General relativity is the global solution

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ABSTRACT
General relativity is a constrained theory built upon a handful of postulates including general covariance, diffeomorphism invariance, and the relational character of spacetime. These postulates are not optional features. They are the structural commitments that give general relativity physical meaning. When these commitments are enforced exactly, without supplementary assumptions, the theory selects a unique static geometry governed by a single measured scale. This geometry reproduces the linear Hubble law, the nonlinear curvature of the Type Ia supernova distance modulus, and the CMB monopole temperature, without a cosmological constant and without dark energy. The geometry predicts zero redshift drift. Any expanding model predicts nonzero drift. The ELT-ANDES measurement is a clean, binary test.

Key words: cosmology: theory – gravitation – large-scale structure of Universe – dark energy

1 INTRODUCTION

Lovelock's theorem states that in four dimensions, the Einstein field equations (with cosmological constant) are the unique second-order, divergence-free equations of motion derivable from a local action constructed solely from the metric tensor (Lovelock 1971, 1972). Any attempt to modify gravity while keeping those conditions necessarily gives you back the Einstein equations.

The completeness of these equations is routinely ignored. Every redshift, every geodesic, every potential gradient that matter produces is encoded in the solution. There is nothing to add and nothing to subtract. If a physical effect is present in the geometry, it is present in the observables. If an effect is absent from a model built on these equations, the model is incomplete.

General covariance is the structural commitment that gives these equations physical meaning. It requires that the laws of physics retain their form under arbitrary smooth coordinate transformations. Coordinates carry no physical content. Diffeomorphism invariance is the deeper requirement. The theory is invariant under active diffeomorphisms of the manifold, which means spacetime points have no identity apart from the fields they host (Norton 2019). The geometry encodes relationships between events, not the properties of an independently existing substrate.

Whitehead's philosophy is relevant here.

> "The misconception which has haunted philosophic literature throughout the centuries is the notion of 'independent existence.' There is no such mode of existence; every entity is to be understood in terms of the way it is interwoven with the rest of the universe." (Whitehead 1978)

General relativity is the mathematical embodiment of this principle. The field equations are local. They relate the curvature at a point to the stress–energy at that point. They do not require global boundary conditions, preferred foliations, or universe-wide averaging procedures.

1.1 The cost of the limiting algebra

The Friedmann–Lemaître–Robertson–Walker (FLRW) framework makes three supplementary choices. It adopts a preferred foliation (cosmic time), assumes spatial homogeneity, and averages the matter content to a uniform density before solving the field equations. These choices yield a tractable algebra and have been enormously productive.

They also discard information. The non-commutativity of averaging and solving the field equations has been known since Ellis (1984). Because $G_{\mu\nu}$ is nonlinear in the metric, $\langle G_{\mu\nu}[g]\rangle \neq G_{\mu\nu}[\langle g\rangle]$. The FLRW procedure computes the right-hand side and discards the difference. Buchert (2008) and Wiltshire (2007, 2009) have shown that this difference, the backreaction, can be significant.

The assumed homogeneity is the most consequential of the three choices. Mass density defined as M/V depends on the coordinate volume, which is not a generally covariant quantity. To treat $\rho = M/V$ as a property of a region of space is to grant that region independent existence, precisely what general relativity prohibits. In the actual theory, mass density is a property of an observer's light cone at a given scale, not of a container.

When a limiting algebra discards physical information, the gaps must be filled. The concordance model fills them. Dark energy provides the missing curvature in the Hubble diagram. Dark matter provides the missing gravitational acceleration in galaxies. Inflation solves the horizon and flatness problems created by the global foliation. The pattern is general. Geng et al. (2026) demonstrate that the black hole information paradox arises from artificially restricting the observable algebra. When the full algebraic structure is retained, the apparent mystery dissolves.

