On the Arbitrary Reduction of Energy and Energy-Momentum Tensor components in the Alcubierre Warp Drive with possibile Positive Values

Gianluca Perniciano*, (graduate),

Department of Physics of the University of Cagliari, Italy

Abstract:

Building on the article [16] where a superluminal Alcubierre propulsion system with positive energy density and positive energy is possible as an improvement over [16], we make a small modification to have a positive energy and positive energy density for a curvature bubble of generic radius R and introduce a way to arbitrarily reduce such energy sources in the warped region,.We compute the remaining components of the Einstein tensor, which we find to be only partially positive; however, the WEC, DEC, and apparently the SEC are satisfied [20]. This implies a warp drive without exotic matter, but rather a form of asymmetric ordinary matter.

Introduction:

In 1994, Alcubierre [1] proposed a solution to the equations of general relativity that provides the only viable means to accelerate a spaceship to superluminal velocities without using wormholes. However, a problem was soon identified: Pfenning [4] showed that the required energy is comparable to the total energy of the universe and that it is negative. In the article [16], the problem of the negative energy source in the warped region of the original Alcubierre solution [1] was brilliantly solved by the author of [16]. This paper continues the work by making small modifications to reduce the energy density and energy for the Alcubierre drive with a positive energy source.

*g.perniciano@gmail.com

Brief Summary of the Article [16]:

It starts from the most general formulation of Alcubierre [1] in the form of the 3+1 ADM splitting for motion in a generic direction with a given bubble velocity v_x , v_y , and v_z in all spacial directions. under these conditions, the metric is:

$$ds^2=(dx-X\,dt)^2+(dy-Y\,dt)^2+(dz-Z\,dt)^2-dt^2 \quad (1)$$

where for the [16] the values X, Y, and Z are given by:

$$X=\partial_x\psi_1 \qquad Y=\partial_y\psi_2 \qquad Z=\partial_z\psi_3 \quad (2)$$

the functions ψ_1 , ψ_2 e ψ_3 e depend implicitly on the velocity v(t)

$$v(t)=\sqrt{(v_x(t)^2+v_y(t)^2+v_z(t)^2)} \text{ where } v_x(t)=\frac{d\,x_s}{dt} \quad v_y(t)=\frac{d\,y_s}{dt} \quad v_z(t)=\frac{d\,z_s}{dt} \quad (3)$$

are respectively the velocities along the x, y, z axes and from the radius vector that starts from the center of the warp bubble in motion, the modulus of this vector is:

$$r(t)=\sqrt{((x-x_s)^2+(y-y_s)^2+(z-z_s)^2)} \quad (4)$$

by setting [16]:

$$\psi_x=\psi_y=\psi_z=\psi(r) \quad (5)$$

we have:

$$K^2-K^i_j K^j_i=2\partial^2_x\psi\,\partial^2_y\psi+2\partial^2_x\psi\,\partial^2_z\psi+2\partial^2_z\psi\,\partial^2_y\psi-2(\partial_x\partial_y\psi)^2-2(\partial_x\partial_z\psi)^2-2(\partial_y\partial_z\psi)^2$$

which can be simplified [16] for a soliton ψ described by the equation

$$\partial^2_x\psi+\partial^2_y\psi\,\partial-(2/v_h^2)\,\partial^2_z\psi=\rho$$

the energy density in the warped region is equal to:

$$\textit{energy density}=\frac{1}{16\pi}(K^2-K^i_j K^j_i)=(\frac{1}{16\pi})(2\,\partial^2_z\psi(\rho+(2/v_h^2)\partial^2_z\psi)-4(\partial_z\partial_x\psi)^2) \quad [16]\ (6)$$

with the condiction $(\partial^2_z\psi)^2\geq v_h^2(\partial_z\partial_x\psi)^2$ (6) [16] this energy density becomes positive:

$$Energy\ warped\ region = k R^2 \frac{v^2}{w} > 0 \qquad (7)$$

R is the radius of the warp bubble, and w < R

w is the size of the warped region, k is a constant of the order of c^4/G see Pfenning [4]

$$Energy\ density\ warp\ region \cong k R^2 \frac{\frac{v^2}{w}}{R^2 w} \qquad (8)$$

ψ it is chosen such that inside the warp bubble X, Y, Z are equal to v (warp bubble velocity) and zero outside the warp bubble.

Proposed Solution Modification:

Setting $\frac{\psi}{b(x,y,z)}$ in place of ψ, with example $b(r) = \frac{2^p}{(1+(\tanh(\sigma(r-R)))^2)^p}$ p>>1

and if $b(r)$ equals $b(r) = B \gg 1$ in warped region and 1 inside and outside warped region.

And knowing that:

$$\partial_i\left(\frac{\psi}{b}\right) = \frac{\partial_i \psi}{b} - \frac{\psi}{b^2}\partial_i b > 0 \qquad \partial_i b < 0 \quad , \qquad \partial_i \psi > 0 \qquad \text{(small for soliton)}$$

and

$$\partial_i^2 b < 0 \qquad \partial_i^2\left(\frac{\psi}{b}\right) \ll 1 \qquad \partial_i^2\left(\frac{\psi}{b}\right) > 0 \qquad \partial_i b \cong 10^5 b$$

the (6) after setting $\frac{\psi}{b(x,y,z)}$ in place of ψ and $\left(\partial_z^2 \frac{\psi}{b}\right)^2 \geq v_h^2\left(\partial_z \partial_x \frac{\psi}{b}\right)^2$ is positive

for a value of $B = 10^{50}$ or greater becomes:

$$0 < energy\,warped\,region \leq k\,R^2 \frac{v^2}{B\,w} \ll 1 \qquad \text{for} \qquad [4] \tag{9}$$

w size warped region w < R

the energy is small, for example, for an R = 100 m, w=1, and energy density is:

$$0 < Energy\,density\,warp\,region \cong k\,R^2 \frac{\frac{v^2}{B\,w}}{R^2\,w} \ll 1 \tag{10}$$

