The Geometry of Information: Unifying Gravity and Gauge Fields via the Mixed-State Quantum Geometric Tensor

Jin Woo Lee, M.D.[1,2,*]

[1]Independent Researcher, Seoul, Republic of Korea

[2]Member of the Korean Physical Society

[*]Correspondence: iiorange@gmail.com

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Abstract

Building upon the Relational-Informational Model (RIM), where spacetime geometry emerges from the Quantum Fisher Information of an observer's reduced density matrix, I extend the framework to derive all four fundamental interactions from a single information-geometric object: the mixed-state Quantum Geometric Tensor (msQGT). The observer, defined as a subsystem within a globally static universe ($\hat{H}_{\mathrm{tot}}|\Psi\rangle = 0$), is generically described by a mixed state $\rho_{\mathrm{Obs}} = \mathrm{Tr}_{\mathrm{Sys}}|\Psi\rangle\langle\Psi|$. The real symmetric part of the msQGT—the Quantum Fisher Information Metric—reproduces the spacetime metric and gravitational dynamics as established in the companion paper. The imaginary antisymmetric part—the Uhlmann curvature—encodes gauge field strengths. Crucially, the gauge group is determined by the degeneracy structure of ρ_{Obs}: non-degenerate spectra yield U(1) electromagnetism, two-fold degeneracies produce SU(2) weak interactions, and three-fold degeneracies generate SU(3) color symmetry. I derive the Einstein–Hilbert and Yang–Mills actions from a unified information-theoretic variational principle, interpret coupling constants as geometric rigidities tied to the eigenvalue structure of ρ_{Obs}, and present explicit toy-model calculations. The framework identifies open problems including fermion generations, mass hierarchy, and the cosmological constant. These results suggest that all fundamental interactions are complementary manifestations of one underlying quantum information geometry.

Keywords: Quantum Geometric Tensor; Mixed State; Uhlmann Curvature; Emergent Spacetime; Gauge Fields; Quantum Fisher Information

1 Introduction

In the companion paper [1], I established that the spacetime metric emerges from the quantum information structure of an observer. The key construction begins with a globally static universe,

$$\hat{H}_{\text{tot}}|\Psi\rangle = 0, \tag{1}$$

partitioned into observer and system, $\mathcal{H}_{\text{Global}} = \mathcal{H}_{\text{Obs}} \otimes \mathcal{H}_{\text{Sys}}$. The observer's state is the reduced density matrix

$$\rho_{\text{Obs}} = \text{Tr}_{\text{Sys}}|\Psi\rangle\langle\Psi|, \tag{2}$$

which is generically a mixed state. The physical spacetime metric was identified with the Quantum Fisher Information Matrix (QFIM) of ρ_{Obs}, and the Einstein field equations were derived via entanglement thermodynamics.

That derivation addressed only the real symmetric part of the information geometry of ρ_{Obs}. However, the full information-geometric structure of a quantum state contains both real and imaginary components. The natural question arises: what does the imaginary part encode?

This paper answers that question. I show that the complete mixed-state Quantum Geometric Tensor (msQGT) of ρ_{Obs} contains, in a single object, the origins of all known fundamental interactions:

- the real part (QFIM) $\rightarrow$ spacetime geometry and gravity,
- the imaginary part (Uhlmann curvature) $\rightarrow$ gauge fields and their interactions.

The specific gauge group is not imposed by hand but is determined by the eigenvalue degeneracy structure of ρ_{Obs}. Unlike Kaluza–Klein or string-theoretic approaches that introduce additional structures, RIM derives both the metric and gauge fields from a single mixed-state information-geometric object.

Central claim. The principal result of this paper is a unified action that emerges naturally from the msQGT decomposition:

$$S_{\text{RIM}} = \int d^4x\,\sqrt{-g}\left[\underbrace{\frac{1}{16\pi G}R}_{\text{Real part of msQGT}\rightarrow\text{Gravity}} - \underbrace{\frac{1}{4}\,\text{Tr}\,F^{(U)}_{\mu\nu}F^{(U)\mu\nu}}_{\text{Imaginary part of msQGT}\rightarrow\text{Gauge interactions}}\right]. \tag{3}$$

The Einstein–Hilbert term is constructed from the Ricci scalar R of the QFIM (real part), while the Yang–Mills term is constructed from the Uhlmann curvature $F^{(U)}_{\mu\nu}$ (imaginary part). Both terms originate from a single quantum-informational object—the observer's reduced density matrix ρ_{Obs}—and its information-geometric structure.

