of faces	Normal distance of each face from the centre of the given icosidodecahedron	Solid angle subtended by each face at the centre of the given icosidodecahedron
Equilateral triangle	20	$\frac{(3+\sqrt{5})a}{2\sqrt{3}} \approx 1.511522628a$	$2\pi - 6\sin^{-1}\left(\frac{3+\sqrt{5}}{2\sqrt{5+2\sqrt{5}}}\right) \approx 0.179853499\ sr$

Regular pentagon	12	$\frac{(\sqrt{5}+1)a}{\sqrt{10-2\sqrt{5}}} = acot36^o \approx 1.37638192a$	$2\pi - 10\sin^{-1}\left(\frac{1}{2}\sqrt{\frac{10-2\sqrt{5}}{5}}\right) \approx 0.747441718\ sr$

Inner (inscribed) radius $(\boldsymbol{R_i})$	$R_i = \frac{(\sqrt{5}+1)a}{\sqrt{10-2\sqrt{5}}} \approx 1.37638192a$
Outer (circumscribed) radius $(\boldsymbol{R_o})$	$R_o = \frac{(\sqrt{5}+1)a}{2} \approx 1.618033989a$
Mean radius $(\boldsymbol{R_m})$	$R_m = a\left(\frac{45+17\sqrt{5}}{8\pi}\right)^{\frac{1}{3}} \approx 1.489254843a$
Surface area $(\boldsymbol{A_s})$	$A_s = (5\sqrt{3} + 15cot36^o)a^2 \approx 29.30598285a^2$
Volume $(\boldsymbol{V})$	$V = \frac{(45+17\sqrt{5})a^3}{6} \approx 13.83552594a^3$

Note: Above articles had been developed *& illustrated by Mr H.C. Rajpoot (B Tech, Mechanical Engineering)*

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