The Čerenkov effect in the altermagnetic medium

Miroslav Pardy
Department of Physical Electronics
Masaryk University
Kotlářská 2, 611 37 Brno, Czech Republic
e-mail:pamir@physics.muni.cz

February 28, 2026

Abstract

We derive the spectral formula of the Čerenkov radiation in dielectric medium with the index of refraction n and magnetic permeability μ. This formula is valid also for the altermagnetic medium, where the index of diffraction and permeability must be calculated by the adequate methods which are analogical to the methods published by author (2018).

1 Introduction

The Čerenkov electromagnetic radiation is generated by a fast moving charged particle in a medium when its speed is faster than the speed of light in this medium. This radiation was first observed experimentally by Čerenkov (1934) and theoretically interpreted by Tamm and Frank (1937) in the framework of classical electrodynamics. A source theoretical description of this effect was given by Schwinger, Tsai and Erber (1976) at the zero-temperature regime and the classical spectral formula was generalized to the finite temperature situation, Lorentz contraction, massive photons, or, for the 2-dimensional system in the framework of the source theory by author (Pardy, 1989; 1997; 2002; 2015).

Now, the question arises: What is the relation of the Čerenkov radiation to the spin waves in altermagnets?

2 Altermagnetism

Altermagnetism (AM) is a novel form of magnetic order, originating from composite symmetries that couple real space and spin space operations (Šmejkal et al., 1922a; 1922b) Altermagnets combine key features of both ferromagnets (FM) and antiferromagnets (AFM): on the one hand, their band structure exhibits a pronounced spin splitting reminiscent of ferromagnetisms; on the other hand, the total magnetization strictly cancels out, resulting in zero net magnetization, as in AFM. This unique combination eliminates stray-field perturbations while still providing highly efficient spin polarization, making AM an appealing platform for low-power spintronic devices. Both theoretical and experimental studies further suggest that AM materials can host a variety of emergent phenomena, including non-relativistic spin splitting (Šmejkal et al., 1922a; 1922b), topological quantum states (Šmejkal et al., 1922a; 1922b), spin-charge coupling, (Šmejkal et al., 1922a; 1922b) and unconventional transport properties, thereby carrying significant implications for both fundamental physics and applications (Šmejkal et al., 1922a; 1922b).

The integral part of AM is altermagnon being a quasiparticle, a collective excitation of the spin structure of an electron in a crystal lattice . In the equivalent wave picture of quantum mechanics, an altermagnon can be viewed as quantized altermagnetic spin wave. Altermagnons carry a fixed amount of energy and lattice momentum, and are spin-1, indicating they obey boson behavior.

Altermagnon behavior can be studied with a variety of scattering techniques. Altermagnon gas behaves as a Bose gas with no chemical potential. Microwave pumping can be used to excite spin waves and create additional non-equilibrium altermagnons which thermalize into phonons. At a critical density, acondensate is formed, which appears as the emission of monochromatic microwaves. This microwave source can be tuned with an applied magnetic field. The feasible experiments can be suggested for the verification of altermagnons by the Čerenkov radiation of altermagnetic media.

3 The quantum theory of the Čerenkov radiation

Source theory, being the new quantum theory of fields (Schwinger, 1969; 1970; 1973; 1989; Dittrich, 1978), is the theoretical construction that uses quantum-mechanical particle language. Initially it was constructed for a description of the particle physics situations occurring in high-energy physics experiments. However, it was found that the original formulation simplifies the calculations in the electrodynamics and gravity where the interactions are mediated by the photon or graviton, respectively.

The basic formula in the source theory is the vacuum to vacuum amplitude (Schwinger et al., 1976):

$$\langle 0_+|0_-\rangle = e^{\frac{i}{\hbar}W(S)}, \tag{1}$$

where the minus and plus signs on the vacuum symbol are causal labels, referring to any time before and after the space-time region where sources are manipulated. The exponential form is introduced with regard to the existence of the physically independent experimental arrangements, which has a simple consequence that the associated probability amplitudes multiply and corresponding W expressions add (Schwinger, 1969; 1970; 1973; 1989).

The electromagnetic field is described by the amplitude (1) with the action

$$W(J) = \frac{1}{2c^2}\int (dx)(dx')J^\mu(x)D_{+\mu\nu}(x-x')J^\nu(x'), \tag{2}$$

where the dimensionality of $W(J)$ is the same as the dimensionality of the Planck constant $\hbar$. J_μ is the charge and current densities. The symbol $D_{+\mu\nu}(x-x')$ is the photon propagator and its explicit form will be determined later.