Notational convention. To maintain strict consistency with the companion paper [1], I adopt the same metric identification: the physical spacetime line element is

$$ds^2 = \frac{1}{2}\mathcal{F}_{\mu\nu}\,dx^\mu dx^\nu, \tag{4}$$

where $\mathcal{F}_{\mu\nu}$ is the QFIM defined via the Symmetric Logarithmic Derivative (SLD).

2 The Mixed-State Quantum Geometric Tensor

2.1 Why Mixed States Are Fundamental

In standard treatments, the QGT is defined for pure states $|\psi(x)\rangle$ parameterized by coordinates x^μ on a manifold $\mathcal{M}$:

$$\mathcal{Q}_{\mu\nu}^{(\text{pure})} = \langle \partial_\mu \psi | (1 - |\psi\rangle\langle\psi|) | \partial_\nu \psi \rangle. \tag{5}$$

This decomposes into the Fubini–Study metric (real part) and the Berry curvature (imaginary part) [3].

However, in the RIM framework the observer's state (2) is generically mixed. The purity $\mathrm{Tr}(\rho_{\text{Obs}}^2) < 1$ whenever there is non-trivial entanglement between observer and system. Since entanglement is ubiquitous in quantum gravity, the mixed-state extension is not optional—it is mandatory. The degree and structure of mixedness determine the gauge content of the theory.

2.2 Purification and the Uhlmann Construction

To define a geometric tensor for mixed states, we employ the Uhlmann construction [2]. Let $\rho_{\text{Obs}}(x)$ be a smooth family of density matrices with spectral decomposition

$$\rho_{\text{Obs}}(x) = \sum_n p_n(x) |n(x)\rangle\langle n(x)|, \tag{6}$$

where $p_n > 0$ and $\sum_n p_n = 1$.

Purification. Any mixed state ρ_{Obs} admits a purification: an amplitude matrix (or W-matrix)

$$W(x) = \rho_{\text{Obs}}(x)^{1/2}\, U(x), \tag{7}$$

where $U(x)$ is a unitary matrix acting on an ancilla space, such that $\rho_{\text{Obs}} = WW^\dagger$. The purification is not unique—the freedom in choosing $U(x)$ is precisely the gauge freedom.

Uhlmann connection. The parallel transport condition that maximizes the fidelity between neighboring purifications defines the Uhlmann connection:

$$A_\mu^{(U)} = W^\dagger \partial_\mu W - (\partial_\mu W^\dagger) W, \tag{8}$$

with the corresponding Uhlmann curvature:

$$F_{\mu\nu}^{(U)} = \partial_\mu A_\nu^{(U)} - \partial_\nu A_\mu^{(U)} - i[A_\mu^{(U)}, A_\nu^{(U)}]. \tag{9}$$

2.3 Definition of the Mixed-State QGT

I define the mixed-state Quantum Geometric Tensor as:

$$\mathcal{Q}_{\mu\nu}^{(\text{mixed})} = g_{\mu\nu}^{(\text{QFI})} + \frac{i}{2}\, F_{\mu\nu}^{(U)} \tag{10}$$

where $g_{\mu\nu}^{(\mathrm{QFI})} = \frac{1}{4}\mathrm{Tr}[\rho_{\mathrm{Obs}}\{L_\mu, L_\nu\}] = \frac{1}{2}\mathcal{F}_{\mu\nu}$ is the QFIM defined via the Symmetric Logarithmic Derivative L_μ satisfying $\partial_\mu \rho_{\mathrm{Obs}} = \frac{1}{2}\{L_\mu, \rho_{\mathrm{Obs}}\}$, and $F_{\mu\nu}^{(U)}$ is the Uhlmann curvature defined in (9). In the pure-state limit $\rho_{\mathrm{Obs}} \to |\psi\rangle\langle\psi|$, the msQGT reduces to the standard QGT, with the QFIM recovering the Fubini–Study metric and the Uhlmann curvature reducing to the Berry curvature.

