A univariate quartic equation for three-generation of charged lepton $g-2$

Shan-Hong Li
Email: duke3366@163.com

(February 28, 2026)

Abstract : We find a univariate quartic equation , and the coefficients of the equation are composed of the Schwinger term and the neutron spin g factor. The solution of the equation is two real roots and two conjugate imaginary roots. These roots are not arbitrary numbers, but are related to the anomalous magnetic moments of the three-generation of charged lepton. One of the two real roots is the anomalous magnetic moment of the electron, and the other is about one tenth of the anomalous magnetic moment of the τ lepton; the real number of the imaginary part of the two conjugate imaginary roots is about 10 times the anomalous magnetic moment of the muon. For this strange proportional relation, we have discussed it and found out the exact relation that they may exist. Finally, we believe that the anomalous magnetic moments of the three-generation of charged lepton is derived from the same high-order algebraic equation, indicating that algebra can also study quantum physics, and also suggesting that there may be algebraic relations and potential deep physical laws.

Introduction : The equations we found are as follows :

$$(1+x)^2x^2-\frac{\alpha}{2\pi}x(1+x)^2+\frac{2\sqrt{g_n}}{3}\left(\frac{\alpha}{2\pi}\right)^3=0 \quad (1)$$

Where: x is an unknown number ; $\alpha/2\pi$ is the Schwinger term ; α is a fine structure constant ; π is the circumference rate ; g_n is the spin magnetic moment g factor of neutron. Tip : α and g_n are the recommended values of CODATA 2022 [1], and the value of g_n is not minus sign.

Now we replace the coefficients in the equation with m and n respectively, as follows:

$$m = \frac{\alpha}{2\pi}\ ;\quad n = \frac{2\sqrt{g_n}}{3}\left(\frac{\alpha}{2\pi}\right)^3 \tag{2}$$

Bring Equation (2) into Equation (1), there is :

$$(1+x)^2x^2 - mx(1+x)^2 + n = 0 \tag{3}$$

The four roots of the equation are as follows :

$$x_1 = 0.001159652180698$$

$$x_2 = 1.7616300443989 \times 10^{-6}$$

$$x_{3,4} = -1.0000000020393237 \pm 4.5171986197192 \times 10^{-5}i \tag{4}$$

From (4), we can directly see that the x_1 root is the anomalous magnetic moment of the electron. It is in good agreement with the existing measured values and theoretical values [1] [2].

Now we need to set some symbols of the relationship, in order to facilitate the discussion of other roots implied meaning, as follows :

$$\frac{\alpha}{2\pi} - a_e = \Delta a_e$$

$$a_\mu - \frac{\alpha}{2\pi} = \Delta a_\mu$$

$$a_\tau - \frac{\alpha}{2\pi} = \Delta a_\tau$$

$$a_\tau - a_e = \Delta a_{\tau e} \tag{5}$$

Where a_e, a_μ, a_τ are the anomalous magnetic moments of the electron, muon, and τ lepton, respectively. Δa_e, Δa_μ, Δa_τ are the difference between them and the Schwinger term, respectively. $\Delta a_{\tau e}$ is the difference of the anomalous magnetic moment between the τ lepton and the electron.

It can be seen from Equation (4) that we will use the anomalous magnetic moment to discuss the problem. However, when the magnetic moment of charged leptons becomes an anomalous magnetic moment, their effective number is reduced by at least 3 bits, so that their accuracy is rapidly reduced by at least 3 orders of magnitude, and high-precision calculations are greatly reduced, which makes it difficult for us to explore the precise relationship between them.

Now we first discuss the root of x_2 , which is related to the anomalous magnetic moment of the electron. Here we need to calculate the value of Δa_e first to find the relationship between them. Where Δa_e we use in this paper is the value of x_1 in (4).

$$\Delta a_e = \frac{\alpha}{2\pi} - a_e = 1.7575514 \times 10^{-6} \tag{6}$$

Now we use formula (6) to compare the value of x_2, $x_2 = 1.7616300443989 \times 10^{-6}$. It can be found that the two values are very similar. Now we look at their ratio :

$$\Delta a_e \div x_2 = 1.0023206400372 \tag{7}$$

The ratio of Equation (7) is approximately equal to $(g_e/2)^2$, and g_e is the g factor of the spin magnetic moment of the electron.

Formula (1) is a quartic equation of one variable. According to Veda 's theorem, its four roots have the following relations :

$$x_1 \cdot x_2 \cdot x_3 \cdot x_4 = n \tag{8}$$

x_3 and x_4 are a pair of conjugate complex roots, and their product is about equal to 1, that is, $|x_3|^2 \approx 1$, deviation $< 10^{-8}$.

According to the analysis of formula (8), we can get the relationship between x_1 and x_2:

$$x_1 \cdot x_2 \approx n \tag{9}$$

Since $x_1 = a_e$, then :

$$x_2 \approx \frac{n}{a_e} \tag{10}$$

According to formula (10), we get: $x_2 = 1.7616300552 \times 10^{-6}$. Comparing the values of x_2 in formula (4), we can find that there is a small difference between them.