The same logic applies here. The observed redshift has two contributions permitted by general relativity, kinematic recession ($z_{\rm exp}$) and the gravitational potential difference between emitter and observer ($z_{\rm grav}$). The FLRW framework sets $z_{\rm grav} = 0$ by averaging the density to a constant. This paper retains the information that averaging discards and finds that $z_{\rm grav}$ alone reproduces the key observables.

This paper enforces the existing rules of general relativity without supplementary assumptions and derives the consequences for null geodesics. The only choice made is a removal. We do not assume that the universe is expanding. Expansion is an input to ΛCDM, not an output. Removing it is the subtraction of an assumption. What remains is general relativity applied to a static, statistically isotropic spacetime, and the mass profile itself is not chosen but derived from general covariance.

2 MASS DENSITY AND GENERAL COVARIANCE

Consider an observer and the mass enclosed within a sphere of coordinate radius r centred on that observer. How should this enclosed mass be characterised in a way that respects general covariance?

The volume of a coordinate sphere, $V = \frac{4}{3}\pi r^3$, is not a geometric invariant. The proper volume of a spacelike region is well-defined but requires a choice of hypersurface, a foliation, which general covariance does not fix. Volume-based characterisations of enclosed mass are therefore not unique without supplementary structure. They depend on a choice the theory leaves open. The area of a 2-sphere requires no such choice. It is constructed from the induced metric on the surface alone, making it quasi-local, observer-operational, and invariant under reparameterisation of the radial coordinate. Surface density is therefore the only characterisation of enclosed mass at a given scale that is definable from data on a single observer's past light cone, requires no global slicing, and reduces to the standard quasi-local mass (Misner–Sharp) in the spherically symmetric limit.

The surface density

$$\sigma(r) \equiv \frac{m(r)}{\pi r^2} \tag{1}$$

is therefore the natural, covariant characterisation of enclosed mass at a given scale.

This is the holographic principle, not as a conjecture imported from quantum gravity, but as the unique consequence of taking general covariance seriously when characterising mass. The Bekenstein–Hawking entropy bound $S_{\rm BH} = \pi r^2/\ell_{\rm P}^2$ (Bekenstein 1973; Hawking 1975) encodes the same content. The information capacity of a region scales with its bounding area, not its volume, because area is the invariant quantity.

Scale independence requires that σ does not single out a preferred radius. Writing $m(r) \propto r^{1+\alpha}$,

$$\sigma \propto r^{\alpha-1}. \tag{2}$$

Three cases arise.

$$\alpha < 1: \quad \sigma \to 0 \text{ at large } r \quad \text{(preferred small scale)}, \tag{3}$$

$$\alpha = 1: \quad \sigma = \text{const} \quad \text{(no preferred scale)}, \tag{4}$$

$$\alpha > 1: \quad \sigma \to \infty \text{ at large } r \quad \text{(preferred large scale)}. \tag{5}$$

A surface density that varies with r introduces a characteristic scale not provided by the theory. Specifying that scale requires supplementary initial or boundary conditions beyond the postulates of general relativity. The constant profile is the unique solution that closes the algebra without restricting it. The mass profile is

$$\boxed{m(r) = 2\pi k\, r^2, \qquad \sigma = 2k = \text{const},} \tag{6}$$

where k has dimensions of mass per area. Steeper profiles ($\alpha > 1$) violate the entropy bound. Shallower profiles ($\alpha < 1$) produce a vanishing field at large r and require a boundary. Only the marginal case is self-consistent. The implied mean density and gravitational acceleration are

$$\bar{\rho}(r) = \frac{3k}{2r}, \qquad a(r) = \frac{Gm(r)}{r^2} = 2\pi G k \equiv a_0 = \text{const}. \tag{7}$$

This profile is empirically accessible. The monotonic decrease of average enclosed density with scale, from stellar interiors ($\sim 10^5\ \text{kg m}^{-3}$) through galaxies ($\sim 10^{-21}$), clusters ($\sim 10^{-24}$), to the cosmic web ($\sim 10^{-26}$), is among the most robustly established facts in observational astronomy. The $1/r$ scaling is the simplest functional form consistent with this hierarchy.

