On the Origin of the Entropy of Black Holes

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Abstract

Using the c-SRQM framework, a quantum description of the black hole singularity was introduced previously. According to the theory, a black hole singularity is made of a series of concentric spherical shells of a constant diametral step size A. Each shell was shown to be occupied with a fixed set of Primordial Stem Particles (PSP) of rest mass $\bar{m} = h/Ac$. The PSP appears to be defining a regime where the point-like and field-like particle physics meet. The innermost shell with diameter A, if it were to be found standing alone by itself, would be the smallest possible black hole in nature with mass of $M_1 = Ac^2/4G$, named the Unit Black Hole (UBH). In this article we have shown that the Beckenstein-Hawking entropy of black holes can be traced back to the available microstate arrangements of the PSPs on the singularity shells. The Page time and total evaporation time obtained from this theory are shown to be consistent with the existing theoretical predictions, a non-trivial hint to the consistency of the proposed theory.

1 Background

The combined theory of Special Relativity and Quantum Mechanics (c-SRQM) studies the state of an abstract particle under a definite momentum in absolute vacuum - isolated from any external influences [1]. The spacetime coordinate of the abstract particle is shown to be describable by being in an infinite series of *boxes* that are aligned along the particle's trajectory, plus a periodicity condition that guarantees there are no *gaps* between any two walls of adjacent boxes. In the c-SRQM theory, it is postulated that the length of these boxes must obey Lorentz transformation between the inertial observers. The invariant of the Lorentz transformation of the spacetime coordinate uncertainties, ie. the interval A, is found to be the diameter of the smallest possible black hole in nature, named UBH. The invariant A is also the Compton wavelength of a Primordial Stem Particle (PSP) which the UBH singularity thought to be comprised of. Since at its ground state the spatial uncertainty of the PSP is also A, the stem particle seems to be defining a regime where the very notion of "localized particle" is transitioning into the notion of "field". In other words, for a particle whose Compton wavelength is greater than A its rest mass gets so feeble that it physically becomes unresolvable (or immeasurable). Therefore, the invariant A of spacetime uncertainties should be thought as a physical constant which defines the *resolution interval* in nature. A particle with rest mass less than that of stem particle $\bar{m} = h/Ac$ cannot be considered as a localized particle. Having the resolution interval constant A defined, the c-SRQM then ascertains that there must be an upper limit to acceleration $a_u = c^2/A$ in nature. It is further shown in [2] that:

$$\lim_{a \to a_u} \frac{d}{dt'}(\delta x') = c. \tag{1}$$

where t' is coordinate time, a is local acceleration and $\delta x'$ is the spatial uncertainty along the direction of the acceleration a. This physically means that any acceleration higher than the limit a_u would result in a superluminal expansion of the spatial uncertainty of the accelerating object, in a direct violation of GR that prohibits superluminal phenomenon. The limit acceleration must also represent the *gravitational strength at the surface of the physical singularity of the UBH*. This then leads to the UBH mass $M_1 = Ac^2/4G$, where the gravitational strength at the surface of physical singularity (or equivalently at the event horizon) is $a_u = c^4/(4GM_1)$.

The theory is then extended to show that the mass M_b and event horizon D_b of larger black holes with the index b can be quantized using [3]:

$$M_b = M_1 b^3 \tag{2}$$

and:

$$D_b = A\, b^3 \tag{3}$$

Subsequently, the number of the particles $\bar{N}_j$ on each shell j was shown to be given by:

$$\bar{N}_j = (3j^2 - 3j + 1)\bar{N}_1 \qquad j = 1, 2, ..., b \tag{4}$$

where $\bar{N}_1$ is the number of PSP constituents of quantum index $n = 1$ on the innermost UBH shell given by :

$$\bar{N}_1 = \frac{M_1}{\bar{m}} = \frac{A^2 c^3}{4Gh} = \frac{1}{4}(\frac{A}{L_p})^2 \tag{5}$$

From Eqn 5 we noted that since *perfect squares of odd numbers are never divisible by* 4, the necessity of $\bar{N}_1 \in \mathbb{N}$ then demands that *the Planck length* L_p *must be an even divisor of UBH diameter* A. Finally, recalling that $\bar{N}_1$ is the *quanta* of particle count in black hole singularity, the number of total particles $\bar{N}_b$ in a black hole with index b is given by:

$$\bar{N}_b = \sum_{j=1}^{b} (3j^2 - 3j + 1)\bar{N}_1 = \bar{N}_1\, b^3 \tag{6}$$

