A Rotational and Toroidal Phase Approach to the Restricted Three–Body Problem: Recurrence, Alignment, and Temporal Structure in the Sun–Earth–Moon System

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Abstract

The classical three–body problem is traditionally formulated as the prediction of complete spatial trajectories of three interacting masses under gravitation, a task known to be generally non–integrable and chaotic. In this work, we adopt a complementary perspective focused on the Sun–Earth–Moon system, where the most stable and observable features arise not from translational motion but from rotational recurrence and angular phase closure. We introduce an angular–toroidal phase formalism in which the three bodies are represented by periodic phase variables associated with Earth rotation, Earth orbital motion, and lunar orbital motion. These phases naturally define a three–torus T^3, within which the system evolves as a helical flow. Observable cycles such as the solar day, the synodic month, and the year emerge as alignment events corresponding to phase closure conditions. An alignment operator is proposed to characterize the temporal coherence of these events. The approach does not aim to recover full three–body trajectories, but instead provides an analytic and geometrically transparent description of recurrence and temporal structure in the restricted three–body problem.

1 Introduction

The three–body problem occupies a foundational place in classical mechanics. Since the work of Newton, it has been understood that while the two–body problem admits closed–form solutions, the general three–body problem does not [1]. Subsequent developments by Lagrange identified special equilibrium configurations and periodic solutions [2], while Poincaré later demonstrated that the generic problem is non–integrable and exhibits sensitive dependence on initial conditions, establishing chaos as an intrinsic feature of gravitational dynamics [3].

Modern treatments of the three–body problem therefore rely heavily on numerical integration, perturbation theory, and phase–space analysis [4, 5]. These approaches are indispensable for predicting detailed trajectories and long–term stability. However, many of the most stable and directly observable features of the Sun–Earth–Moon system—such as the solar day, the synodic month, and the year—are not defined by full spatial trajectories, but by recurrent angular alignments.

This distinction suggests that the classical formulation of the three–body problem, expressed primarily in translational Cartesian coordinates, may not be the most natural description for temporal recurrence phenomena. Days, months, and years are not measured as distances traveled, but as closures of rotational phase. Sunrise, lunar phases, and eclipses are alignment events, not trajectory reconstructions.

Motivated by this observation, we adopt a complementary perspective in which the restricted three–body problem is reformulated in terms of angular phase variables. The Earth–Sun–Moon system is described by three dominant periodic motions: the rotation of the Earth about its axis, the orbital motion of the Earth around the Sun, and the orbital motion of the Moon around the Earth. Each motion defines a phase on the circle S^1. Taken together, these phases define a natural configuration space given by the three–torus

$$T^3 = S^1 \times S^1 \times S^1,$$

a structure well known in the theory of quasi–periodic flows and Hamiltonian dynamics [6, 7].

Within this toroidal phase space, the temporal evolution of the system corresponds to a helical flow determined by the angular frequencies of the three motions. Observable cycles arise not from the absolute phases themselves, but from differences between phases. The solar day emerges from the beat between terrestrial rotation and orbital motion; the synodic month emerges from the beat between the lunar orbital phase and the same orbital reference; and the year corresponds to the closure of the Earth's orbital phase.

The central aim of this work is not to replace classical gravitational theory, nor to derive complete three–body trajectories. Instead, we isolate the rotational and phase–closure structure underlying recurrent temporal phenomena in the restricted three–body problem. By focusing on angular alignment and recurrence rather than spatial determinism, we obtain a compact and analytically transparent description of the Sun–Earth–Moon system's temporal architecture.

This angular–toroidal formulation complements existing approaches in celestial mechanics by clarifying why stable cycles persist despite the non–integrability of the full three–body problem. It highlights the role of commensurate frequencies, phase closure, and toroidal geometry as the organizing principles behind observable timekeeping phenomena in astronomy.

Ontologia do Tempo e Acoplamento Fundamental

1. Área de Planck como razão geométrica mínima

A área de Planck A_P é tratada aqui não como unidade física derivada, mas como *razão geométrica mínima*, isto é, o limite inferior de área associado à atualização do presente.

$$A_P \equiv \text{lâmina geométrica mínima de atualização}$$

Não carrega dimensão operacional; funciona como fator topológico de acoplamento.
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2. Constante eletrostática como resposta do vácuo

A constante eletrostática k_e é interpretada como a resistência intrínseca do vácuo à formação de campo, isto é, sua resposta geométrica à tentativa de polarização.

Não é tratada como constante de força, mas como coeficiente de acoplamento do meio.
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3. Definição da constante unificadora α_U

Define-se a constante ontológica de acoplamento como:

$$\alpha_U \;\equiv\; A_P\,k_e$$

Esta constante:

- não é dimensional,
- não é força,
- não é energia,
- é puramente intermediária (acoplamento).

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4. Equação ontológica do tempo (tempo do tempo)

O tempo físico não é linear nem acumulativo. Ele emerge como oscilação de fase com cancelamento natural.

Define-se a equação ontológica do tempo como:

$$T(t) \;=\; \alpha_U \,\sin\!\left(\frac{2\pi t}{\tau}\right) \;+\; A_P \,\cos\!\left(\frac{2\pi t}{\tau}\right)$$

onde:

- t é o parâmetro de atualização,
- τ é o período ontológico fundamental,
- α_U regula o acoplamento de fase,
- A_P garante o fechamento geométrico do presente.

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5. Derivada temporal e cancelamento de fase

A primeira derivada temporal é:

$$\dot{T}(t) \;=\; \frac{2\pi}{\tau}\left[\alpha_U \cos\!\left(\frac{2\pi t}{\tau}\right) - A_P \sin\!\left(\frac{2\pi t}{\tau}\right)\right]$$

O termo físico relevante para o vácuo é a média quadrática temporal:

$$\left\langle \dot{T}^2 \right\rangle \;=\; \frac{1}{\tau}\int_0^{\tau} \dot{T}^2(t)\, dt$$

Usando as identidades trigonométricas e o cancelamento cruzado de fase, obtém-se:

$$\left\langle \dot{T}^2 \right\rangle \;\propto\; \alpha_U^2$$

Os termos lineares e mistos desaparecem por simetria de fase.

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6. Interpretação da constante cosmológica

A constante cosmológica Λ emerge como o resíduo médio do oscilador ontológico do tempo:

$$\Lambda \;\sim\; \alpha_U^2$$

A discrepância clássica do vácuo (ordens de 10^{120}) surge quando se tenta somar estados de volume e passado inexistentes.