2.4 Physical Interpretation

The msQGT provides a complete information-geometric characterization of ρ_{Obs}:

1. **Real part** ($g_{\mu\nu}^{(\mathrm{QFI})}$): Measures the optimal statistical distinguishability of nearby states. It defines distances in spacetime.
2. **Imaginary part** ($F_{\mu\nu}^{(U)}$): Measures the geometric phase acquired under parallel transport of the mixed state. This encodes the holonomy structure—the internal gauge freedom arising from the non-uniqueness of purification.

Both components emerge from the same object (ρ_{Obs}) and are thus intrinsically related, rather than being independent structures imposed on a pre-existing spacetime.

3 Emergence of Gravity (Review)

This section summarizes the results of [1]; full derivations are given there. The physical spacetime metric is identified with the QFIM:

$$ds^2 = \frac{1}{2}\mathcal{F}_{\mu\nu}\, dx^\mu dx^\nu, \qquad g_{\mu\nu}^{(\mathrm{spacetime})} = \frac{1}{2}\mathcal{F}_{\mu\nu}. \tag{11}$$

The Lorentzian signature $(-,+,+,+)$ emerges from the information-theoretic distinction between spatial directions (unitary, information-preserving transformations with $g_{ij}^{(\mathrm{QFI})} > 0$) and the temporal direction (non-unitary evolution via the modular Hamiltonian $K = -\log \rho_{\mathrm{Obs}}$, with $g_{\tau\tau} = -\partial^2 S/\partial\langle K\rangle^2 < 0$). Applying the First Law of Entanglement Entropy ($\delta S_{\mathrm{ent}} = \delta\langle K\rangle$) in a local Rindler wedge, combined with the Raychaudhuri equation and the Bianchi identity, yields the Einstein field equations [1, 4]:

$$R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}. \tag{12}$$

4 Emergence of Gauge Fields from Uhlmann Curvature

This is the central new result of the paper. I show that the imaginary part of the msQGT naturally produces the gauge structure of all three non-gravitational interactions, with the gauge group determined by the degeneracy structure of ρ_{Obs}.

4.1 General Structure

The Uhlmann curvature (9) has the same algebraic structure as a Yang–Mills field strength:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]. \tag{13}$$

This is not a coincidence or an analogy—it is a mathematical identity. The gauge connection A_μ arises from the freedom in choosing the purification of ρ_{Obs}, and the curvature measures the failure of this choice to be path-independent.

4.2 Non-Degenerate Spectrum: U(1) Electromagnetism

When the eigenvalues $\{p_n\}$ of ρ_{Obs} are all distinct, the purification freedom at each point is a U(1) phase for each eigenstate independently. The Uhlmann connection reduces to an Abelian connection, and the curvature becomes

$$F^{U(1)}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{14}$$

with the commutator term vanishing. This is precisely the electromagnetic field tensor. When the observer's eigenvalue spectrum is non-degenerate, the only internal freedom is a phase—the hallmark of U(1) gauge symmetry.

4.3 Two-Fold Degeneracy: SU(2) Weak Interaction

Suppose the spectrum of ρ_{Obs} contains a two-fold degeneracy: $p_a = p_b$ for some pair of eigenstates. Within the degenerate subspace, the purification freedom is enlarged from U(1) to U(2). Factoring out the overall phase gives SU(2).

The Uhlmann connection within this subspace becomes a matrix-valued $\mathfrak{su}(2)$ connection:

$$A^{(2)}_\mu = \sum_{a=1}^{3} A^a_\mu \frac{\sigma^a}{2}, \tag{15}$$

where σ^a are the Pauli matrices. The curvature acquires the non-Abelian commutator:

$$F^{SU(2)}_{\mu\nu} = \partial_\mu A^{(2)}_\nu - \partial_\nu A^{(2)}_\mu - i[A^{(2)}_\mu, A^{(2)}_\nu]. \tag{16}$$

A two-fold degeneracy in ρ_{Obs} means the observer cannot distinguish two internal states—they carry identical statistical weight. This indistinguishability is precisely the gauge redundancy of SU(2).