It may be easy to show that the probability of the persistence of vacuum is given by the following formula (Schwinger et al., 1976):

$$|\langle 0_+|0_-\rangle|^2 = \exp\left\{-\frac{2}{\hbar}\mathrm{Im}\,W\right\} \stackrel{d}{=} \exp\left\{-\int dt d\omega \frac{P(\omega,t)}{\hbar\omega}\right\}, \tag{3}$$

where we have introduced the so-called power spectral function $P(\omega, t)$ (Schwinger et al., 1976). In order to extract this spectral function from $\mathrm{Im}\,W$, it is necessary to know the explicit form of the photon propagator $D_{+\mu\nu}(x - x')$.

The electromagnetic field is described by the four-potentials $A^\mu(\phi, \mathbf{A})$ and it is generated by the four-current $J^\mu(c\varrho, \mathbf{J})$ according to the differential equation (Schwinger et al., 1976)

$$\left(\Delta - \frac{\mu\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}\right) A^\mu = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right) J_\nu \tag{4}$$

with the corresponding Green function $D_{+\mu\nu}$:

$$D_+^{\mu\nu} = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2-1}{n^2}\eta^\mu\eta^\nu\right) D_+(x - x'), \tag{5}$$

where $\eta^\mu \equiv (1, \mathbf{0})$, μ is the magnetic permeability of the dielectric medium with the dielectric constant ε, c is the velocity of light in vacuum, n is the index of refraction of this medium, and $D_+(x - x')$ was derived by Schwinger, Tsai and Erber (Schwinger et al., 1976) in the following form:

$$D_+(x - x') = \frac{i}{4\pi^2 c}\int_0^\infty d\omega \frac{\sin\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} e^{-i\omega|t-t'|}. \tag{6}$$

Using formulas (2), (3), (5), and (6), we get for the power spectral formula the following expression (Schwinger et al., 1976):

$$P(\omega, t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'dt' \frac{\sin\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|}\cos[\omega(t - t')]\times$$

$$\times\left\{\varrho(\mathbf{x}, t)\varrho(\mathbf{x}', t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x}, t)\cdot\mathbf{J}(\mathbf{x}', t')\right\}. \tag{7}$$

The charge and current density of electron moving with the velocity $\mathbf{v}$ and charge e is as it is well known:

$$\varrho = e\delta(\mathbf{x} - \mathbf{v}t) \tag{8}$$

$$\mathbf{J} = e\mathbf{v}\delta(\mathbf{x} - \mathbf{v}t). \tag{9}$$

Now, we are prepared to apply the last formula to the situations of the charge moving in the altermagnetic medium.

4 The quantum theory of dispersion

We follow the author article (Pardy, 2018). We suppose that electrons in atoms are in the same quantum state. The perturbation method is adequate for the application in a medium because the interaction energy of atoms with the external field is very small in comparison with the energy of electrons in atoms.

The impinging wave of electromagnetic field is of the form $E = E_0 \cos(\omega t - 2\pi x/\lambda)$. With regard to the fact that $\lambda \sim 10^{-5}$cm and atom is of the size $a \sim 10^{-8}$cm, the quantity x/λ can be neglected in the electromagnetic wave and we write

$$E = E_0 \cos(\omega t - 2\pi x/\lambda) \to E = E_0 \cos(\omega t). \tag{10}$$

So, the quantum theory of dispersion can be derived in the framework of the nonrelativistic Schrödinger equation (Sokolov et al., 1962) for an electron moving in dielectric medium and in the field with the periodic force

$$F_x = -eE_0 \cos \omega t, \quad F_y = F_z = 0. \tag{11}$$

Then, the corresponding potential energy is

$$V' = exE_0 \cos \omega t \tag{12}$$

and this potential energy is the perturbation energy in the Schrödinger equation

$$\left(i\hbar \frac{\partial}{\partial t} - H_0 - V' \right) \psi_k(t) = 0, \tag{13}$$

where for $V' = 0$ it is $\psi_k(t) \to \psi_k^0(t)$ and

$$\psi_k^0(t) = \psi_k^0 e^{-\frac{i}{\hbar} E_k t} = \psi_k^0 e^{-i\omega_k t}, \tag{14}$$

where ψ_k^0 is the solution of the Schrödinger equation without perturbation, or,

$$\left(i\hbar \frac{\partial}{\partial t} - H_0 \right) \psi_k^0(t) = 0. \tag{15}$$

We are looking for the solution of the Schrödinger equation involving the perturbation potential in the form

$$\psi_k(t) = \psi_k^0(t) + \psi_k^1(t), \tag{16}$$

where $\psi_k^1(t)$, is the perturbation wave function correction to the non-perturbation wave function.