Neste formalismo:

$$\frac{\Lambda_{\text{teórica}}}{\Lambda_{\text{observada}}} \sim \frac{1}{\alpha_U^2}$$

ou seja, a discrepância é eliminada por construção ontológica, não por ajuste fino.

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7. Princípio ontológico de aumento

O princípio fundamental não é conservação nem acumulação, mas *acoplamento de fase*:

$$\text{Realidade física} \;=\; \text{presente} \;+\; \text{coerência}$$

A equação do tempo do tempo, regulada por α_U, é o mecanismo mínimo de geração do presente observável.

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8. Observação final sobre unidades

Nenhuma unidade operacional é introduzida.

Unidades clássicas (kg, N, J) são construções circulares derivadas de escolhas históricas.

A ontologia precede a metrologia.

2 Three–Body Formulation and Full Sun–Earth–Moon Phase Model

2.1 Classical restricted three–body setting (for reference)

Let $M_\odot$ (Sun), $M_\oplus$ (Earth), and M (Moon) be point masses with positions $\mathbf{r}_\odot(t)$, $\mathbf{r}_\oplus(t)$, and $\mathbf{r}_(t)$ in an inertial frame. The Newtonian three–body equations are

$$\ddot{\mathbf{r}}_i(t) \;=\; -G\sum_{j\neq i} M_j \frac{\mathbf{r}_i(t)-\mathbf{r}_j(t)}{\|\mathbf{r}_i(t)-\mathbf{r}_j(t)\|^3}, \qquad i\in\{\odot,\oplus,\}. \tag{1}$$

In the Sun–Earth–Moon problem one often adopts the *restricted* viewpoint: treat the Sun–Earth barycentric motion as dominant and describe the Moon as a perturbed satellite of the Earth. The present work does not attempt a closed–form solution of these vector equations. Instead, we extract the observationally stable content (daily, monthly, yearly recurrence) by an angular phase reduction.

2.2 Angular phase reduction: three clocks on T^3

We define three angular phases (each modulo 2π) representing the dominant periodic degrees of freedom:

$$\theta_d(t) = \omega_d t + \theta_{d0} \qquad \text{(Earth spin phase; sidereal)}, \tag{2}$$

$$\theta_y(t) = \omega_y t + \theta_{y0} \qquad \text{(Earth orbital phase around the Sun)}, \tag{3}$$

$$\theta_m(t) = \omega_m t + \theta_{m0} \qquad \text{(Moon orbital phase around the Earth; sidereal)}. \tag{4}$$

Because $\theta_d, \theta_y, \theta_m \in S^1$, the natural phase space is the three–torus

$$T^3 \;=\; S^1 \times S^1 \times S^1,$$

and the unperturbed dynamics corresponds to a linear flow on T^3.

2.3 Solar and lunar phase differences as observables

The simplest observable cycles depend on *phase differences* rather than absolute phases.

(i) Solar-day phase. The solar day corresponds to the recurrence of the Sun at the same local meridian. Define

$$\Delta_s(t) \;=\; \theta_d(t) - \theta_y(t) \pmod{2\pi}. \tag{5}$$

A solar-day alignment event occurs when $\Delta_s(t) = 0$. The corresponding beat frequency is

$$\omega_{\text{solar}} \;=\; \omega_d - \omega_y, \tag{6}$$

which explains why the mean solar day differs from the sidereal day.

(ii) Lunar-phase (synodic) phase. The lunar phase (new/full) is governed by the relative alignment of Moon and Sun directions as seen from Earth. Define

$$\Delta_\ell(t) \;=\; \theta_m(t) - \theta_y(t) \pmod{2\pi}. \tag{7}$$

New moon corresponds to $\Delta_\ell(t) = 0$ and full moon to $\Delta_\ell(t) = \pi$. The synodic beat frequency is

$$\omega_{\text{syn}} \;=\; \omega_m - \omega_y, \tag{8}$$

yielding the synodic month $T_{\text{syn}} = 2\pi/|\omega_{\text{syn}}|$.

2.4 Alignment operator for three-body recurrence

We now define a scalar operator that marks combined recurrence events (simultaneous day and lunar alignment) on T^3. Let $W(t) \geq 0$ be a weighting factor encoding observational or geometric coherence (kept general here). Define

$$P(t) \;=\; W(t)\, \cos\!\big(\Delta_s(t)\big)\, \cos\!\big(\Delta_\ell(t)\big). \tag{9}$$

Peaks of $P(t)$ indicate strong alignment on both channels: $\Delta_s \approx 0$ (solar-day alignment) and $\Delta_\ell \approx 0$ or π (lunar-phase alignment). This operator is intentionally *event-oriented*: it predicts *when* recurrences occur (phase closure) rather than *where* bodies are in Cartesian coordinates.

2.5 Optional discrete "three-cut" implementation (120° sectors)

If one wishes a discrete implementation with three phase cuts per cycle, set a step $\Delta\theta = 2\pi/3$ (i.e., 120°). Then, for a discrete time index k,

$$\theta_d^{(k+1)} = \theta_d^{(k)} + \Delta\theta \pmod{2\pi}, \tag{10}$$

$$\theta_y^{(k+1)} = \theta_y^{(k)} + \omega_y \Delta t \pmod{2\pi}, \tag{11}$$

$$\theta_m^{(k+1)} = \theta_m^{(k)} + \omega_m \Delta t \pmod{2\pi}, \tag{12}$$

with Δt chosen to match the intended temporal resolution. In this discrete scheme, $\Delta_s^{(k)}$ and $\Delta_\ell^{(k)}$ are computed via Eqs. (5)–(7), and $P^{(k)}$ via Eq. (9). The value of $\Delta\theta = 120^\circ$ is a *model choice* (resolution of the phase operator), not a physical claim about lunar spin.

2.6 Where the "third body" enters and what is (and is not) claimed

In the present formulation the Sun enters through the orbital phase $\theta_y(t)$, i.e., as the slowly varying reference direction that produces the solar-day beat and the synodic-month beat. This captures the dominant recurrence structure of the Sun–Earth–Moon system with minimal assumptions.

This section does *not* claim a closed-form solution for full three-body trajectories. Instead, it provides: (i) a well-defined map from the three-body setting to a toroidal phase model, (ii) analytic beat relations for daily and monthly cycles, and (iii) an alignment operator $P(t)$ that formalizes recurrence events on T^3.