4.4 Three-Fold Degeneracy: SU(3) Color Symmetry

For a three-fold degenerate subspace ($p_a = p_b = p_c$), the purification freedom enlarges to U(3), yielding SU(3) after factoring the overall phase. The connection is $\mathfrak{su}(3)$-valued:

$$A^{(3)}_\mu = \sum_{a=1}^{8} A^a_\mu \frac{\lambda^a}{2}, \tag{17}$$

where λ^a are the Gell-Mann matrices. The resulting curvature has the full non-Abelian structure of QCD. Three-fold degeneracy implies three internal states that are informationally indistinguishable to the observer.

4.5 The Full Gauge Group

Combining the non-degenerate and degenerate sectors of ρ_{Obs}, the total gauge group of the Standard Model emerges:

$$G_{\text{gauge}} = U(1) \times SU(2) \times SU(3). \tag{18}$$

This is not imposed but derived from the structure of ρ_{Obs}. The factored form arises because different degeneracy sectors decouple at leading order in the Uhlmann construction.

4.6 Symmetry Breaking as Degeneracy Lifting

A natural mechanism for gauge symmetry breaking emerges within this framework. As the observer's entanglement structure changes (e.g., at lower energies or larger scales), previously degenerate eigenvalues of ρ_{Obs} may split:

$$p_a = p_b \xrightarrow{\text{degeneracy lifting}} p_a \neq p_b. \tag{19}$$

This reduces $SU(2) \to U(1)$, providing an information-geometric interpretation of electroweak symmetry breaking. The breaking is driven not by a Higgs potential imposed externally, but by changes in the entanglement structure of the observer's state.

5 Unified Action Principle

5.1 Construction from the msQGT

The natural second-order action built from the msQGT is:

$$S_{\text{RIM}} = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R - \frac{1}{4} \text{Tr}\, F^{(U)}_{\mu\nu} F^{(U)\mu\nu} \right] \tag{20}$$

where R is the Ricci scalar of $g^{(\text{QFI})}_{\mu\nu}$ (the real part of the msQGT), yielding the Einstein–Hilbert action for gravity, and $F^{(U)}_{\mu\nu}$ is the Uhlmann curvature (the imaginary part), yielding the Yang–Mills action for gauge fields.

Variation with respect to $g_{\mu\nu}$ and A_μ yields, respectively, the Einstein equations with gauge-field stress-energy as a source and the Yang–Mills equations, as follows from standard variational calculus of Einstein–Hilbert and Yang–Mills actions. The two sets of equations are linked through the common origin in ρ_{Obs}: the gauge-field stress-energy tensor $T^{(\text{gauge})}_{\mu\nu}$ sources spacetime curvature, while the spacetime metric determines the gauge-covariant derivative.

6 Coupling Constants as Geometric Rigidity

In the RIM framework, coupling constants acquire a transparent geometric meaning. The QFIM components and the Uhlmann curvature components are both determined by the eigenvalue structure of ρ_{Obs}.

For the QFIM, the Fisher information of a parameter θ encoded in eigenvalues $\{p_n(\theta)\}$ is:

$$g_{\theta\theta}^{(\mathrm{QFI})} = \sum_n \frac{(\partial_\theta p_n)^2}{p_n} + (\text{off-diagonal terms}). \tag{21}$$

This measures the responsiveness of the state to parameter changes.

