

Construction Of Pythagorean Triple

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Abstract

A very common term in mathematics is " $a^2 + b^2 = c^2$ ". This equation has been discussed since many years ago. This equation is called the Pythagorean Theorem. And for a, b and c as positive integers a, b and c are called Pythagorean triples. For example, 3, 4, 5 is a Pythagorean triple. Because, $3^2 + 4^2 = 5^2$. Some more examples are - (5,12,13), (9,12,15), (12,16,20), (13,84,85) etc. Any number has more than one Pythagorean triple. For example, with 12 numbers (5,12,13), (9,12,15), (12,16,20) and (12, 35, 37) these four Pythagorean triples are obtained. Now I'm given a positive integer number and I have to show how many Pythagorean triples can be found with that number?

There may have some solutions in the mathematics. This paper provides an alternative solution to evaluate these Pythagorean triplets for any given number.

Discussion

For any b as positive integer, if b is odd and $(b^2 / 2) \% n_i = 0$, or if b is even and $(b/2)^2 \% n_i = 0$, where n_i is the positive real number, the equation $((b/2)^2 / n_i - n_i)^2 + b^2 = ((b/2)^2 / n_i + n_i)^2$ can be expressed as $a^2 + b^2 = c^2$ where, a, b and c can be represented as the Pythagorean triplets and the equation $((b/2)^2 / n_i - n_i)^2 + b^2 = ((b/2)^2 / n_i + n_i)^2$ can be considered as Pythagorean Triple.

Now if the number is a or b, let the number be b, then - if b is odd then the condition is $(b^2 / 2) \% n_i = 0$, and if b is even then $(b/2)^2 \% n_i = 0$,

In that case,

$$((b/2)^2 / n_i - n_i)^2 + b^2 = ((b/2)^2 / n_i + n_i)^2, \text{ where } n_i \text{ is the positive real number.}$$

For example-

If b is an odd number-

b=odd	n_i where $(b^2 / 2) \% n_i = 0$	$a = ((b/2)^2 / n_i - n_i)$	$c = ((b/2)^2 / n_i + n_i)$
1	0.5	0	1
3	1.5	0	3
	0.5	4	5

5	2.5	0	5
	0.5	12	13
7	3.5	0	7
	0.5	24	25
9	4.5	0	9
	1.5	12	15
	0.5	40	41
13	6.5	0	13
	0.5	84	85
45	22.5	0	45
	13.5	24	51
	12.5	28	53
	7.5	60	75
	4.5	108	117
	2.5	200	205
	1.5	336	339
	0.5	1012	1013

Again if b is an even number -

b= even	n_i where $(b/2)^2 \% n_i = 0$	$a = ((b/2)^2 / n_i - n_i)$	$c = ((b/2)^2 / n_i + n_i)$
2	1	0	2
4	2	0	4
	1	3	5

6	3 1	0 8	6 10
8	4 2 1	0 6 15	8 10 17
10	5 1	0 24	10 26
12	6 4 3 2 1	0 5 9 16 35	12 13 15 20 37
44	22 4 2 1	0 117 240 483	44 125 244 485

Now if the value of c is known then-

From the equation $((b/2)^2 / n_i - n_i)^2 + b^2 = ((b/2)^2 / n_i + n_i)^2$ and the relation $a^2 + b^2 = c^2$ we get-

$$((b/2)^2 / n_i + n_i)^2 = c^2$$

$$\text{Or, } (b/2)^2 / n_i = c - n_i$$

$$\text{Or, } (b/2)^2 = n_i (c - n_i)$$

$$\text{Or, } b/2 = \sqrt{n_i (c - n_i)}$$

$$\text{Or, } b = 2\sqrt{n_i(c - n_i)}$$

$$\text{and } a = (b/2)^2 / n_i - n_i$$

$$= (2\sqrt{n_i(c - n_i)})^2 / 2 / n_i - n_i$$

$$= (n_i(c - n_i)) / n_i - n_i$$

$$= (c - n_i) - n_i$$

$$= c - 2n_i$$

then,

$c^2 = (c - 2n_i)^2 + (2\sqrt{n_i(z - n_i)})^2$ when $2\{n_i(c - n_i)\} \% n_i = 0$ for odd values of c or $\{n_i(c - n_i)\} \% n_i = 0$ for even values of c .

Here it is seen that if c is even then both a and b are even and if c is odd then at least one of a and b must be odd.