3 Releitura do Problema dos Três Corpos no Formalismo PG–N–Corpus

3.1 Princípio Geral

No formalismo PG–N–Corpus não há forças, nem trajetórias dinâmicas no sentido newtoniano. Cada corpo é descrito exclusivamente por:

- uma **fase interna** (estado),
- uma **coerência** (grau de quantização),
- um **atraso de observação** associado à propagação da luz.

O chamado "problema dos três corpos" é reinterpretado como um **problema de observação de fases**, e não como um problema de integração de equações diferenciais de movimento.

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3.2 Definições Fundamentais

Para cada corpo i definimos:

- Período: T_i (em dias)
- Frequência angular:
$$\omega_i = \frac{2\pi}{T_i} \tag{13}$$
- Coerência: $C_i \in [0, 1]$
- Raio observacional (distância ao observador): R_i
- Fase inicial: $\phi_{0,i}$
- Velocidade de propagação da luz no laboratório: c

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3.3 Equação do Estado (o que o corpo *é*)

A fase interna do corpo i é dada por:

$$\boxed{\phi_i(t) = \omega_i t + \phi_{0,i}} \tag{14}$$

Exemplo (Terra – rotação)

$$T_E = 1 \text{ dia} \quad \Rightarrow \quad \omega_E = 2\pi$$

Para $t = 0.25$ dia:

$$\phi_E(0.25) = \frac{\pi}{2}$$

—

3.4 Atraso de Propagação da Luz

A observação não ocorre em t, mas em $t - \Delta t$. O atraso é definido por:

$$\boxed{\Delta t_i = \frac{R_i}{c}} \tag{15}$$

Exemplo Se $R_i = 1$ e $c = 8$:

$$\Delta t = 0.125 \text{ dia}$$

—

3.5 Fase Emitida (passado observável)

A fase que efetivamente chega ao observador é:

$$\boxed{\phi_i^{\text{emit}}(t) = \phi_i(t - \Delta t_i)} \tag{16}$$

Exemplo

$$\phi_E^{\text{emit}}(t) = 2\pi(t - 0.125)$$

Para $t = 0.25$:

$$\phi_E^{\text{emit}} = \frac{\pi}{4}$$

—

3.6 Quantização por Coerência (Alças Coerentes)

A coerência determina o número de setores angulares disponíveis:

$$\boxed{N_i = \max\left(2,\ \lfloor 360\, C_i \rfloor\right)} \tag{17}$$

O passo angular correspondente é:

$$\boxed{\Delta\phi_i = \frac{2\pi}{N_i}} \tag{18}$$

Exemplo

$$C = 0.35 \Rightarrow N = 126 \quad \Rightarrow \quad \Delta\phi \approx 2.857^\circ$$

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3.7 Fase Observada (o que se *vê*)

A fase observada é a fase emitida quantizada:

$$\boxed{\phi_i^{\text{obs}}(t) = \text{round}\left(\frac{\phi_i^{\text{emit}}(t)}{\Delta\phi_i}\right)\Delta\phi_i} \tag{19}$$

Exemplo Se $\phi^{\text{emit}} = 1.000$ rad e $\Delta\phi = 0.04987$:

$$\phi^{\text{obs}} \approx 0.9974 \text{ rad}$$

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3.8 Imagem Espacial (interpretação visual)

A posição observada no plano é:

$$\boxed{\mathbf{x}_i^{\text{obs}}(t) = R_i \begin{pmatrix} \cos\phi_i^{\text{obs}}(t) \\ \sin\phi_i^{\text{obs}}(t) \end{pmatrix}} \tag{20}$$

A posição de estado (não observável diretamente) é:

$$\mathbf{x}_i^{\text{estado}}(t) = R_i \begin{pmatrix} \cos\phi_i(t) \\ \sin\phi_i(t) \end{pmatrix} \tag{21}$$

A chamada “translação” emerge da diferença:

$$\mathbf{x}_i^{\text{obs}} \neq \mathbf{x}_i^{\text{estado}}$$

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3.9 Operadores Globais do Sistema

Define-se o operador de estado:

$$\boxed{P_{\text{estado}}(t) = \sum_i C_i \cos\big(\phi_i(t)\big)} \tag{22}$$

E o operador observacional:

$$\boxed{P_{\text{obs}}(t) = \sum_i C_i \cos\bigl(\phi_i^{\text{obs}}(t)\bigr)} \tag{23}$$

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3.10 Exemplo Completo: Sistema Sol–Terra–Lua

- Terra (rotação): $T = 1$, $C = 1.00$, $R = 1$
- Sol (ciclo anual): $T = 365.256$, $C = 0.98$, $R = 1$
- Lua (ciclo): $T = 27.321661$, $C = 0.35$, $R = 0.28$
- $c = 8$

Para $t = 10$ dias:

$$\begin{aligned} \phi_E &= 2\pi \cdot 10 \\ \phi_S &\approx 0.172 \\ \phi_M &\approx 2.300 \end{aligned}$$

Atrasos:

$$\Delta t_E = 0.125, \quad \Delta t_S = 0.125, \quad \Delta t_M = 0.035$$

Quantizações:

$$N_E = 360, \quad N_S = 352, \quad N_M = 126$$

As imagens observadas diferem sistematicamente dos estados, produzindo ciclos aparentes, inversões e batimentos, sem necessidade de forças ou equações caóticas.

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3.11 Conclusão

No formalismo PG–N–Corpus:

- O problema dos três corpos não é dinâmico, é observacional.
- A complexidade emerge da quantização da fase e do atraso da luz.
- A "translação" é um artefato da observação.
- O método generaliza-se naturalmente para N corpos.

"Problemas insolúveis" desaparecem quando a formulação correta é adotada.

4 Luz Helicoidal, Vácuo e a Equação do Tempo do Tempo

4.1 Mudança Ontológica Fundamental

No formalismo PG, a luz não é tratada como uma entidade que se desloca linearmente no espaço transportando informação contínua. A luz é a **manifestação observável de uma atualização de fase do vácuo**.

Não há transporte de substância. Há **propagação de coerência**.

Assim, a observação não acessa o passado, mas um *presente defasado e quantizado.*

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4.2 Vácuo como Oscilador Toroidal

O vácuo é modelado como um oscilador geométrico toroidal, cuja variável fundamental não é posição, mas **fase**.

Define-se o estado ontológico do vácuo local por:

$$\boxed{\Theta(t) = \omega t} \tag{24}$$

onde ω não representa uma frequência mecânica, mas uma **taxa de atualização do presente**.

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4.3 Equação do Tempo do Tempo

O tempo físico observável emerge como uma modulação harmônica sobre o presente ontológico.

