Emergence of the Electroweak Scale from Continuous Dimensions

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Abstract

In this sequel to [6], we show that the electroweak vacuum v emerges as infrared scale generated by dimensional deviation $\epsilon \ll 1$. The mechanism detailed here mirrors the mass-gap generation mechanism of non-Abelian gauge theory.

Key words: electroweak scale, continuous spacetime dimensions, Renormalization Group flows, Beyond the Standard Model Physics,

1. Introduction

The origin of the electroweak scale,

$$v = 246 \text{ GeV},$$

is typically framed as a hierarchy problem: why is $v \ll M_{\rm Pl}$ (where $M_{\rm Pl}$ is the Planck scale) and stable under quantum corrections? Discussions and debate focus on naturalness, fine-tuning, or physics beyond the Standard Model range (BSM).

In line with [6], we adopt here a different perspective. We treat the Renormalization Group (RG) as a *dynamical system in theory space* and ask whether the electroweak scale can arise as an *infrared bifurcation scale*, analogous to the QCD mass scale.

The key observation is that the Higgs mass parameter is marginal at the critical spacetime dimension $D_c = 4$ where it sits at the boundary between a hyperbolic and non-hyperbolic RG flow. An infinitesimal deviation from $D = 4$ destabilizes the fixed point and generates a finite scale.

2. Higgs Sector in $D = 4 + \epsilon$ Dimensions

Consider the Higgs sector of the Standard Model,

$$\mathcal{L}_H = (D_\mu \Phi)^\dagger (D^\mu \Phi) - m_0^2\, \Phi^\dagger \Phi - \lambda_0 (\Phi^\dagger \Phi)^2.$$

In $D = 4 + \epsilon$ spacetime one has the following mass dimensions,

$$[m_0^2] = \text{mass}^2, \qquad [\lambda_0] = \text{mass}^{-\epsilon}.$$

which means that the quartic coupling is no longer dimensionless. To restore renormalized (dimensionless) parameters, we use the parameterization

$$\boxed{\lambda_0 = \mu^{-\epsilon} \lambda(\mu), \qquad m_0^2 = m^2(\mu).}$$

3. RG Time and Reference Scale

The RG "time" is defined by

$$\boxed{t = \ln\left(\frac{\mu}{\mu_0}\right),}$$

where μ_0 is an arbitrary but fixed reference scale.

4. RG Equations in $D = 4 + \epsilon$ Dimensions

At one loop, the Higgs sector obeys

$$\boxed{\frac{d\lambda}{dt} = \epsilon\,\lambda - \beta_\lambda(\lambda, g_i, y_t),} \qquad (1)$$

$$\boxed{\frac{dm^2}{dt} = [2 - \gamma_m(\lambda, g_i, y_t)]m^2.} \qquad (2)$$

Here:

- g_i are gauge couplings,
- y_t is the top Yukawa coupling,
- γ_m is the Higgs mass anomalous dimension.

The canonical term "2" in Eq. (2) is purely dimensional and independent of dynamics.

5. Marginality and Loss of Hyperbolicity

At the critical dimension $D_c = 4$,

$$[\lambda] = 0, \qquad [m^2] = 2.$$

The quartic coupling is marginal, while the mass term is relevant. Near the Gaussian fixed point, the anomalous dimension behaves as,

$$\gamma_m \sim \mathcal{O}(\lambda, g_i^2, y_t^2).$$

When radiative corrections nearly cancel the canonical scaling,

$$2 - \gamma_m \approx 0,$$

the RG flow becomes *non-hyperbolic*. This is the electroweak analogue of the Yang–Mills critical surface.

6. RG Bifurcation of the Higgs Mass Parameter

Retaining the leading contribution,

$$\gamma_m \simeq C\,\lambda,$$

eq. (2) becomes

$$\frac{dm^2}{dt} = (2 - C\lambda)\, m^2. \qquad (3)$$

The quartic coupling obeys

$$\frac{d\lambda}{dt} = \epsilon\lambda - \beta_0\lambda^2. \qquad (4)$$

Equations (3) – (4) define a *coupled dynamical system*. The critical surface is defined by setting (4) to zero, which leads to

$$\lambda_* = \frac{\epsilon}{\beta_0}.$$

At this point,

$$2 - C\lambda_* = 0,$$

and hyperbolicity is lost.

7. Emergence of the Electroweak Scale

Integrating Eq. (3) we obtain,

$$m^2(\mu) = m^2(\mu_0)\ \exp\left[\int_0^{\ln(\mu/\mu_0)} dt'\ (2 - C\lambda(t'))\right]$$

As $\lambda(t)$ approaches λ_*, the exponent saturates and defines a finite scale $\mu = v$ as in

$$\boxed{v = \mu_0 \exp\left[-\frac{2}{C\,\lambda(\mu_0)}\right]} \qquad (5)$$

This scale is:

- RG invariant,
- finite as $\epsilon \to 0$,
- generated from the inherent bifurcation produced by instability of (3) at $D_c = 4$.

8. Interpretation

- The Higgs vacuum expectation value v is not a fundamental input,
- It is the infrared bifurcation scale of a marginally unstable RG flow.
- The mechanism is mathematically identical to the emergence of Λ_{QCD}.

These findings place electroweak symmetry breaking and QCD mass generation within a single universality class.

9. Relation to Fractal Spacetime

In fractal spacetime models, the effective dimension flows with scale,

$$D_{\text{eff}}(\mu) = 4 + \epsilon(\mu).$$

The electroweak scale corresponds to the point where dimensional flow destabilizes the scale-invariant fixed point.

9. Conclusions

We have shown that the electroweak scale emerges dynamically from a bifurcation triggered by arbitrarily small dimensional deviations. When RG equations are written with explicit reference scales and dimensionless couplings, the Higgs sector exhibits the same instability mechanism responsible for the QCD mass gap.

In this picture, the electroweak symmetry breaking is reinterpreted as the infrared instability of a marginal fixed point, not as evidence for new physics thresholds. Interested readers are invited to consult [1 - 5] for additional technical insights and clarifications.

We close by bringing up an intriguing result emerging from (5) combined with

$$2 - C\lambda_* = 0 \qquad (6)$$

and the Higgs boson mass computed at the tree level, i.e.,

$$m_H = \sqrt{2\lambda}\, v \Rightarrow \lambda = \frac{m_H^2}{2v^2} \qquad (7)$$

Under the reasonable assumption $\lambda \approx \lambda(\mu_0) \approx \lambda_*$, by (5), (6) and (7) one arrives at

$$C = \frac{4v^2}{m_H^2} \qquad (8)$$

and

$$\boxed{v = \frac{\mu_0}{e}} \qquad (9)$$

Under the further assumption that $\mu_0 \approx 668\,\text{GeV}$ represents the *experimental exclusion limit* for BSM models (such as extended Higgs sectors, leptoquarks or supersymmetric particles decaying into heavy Standard Model fermions), (9) gives the estimate

$$v \approx 246\ \text{GeV}$$

in agreement with the value of the electroweak vacuum. It is, however, unclear whether the derivation of v from $\mu_0 \approx 668$ GeV has any physical significance in BSM searches or it is simply a numerical coincidence.

References

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