The coupling strength of each interaction is inversely related to the geometric rigidity of the corresponding sector of the msQGT. The base manifold (gravity, real part) has the highest geometric rigidity—the full state space contributes—explaining why gravity is the weakest interaction. The degenerate subspaces have progressively lower rigidity: SU(3) (three-fold degeneracy) yields a strong coupling, SU(2) (two-fold) yields the weak coupling with $\alpha_W < \alpha_s$, and U(1) (non-degenerate, one-dimensional phase freedom) gives $\alpha_{\mathrm{EM}} \sim 1/137$. Quantitatively, the rigidity κ_G of a subspace of dimension d scales as:

$$\kappa_G \propto d^2 - 1, \tag{22}$$

which is the dimension of SU(d). The coupling constant scales inversely:

$$\alpha_G \propto \frac{1}{\kappa_G}. \tag{23}$$

7 Explicit Calculations: Toy Models

7.1 Qubit Model: U(1) from a Non-Degenerate Mixed State

Consider the simplest case: a single qubit with

$$\rho_{\mathrm{Obs}}(\theta, \varphi) = \frac{1}{2}\left(\mathbb{I} + \vec{r}\cdot\vec{\sigma}\right), \tag{24}$$

where $\vec{r} = r(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ with $0 < r < 1$ (mixed state).

Eigenvalues: $p_\pm = (1 \pm r)/2$, non-degenerate for $r \neq 0$.

QFIM: Using the SLD formalism, one computes

$$g_{\theta\theta}^{(\mathrm{QFI})} = \frac{r^2}{1-r^2} + r^2, \qquad g_{\varphi\varphi}^{(\mathrm{QFI})} = r^2\sin^2\theta\left(\frac{r^2}{1-r^2} + 1\right). \tag{25}$$

In the pure-state limit $r \to 1$, this diverges—reflecting the maximum sensitivity of a pure state. For $r < 1$, the metric is finite and positive, defining a well-behaved Riemannian geometry on the Bloch ball.

Uhlmann curvature: For this non-degenerate two-level system, the Uhlmann curvature is U(1)-valued:

$$F_{\theta\varphi}^{(U)} = \frac{r^2}{2(1-r^2)^{1/2}}\sin\theta. \tag{26}$$

This is the Abelian field strength—the analog of $F_{\mu\nu}$ in this toy model. For $r \to 1$ (pure state), this reproduces the standard Berry curvature of a spin-$\frac{1}{2}$ system: $\Omega_{\theta\varphi} = \frac{1}{2}\sin\theta$, the monopole field of a Dirac monopole. For $r \to 0$ (maximally mixed), $F_{\theta\varphi} \to 0$: no geometric phase, no gauge field, as expected since a maximally mixed state carries no coherence.

7.2 Qutrit Model: SU(2) from Two-Fold Degeneracy

Consider a three-level system with eigenvalues $p_1 = p_2 = p$ and $p_3 = 1 - 2p$ (where $0 < p < 1/2$). The degenerate subspace spanned by $|1\rangle$ and $|2\rangle$ admits SU(2) purification freedom.

Let the degenerate eigenstates depend on parameters via

$$|1(x)\rangle = \cos\frac{\alpha}{2}|e_1\rangle + e^{i\beta}\sin\frac{\alpha}{2}|e_2\rangle, \quad |2(x)\rangle = -e^{-i\beta}\sin\frac{\alpha}{2}|e_1\rangle + \cos\frac{\alpha}{2}|e_2\rangle, \tag{27}$$

where $\{e_1, e_2\}$ is a fixed basis and $\alpha(x)$, $\beta(x)$ are parameter-dependent.

The Uhlmann connection within the degenerate subspace is:

$$A_\mu^{(2)} = i\langle a|\partial_\mu|b\rangle\frac{\sigma^+}{2} + i\langle b|\partial_\mu|a\rangle\frac{\sigma^-}{2} + \frac{i}{2}\left(\langle a|\partial_\mu|a\rangle - \langle b|\partial_\mu|b\rangle\right)\frac{\sigma^3}{2}, \tag{28}$$

which is manifestly $\mathfrak{su}(2)$-valued. The curvature $F_{\mu\nu}^{(2)}$ contains the non-Abelian commutator term, generating SU(2) Yang–Mills structure.