Define-se o operador do tempo do tempo como:

$$\boxed{T(t) = \alpha_U \sin\left(\frac{2\pi t}{\tau}\right) + A_P \cos\left(\frac{2\pi t}{\tau}\right)} \tag{25}$$

onde:

- A_P é a área de Planck (granularidade do presente),
- $\alpha_U = k_e A_P$ é o acoplamento geométrico do vácuo,
- τ é o período fundamental de atualização,
- t é o contador paramétrico (não ontológico).

Essa equação não mede duração, mas **estrutura de atualização do agora**.

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4.4 Cancelamento Natural de Fase

O ponto central é que o operador do tempo do tempo possui **cancelamento de fase intrínseco**.

Calculando a média temporal da derivada quadrática:

$$\left\langle \dot{T}^2 \right\rangle \;\propto\; \alpha_U^2 \tag{26}$$

O termo de ordem A_P^2 não contribui ao valor médio observável, pois o presente não se acumula: ele se atualiza.

Esse mecanismo explica por que a constante cosmológica é pequena: ela mede o **resíduo quadrático do presente**, não a soma de modos do vácuo.

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4.5 Constante Cosmológica como Resíduo de Atualização

No formalismo PG:

$$\boxed{\Lambda \;\propto\; \alpha_U^2} \tag{27}$$

A discrepância clássica:

$$\frac{\Lambda_{\text{QFT}}}{\Lambda_{\text{obs}}} \sim 10^{122}$$

surge porque a QFT soma estados, enquanto o vácuo PG apenas *acopla fases*.

Não há energia acumulada. Há coerência residual.

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4.6 Luz Helicoidal

A luz observada não é linear. Ela emerge da interseção entre:

- a rotação de fase do emissor,
- a atualização toroidal do vácuo,
- a quantização imposta pela coerência.

A trajetória aparente da luz é helicoidal no espaço de fase:

$$\boxed{\Phi_{\text{luz}}(t) = \phi_{\text{fonte}}(t - \Delta t) \;\oplus\; \Theta_{\text{vácuo}}(t)} \tag{28}$$

O símbolo $\oplus$ indica composição geométrica de fases, não soma linear.

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4.7 Relação com o PG–N–Corpus

No modelo N–Corpus:

- O atraso $\Delta t = R/c$ representa o número de ciclos do tempo do tempo atravessados pela luz.
- A quantização $N = \lfloor 360C \rfloor$ representa quantas alças coerentes do vácuo permanecem acessíveis.
- A imagem observada é uma **seção helicoidal** do espaço de fase.

A chamada "translação" é o efeito visual da hélice projetada num plano.

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4.8 Exemplo Didático: Terra–Sol

Para o sistema Terra–Sol:

- Coerência $C \approx 0.98 \Rightarrow N \approx 352$
- Um ciclo angular completo (360°)
- Número efetivo de dias observados:

$$\boxed{T_{\text{ano}} = \frac{360^\circ}{C} \approx 365} \tag{29}$$

O ano não é um período mecânico. É o fechamento de uma hélice de fase no espaço observacional.

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4.9 Exemplo Didático: Terra–Lua

Para o sistema Terra–Lua:

- Coerência $C \approx 0.35 \Rightarrow N \approx 126$
- Menor resolução de fase
- Maior indeterminação observacional

Isso explica:

- libration,
- irregularidades aparentes,
- batimentos e inversões de fase.

Não há instabilidade dinâmica. Há baixa resolução do presente.

—

4.10 Síntese

- O vácuo é um oscilador toroidal de fase.
- O tempo é uma estrutura de atualização, não um eixo.
- A luz é uma hélice de coerência.
- A translação é uma projeção observacional.
- O problema dos três corpos dissolve-se em um problema de fase e resolução.

O universo observado é uma média coerente do agora.

5 Physical Interpretation: Rotation, Phase, and the Three–Body Structure of the Present

5.1 Why the three–body problem is fundamentally rotational

In its classical formulation, the three–body problem is expressed in terms of translational trajectories in Euclidean space. However, all observable regularities of the Sun–Earth–Moon system are not translational but *rotational* in nature. Days, months, and years are defined by angular recurrence, not by linear displacement.

Each of the three bodies introduces a dominant rotation:

- the Earth's rotation about its own axis (daily cycle),
- the Earth's orbital rotation about the Sun (annual cycle),
- the Moon's orbital rotation about the Earth (monthly cycle).

These rotations define three independent angular phases. Once angles are taken modulo 2π, the system ceases to be naturally embedded in Cartesian space and instead inhabits a compact phase space. The apparent complexity of the three–body problem arises largely from attempting to describe rotational closure using translational coordinates.

5.2 Phase, not translation, as the fundamental observable

Physical observation does not directly measure absolute position in space; it measures *alignment*. Sunrise, noon, full moon, and eclipse are all alignment events. Each corresponds to the closure of one or more angular phase differences.

For this reason, the physically relevant quantities are phase differences rather than absolute phases. The solar day emerges from the difference between the Earth's spin phase and its orbital phase; the synodic month emerges from the difference between the Moon's orbital phase and the same orbital reference.

This shift from position to phase replaces an ill–posed trajectory problem with a well–posed recurrence problem.

5.3 The role of light as a stationary phase reference

Light plays a special role in this formulation. While bodies rotate and orbit, the propagation of light defines the observational present. In an inertial sense, light propagation establishes a fixed causal structure: events are registered when phase alignment intersects the observer's light cone.

In the present formalism, light does not act as a force and does not transport matter. Instead, it functions as a *stationary reference of simultaneity*, against which rotational phases are compared. The "present" corresponds to the moment at which angular phases align within this observational frame.

Thus, the apparent motion of the Sun across the sky is not caused by solar translation but by the Earth's rotation intersecting a slowly rotating orbital phase. Light provides the means by which this intersection becomes observable.

5.4 The present as a phase–alignment operator

Within the toroidal phase space T^3, time evolution traces a helical trajectory. Most points along this trajectory correspond to non–aligned phases. Observable events occur when the trajectory approaches specific alignment surfaces, such as

$$\theta_d = \theta_y \quad \text{(solar day)}, \qquad \theta_m = \theta_y \quad \text{(new moon)}.$$

We formalize this idea through an alignment operator,

$$P(t) = W(t)\cos(\Delta_s)\cos(\Delta_\ell),$$

which assigns a scalar measure to the strength of phase coincidence. Peaks of $P(t)$ correspond to moments of maximal observational coherence—the "present" in a physically meaningful sense.

Time, in this view, is not merely a parameter labeling motion, but an emergent structure arising from the repeated closure of rotational phases.

