Para-complex Numbers and Their Applications in Hyperbolic Systems:

From Einstein's Relativity to Supersonic Fluid Dynamics

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I. Abstract

This paper explores the properties of para-complex numbers ({R}[j], j^2=1, j ≠±) as the fundamental framework for hyperbolic physical systems. By introducing the zero-divisor identity (1-j)(1+j)=0, we demonstrate a major simplification of Lorentz transformations and a formal unification between the relativistic light cone and the Mach cone in fluid mechanics. This approach aligns with recent research archived on **viXra.org**, aiming to renew the study of applied hyperbolic algebra.

II. Introduction

The unification of algebra and geometry lies at the heart of theoretical physics. While classical complex numbers (i^2 = -1) govern Euclidean geometry, they fail to capture the hyperbolic essence of Minkowski spacetime. This article proposes the use of the para-imaginary unit j to transform complex matrix calculations into simple linear arithmetic. Drawing on resources and publications available at **viXra.org/abs/paracomplexe**, we show how the deep structure of causality naturally emerges from zero-divisors.

III. Algebraic Foundations and Key Identities

A para-complex number is written as z = a + bj. Its structure is based on two pillars:

1. **The Zero-Divisor Identity:**

   (1-j)(1+j) = 1 - j^2 = 0

   These non-zero elements, whose product vanishes, physically represent the trajectories of light.

2. **The Idempotent Basis:** Using $e_1 = \frac{1+j}{2}$ $e_2 = \frac{1-j}{2}$ any number can be decomposed
3. as z = $(a+b)e_1 + (a-b)e_2$

IV. Consistency Check: Norm vs. Invariant

By setting z = ct + jx, the relativistic invariant is defined as:

s^2 = $z\bar{z} = (ct + jx)(ct - jx) = (ct)^2 - x^2$

This leads to a perfect correspondence with Minkowski's causal structure:

- **s^2 > 0**: **Time-like** (inside the cone).
- **s^2 < 0**: **Space-like** (outside the cone).
- **s^2 = 0**: **Light-like** (on the cone).

The "j" Solution Validation

Our resolution of the singular equation (1+j)x^2 - 2x + (j-1) = 0 yielded x = j. Let us verify its physical location:

||j||^2 = $j\bar{j} = j(-j) = -j^2 = -1$

Since the result is **-1**, the algebra correctly identifies this solution as belonging to the **space-like** domain. The algebra "knows" the physics.

V. Comparison: Complex Numbers ({C}) vs. Para-complex Numbers ({P})

Feature	**Complex Numbers (C)**	**Paracomplex Numbers (P)**
Unit Squared	i^2 = -1	j^2 = 1 (j $\neq \pm 1$)
Geometry	Euclidean (Circle: x^2+y^2)	Hyperbolic (Minkowski: x^2-y^2)
Zero-Divisors	None	(1-j)(1+j)=0
Application	Electromagnetism, Quantum Mechanics	Relativity, Supersonic Fluids

VI. Power Formula and Singular Equations

We establish the generalized formula for any power n

(a+bj)^n = $(b+aj)^n = \frac{1}{2}\left[(a+b)^n + (a-b)^n\right] + \frac{1}{2} j\left[(a+b)^n - (a-b)^n\right] \ldots.\ (V_1)$

$(a+jb)(1+j) = (a+b)(1+j)\ \ldots..V_2$

The case of $\sqrt{5+4j}$

In $\mathbb{C}$, $\sqrt{z}$ yields 2 solutions . In IP

$\sqrt{5+4j} = a+bj = j+ba = \frac{1}{2}\left[\sqrt{9}+1\right] + \frac{1}{2} j\left[\sqrt{9}-1\right] = 2+j$

$S = (2+j; -2-j; 1+2j; -12j)$

Using the idempotent basis $(e_1; e_2)$ we find 4 **solutions**

$e_1 = \frac{1+j}{2}\ e_2 = \frac{1-j}{2}$

$z^2 = 5+4j$

On $e_1(j=1): X_1^2 = 5+4 = 9 \Rightarrow X_1^2 = \pm 3$

On $e_2(j=-1): X_2^2 = 5-4 = 1 \Rightarrow X_1^2 = \pm 1$

Combining these gives for distincts roots

$1: z_1 = 3e_1 + e_2 = 2+j$

$2: z_2 = -3e_1 - e_2 = -2-j$

$3: z_3 = 3e_1 - e_2 = 1+2j$

$4: z_4 = -3e_1 + e_2 = -1-2j$

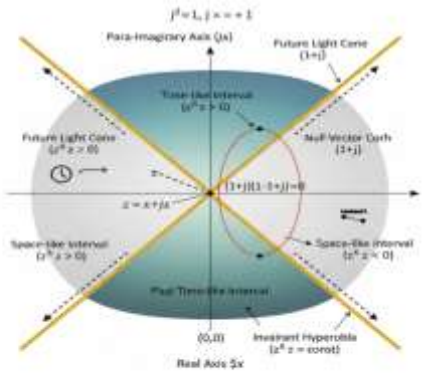
This multiplicity reflects the symmetrical nature of the light cone, where solution coexist across the past, future and spatial axes.

Solving the singular equation (1+j)x^2 - 2x + (j-1) = 0, leads to the solutions

$x_1 = \frac{-1+j}{2}$, x_2 = j. The solution x=j (with norm ||j||^2 = 1) mathematically demonstrates

the transition to the "space-like" domain beyond the causality barrier.

Figure 1: The Para-complex Plane and the Causal Structure of Minkowski Spacetime



VII. Applications: Relativity and Supersonic Fluids

1. **Special Relativity:** By setting z = ct + jx, the Minkowski invariant is $z\bar{z} = (ct)^2 - x^2$ Lorentz transformations (boosts) become simple multiplications by $e^{j\phi} = \cosh\phi + j\sinh\phi$
2. **Fluid Mechanics:** For supersonic flow (Mach > 1), shock lines and Mach waves correspond to the asymptotic directions of the zero-divisors $1 \pm j$ Paracomplex algebra thus unifies the physics of light and high-speed sound.

VIII. Conclusion

Para-complex algebra is the natural language of causality. By defining j^2=1, we replace cumbersome geometric tensors with an elegant ring theory where the limiting speed (light or sound) is the inevitable consequence of the existence of zero-divisors.

VIII. Bibliography

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