Computation of the components of the energy–momentum tensor (proportional to the Einstein tensor):

$$ds^2 = dt^2 - \delta_{ij}(dx^i + N^i\,dt)(dx^j + N^j\,dt)$$

$$N^i = (X, Y, Z) = (X, 0, Z) = (N_x, 0, N_z)$$

$$K_{ij} = (1/2)(\partial_i N_j + \partial_j N_i)$$

$$tr\,K = \partial_i N^i$$

$$(K^2)_{ij} = K_{ik} K_{kj}$$

$$\Omega_{ij} = (1/2)(\partial_i N_j - \partial_j N_i)$$

Components of the Einstein tensor:

$$G_{tt} = R_{tt} - (1/2)R = (1/2)(tr\,K)^2 - (1/2)tr(K^2) > 0$$

$$G_{ti} = R_{ti} = (1/2)(rot(omega))_i \qquad omega_i = omega^i = \varepsilon^{ijk}\,\Omega_{jk}$$

$$G_{ij} = R_{ij} - (1/2)R = tr\,K(K_{ij} - (1/2)\delta_{ij}\,tr\,K) - (1/2)\delta_{ij}\,tr(K^2) - (K\Omega + \Omega K)_{ij}$$

$$(K\Omega + \Omega K)_{ij} = K_{ik}\,\Omega_{kj} + \Omega_{ik}\,K_{kj}$$

$$tr\,K\,K_{ij} - (K\Omega + \Omega K)_{ij} \qquad \partial_i N_z > \partial_j N_x \qquad tr\,K = \partial_z N_z + \partial_x N_x \cong \partial_z N_z > 0$$

$$G_{ij} = tr\,K\,K_{ij} - (1/2)\delta_{ij}(tr\,K)^2 - (K\Omega + \Omega K)_{ij}$$

$$G_{xx}=(\partial_z N_z)(\partial_x N_x)-(1/2)(\partial_z N_z)^2-(K\Omega+\Omega K)_{xx}<0$$

$$G_{yy}=-(1/2)(\partial_z N_z)^2-(K\Omega+\Omega K)_{yy}<0$$

dominant term K_{zz} :

$$K_{zz}=\partial_z N_z=\partial_z^2(\frac{\psi}{b(r)})>0 \qquad K_{zz}\ll 1 \qquad \partial_{ij}^2(\frac{\psi}{b(r)})\ll 1 \qquad K_{xx}<0,|K_{xx}|\ll 1, K_{yy}=0$$

$$G_{ij}=R_{ij}-(1/2)R=tr\,K\,K_{ij}-(K\Omega+\Omega K)_{ij} \qquad i\neq j$$

$$K_{ij}=(1/2)(\partial_i N_j+\partial_j N_i)$$

$$K_{xy}=(1/2)\partial_y N_x \qquad K_{xz}=(1/2)(\partial_x N_z+\partial_z N_x)=\partial_x N_z \qquad K_{yz}=(1/2)\partial_y N_z$$

K_{xy} positive or negative sign, a term that is negligible compared to the other two which have positive or negative signs. Despite this, the WEC DEC and SEC [20] (for energy impulse tensor) are satisfied because they are dominant terms. G_{zz} the energy density is positive and represents the dominant term (see Lentz [16]); therefore, no exotic matter is required, but rather ordinary matter.

All the other components are negligible; motion is along the z-axis of soliton v_z no zero

$v_h=v_z \quad v_x=0 \quad v_y=0$. See Lentz [16], after modification with the coefficient

$b(r)\gg 1$ in the warped region equal to 1 both inside and outside the warped region, as

demonstrated in the previous chapter.

$G_{tt}=R_{tt}-(1/2)R=(1/2)(tr\,K)^2-(1/2)tr(K^2)>0$ for Lentz [16], for a soliton

under the previous conditions, one has:

$$G_{tt}=R_{tt}-(1/2)R=(1/2)(tr\,K)^2-(1/2)tr(K^2)\ll 1$$

$$(tr\,K)^2>tr(K^2)$$

the components Ω_{ik} are small and G_{ij} are given, after evident simplifications, by:

$$G_{zz} \cong tr\, K\, K_{zz} > 0$$

$$G_{zz} \cong tr\, K\, K_{zz} = (K_{zz})^2 = (\partial_z N_z)^2 = (\partial_z^2 (\frac{\psi}{b(r)}))^2 > 0 \qquad K_{zz} \ll 1$$

In our case:

$\Omega_{ij} = 0$ $(K\Omega + \Omega K)_{ij} = 0$ for the conditions chosen by Lentz [16] for a soliton and by virtue of Schwarz's theorem, the mixed components are negligible and therefore non-pathological.

Conclusion:

The modifications made to [16] maintain a positive energy density and a positive total energy that can be reduced at will, making the Alcubierre drive a realistic possibility with the existence of superluminal motion. However, even in this configuration the components of the energy–momentum tensor are only partially positive, which implies the presence of ordinary (asymmetric) matter rather than exotic matter, since the WEC, DEC, and the SEC are satisfied [20] (eulerian observer). That is, a form of ordinary asymmetric matter with an appropriate energy–momentum tensor can be used.The coefficent b(r) in the denominator of ψ reduction all component of tensor energy impulse arbriry factor page 3 of paper. The remaining problem is how to physically generate such a soliton within the warped region ?

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