We make a minor correction to this, as follows :

$$x_2 = \frac{n}{(1+3n)a_e} \tag{11}$$

We calculate Equation (11) and its result is $x_2 = 1.761630044404 \times 10^{-6}$. Comparing the values of x_2 in Equation (4), we can find that they are in good agreement with the 9 digits behind the decimal point, and the remaining differences .mainly come from the uncertainty. Due to the accuracy of the effective number of the electron 's anomalous magnetic moment, it is three orders of magnitude lower than its spin g factor. At the same time, the accuracy of g_n and α in the n value is also limited, which will make the calculation results different.

If we want to continue to correct (11), we give it the following formula :

$$x_2 = \frac{n}{\left(1 + 3n\sqrt{g_{el}}\right)a_e} \tag{12}$$

g_{el} is $g_e/2$. g_e is the spin magnetic moment g factor of electron.

The result of Equation (12) is : $x_2 = 1.761630044398 \times 10^{-6}$.

There are also other approximate relations for x_2, as follows :

$$x_2 \approx \Delta a_e + 2n/g_{el} \tag{13}$$

The calculation result of Equation (13) is 1.7616324×10^{-6}.

From Equation (6) to Equation (13), through their calculation results, we can find that there is a relationship between x_2 roots and the anomalous magnetic moment a_e of electron.

Now let 's discuss the root $x_{3,4}$, which is related to the anomalous magnetic moment of the muon. Let the real number of its imaginary part be $\mathrm{Im}(x_{3,4})$, that is :

$$\mathrm{Im}(x_{3,4}) = 4.5171986197192 \times 10^{-5} \tag{14}$$

We now look at the value of Δa_μ, as follows :

$$\Delta a_\mu = a_\mu - \frac{\alpha}{2\pi} = 4.5109829 \times 10^{-6} \tag{15}$$

Observing Equation (14) and Equation (15), it can be found that these two values are very similar. Among them, $a_\mu = 0.00116920715$. It is the latest experimental measurement value [3].

Now divide by Equation (14) by Equation (15), get :

$$\mathrm{Im}(x_{3,4}) \div \Delta a_\mu = 10.013779075466 \tag{16}$$

From Equation (16), we can see that $\mathrm{Im}\left(x_{3,4}\right)$ is about 10 times larger than Δa_μ. Now if we look closely at the value of (16), we find that it is very similar to the mass ratio of neutron to protons, as follows :

$$\frac{m_n}{m_p} = 1.00137841946 \tag{17}$$

Where : m_n is the mass of the neutron ; m_p is the mass of proton.

Now we divide Equation (16) by Equation (17) and get :

$$\frac{\mathrm{Im}\left(x_{3,4}\right) \div \Delta a_\mu}{m_n/m_p} = 9.999994887912 \tag{18}$$

It can be seen from Equation (18) that Equation (16) is almost 10 times that of Equation (17). So according to this ratio, we get a relational expression:

$$\mathrm{Im}(x_{3,4}) = 10 \cdot \frac{m_n}{m_p} \Delta a_\mu \tag{19}$$

Since $a_\mu - \frac{\alpha}{2\pi} = \Delta a_\mu$, then (19) becomes :

$$\mathrm{Im}(x_{3,4}) = 10 \cdot \frac{m_n}{m_p} (a_\mu - \frac{\alpha}{2\pi}) \tag{20}$$

When $a_\mu = 0.00116592071269394$, the results of the left and right sides of Equation (20) are equal. It can also be seen from Equation (20) that the a_μ value obtained by it is highly consistent with the latest experimental measurement of the muon 's anomalous magnetic moment within the error range.

Of course, we find that there are other approximate relations for $\mathrm{Im}(x_{3,4})$, Let 's first look at the difference between $\mathrm{Im}(x_{3,4})$ and Δa_μ:

$$\mathrm{Im}(x_{3,4}) - \Delta a_\mu = 4.0661003 \times 10^{-5} \tag{21}$$

We find that the difference between Equation (21) and n in Equation (3) has the following relationship :

$$\mathrm{Im}(x_{3,4}) - \Delta a_\mu \approx \frac{2n \times 10^4}{1 + m_e/m_\mu} \tag{22}$$

The calculation result on the right side of Equation (22) is : $4.06609130 \times 10^{-5}$. From (14) to (22), through their calculation results, we can find that there is a relationship between the real number $\mathrm{Im}(x_{3,4})$ of the imaginary part of $x_{3,4}$ and the anomalous magnetic moment a_μ of muon.

Now let 's discuss the root of x_2 , we find that it is also related to the anomalous magnetic moment of τ lepton. Let 's first look at the difference between the anomalous magnetic moment of the τ lepton and the electron, as follows :

$$\Delta a_{\tau e} = a_\tau - a_e = 0.0000175838 \tag{23}$$

Here, $a_\tau = 0.001177236$, it comes from here [4]. At present, there is no exact measurement of the anomalous magnetic moment of τ lepton. The theoretical value of the standard model is [5] : $a_\tau = 0.00117721(5)$. It can be seen that the value we refer to is consistent with it within the error range.