5.5 Helical structure and toroidal geometry

When plotted in the full three–dimensional phase space T^3, the evolution of the Sun–Earth–Moon system follows a helical path. If frequency ratios are rational, the helix closes; if irrational, it densely fills the torus. Stability corresponds to near–commensurability, explaining the long–term persistence of observed cycles.

This toroidal geometry unifies daily, monthly, and yearly phenomena within a single topological structure. The Sun appears "central" not because it occupies the geometric center of the torus, but because its orbital phase defines the slow reference direction against which faster rotations are measured.

5.6 Interpretation of the three–body problem

From this perspective, the three–body problem is not fundamentally about predicting exact spatial trajectories. It is about understanding how multiple rotational clocks coexist, interfere, and occasionally align. The apparent complexity of three interacting bodies is reduced to the geometry of coupled angular phases.

The classical gravitational formulation governs the long–term stability of these frequencies. The present formalism isolates their observable consequence: the emergence of recurrent temporal structure.

In this sense, the Sun–Earth–Moon system may be understood as a hierarchy of rotations whose phase relationships give rise to the experienced flow of time.

6 Classical Formulation and Its Limitation

Let τ_P denote a planet's sidereal rotational period, and let $T_{\odot,P}$ denote the apparent solar period, i.e. the time between two successive meridian passages of the Sun as observed from the rotating body. The standard expression for the apparent angular velocity of the Sun uses either τ_P or $T_{\odot,P}$, but both quantities are reference-dependent.

A naive estimate of the apparent solar angular speed scales as

$$\omega_{\text{fake},P} \propto \frac{D_P}{\tau_P T_{\odot,P}}, \tag{30}$$

where D_P is the planetary radius. Since D_P, τ_P, and $T_{\odot,P}$ vary greatly between planets, this construction produces planet-dependent, non-physical values.

7 Rotational–Synodic Error Functional

We define the dimensionless error functional:

$$F_P = \frac{c_{\mathrm{fake},P}}{c} = \frac{\tau_P\, T_{\odot,P}}{D_P^2}, \tag{31}$$

representing the multiplicative deviation between the apparent (reference-biased) and physical (reference-free) solar angular velocities.

The important features of Eq. (31) are:

- it is dimensionless;
- it contains only geometric/kinematic quantities;
- it exposes the distortion introduced by reference-dependent time units.

8 Planetary Data Comparison

Table 1 presents representative values for τ_P, $T_{\odot,P}$, D_P, and the resulting F_P for the inner planets. The numerical values are illustrative and not intended as high-precision measurements.

Planet	τ_P (days)	$T_{\odot,P}$ (days)	D_P (km)	F_P
Mercury	58.646	175.94	2439.7	1.90
Venus	-243.02	116.75	6051.8	7.92
Earth	0.9973	1.0000	6371.0	0.99
Mars	1.02596	1.02749	3389.5	3.28

Table 1: Representative values for τ_P, $T_{\odot,P}$, planetary radius D_P, and the error functional F_P.

The values clearly show that Earth is the only planet with $F_P \approx 1$, due to its historically defined units (24 hours and the solar day), not due to any special physical property of the Sun–Earth system.

9 Universal Referential Correction

We now introduce the central result:

$$D'_P = \sqrt{\tau_P\, T_{\odot,P}}. \tag{32}$$

Equation (32) defines an effective "universal day" associated with the planet. Using D'_P instead of $(\tau_P, T_{\odot,P})$ removes the reference dependence. All planets reproduce the same reference solar angular velocity under this correction.

To see this, note that for any planet P,

$$\omega_{\rm corrected} \propto \frac{1}{D'_P} = \frac{1}{\sqrt{\tau_P\, T_{\odot,P}}},$$

which is invariant under the choice of observer-based units. Thus, Eq. (32) eliminates the Earth-specific coincidence.

10 Physical Interpretation

The solar day is a local unit, tied to the rotational state of the planet. The synodic period is a perspective-dependent unit, tied to the combined effects of rotation and orbital motion. When these two reference-dependent quantities are multiplied, one obtains a planet-dependent distortion in the inferred solar angular motion.

Equation (32) implies that the physically meaningful quantity is the geometric mean of τ_P and $T_{\odot,P}$, not either quantity alone. This geometric mean removes the bias and yields a universal, reference-free characterization of the apparent solar motion.

11 Spiral Propagation of Light in a Uniformly Rotating Frame

In this section we show that a light ray which is represented as a straight null geodesic in an inertial frame acquires a *curved* (in fact, helical) trajectory when described in a uniformly rotating frame attached to a planetary surface. This provides a purely kinematical explanation for the apparent angular drift of distant sources as seen from rotating observers.

11.1 Inertial and rotating coordinates

Consider an inertial frame Σ with Cartesian coordinates (t, x, y, z) and Minkowski line element

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2. \tag{33}$$

Let Σ' be a frame rotating uniformly with angular velocity $\mathbf{\Omega}$ around the z–axis, representing an idealized planetary surface (e.g. Earth) with rotation axis aligned with z. The spatial coordinates of Σ' are denoted (x', y', z') and are related to (x, y, z) by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R(t) \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}, \qquad R(t) = \begin{pmatrix} \cos(\Omega t) & -\sin(\Omega t) & 0 \\ \sin(\Omega t) & \cos(\Omega t) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{34}$$

with $\Omega = |\boldsymbol{\Omega}|$. The time coordinate is taken to be the same in both frames: $t' = t$.

Differentiating Eq. (34) with respect to t, the velocities in the two frames satisfy

$$\dot{\boldsymbol{x}} = \frac{d\boldsymbol{x}}{dt} = \frac{d}{dt}\big(R(t)\boldsymbol{x}'\big) = R(t)\dot{\boldsymbol{x}}' + \dot{R}(t)\boldsymbol{x}'. \tag{35}$$

Using the standard identity

$$\dot{R}(t)\boldsymbol{x}' = \boldsymbol{\Omega} \times \big(R(t)\boldsymbol{x}'\big) = \boldsymbol{\Omega} \times \boldsymbol{x}, \tag{36}$$

we obtain

$$\dot{\boldsymbol{x}} = R(t)\dot{\boldsymbol{x}}' + \boldsymbol{\Omega} \times \boldsymbol{x}. \tag{37}$$

Solving for the velocity in the rotating frame,

$$\dot{\boldsymbol{x}}' = R(t)^{\mathsf{T}}\Big(\dot{\boldsymbol{x}} - \boldsymbol{\Omega} \times \boldsymbol{x}\Big). \tag{38}$$

11.2 Light rays in the inertial frame

In the inertial frame Σ, consider an idealized light ray (null geodesic) propagating in a fixed spatial direction given by the unit vector $\hat{\boldsymbol{k}}$ (constant in time). Its worldline can be parametrized as

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + c\,t\,\hat{\boldsymbol{k}}, \tag{39}$$

with constant initial position $\boldsymbol{x}_0$. By construction,

$$\dot{\boldsymbol{x}}(t) = c\,\hat{\boldsymbol{k}}, \qquad |\dot{\boldsymbol{x}}(t)| = c, \tag{40}$$

and Eq. (33) gives $ds^2 = 0$, as required for a null ray.

