The Proton as a Phase Resonator: Quarter-Wave Origin of the Compton Wavelength

"Wave Interpretation of the $\pi/2$ Factor Linking Charge Radius and Compton Frequency"

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Abstract
In a previous work [1], we highlighted a kinematic relation linking the measured charge radius of the proton R_p to a frequency f, whose numerical value coincides remarkably well with the Compton frequency of the proton:

$$f = \frac{c}{(\pi/2)\, R_p}$$

This relation can also be rearranged into a compact geometric form: the non-reduced Compton wavelength of the proton is equal to one quarter of the equatorial perimeter associated with R_p, namely

$$\lambda_{C,p} = \frac{\pi}{2} R_p.$$

The present article continues this analysis by proposing a wave interpretation of the appearance of the factor $\pi/2$. Relying on standard results from wave physics (standing waves, mixed boundary conditions, resonators), we show that in a wave-based reading this factor is neither arbitrary nor fitted, but constitutes the universal signature of a fundamental quarter-wave mode associated with a central phase constraint. Within this framework, the nucleus is no longer interpreted as a rigid boundary, but as a phase tuner.
The same structure is immediately extended to the neutron by simple substitution, providing a coherent length scale associated with its Compton frequency.

Context and starting point

$$f_{(c \cdot p)} = \frac{c}{(\pi/2)\, R_p}$$

Substituting numerical values:

$$\text{Proton radius} = 0.8414 \times 10^{-15}\ \text{m(CODATA 2018 value)}$$
$$\pi/2 \approx 1.570796327$$

The denominator evaluates to:

$$R_p \cdot \pi/2 = (0.8414 \times 10^{-15}) \times 1.570796327 \approx 1.321668029 \times 10^{-15}\ \text{m}$$

Thus,

$$f = \frac{299\,792\,458}{1.321668 \times 10^{-15}} \approx 2.2682886 \times 10^{23}\ \text{Hz.}$$

Comparison with the theoretical Compton frequency

Let us compute f_C from the CODATA 2018 fundamental constants:

$$m_p = 1.67262192369 \times 10^{-27}\ \text{kg}$$

$$f_{(C\cdot p)} = \frac{1.50327759 \times 10^{-10}}{6.62607015 \times 10^{-34}}$$
$$f_{(C\cdot p)} = 2.2687318 \times 10^{23} \text{ Hz.}$$

The precision is of the order of 0.019% with respect to the CODATA 2018 value. This correspondence is therefore numerically very good within the precision of the constants used.

The Compton wavelength,

$$\lambda_C = \frac{h}{mc},$$

is classically introduced as a relativistic scale associated with the phase of a massive particle. Independently, the proton charge radius $R_p$is an experimental quantity derived from electromagnetic form factors. In a previous article, we highlighted the empirical frequency relation

$$f = \frac{c}{(\pi/2)\, R_p},$$

whose numerical value coincides, within experimental precision, with the Compton frequency of the proton

$$f_{(C\cdot p)} = \frac{m_p c^2}{h}.$$

This observation can be rearranged into the equivalent geometric form

$$\lambda_{(C\cdot p)} = \frac{c}{f_{(C\cdot p)}} = \frac{\pi}{2} R_p = \frac{1}{4}(2\pi R_p),$$

that is: the Compton wavelength corresponds to one quarter of the equatorial perimeter associated with the measured charge radius. A previous article by Danish researchers O. L. Trinhammer and H. G. Bohr in 2019 had already pointed out this numerical correspondence [6].
The central question is then: **why the factor $\pi/2$?**

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2. The quarter-wave as minimal phase accumulation

In any linear wave system, a sinusoidal wave accumulates a phase 2πover one wavelength λ. The phase accumulated over a distance $L$is

$$\Delta\phi = kL = \frac{2\pi}{\lambda} L.$$

Imposing a phase accumulation $\Delta\phi = \pi/2$immediately leads to

$$L = \lambda/4.$$

The quarter-wave thus corresponds to the smallest non-trivial length capable of imposing a phase $\pi/2$. This property is independent of the physical support of the wave: it depends only on the boundary conditions.

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3. Mixed boundary conditions and quarter-wave resonators

In wave physics, quarter-wave resonators appear systematically when the boundary conditions are mixed:

- a constraint imposing a zero derivative (Neumann condition, antinode of the wave),
- coupled to a free or open end.

This framework is treated in a standard way in the chapters devoted to standing waves, cavities, and resonators in *Physics of Waves* [2].

In such a system, the spectrum is quantized according to

$$k = \frac{(n + 1/2)\pi}{L},$$

and the fundamental mode ($n = 0$) is precisely a quarter-wave.
The crucial point is that the $+1/2$term—and therefore the $\pi/2$phase—is not fitted: it appears automatically as soon as the boundary conditions are mixed.