After insertion of formula (16) to eq. (13), we get

$$\left(i\hbar \frac{\partial}{\partial t} - H_0 \right) \psi_k^1(t) = \frac{1}{2} exE_0 \psi_k^0 \left(e^{-it(\omega_k - \omega)} + e^{-it(\omega_k + \omega)} \right). \tag{17}$$

Let us look for the solution of eq. (17) in the form:

$$\psi_k^1(t) = ue^{-it(\omega_k - \omega)} + ve^{-it(\omega_k + \omega)}. \tag{18}$$

After insertion of (18) into (17), we get two equations for u and v:

$$\left(\hbar(\omega_k - \omega) - H_0 \right) u = \frac{1}{2} exE_0 \psi_k^0, \tag{19}$$

$$\left(\hbar(\omega_k + \omega) - H_0 \right) v = \frac{1}{2} exE_0 \psi_k^0. \tag{20}$$

Then, using the formal expansion

$$u = \sum_{k''} C_{k''} \psi^0_{k''}, \tag{21}$$

we get from eq.

$$\left(E_{k''} - H_0\right) \psi^0_{k''} = 0 \tag{22}$$

the following equation

$$\hbar \sum_{k''} C_{k''} \left(\omega_{kk''} - \omega\right) \psi^0_{k''} = \frac{exE_0}{2} \psi^0_k \tag{23}$$

with

$$\omega_{kk''} = \frac{E_k - E_{k''}}{\hbar}. \tag{24}$$

Using the orthogonal relation

$$\int \psi^{0*}_{k'} \psi^0_{k''} d^3x = \delta_{k'k''}, \tag{25}$$

we get the following relation for C_k and u as follows:

$$C_k = -\frac{eE_0}{2\hbar} \cdot \frac{x_{k'k}}{\omega_{k'k} + \omega}, \tag{26}$$

$$u = \sum_{k'} \left(-\frac{eE_0}{2\hbar}\right) \cdot \frac{x_{k'k}}{\omega_{k'k} + \omega} \psi^0_{k'} \tag{27}$$

and $v = u(-\omega)$, or

$$v = \sum_{k'} \left(-\frac{eE_0}{2\hbar}\right) \cdot \frac{x_{k'k}}{\omega_{k'k} - \omega} \psi^0_{k'} \tag{28}$$

and

$$x_{k'k} = \int \psi^{0*}_{k'} x \psi^0_k d^3x. \tag{29}$$

The general wave function can be obtained from eqs. (16), (18), (27) and (28) in the form:

$$\psi_k(t) = e^{-i\omega_k t} \quad \times$$

$$\left\{\psi^0_k - \frac{eE_0}{\hbar} \sum_{k'} \frac{x_{k'k}}{\omega^2_{k'k} - \omega^2} \psi^0_{k'} \left[\omega_{k'k} \cos \omega t - i\omega \sin \omega t\right]\right\}. \tag{30}$$

The classical polarization of a medium is given by the well known formula

$$P = Np = -Nex, \tag{31}$$

where N is the number of atom in the unite volume of dielectric medium. So we are able to define the quantum analogue form of the polarization as it follows:

$$P = N\bar{p} = -Ne \int \psi^*_k(t) x \psi_k(t) d^3x, \tag{32}$$

or, with

$$\int \psi_k^{0*} x \psi_k^0 d^3x = 0, \tag{33}$$

we have

$$P = \sum_{k'} \left(2\frac{Ne^2 E_0}{\hbar} \right) \cdot \frac{\omega_{k'k}|x_{k'k}|^2}{\omega_{k'k}^2 - \omega^2} \cos \omega t. \tag{34}$$

Using the classical formula for polarization P,

$$\mathbf{P} = \frac{n^2 - 1}{4\pi} \mathbf{E}, \tag{35}$$

we get for the quantum model of polarization

$$\frac{n^2 - 1}{4\pi} = \sum_{k'} \left(2\frac{Ne^2}{\hbar} \right) \cdot \frac{\omega_{k'k}|x_{k'k}|^2}{\omega_{k'k}^2 - \omega^2}. \tag{36}$$

Using the definition of the coefficients $f_{k'k}$ by relation

$$f_{k'k} = \frac{2m}{\hbar} \omega_{k'k} |x_{k'k}|^2, \tag{37}$$

we get the modified equation (36) as follows:

$$\frac{n^2 - 1}{4\pi} = \frac{Ne^2}{m} \sum_{k'} \frac{f_{k'k}}{\omega_{k'k}^2 - \omega^2}. \tag{38}$$

The last formula should be compared with the classical one:

$$\frac{n^2 - 1}{4\pi} = \frac{e^2}{m} \sum_{k} \frac{N_k}{\omega_k^2 - \omega^2}, \tag{39}$$

where N_k is number of electrons moving with frequency ω_k in the unit volume.