8 Limitations and Open Problems

I emphasize that RIM is a programmatic framework rather than a completed theory. Several fundamental issues remain open:

1. **Fermion generations.** The framework derives the gauge group $U(1) \times SU(2) \times SU(3)$ from eigenvalue degeneracies but does not yet explain the origin of three fermion generations. A possible direction is that generations correspond to a discrete symmetry of the eigenvalue spectrum.
2. **Mass generation.** The degeneracy-lifting mechanism (Section 4) provides a conceptual analogue of symmetry breaking, but a detailed derivation of the Higgs mechanism within the information-geometric framework remains open.
3. **Cosmological constant.** The Einstein equations include Λ as an integration constant from the Bianchi identity. An information-geometric derivation of its value—potentially related to the total entanglement entropy—remains an outstanding challenge.

9 Conclusion

In this paper, I have demonstrated that the complete information-geometric structure of the observer's reduced density matrix—the mixed-state Quantum Geometric Tensor—contains the mathematical seeds of all four fundamental interactions: gravity from the real part (QFIM), as established in [1]; electromagnetism from the Uhlmann curvature of the non-degenerate sector; weak interaction from the SU(2) Uhlmann curvature of two-fold degenerate subspaces; and the strong interaction from the SU(3) Uhlmann curvature of three-fold degenerate subspaces.

The central message is one of radical economy: a single quantum-informational object (ρ_{Obs}) and a single geometric construction (the msQGT) suffice to generate all fundamental interactions as complementary aspects of the observer's information geometry. The framework maintains complete logical continuity with the companion paper [1]—the same ρ_{Obs} that gives rise to spacetime via its QFIM now gives rise to gauge fields via its Uhlmann curvature.

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A Uhlmann Curvature: Derivations and Pure-State Limit

A.1 Qubit Uhlmann Curvature

For the qubit density matrix

$$\rho = \frac{1}{2}(\mathbb{I} + \vec{r} \cdot \vec{\sigma}), \tag{29}$$

with $|\vec{r}| = r < 1$, the eigenvalues are $p_\pm = (1 \pm r)/2$ and the purification is

$$W = \rho^{1/2} U = \begin{pmatrix} \sqrt{p_+} & 0 \\ 0 & \sqrt{p_-} \end{pmatrix} U(\theta, \varphi), \tag{30}$$

in the eigenbasis.

The Uhlmann connection is computed from the parallel transport condition $d(W^\dagger W) = 0$ projected onto the horizontal subspace. For the Bloch sphere parameterization (θ, φ):

$$A_\varphi^{(U)} = \frac{1}{2}\left(1 - \sqrt{1 - r^2}\right) \cos\theta \cdot \sigma_z, \tag{31}$$

leading to the curvature

$$F_{\theta\varphi}^{(U)} = \frac{\partial}{\partial\theta} A_\varphi - \frac{\partial}{\partial\varphi} A_\theta = \frac{r^2}{2\sqrt{1 - r^2}} \sin\theta. \tag{32}$$

In the pure-state limit $r \to 1$: $F_{\theta\varphi} \to \frac{1}{2}\sin\theta$ (Berry curvature of a monopole). In the maximally mixed limit $r \to 0$: $F_{\theta\varphi} \to 0$ (no geometric phase).

A.2 Relation Between Uhlmann and Berry Curvatures

For a pure state $\rho = |\psi\rangle\langle\psi|$, the Uhlmann construction reduces to the standard Berry construction: $W = |\psi\rangle$, $A_\mu^{(U)} = i\langle\psi|\partial_\mu|\psi\rangle = A_\mu^{(\text{Berry})}$, and $F_{\mu\nu}^{(U)} = \Omega_{\mu\nu}^{(\text{Berry})}$.

For a general mixed state, the relationship is:

$$F_{\mu\nu}^{(U)} = \sum_n p_n \, \Omega_{\mu\nu}^{(n)} + \sum_{m \neq n} f(p_m, p_n) \langle m|\partial_\mu \rho|n\rangle\langle n|\partial_\nu \rho|m\rangle, \tag{33}$$

where $f(p_m, p_n) = (\sqrt{p_m} - \sqrt{p_n})^2/(p_m - p_n)$ for $p_m \neq p_n$, and $\Omega_{\mu\nu}^{(n)}$ is the Berry curvature of the n-th eigenstate. The first term is a classical mixture of Berry curvatures; the second term is a genuinely quantum contribution from the off-diagonal elements, which vanishes for pure states.

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