3 THE METRIC

The mass profile $m(r) = 2\pi k\, r^2$ determines a static, spherically symmetric geometry through the standard machinery of general relativity.

3.1 Radial component

The Misner–Sharp relation gives the radial metric coefficient

$$g_{rr} = \left(1 - \frac{2Gm(r)}{c^2 r}\right)^{-1} = \left(1 - \frac{r}{r_{\rm h}}\right)^{-1}, \tag{8}$$

where

$$r_{\rm h} \equiv \frac{c^2}{4\pi G k} \tag{9}$$

is the radius at which the Schwarzschild condition $2Gm(r_{\rm h})/c^2 = r_{\rm h}$ is satisfied. It is a consequence of the mass profile.

3.2 Temporal component

With g_{rr} fixed by the mass profile, the remaining freedom is the temporal component g_{tt}. The TOV equation determines g_{tt} once the radial pressure is specified. Vanishing radial pressure ($p_r = 0$) is the minimal closure for a static configuration. It states that matter is not being pushed or pulled radially, which is what static equilibrium means. Any nonzero p_r would be additional physics imposed on the solution. The tangential pressure is then determined by the field equations as an output.

For a general static metric $ds^2 = -e^{2\Phi} c^2\, dt^2 + (1 - r/r_{\rm h})^{-1} dr^2 + r^2\, d\Omega^2$, the TOV equation with $p_r = 0$ gives

$$\frac{d\Phi}{dr} = \frac{Gm(r)}{c^2\, r\left(r - 2Gm/c^2\right)} = \frac{1}{2(r_{\rm h} - r)}. \tag{10}$$

Integrating with $\Phi(0) = 0$,

$$e^{2\Phi} = (1 - r/r_{\rm h})^{-1}. \tag{11}$$

3.3 The line element

The complete metric is

$$\boxed{ds^2 = -\frac{c^2\, dt^2}{1 - r/r_{\rm h}} + \frac{dr^2}{1 - r/r_{\rm h}} + r^2\, d\Omega^2,} \tag{12}$$

valid for $0 \le r < r_{\rm h}$. Three structural properties distinguish it from the Schwarzschild and de Sitter metrics.

First, $g_{tt} \cdot g_{rr} = -c^2/(1 - r/r_{\rm h})^2$, not $-c^2$. The temporal and radial components do not simply invert. This is the unique signature of distributed matter with $p_r = 0$.

Second, radial null geodesics satisfy $dr/dt = \pm c$ everywhere. The factors of $(1 - r/r_{\rm h})^{-1}$ cancel identically for $ds^2 = 0$. The coordinate speed of light is c at every radius.

Third, the proper radial distance from the origin to the horizon is finite.

$$d_{\rm prop} = \int_0^{r_{\rm h}} \frac{dr}{\sqrt{1 - r/r_{\rm h}}} = 2r_{\rm h}. \tag{13}$$

The coordinate singularity at $r = r_{\rm h}$ is removable.

3.4 Stress–energy content

The Einstein field equations with the metric (12) yield

$$\rho(r) = \frac{k}{r}, \qquad p_r = 0, \qquad p_t = \frac{c^4}{16\pi G\, r_\mathrm{h}(r_\mathrm{h} - r)}. \tag{14}$$

The stress–energy tensor is an *output* of the Einstein equations for this metric. It describes the matter content consistent with the geometry along one observer's light cone, not a global inventory of the contents of the universe. No such inventory exists in general relativity. The field equations are local. A different observer at a different location writes down a different coordinate patch, solves the same equations, and obtains the same functional form, but the specific $T_{\mu\nu}$ is theirs.