2 Quantized Hawking temperature

Starting directly from Hawking radiation temperature and substituting for the black hole mass $M_b = M_1 b^3$ from Eqn 2 we have:

$$T_b = \frac{\hbar c^3}{8\pi\kappa G M_1 b^3} \tag{7}$$

and substituting further for the UBH mass $M_1 = Ac^2/4G$ and simplifying using Planck temperature $T_p = m_p c^2/\kappa$, Planck mass $m_p = \sqrt{ch/G}$ and Planck length $L_p = \sqrt{Gh/c^3}$, all "non-reduced" Planck constants, we will eventually arrive at the quantized form of Hawking radiation of black holes as follows:

$$T_b = \frac{1}{4\pi^2} \frac{L_p}{D_b} T_p \tag{8}$$

3 Quantized entropy

For the black hole entropy S, from Beckenstein-Hawking equation, we have:

$$S = \frac{\kappa}{4l_p^2} H \tag{9}$$

where κ is the Boltzmann constant, l_p is the reduced Planck length and H is the surface area of the event horizon. Knowing the event horizon diameter $D_b = b^3 A$, we then have the surface area H quantized as :

$$H_b = \pi A^2 b^6 \tag{10}$$

Substituting for the latter in the Beckenstein-Hawking equation above, we then have the entropy of black holes quantized as:

$$S_b = \pi\kappa\,(\frac{A^2}{4l_p^2})\,b^6 \tag{11}$$

The rate of change in black hole entropy ΔS_b in terms of the quantum index b is therefore given by:

$$\Delta S_b = \frac{3\pi\kappa A^2}{2l_p^2}\,b^5 \tag{12}$$

For the total energy of a black hole from Eqn 2 we have:

$$E_b = M_1 b^3 c^2 \tag{13}$$

Similarly, the rate of change in black hole energy ΔE_b in terms of the quantum index b is therefore given by:

$$\Delta E_b = 3M_1 c^2 b^2 \tag{14}$$

It is trivial to show that the quantized Hawking temperature T_b of Eqn 8 can be retrieved by substituting for the entropy and energy terms from Eqns 12 and 14 in $\Delta S_b = \Delta E_b / T_b$. More importantly, however, by substituting for the term $A^2/4l_p^2$ in Eqn 11 in terms of $\bar{N}_1$ from Eqn 5, the entropy of black holes can be expressed in terms of the of their constituents as follows:

$$S_b = 2\pi^2 \kappa \bar{N}_1 b^6 \tag{15}$$

Comparing Eqn. 15 with the general definition of entropy $S_b = \kappa \ln \Omega_b$, the number of microstates Ω_b of a black hole with quantum index b will be:

$$\Omega_b = e^{2\pi^2 \bar{N}_1 b^6} \tag{16}$$

In the next section we show that the total number of quanta emitted in the emission process of a black hole is directly proportional to its entropy.

4 Evaporation quanta

The backward difference in a black hole energy from Eqn 13 under a single quantum index dropping from b to $b-1$ is then given by:

$$\Delta E_b = M_1 c^2 \left[b^3 - (b-1)^3\right] \tag{17}$$

Similarly, the backward difference in a black hole entropy from Eqn 15 under a single quantum index dropping from b to $b-1$ is given by:

$$\Delta S_b = 2\pi^2 \kappa \bar{N}_1 \left[b^6 - (b-1)^6 \right] \tag{18}$$

Using above, the quanta of Hawking radiation energy $Q_b = \kappa \Delta E_b / \Delta S_b$ is given by:

$$Q_b = \frac{1}{2\pi^2(2b^3 - 3b^2 + 3b - 1)} \frac{h}{Ac} \tag{19}$$

Therefore, for the total number of emitted quanta N_{evp} during complete evaporation of a black hole we will have:

$$N_{\text{evp}} = \frac{\text{total energy emitted}}{\text{emitted energy per quanta}} = \frac{E_b}{Q_b} \tag{20}$$