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4. Wave interpretation of the proton

The observed relation for the proton,

$$\lambda_{(C,p)} = \frac{\pi}{2} R_p,$$

can then, in this reading, be interpreted as a phase relation.
A minimal interpretation consists in modeling the system as a radial mode for which:

- the center $r = 0$is not a rigid wall, but a symmetry point imposing a Neumann condition

$$\frac{\partial \psi}{\partial r} |_{r=0} = 0,$$

- the exterior is open (radiation, coupling, dissipation).

In such a configuration, the selected fundamental mode is naturally the one that accumulates a phase $\pi/2$over a characteristic length. The charge radius $R_p$is then not a material boundary, but a tuning length, that is, the minimal distance imposing this phase constraint. The nucleus thus appears as a phase tuner, not as a geometrically rigid object.

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5. Immediate extension to the neutron

The previous structure does not depend on charge, but only on mass via the Compton frequency. It is therefore natural to apply the same relation to the neutron by simple substitution:

$$R = \frac{2c}{\pi f_C} = \frac{2}{\pi}\lambda_C.$$

For the neutron,

$$f_{(C,n)} = \frac{m_n c^2}{h} = 2.2718590 \times 10^{23} \text{ Hz}.$$

Let us find the radius from the kinematic equation $f = c/((\pi/2)R)$.

Solving for the radius R:

$$(\pi/2)R = \frac{c}{f}$$
$$R = \frac{2c}{\pi f}.$$
$$R_n = \frac{599\,584\,916}{7.13657 \times 10^{23}} = 8.402 \times 10^{-16} \text{ m}.$$
$$R_n = 0.8402 \text{ fm}.$$

The natural extension of the model to the neutron suggests a radius of 0.84 fm, i.e. very slightly smaller than that of the proton.

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6. Insight into the Proton Radius Puzzle

The proposed wave framework provides an interpretation of the Proton Radius Puzzle.
If the radius $R_p$corresponds to a $\pi/2$phase position, its measurement depends on the ability of the probe particle to couple to the fundamental mode:

- the electron, with a large Compton wavelength, mainly couples to the external region of the mode,
- the muon, being much more massive, penetrates more deeply and probes more directly the fundamental phase structure.

The convergence of muonic hydrogen measurements toward $R_p \simeq 0.841$ fmsuggests that this method captures more faithfully the tuning length of the fundamental mode, whereas electronic measurements may undergo an effective phase shift.

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7. Status and scope of the proposal

It is essential to recall the status of this approach. The relation

$$\lambda_C = \frac{\pi}{2} R$$

is first and foremost a phenomenological observation, and not a law derived from first principles of QCD.
Its interest lies in the fact that it:

- naturally explains the appearance of the factor $\pi/2$,
- links mass, frequency, and geometry through a minimal phase condition,
- fits entirely within the standard framework of wave physics.

It suggests that certain nuclear quantities can be interpreted as phase tuning lengths, opening a complementary path to the usual microscopic descriptions.

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8. Mass, energy, and the speed of light as phase phenomena

Within the proposed quarter-wave interpretation of the proton, the relativistic–quantum relation

$$E = mc^2 = hf$$
$$E/h = f = \frac{c}{(\pi/2)R_p} = 2.268 \times 10^{23}\ \text{Hz}$$

acquires a precise geometric and dynamical meaning in the case of the proton. Energy is no longer considered a primitive quantity, but as the expression of a phase frequency. Energy Ecan indeed be associated with a frequency $f = E/h$, and mass then appears as a locked frequency, characterizing a rate of phase accumulation per unit proper time.

In this reading, energy is a frequency, mass is a locked frequency, and the proton is no longer interpreted as a static material object, but as a fundamental phase clock. Its stability follows from the fact that it adopts the minimal fundamental mode compatible with an open system, characterized by a phase accumulation equal to $\pi/2$. The proton radius thus ceases to be a material boundary and becomes a tuning length, that is, the minimal distance required to impose a quarter-cycle of phase.

Within this framework, the factor $c$no longer has the status of a mysterious constant arbitrarily relating space and time. It appears as the speed of phase propagation, linking a geometric length to a frequency. The relation $E = mc^2$can then be interpreted as the expression of a resonance phenomenon: a fundamental frequency, imposed by a quarter-wave phase condition, manifests macroscopically in the form of inertial mass.
The proton thus appears as an elementary phase lock, providing an internal clock from which atomic and electromagnetic scales inherit by amplification.

Conclusion

In a wave interpretation, the factor $\pi/2$linking the proton charge radius to its Compton wavelength appears as the universal signature of a fundamental quarter-wave mode.
This reading transforms a numerical coincidence into a resonance structure: mass becomes a locked frequency, radius a tuning length, and the nucleus a phase tuner.
This approach remains deliberately phenomenological and does not claim to replace microscopic descriptions arising from QCD or the Standard Model. It does, however, propose a complementary reading, based on general wave principles, likely to shed light on the relation between mass, frequency, and spatial extension in relativistic quantum systems. Natural extensions of this framework concern the neutron, other baryons, as well as experimental tests based on probes of different masses.

This manuscript was originally written in French and translated into English by the author. The author apologizes for any remaining linguistic imperfections and welcomes any suggestions for improving clarity.

References

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