The vanishing radial pressure describes matter in radial gravitational equilibrium, supported against collapse by tangential stresses. The field equations produce this for the given mass profile. It is also consistent with the observed large-scale structure. Matter is organised into sheets, filaments, and walls under tangential stress, not radially infalling dust. The geometry does not collapse for the same reason the large-scale structure does not collapse. The tangential stresses that the field equations require are the tangential stresses that the universe exhibits.

All classical energy conditions are satisfied for $0 < r < r_\mathrm{h}$.

4 GRAVITATIONAL REDSHIFT AND THE HUBBLE LAW

A photon emitted at coordinate radius r and received at the origin undergoes a gravitational redshift

$$1 + z = \left(1 - \frac{r}{r_\mathrm{h}}\right)^{-1/2}. \tag{15}$$

Expanding to first order at small r/r_h,

$$z \simeq \frac{r}{2r_\mathrm{h}}. \tag{16}$$

This is the linear Hubble law $cz = H_0\, d$ with the identification

$$H_0 \equiv \frac{c}{2r_\mathrm{h}} = \frac{2\pi G k}{c}. \tag{17}$$

An observer measuring the linear redshift–distance relation cannot distinguish between kinematic recession and this gravitational gradient. The measured value $H_0 \simeq 67$–$73\ \mathrm{km\,s^{-1}Mpc^{-1}}$ (Planck Collaboration 2020; Riess et al. 2022) fixes all geometric scales.

$$k = \frac{cH_0}{2\pi G}, \qquad r_\mathrm{h} = \frac{c}{2H_0}. \tag{18}$$

At higher order, the redshift grows faster than linearly. Inverting equation (15),

$$\frac{r}{r_\mathrm{h}} = 1 - \frac{1}{(1+z)^2} = \frac{z(z+2)}{(1+z)^2}. \tag{19}$$

The nonlinearity produces an upward curvature in the Hubble diagram. In ΛCDM, this curvature requires a cosmological constant. Here it is the natural behaviour of a gravitational potential that deepens with distance.

5 DISTANCE MEASURES AND THE HUBBLE DIAGRAM

For the metric (12), the Sachs optical equations (Appendix A) give the angular-diameter distance through a filled beam of distributed matter (Hadrovic & Binney 1997)

$$D_A(r) = r\sqrt{1 - \frac{r}{r_\mathrm{h}}}. \tag{20}$$

Etherington reciprocity (Etherington 1933; Ellis 2007) gives the luminosity distance

$$D_L = (1+z)^2 D_A = \frac{r}{\sqrt{1 - r/r_\mathrm{h}}}. \tag{21}$$

Substituting equation (19),

$$D_L(z) = r_\mathrm{h}\, z(z+2) = \frac{c}{2H_0}\, z(z+2). \tag{22}$$

This is a closed-form, single-parameter prediction with no free functions, no fitting, and no dark components.

The distance modulus is $\mu = 5\log_{10}(D_L/10\ \mathrm{pc})$. Table 2 compares the shape residual $\Delta\mu \equiv \mu_\mathrm{Brown} - \mu_{\Lambda\mathrm{CDM}}$ with flat ΛCDM ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$) after absorbing the absolute-magnitude degeneracy. For $z < 0.5$ the RMS residual is 0.05 mag. For $z < 1.0$ the maximum excursion is 0.07 mag. Both are well within the intrinsic scatter of Type Ia supernovae (∼ 0.15 mag; Scolnic et al. 2022).

The nonlinear curvature of the Hubble diagram, which ΛCDM attributes to accelerating expansion, arises here from the gravitational potential gradient. The two predictions are observationally indistinguishable at $z < 1$ with present data.

Table 1. Geometric scales for three values of H_0.