Substituting from Eqns 13 and 19 we then have:

$$N_{\text{evp}} = 2\pi^2 \bar{N}_1 (2b^6 - 3b^5 + 3b^4 - b^3) \tag{21}$$

Comparing with the equation of entropy $S_b = 2\pi^2 \kappa \bar{N}_1 b^6$ we then conclude:

$$N_{\text{evp}} = \xi \frac{S_b}{\kappa} = \xi \ln(\Omega_b) \tag{22}$$

where the bounds of coefficient $1 \leq \xi < 2$ correspond to bounds of index $1 \leq b < \infty$, covering the entire range of black holes mass, from that of the UBH to the larger ones. From Eqn 22 it is clear that total number of emitted quanta in black hole radiation is proportional to the logarithm of the number of microstates Ω_b in the singularity. This effectively means *evaporation of black holes is a slow process in which all the internal microstates of the constituents of the singularity are eventually exhausted.*

5 Evaporation time

Starting from $M_b = M_1 b^3$, we then have:

$$\frac{dM_b}{dt} = 3b^2 M_1 \frac{db}{dt} \tag{23}$$

Knowing $\dot{M}_b = -\beta / M_b^2$ and the constant $\beta = \hbar c^4 / (15360 \pi G^2)$, we therefore have:

$$\frac{-\beta}{M_1^2 b^6} = 3b^2 M_1 \frac{db}{dt} \tag{24}$$

From above we then have:

$$b^8 \, db = \frac{-\beta}{3M_1^3} \, dt \tag{25}$$

Integrating the above from the black hole's initial quantum index $b_i = b$ at time zero to quantum index 0 after t_{evp} time:

$$\int_b^0 b^8 db = \frac{-\beta}{3M_1^3} \int_0^{t_{evp}} dt \tag{26}$$

we will have:

$$t_{evp} = \frac{M_1^3}{3\beta} \, b^9 \tag{27}$$

which indicates the evaporation time $t_{evp} \propto M_b^3$. [4]

6 Page parameters

In this section we now find the "Page index" b_P which by definition corresponds to an epoch wherein half of the initial black hole entropy S_b is radiated away [5], i.e. $S_{b_P} = 0.5S_b$. From Eqn 15, we have $b_P^6 = 0.5b^6$, therefore:

$$b_P = 2^{-\frac{1}{6}}b \tag{28}$$

Given $2^{-1/6} \approx 0.89$, from above it is concluded that about 11 % reduction in the initial quantum index corresponds to loss of half of the black hole entropy. This further confirms that the PSP constituents comprising the outer layers of the singularity are more closely packed compared to those of the inner shells and half of the available degrees of freedom in the microstates are coming from top 11 % of the shells. With this, the mass corresponding to half entropy is given by:

$$M_P = M_1(2^{-1/6}b)^3 = \frac{1}{\sqrt{2}}\,M_1b^3 = 0.707M_b \tag{29}$$

meaning the Page time occurs when about 30 % of the original mass of the black hole is radiated away. Given Hawking temperature is inversely proportional to the mass M_b, we therefore have:

$$T_P = \sqrt{2}\,T_b \tag{30}$$

Finally, from Eqn 27 the time corresponding to half entropy can be simply obtained from $t_P = M_1^3(b^9 - b_P^9)/3\beta$. Substituting for b_P from Eqn 28, the time needed to arrive at half entropy condition is given by:

$$t_P = \frac{M_1^3}{3\beta}(1 - \frac{1}{2\sqrt{2}})b^9 \approx 0.65\,t_{evp} \tag{31}$$

Substituting the age of universe for t_P in above and solving for the required index to meet the condition we found that $\bar{b} << 1$. This simply means the universe is way too young for any of its primordial black holes to have crossed the Page stage.

7 Conclusion

The combined theory of Special Relativity and Quantum Mechanics, c-SRQM, provides a framework through which the singularity of black holes can be described by a set of concentric spherical shells each comprised of a distinct number of primordial stem particles at a distinct quantum state. It is shown that during the Hawking evaporation of a black hole, the total number of emitted quanta can be traced back to the number of internal microstates of the PSPs in the singularity. Half of the entropy of a black hole is at the outer 11 % shells of its singularity.

References

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