“Minus Two Fish” Paradox in Practice and Possible Reality of “Strange” Mathematical Solutions

Roman Vinokur
Independent Consultant, Woodland Hills, CA, USA

A feasible scenario to the famous “minus two fish” paradox is shown in this paper. Not rarely theoretical models produce ‘strange’ mathematical solutions which look incredible to common sense but describe real phenomena. Such examples teach engineers and scientists not to dismiss ‘strange’ mathematical results without verifying their feasibility. In particular, the prediction of the positron by Paul Dirac confirms this pattern.

There were three of us on a fishing trip - John, Paul, and I (Harris). The Thames here abounds in pike, roach, eels, and bleak. Walking along the bank, you can see whole shoals of them but how get them to take the hook? Time passed, the fish refused to bite, and then Paul told John and me about a mathematical problem he had solved in a Christmas competition organized by the Cambridge Student Society.

Three fishermen lay down to sleep without dividing their catch. At one o’clock in the morning one of them woke up and decided to go home, taking with him one third of the catch. When he divided it into three equal parts, there was one extra fish left over, which he threw back into the river. At two o’clock the second fisherman woke up and, not knowing that one of his companions had already left, again divided the catch into three equal parts. He too had one extra fish, which he threw into the river. At three o’clock the third fisherman woke up and did the same thing by dividing the catch by three and throwing away the “extra” fish.

The problem asked how many fish the fishermen had caught.

John and I took out pencils and paper and obtained the general solution:
the fishermen caught 27N - 2 fish, where N is an integer.
For N = 1 the formula gives 25 fish, for N = 2 it turns out that the fishermen caught 52 fish, etc.
We decided that 25 fish was the most realistic answer, but it turned out that Paul proposed the following strange solution:
minus two fish (for N = 0).

John and I burst out laughing together. In their time, the members of the competition jury also laughed when they saw such a mathematical solution…

Meanwhile, the three of us had caught only one fish, and after that the biting stopped altogether. We scattered along the bank, each looking for a good spot on his own. But luck did not favor us any further.

John was the first to return to the bucket, remembering that he had to go home earlier than the others. Deciding to take one last look at the fish we had caught earlier, he pulled it out of the bucket. Suddenly the fish slipped from his hands and dived back into the river.

The upset John decided to make up for our common loss. He ran to a nearby inn, borrowed a similar fish from the innkeeper's fresh catch, threw it into our bucket, and left for home without saying goodbye (like a true Englishman).

It should be noted that the innkeeper made it a condition that he be repaid later not with money, but with a fish. Then Paul appeared, and the same thing happened to him: he too ended up owing the innkeeper one fish. The same happened to me later, after John and Paul had already left.

When we met the next day, we figured out what had happened and laughed for a long time, because in the end we had caught exactly minus two fish. One fish was caught at the very beginning of the fishing trip, and three fish still had to be caught in order to repay the debt to the innkeeper. As a result, one fish caught minus three that still had to be caught equals minus two fish. The "strange" mathematical solution turned out to be quite real.

Paul's last name was Dirac, and his future specialty was theoretical physics. In 1928, Dirac predicted the positron when he formulated a new equation for the relativistic electron. This equation naturally allowed "strange" solutions with positive charge, which Dirac interpreted as a new particle - the antiparticle of the electron, which had a negative charge. Seven years later, the positron was discovered by Carl Anderson.

As for the "three fishermen problem" (the famous puzzle about dividing fish, often jokingly linked to Dirac, Jerome K. Jerome, or "physicists' folklore"), there is no reliable historical evidence that Dirac ever formally "solved" it or published a solution at a specific time. Most likely, it is a recreational math anecdote, later attributed to him because of his style of thinking and reputation, rather than a scientific result with a concrete date.

On the other hand, in this paper the author made up the case where the scenario with "minus two fish" could have occurred in practice.

The history of science contains many 'strange' cases in which "strange" solutions of mathematical equations turned out to describe real natural phenomena. Such stories teach engineers and scientists not to disregard 'strange' mathematical solutions before verifying whether they could be real. The prediction of the existence of the positron confirms this pattern.

References

1. Farmelo, G. The Strangest Man: The Hidden Life of Paul Dirac, Mystic of the Atom. New York: Basic Books, 2009.

2. Close, F. Antimatter. Oxford: Oxford University Press, 2010.