H_0 ($\mathrm{km\,s^{-1}\,Mpc^{-1}}$)	r_h (Gpc)	k ($\mathrm{kg\,m^{-2}}$)	a_0 ($\mathrm{m\,s^{-2}}$)
67	2.237	1.552	6.51×10^{-10}
70	2.141	1.622	6.80×10^{-10}
73	2.053	1.691	7.09×10^{-10}

Table 2. Shape residual $\Delta\mu = \mu_\mathrm{Brown} - \mu_{\Lambda\mathrm{CDM}}$ after removing a constant magnitude offset. ΛCDM uses $\Omega_m = 0.3$, $H_0 = 73\ \mathrm{km\,s^{-1}Mpc^{-1}}$.

z	$\Delta\mu$ (mag)	Note
0.05	+0.023	Indistinguishable
0.10	+0.000	Indistinguishable
0.20	−0.034	Indistinguishable
0.30	−0.056	Within SN Ia scatter
0.50	−0.072	Within SN Ia scatter
0.70	−0.060	Within SN Ia scatter
1.00	−0.010	Indistinguishable
1.50	+0.117	Marginal
2.00	+0.264	Regime where beam-filling matters

6 HORIZON TEMPERATURE AND THE THERMAL BACKGROUND

The horizon at $r = r_\mathrm{h}$ has surface gravity

$$\kappa = \frac{c^2}{2r_\mathrm{h}} = cH_0 = a_0. \tag{23}$$

The surface gravity, the background acceleration, and the Hubble scale are the same quantity. The associated Hawking temperature is

$$T_\mathrm{H} = \frac{\hbar\kappa}{2\pi k_\mathrm{B} c} = \frac{\hbar H_0}{2\pi k_\mathrm{B}}. \tag{24}$$

For $H_0 = 70\ \mathrm{km\,s^{-1}Mpc^{-1}}$, $T_\mathrm{H} \simeq 2.76 \times 10^{-30}$ K.

Table 3. Emission offset ϵ from the horizon for $T_{\rm CMB} = 2.7255$ K.

H_0 (km s^{-1} Mpc^{-1})	$r_{\rm h}$ (m)	$T_{\rm H}$ (K)	ϵ (m)	$\epsilon/\ell_{\rm P}$
67	6.90×10^{25}	2.64×10^{-30}	6.48×10^{-35}	4.01
70	6.61×10^{25}	2.76×10^{-30}	6.77×10^{-35}	4.19
73	6.34×10^{25}	2.88×10^{-30}	7.06×10^{-35}	4.37

Radiation emitted at $r = r_{\rm h} - \epsilon$ is blueshifted to the observer at the origin by $(1 - r/r_{\rm h})^{-1/2} = (r_{\rm h}/\epsilon)^{1/2}$. The observed temperature is

$$T_{\rm obs} = T_{\rm H}\sqrt{\frac{r_{\rm h}}{\epsilon}}. \tag{25}$$

Setting $T_{\rm obs} = T_{\rm CMB} = 2.7255$ K (Fixsen 2009) fixes the emission offset.

$$\epsilon = r_{\rm h}\left(\frac{T_{\rm H}}{T_{\rm CMB}}\right)^2 = \frac{(\hbar c)^2}{16\pi^2 k_{\rm B}^2 T_{\rm CMB}^2 r_{\rm h}}. \tag{26}$$

Table 3 evaluates this for three values of H_0. The emission surface sits at a few Planck lengths from the horizon in every case.

This is not arranged. The ratio $\epsilon/\ell_{\rm P} \approx 4$ is not tunable. It is fixed by H_0 and $T_{\rm CMB}$, both measured independently. In a static geometry with a horizon, the Tolman relation determines the temperature gradient. The Planck-scale offset is a consequence of that thermodynamics.

The blackbody spectrum requires no separate explanation. Hawking radiation is thermal by construction. The CMB is the Hawking spectrum of the horizon, blueshifted to the observer.

7 THE DEGENERACY

The gravitational redshift derived in Section 4 is a consequence of applying general relativity to any mass distribution with $\bar{\rho} \propto 1/r$. The potential exists regardless of whether the universe is expanding, and photons traversing it acquire a redshift regardless of the kinematic state of the background.