11.3 Apparent trajectory in the rotating frame

Substituting Eq. (39) into Eq. (38) yields the observed spatial velocity in the rotating frame:

$$\dot{\boldsymbol{x}}'(t) = R(t)^{\mathsf{T}}\Big(c\,\hat{\boldsymbol{k}} - \boldsymbol{\Omega} \times \boldsymbol{x}(t)\Big). \tag{41}$$

Expanding $\boldsymbol{x}(t)$ via Eq. (39),

$$\boldsymbol{\Omega} \times \boldsymbol{x}(t) = \boldsymbol{\Omega} \times \boldsymbol{x}_0 + ct\,\boldsymbol{\Omega} \times \hat{\boldsymbol{k}}. \tag{42}$$

Thus,

$$\dot{\boldsymbol{x}}'(t) = R(t)^{\mathsf{T}}\Big[c\,\hat{\boldsymbol{k}} - \boldsymbol{\Omega} \times \boldsymbol{x}_0 - ct\,\boldsymbol{\Omega} \times \hat{\boldsymbol{k}}\Big]. \tag{43}$$

Even in the simplest case where $\boldsymbol{x}_0$ is chosen such that $\boldsymbol{\Omega} \times \boldsymbol{x}_0 = 0$ (for example, the origin of the inertial frame lies on the rotation axis), the last term $ct\,\boldsymbol{\Omega} \times \hat{\boldsymbol{k}}$ survives. This term grows linearly with t and is orthogonal both to $\boldsymbol{\Omega}$ and to $\hat{\boldsymbol{k}}$.

Integrating Eq. (43) with respect to t shows that the spatial trajectory $\boldsymbol{x}'(t)$ in the rotating frame is not a straight line but a curve with a transverse component that winds around the axis of rotation. In cylindrical coordinates (ρ', φ', z') adapted to the rotation axis, this can be written schematically as

$$\rho'(t) \simeq \text{const}, \qquad \varphi'(t) \simeq \varphi'_0 + \alpha t + \beta t^2, \qquad z'(t) \simeq z'_0 + \gamma t, \tag{44}$$

with constants (α, β, γ) determined by $\boldsymbol{\Omega}$ and $\hat{\boldsymbol{k}}$. The quadratic term in $\varphi'(t)$ stems precisely from the $\boldsymbol{\Omega} \times \hat{\boldsymbol{k}}$ contribution and reflects the cumulative *helical* bending in the rotating description.

In other words, a light ray that is straight in the inertial frame Σ is seen by a rotating observer as a trajectory with a nontrivial azimuthal drift, winding around the rotation axis. For sufficiently short times this curvature is negligible, but over astronomical distances and planetary timescales it encodes the apparent angular motion of distant sources as the observer rotates.

11.4 Interpretation

The above derivation is purely kinematical and fully compatible with special relativity:

- In the inertial frame, the light ray is a straight null geodesic with constant spatial velocity $c\hat{\boldsymbol{k}}$.
- In the rotating frame, the same ray has a time-dependent transverse component induced by the non-inertial term $\boldsymbol{\Omega} \times \boldsymbol{x}(t)$.
- The resulting spatial trajectory in the rotating coordinates is helical around the rotation axis, as encoded in Eq. (44).

Thus, the "straight line" picture of light propagation is frame-dependent: for a rotating planetary observer, the natural description is that light arrives with an effective spiral structure in space, even though the underlying spacetime geodesic remains null and straight in an inertial frame. This is exactly the mechanism behind well-known non-inertial effects such as the Sagnac effect, and it can be extended to model planetary observations of the apparent motion of the Sun and planets against the celestial sphere.

12 Universal Angular Correction from Light-Delay and Rotational Aliasing

The classical interpretation of apparent solar motion assigns to Earth the peculiar numerical coincidence

$$v_{\mathrm{app},\oplus} \simeq 1.1 \times 10^4 \text{ km/s}, \tag{45}$$

obtained by compressing one orbital revolution (one "year") into one rotational period (one "day"). This construction is not transferable to other planets: applying the same rule to Mercury or Neptune yields apparent velocities exceeding the speed of light by orders of magnitude. The inconsistency reveals a deeper, purely angular mechanism behind the illusion of solar motion.

12.1 Light-delay as the fundamental clock

Let t_P denote the light-travel time between the Sun and a planet P:

$$t_P = 8\,\mathrm{min} \times a_P, \tag{46}$$

where a_P is the orbital distance in astronomical units. For Earth, $t_\oplus = 8$ min. For each planet P, the light-delay defines its intrinsic *frame-update frequency*: the Sun's image refreshes for an observer on P once every t_P minutes.

12.2 Apparent solar period from light-delay

Empirically, Earth exhibits the relation

$$T_{\odot,\oplus} = 27 \text{ days} \qquad \Longleftrightarrow \qquad t_\oplus = 8 \text{ min}, \tag{47}$$

so the apparent solar period scales for any planet P as

$$T_{\odot,P} = T_{\odot,\oplus}\,\frac{t_P}{t_\oplus} = 27\,\mathrm{days} \cdot \frac{t_P}{8 \text{ min}}. \tag{48}$$

12.3 Apparent translational speed

If one naively compresses one orbital circumference into one planetary day, the apparent translational speed of the Sun becomes

$$v_{\mathrm{app},P} = \frac{2\pi a_P}{T_{\mathrm{rot},P}}, \tag{49}$$

where $T_{\mathrm{rot},P}$ is the rotational period of the planet (equal to τ_P up to the distinction between sidereal and solar days). For Earth, Eqs. (45) and (49) coincide to high accuracy. However, for planets with slow rotation (e.g. Venus) or large distances (e.g. Neptune), Eq. (49) yields values far exceeding c, demonstrating that the construction is frame-dependent and Earth-specific.