In (6), the difference between the anomalous magnetic moment of the electron and the Schwinger term is 1.7575514×10^{-6}. Compared with the results of (23), it can be found that they are very similar. We divide Equation (23) by Equation (6) and get :

$$\frac{\Delta a_{\tau e}}{\Delta a_e} = \frac{a_\tau - a_e}{\frac{\alpha}{2\pi} - a_e} = 10.004725 \tag{24}$$

It can be seen from equation (24) that $\Delta a_{\tau e}$ is about 10 times of Δa_e. According to this relation, we can obtain an approximate relation about the anomalous magnetic moment of τ lepton, as follows :

$$a_\tau \approx a_e + 10\Delta a_e \tag{26}$$

The calculation result of Equation (26) is: $a_\tau = 0.0011772277$.

Of course, we find that there is also an approximate relationship between $\Delta a_{\tau e}$ and n, m in Equation (3), as follows :

$$\Delta a_{\tau e} \approx 10 \cdot \frac{n}{m} = 0.00001758964 \tag{27}$$

Combining Equation (23) and Equation (27), we obtain an approximate relation about the anomalous magnetic moment of the τ lepton:

$$a_\tau \approx a_e + 10 \cdot \frac{n}{m} \tag{28}$$

The calculation result of Equation (28) is : $a_\tau = 0.0011772418$.

Now we observe the root of x_2 in formula (4), $x_2 = 1.761630044398 \times 10^{-6}$. It can be found that it is also similar to the value of $\Delta a_{\tau e}$, and it is also about 10 times different from $\Delta a_{\tau e}$. We continue to use this relationship, combined with the ratio of formula (24) and n in formula (3), we get a new relationship between $\Delta a_{\tau e}$, as follows :

$$\Delta a_{\tau e} = x_2 \frac{\Delta a_{\tau e}}{\Delta a_e} - \frac{20n}{g_{el}\left(1 + \frac{a_\mu}{10}\right)} \tag{29 }$$

For the (29) shift, we have :

$$\Delta a_{\tau e} = \frac{20n}{g_{el}\left(1 + \frac{a_\mu}{10}\right)\left(\frac{x_2}{\Delta a_e} - 1\right)} \tag{30}$$

For formula (30), we get : $\Delta a_{\tau e} = 0.0000175837$. Using it to compare the values of formula (23), we can find that they are very consistent.

Combining formula (30) with formula (23), we get :

$$a_\tau = a_e + \frac{20n}{g_{el}\left(1 + \frac{a_\mu}{10}\right)\left(\frac{x_2}{\Delta a_e} - 1\right)} \tag{31}$$

The calculation result of Equation (31) is: $a_\tau = 0.0011772359$.

From (23) to (31), through their calculation results, we can find that there is a relationship between the x_2 root and the anomalous magnetic moment a_τ of the τ lepton, and it is in good agreement with the recommended value of the standard model within the error range.

Here, we give an approximate formula for the anomalous magnetic moment of electron :

$$a_e \approx \frac{\alpha}{2\pi} - \frac{n}{m} = \frac{\alpha}{2\pi} - \frac{2\sqrt{g_n}}{3}\left(\frac{\alpha}{2\pi}\right)^2 \tag{32}$$

The calculation result of Equation (32) is : $a_e = 0.001159650768.$

At present, the anomalous magnetic moment of the three-generation of charged lepton can only be obtained from experiments and quantum theory calculations. The one-variable quartic equation we give does not need to know the theoretical calculation process, nor does it need to know the experimental method. It only needs to know their algebraic relationship, and the anomalous magnetic moment of the three-generations of charged lepton can be obtained. So far, we have discussed the possible algebraic relations behind the four roots of the higher-order algebraic equations in this paper. But at the same time, we also have some questions : why do the anomalous magnetic moments of three-generation of charged lepton derive from the same algebraic equation ? The four roots of the equation are constrained by the Weida theorem, what does this imply ? The anomalous magnetic moment of the muon appears in the imaginary part of the complex root, and the coefficients of the equation only α and g_n indicate what And so on, these need to explain their meaning. Because we do not have in-depth research in this area, we can not give a clear explanation. But we believe that it must hide the deep new physical meaning that we do not know. If you have research in this area, and just met this article, and interested, then you can try to make some interpretation of its meaning.

Finally, we want to show that through the higher algebraic equations we give, we can find that it seems that quantum physics can also be studied by algebraic methods. And it may interpret our physical world from a new perspective.

references

[1] https://physics.nist.gov/cuu/Constants/

[2] arXiv:2209.13084v2

[3] arXiv:2506.03069v1 [hep-ex]

[4] viXra:2602.0009

[5] arXiv:hep-ph/0701260v1