Since the density hierarchy exists (it is observed) and since general relativity mandates a gravitational redshift for any potential gradient, the gravitational contribution to the observed redshift cannot be zero. One of three conclusions follows.

(i) The gravitational contribution is the full signal, and expansion is absent (the present geometry).
(ii) The gravitational contribution is significant but not dominant, and the inferred expansion rate is overestimated by an amount that has not been computed.
(iii) The gravitational contribution is negligible, but this must be *demonstrated*, not assumed.

The FLRW framework adopts option (iii) by averaging the density to a constant before solving the field equations, which sets the gravitational contribution to zero by construction. This paper investigates option (i) and finds that it reproduces the key observables.

Table 4 summarises the structural comparison.

8 BREAKING THE DEGENERACY WITH THE SANDAGE–LOEB TEST

The observational degeneracy demonstrated in Section 7 covers the Hubble law and the supernova distance modulus. There is one observable that breaks it, the redshift drift $\dot{z}$ (Sandage 1962; Loeb 1998).

In an expanding universe, the redshift of a comoving source changes with time as the scale factor evolves. The predicted drift is $\dot{z} \sim 10^{-10}$ yr^{-1}, with a specific functional form in z determined by the expansion history (Liske et al. 2008; Trost et al. 2025).

In the present geometry, g_{tt} is time-independent. The redshift of a stationary emitter at coordinate r is $1 + z = (1 - r/r_{\rm h})^{-1/2}$, which has no time dependence.

$$\dot{z} = \partial_t \ln(1 + z) = 0. \tag{27}$$

Peculiar accelerations contribute line-of-sight terms with both signs. For a statistically isotropic sample, $\langle \dot{z} \rangle = 0$ (Appendix B).

This is a clean, binary test.

- A positive detection at the ΛCDM level falsifies this geometry.
- A null result falsifies every expanding model.

The ELT with ANDES is designed to detect drift at $\Delta z \sim 10^{-10}$ over a decade. No astrophysical systematics, no nuisance parameters, no model dependence.

9 DISCUSSION

General covariance requires that enclosed mass be characterised by surface density. In spherical symmetry, the 2-sphere area is fixed by the geometry and requires no foliation. The volume element requires a choice of spacelike hypersurface that the theory does not provide. Scale independence of the surface density then selects a unique mass profile. The resulting static geometry reproduces the linear Hubble law and the nonlinear curvature of the Type Ia supernova distance modulus with one free parameter and no dark components. The dark sector of ΛCDM is the closure bookkeeping of a framework that projected out the gravitational redshift by averaging before solving.

The geometry predicts $\dot{z} = 0$ exactly. The ELT-ANDES redshift-drift measurement will distinguish this from the $\dot{z} \neq 0$ prediction of every expanding model within the next decade.

The broader point is methodological. A limiting algebra that discards physical information can generate apparent mysteries that demand new components. The remedy is to check whether the approximation is doing the explaining. The redshift drift is that check.

General relativity is flexible enough that useful physics can be extracted from approximations, foliations, boundary conditions, and other supplementary choices. This flexibility is a strength. But flexibility is not the same as ambiguity. When no supplementary choices are made, the theory is not underdetermined. It is exact. It selects a single geometry, and that geometry is the one derived here.

DATA AVAILABILITY

No new data are created. Code to reproduce the numerical values in the tables is available on request.

APPENDIX A: ANGULAR-DIAMETER DISTANCE FROM THE SACHS OPTICAL EQUATIONS

Radial null geodesics of equation (12) have tangent $k^\mu = (\dot{t}, \dot{r}, 0, 0)$ with $\dot{r} = c(1 - r/r_{\rm h})\dot{t}$. The expansion of the null congruence obeys

$$\frac{d\theta}{d\lambda} + \tfrac{1}{2}\theta^2 = -\tfrac{1}{2}R_{\mu\nu}k^\mu k^\nu, \tag{A1}$$

with shear and twist zero by symmetry. The area distance D_A satisfies

$$\frac{d^2 D_A}{d\lambda^2} = -\tfrac{1}{2}R_{\mu\nu}k^\mu k^\nu\, D_A, \tag{A2}$$

Table 4. Structural comparison. The two frameworks produce the same primary observables through different physical mechanisms.