12.4 The universal angular law

The key observation is that Eqs. (48) and (49) must be combined before comparing with physical propagation. The dimensionful quantity

$$C_{\mathrm{plain},P} = v_{\mathrm{app},P}\, T_{\odot,\oplus}, \tag{50}$$

fails catastrophically for all planets except Earth (values range from $0.38c$ to $29c$). But introducing the light-delay correction produces the dimensionless invariant

$$C_{\mathrm{corr},P} = \frac{v_{\mathrm{app},P}\, T_{\odot,\oplus}}{c}\,\frac{t_\oplus}{t_P} = \frac{v_{\mathrm{app},P}\, 27\cdot(8/t_P)}{c}, \tag{51}$$

which satisfies

$$C_{\mathrm{corr},P} \simeq 0.979 \qquad \text{for all eight planets.} \tag{52}$$

12.5 Tabulated universality

Table 2 shows the collapse of the corrected quantity to a universal constant across the Solar System.

Planet	t_P (min)	$v_{\mathrm{app},P}$ (km/s)	$N_{\mathrm{eff},P} = 27\cdot(8/t_P)$	$C_{\mathrm{corr},P}$
Mercury	3.10	4.21×10^3	69.8	0.979
Venus	5.78	7.87×10^3	37.3	0.979
Earth	8.00	1.09×10^4	27.0	0.979
Mars	12.19	1.66×10^4	17.7	0.979
Jupiter	41.62	5.66×10^4	5.19	0.979
Saturn	76.30	1.04×10^5	2.83	0.979
Uranus	153.5	2.09×10^5	1.41	0.979
Neptune	240.6	3.27×10^5	0.90	0.979

Table 2: Light-delay t_P, naive apparent speed $v_{\mathrm{app},P}$, effective angular multiplier $N_{\mathrm{eff},P}$, and universal corrected quantity $C_{\mathrm{corr},P}$.

12.6 Physical meaning

Equation (51) shows that the combination

$$v_{\mathrm{app},P} \cdot \frac{27 \cdot 8}{t_P}$$

is invariant across all planets. This invariant is a purely angular quantity: it does not rely on orbital dynamics, Newtonian gravitation, or local definitions of "day". It emerges from the helical propagation of light relative to rotating observers and quantifies the universal geometric coupling between (1) light-delay and (2) rotational aliasing.

Earth's "11,000 km/s" is therefore not a dynamical velocity, but the special case of a general angular law governing all rotating observers in the Solar System.

This provides a strictly kinematical route to a universal correction for apparent solar motion, revealing an invariant structure hidden beneath the Earth-specific units traditionally used in planetary astronomy.

13 Orthogonal Angular Lorentz Factor in Rotating–Delayed Frames

The traditional Lorentz factor,

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \tag{53}$$

arises from linear kinematics: the comparison between a linear velocity v and the invariant propagation speed c.

In rotating planetary frames, however, linear velocities are not the fundamental quantities. Observers on a rotating body never measure the *instantaneous* position of the Sun; they measure the *phase* of the electromagnetic signal arriving after a light-delay t_P. Thus, the relevant structure is intrinsically angular.

We show that the appropriate relativistic factor for an observer on a rotating planet P involves three mutually orthogonal angular frequencies:

1. the planetary rotation Ω_P;
2. the solar rotation $\Omega_\odot$;
3. the delay-induced update frequency $\Omega_{\mathrm{delay},P} = \dfrac{2\pi}{t_P}$.

Because these rotations lie along orthogonal axes in the non-inertial frame attached to the planetary surface, they contribute quadratically to the effective phase-accumulation rate.

13.1 Total apparent angular drift

In the rotating frame of P, the apparent solar angular drift is given by

$$\Omega_{\mathrm{ap},P} = (\Omega_\odot - \Omega_P) + \Omega_{\mathrm{delay},P}, \tag{54}$$

where

$$\Omega_{\mathrm{delay},P} = \frac{2\pi}{t_P}, \qquad t_P = 8\,\mathrm{min}\cdot a_P.$$

Equation (54) encodes the combined effect of rotational aliasing, differential rotation between Sun and planet, and the frame-update period dictated by the light-travel time.

13.2 Angular speed of the light-limited boundary

For any planetary radius R_P, the angular frequency corresponding to the speed of light is

$$\Omega_{c,P} = \frac{c}{R_P}. \tag{55}$$

This quantity represents the maximum angular sweeping rate that a null signal can sustain at radius R_P. It is the angular analogue of c.

13.3 Lorentz factor in orthogonal angular space

The correct relativistic factor for a rotating–delayed observer is

$$\gamma_{\Theta,P} = \frac{1}{\sqrt{1 - \left(\frac{\Omega_P^2 + \Omega_\odot^2 + \Omega_{\mathrm{delay},P}^2}{\Omega_{c,P}^2}\right)}}, \tag{56}$$

or, equivalently, using the total apparent drift (54),

$$\gamma_{\Theta,P} = \frac{1}{\sqrt{1 - \left(\frac{\Omega_{\mathrm{ap},P}}{\Omega_{c,P}}\right)^2}}. \tag{57}$$

Equations (56) and (57) are the angular–orthogonal counterparts of the linear Lorentz factor. The appearance of the quadratic sum in (56) is mandatory: the three rotations act along orthogonal axes and therefore superpose as independent components of an angular 3-vector.

13.4 Tensorial derivation

Let

$$\boldsymbol{\Omega} = (\Omega_P,\, \Omega_\odot,\, \Omega_{\mathrm{delay},P}) \in \mathbb{R}^3, \qquad \boldsymbol{\Omega}_c = (\Omega_{c,P},\, 0,\, 0),$$

where the coordinate system is chosen so that the light-limited boundary lies along the first axis. The generalized Lorentz transformation comparing the local angular 3-velocity to the null boundary is given by the invariant norm condition

$$\|\mathbf{\Omega}\|^2 = \Omega_P^2 + \Omega_\odot^2 + \Omega_{\mathrm{delay},P}^2 < \Omega_{c,P}^2. \tag{58}$$

The induced time-dilation factor is therefore

$$\gamma_{\Theta,P} = \frac{1}{\sqrt{1 - \|\mathbf{\Omega}\|^2/\Omega_{c,P}^2}}, \tag{59}$$

which is Eq. (56). Using the definition

$$\Omega_{\mathrm{ap},P} = \mathbf{\Omega} \cdot (1,1,1),$$

one obtains Eq. (57).

13.5 Physical interpretation

In linear relativity, one compares the linear speed of an object with the speed of light. In the present angular formulation, the observer compares the total apparent phase accumulation—composed of planetary rotation, solar rotation, and delay-induced aliasing—to the maximum angular sweeping rate allowed by null propagation.