Let us remark that the oscillator coefficients $f_{k'k}$ in eq. (38) can have also the negative values. It leads to the special behavior of the dispersion. Namely, dispersion is negative. The negative dispersion was discovered by Ladenburg (1921; 1930).

5 Discussion

We have derived the spectral formula of the Čerenkov radiation in dielectric medium with the index of refraction n and magnetic permeability μ. This formula is valid also for the altermagnetic medium where the index of diffraction and permeability must bee calculated in the adequate methods which are analogical to methods published by author (2018).

The formula (36) is new and it is not excluded that it will play the crucial role in modern optics. It is possible expect the application of it in the altermagnetic graphene physics (Novoselov et al., 2005; Bludov et al., 2012), where every new result in quantum optics is valuable. At the same time we hope that the derived formula will be tested for altermagnetic media by the greatest laser system over the world, called ELI.

References

Bludov, Yu. V., Peres, N. M. R. and Vasilevskiy, M. I. (2012). Graphene-based polaritonic crystal, arXiv:1204.3900v1,[cond-mat.mes-hall].

Čerenkov, P. A. (1934). The visible radiation of pure liquids caused by γ-rays, *Comptes Rendus Hebdomaclaires des Séances de l'Academic des Sciences USSR* **2**, 451.

Dittrich, W., (1978). Source methods in quantum field theory, Fortschritte der Physik **26**, 289.

Ladenburg, R. (1921). Die quantentheoretische Bedeutung der Zahl der Dispersionelektronen, Zeitschrift für Physik, **4**, No. 4, 451-468.

Ladenburg, . and Levy, S. (1930). Untersuchungen ber die anomale Dispersion angeregter Gase VI. Teil: Kontrollversuche für den Nachweis der negativen Dispersion: Absorption, anomale Dispersion, Intensittsverteilung und Intensität verschiedener Neonlinien, Zeitschrift für Physik, **65**, Nos. 3-4. pp. 189-206.

Novoselov, K.S.; Geim, A. K.; Morozov, S.V. et al. (2005). Two-dimensional gas of massless Dirac fermions in graphene, *Nature*, 438, 197.

Pardy, M. (1989). Finite-temperature Čerenkov radiation, *Physics Letters A* **134**(6), 357.

Pardy, M. (1997). Čerenkov effect and the Lorentz contraction, *Phys. Rev. A* **55**, 1647.

Pardy, M. (2002). Čerenkov effect with massive photons, *International Journal of Theoretical Physics*, **41**(5), 887.

Pardy, M. (2015). The Two-Dimensional Vavilov-Cerenkov Radiation in LED, Results in Physics 3, 6971.

Pardy, M. (2018). Quantum theory of the dispersion of the laser light, Intellectual Archive, Vol 7, No. 6.

Tamm, I. E. and Frank, I. M. (1937). The coherent radiation of a fast electron in a medium, *Dokl. Akad. Nauk SSSR* **14**, 109.

Schwinger, J. *Particles and Sources*,
(Gordon and Breach, Science Publishers, New york, London, Paris, 1969).

Schwinger, J. *Particles, Sources and Fields I.*,
(Addison-Wesley Publishing Company, Reading, Mass. 1970).

Schwinger, J. *Particles, Sources and Fields II.*, (Addison-Wesley Publishing Company, Reading, Mass. 1973).

Schwinger, J. *Particles, Sources, and Fields III.*, (Addison-Wesley, Reading, Mass. 1989).

Schwinger, J., Tsai, W. Y. and Erber, T. (1976). Classical and quantum theory of synergic synchrotron-Čerenkov radiation, *Annals of Physics (NY)* **96**, 303.

Šmejkal, L., Sinova, J. and Jungwirth, T. (2022a). Beyond conventional ferromagnetism and antiferromagnetism: A phase with nonrelativistic spin and crystal rotation symmetry. Phys. Rev. X 12, 031042.

Šmejkal, L., Sinova, J. and Jungwirth, T. (2022b). Emerging research landscape of altermagnetism. arXiv:2204.10844v1 [cond-mat.mes-hall] 22 Apr. 2022.