Observable	ΛCDM	This geometry
Free parameters	6 ($H_0, \Omega_b, \Omega_c, \tau, n_s, A_s$)	1 (H_0)
Hubble law	Expansion rate	Gravitational redshift gradient
Hubble diagram curvature	Dark energy (Λ)	Nonlinear gravitational potential
CMB monopole temperature	Thermalisation + expansion cooling	Blueshifted Hawking radiation of the horizon
Stress–energy	Input (perfect fluid + Λ)	Output of Einstein equations
Dark energy	Physical component ($\sim$68%)	Absent
Redshift drift $\dot{z}$	$\neq 0$	$= 0$ exactly

with $D_A \to r$ and $dD_A/d\lambda \to 1$ at the observer. Eliminating λ in favour of r gives $D_A(r) = r\sqrt{1 - r/r_\mathrm{h}}$, consistent with equation (20). Etherington reciprocity gives equation (21).

APPENDIX B: REDSHIFT DRIFT IN A STATIC GEOMETRY

For a static metric with time-independent g_{tt}, the redshift of a stationary emitter at coordinate r is $1 + z = (g_{tt}(0)/g_{tt}(r))^{1/2} = (1 - r/r_\mathrm{h})^{-1/2}$. The drift is $\dot{z} = \partial_t \ln(1 + z) = 0$. Peculiar accelerations give line-of-sight contributions with both signs. For an isotropic sample, $\langle \dot{z} \rangle = 0$.

APPENDIX C: EMISSION OFFSET FROM THE HORIZON

This appendix derives equation (26) from first principles and verifies its self-consistency.

The horizon at $r = r_\mathrm{h}$ satisfies the Schwarzschild condition $2Gm(r_\mathrm{h})/c^2 = r_\mathrm{h}$. For a horizon of this radius, the Hawking temperature is

$$T_\mathrm{H} = \frac{\hbar c}{4\pi k_\mathrm{B} r_\mathrm{h}}. \tag{C1}$$

Since $r_\mathrm{h} = c/(2H_0)$, this is equivalent to $T_\mathrm{H} = \hbar H_0/(2\pi k_\mathrm{B})$, which is equation (24).

The Tolman relation requires that in a static gravitational field, a locally thermal distribution at coordinate r is observed at the origin with a temperature blueshifted by $(1 - r/r_\mathrm{h})^{-1/2}$. Radiation emitted at $r = r_\mathrm{h} - \epsilon$ is therefore observed at temperature

$$T_\mathrm{obs} = T_\mathrm{H} \left(\frac{r_\mathrm{h}}{\epsilon}\right)^{1/2}. \tag{C2}$$

Solving for ϵ,

$$\epsilon = r_\mathrm{h} \left(\frac{T_\mathrm{H}}{T_\mathrm{obs}}\right)^2. \tag{C3}$$

Substituting equation (C1),

$$\epsilon = r_\mathrm{h} \cdot \frac{\hbar^2 c^2}{16\pi^2 k_\mathrm{B}^2 T_\mathrm{obs}^2 r_\mathrm{h}^2} = \frac{\hbar^2 c^2}{16\pi^2 k_\mathrm{B}^2 T_\mathrm{obs}^2\, r_\mathrm{h}}. \tag{C4}$$

This is equation (26) with $T_\mathrm{obs} = T_\mathrm{CMB}$.

As a self-consistency check, set $T_\mathrm{obs} = T_\mathrm{H}$. Equation (C3) gives $\epsilon = r_\mathrm{h}$. When the observed temperature equals the Hawking temperature, the emission offset equals the horizon radius, as expected.

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