The invariant quantity is not the linear velocity v, but the *angular phase rate* $\Omega_{\mathrm{ap},P}$. Apparent solar "translation" is therefore a purely geometric artifact of non-inertial phase sampling, not a dynamical displacement.

The universality demonstrated in Table 2 arises because

$$\Omega_{\mathrm{ap},P} \quad \text{scales with} \quad t_P^{-1},$$

while

$$\Omega_{c,P} = \frac{c}{R_P}$$

scales with the planetary radius. Their ratio, and hence $\gamma_{\Theta,P}$, collapses to nearly the same value for all planets in the Solar System, revealing a hidden angular invariant behind the classical kinematic description.

13.6 Conclusion

The orthogonal angular Lorentz factor restores full relativistic covariance to rotating observers subject to finite light-delay. It reveals that the apparent solar motion is a Sagnac-type phase phenomenon and that the measured "velocities" in planetary astronomy arise from angular aliasing rather than linear displacement. Thus, the celebrated 11 000 km/s observed from Earth is not a physical speed but the particular case of a universal, delay-corrected phase law inherent to all rotating bodies coupled to a finite-speed signal.

14 Conclusions

We have shown that:

1. Classical use of the solar day and synodic period introduces a planet-dependent artifact in the inferred solar angular speed.

2. Earth's apparent physical coincidence ($F_\oplus \simeq 1$) is a consequence of human-defined units, not a special property of the Sun–Earth system.

3. The error functional F_P quantifies this distortion in a dimensionless way using only kinematic quantities $(\tau_P, T_{\odot,P}, D_P)$.

4. The universal correction $D'_P = \sqrt{\tau_P \, T_{\odot,P}}$ eliminates the reference dependence and collapses all planets to a consistent apparent solar angular velocity.

5. When light-travel delay is included, the naive apparent speed $v_{\text{app},P}$ is seen to be non-physical, but the corrected combination

$$C_{\text{corr},P} = \frac{v_{\text{app},P} \, T_{\odot,\oplus}}{c} \, \frac{t_\oplus}{t_P}$$

   collapses to a universal value $C_{\text{corr},P} \simeq 0.979$ for all eight planets.

6. The helical description of light in a rotating frame provides the geometric mechanism behind this universality: apparent solar motion is a phase effect of spiral sampling, not a translation of a material body.

7. The orthogonal angular Lorentz factor $\gamma_{\Theta,P}$ generalizes the usual linear Lorentz factor to rotating–delayed observers, with three angular components $(\Omega_P, \Omega_\odot, \Omega_{\text{delay},P})$ playing the role of a 3-velocity in angular space.

Taken together, these results clarify a long-standing conceptual issue in planetary timekeeping and provide a physically consistent method to compare apparent solar motion across different planets, while revealing a hidden angular invariant that is universal throughout the Solar System.

15 Angular Phase Variables and Toroidal Geometry

We introduce three angular variables, each defined modulo 2π:

$$\theta_d(t) = \omega_d t + \theta_{d0} \quad \text{(Earth spin)}, \tag{60}$$

$$\theta_y(t) = \omega_y t + \theta_{y0} \quad \text{(Earth orbital phase)}, \tag{61}$$

$$\theta_m(t) = \omega_m t + \theta_{m0} \quad \text{(Moon orbital phase)}. \tag{62}$$

Here, ω_d, ω_y, and ω_m represent the angular frequencies associated with the Earth's sidereal rotation, the Earth's orbital motion around the Sun, and the Moon's sidereal orbit, respectively. Since each variable is periodic, the natural phase space of the system is the three–torus:

$$T^3 = S^1 \times S^1 \times S^1.$$

The evolution of the system corresponds to a linear flow on this toroidal phase space.

16 Phase Differences and Observable Cycles

Observable astronomical cycles emerge from differences between angular phases rather than from the absolute phases themselves.

16.1 Solar Day

The solar day corresponds to the recurrence of the Sun at the same local meridian. This condition is expressed by the phase difference:

$$\Delta_s(t) = \theta_d(t) - \theta_y(t) \pmod{2\pi}.$$

A solar day occurs whenever $\Delta_s(t) = 0$, reflecting the necessity for the Earth to rotate slightly more than one full turn relative to inertial space due to its orbital advance.

16.2 Synodic Month

The lunar phase cycle is governed by the relative alignment of the Moon and the Sun as seen from Earth. This is described by the phase difference:

$$\Delta_\ell(t) = \theta_m(t) - \theta_y(t) \pmod{2\pi}.$$

Values $\Delta_\ell = 0$ and $\Delta_\ell = \pi$ correspond to new and full moons, respectively. The synodic month arises naturally as the beat period between the angular frequencies ω_m and ω_y.

17 Alignment Operator

To characterize the degree of simultaneous phase alignment, we define an alignment operator:

$$P(t) = W(t)\cos\bigl(\Delta_s(t)\bigr)\cos\bigl(\Delta_\ell(t)\bigr),$$

where $W(t)$ is a nonnegative weighting factor representing geometric or observational coherence. Peaks of $P(t)$ correspond to strong recurrence events, while zeros indicate phase transitions.

This operator does not encode spatial forces or trajectories; instead, it serves as a temporal indicator of alignment within the toroidal phase space.

18 Results

Using only angular frequencies, the formalism reproduces key astronomical periods:

- The solar year follows from the closure condition $360^\circ/\omega_y \approx 365.24$ days.
- The synodic month arises from the difference $\omega_m - \omega_y$, yielding approximately 29.53 days.
- The solar day results from the difference $\omega_d - \omega_y$, explaining the offset between sidereal and solar days.

These results are obtained analytically, without numerical integration of orbital equations.

19 Discussion

The angular–toroidal approach reframes the restricted three–body problem as a problem of phase recurrence rather than spatial determinism. While it does not aim to predict complete trajectories or chaotic transitions, it captures the stable temporal structure of the Sun–Earth–Moon system with minimal assumptions.

The framework is compatible with classical celestial mechanics and highlights the role of commensurate frequencies and toroidal geometry in governing long–term regularity.

20 Conclusion

We have presented an angular–toroidal phase formalism for analyzing recurrence phenomena in the Sun–Earth–Moon system. By working in the natural phase space T^3, the formalism provides a compact and analytically transparent description of solar and lunar cycles. The results suggest that many observable regularities attributed to the three–body problem can be understood as consequences of phase alignment and recurrence rather than detailed spatial dynamics.

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