Quantum Mechanic Results of Bell-test Experiments Explained Classically, Intuitively and Comprehensible.

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Abstract

The fact that particles of an entangled pair are detected from different viewpoints in Bell-test experiments is the reason why no conclusions can be drawn from the comparison of the outcomes of the detections. Bell-test experiments are about correlations between probabilities for certain combinations of (spin-) outcomes and relative angles of settings of the detectors at which the spin of the particles is measured. Quantum Mechanics predicts those correlations correctly. Because repeated experiments show equal results there must be conditions for a pair of entangled particles to show a certain combination of results. This article explains how to find these conditions and the probabilities with which the certain combinations of outcomes occur. The implication of this explanation is that ‘entanglement’ turns out not to be the Quantum Mechanic phenomenon as is usually assumed.

Content

Introduction

A century ago quantum theory was developed in two ways: the wave mechanics by Schrödinger and the matrix mechanics by Heisenberg. They turned out to be equivalent. Additionally Heisenberg discovered the ‘uncertainty relations’. This means that certain properties of a particle cannot be measured at the same time. For instance: one cannot measure the position and the momentum of a particle at the same time. This is only naturel because a particle (as everything else) moves through space and time (spacetime). The position of a particle is the position in space at a certain moment. A moment is time of zero duration. Contrarily momentum depends on speed. Speed is change of position over a certain duration of time. That duration can be arbitrary short, but not zero. So it is obvious that position and momentum cannot be measured at the same time. This does not mean that a particle has no speed at a certain moment. (The fact that time can be arbitrary short in order to measure velocity also means that there is no shortest duration of time, so time is not quantised. This also goes for distance and thus for space, so space too is not quantised).

However, in quantum theory entangled particles can occur. Entangled particles are particles produced at the same time at the same place. Because of conservation laws the particles of such a pair have opposite properties. They move in opposite directions and have opposite spin. Einstein thought that it should be possible to measure the position of one particle and the momentum of the other. Than the position and the momentum of both particles is measured at the same time, albeit not at the same position. Then, afterwards, the position as well as the momentum of each particle is known for the moment of measurement. According to Einstein this means that particles have their properties at each moment of their existence. According to Bohr this was not possible. He stated that when properties of a particle cannot be known or measured at the same time, one could as well treat the particle as not having properties at all before they are measured. This is illogic, of course. Not being able to know or measure different properties of a particle at one moment, doesn't necessarily mean that the particle doesn't have those properties. A pair of entangled particles is not the same as a single particle. Yet Bohr insisted that it was not possible to know both the position and the momentum of a particle at the same time. Heisenberg had shown this with his uncertainty principle. It seemed to be a paradox in quantum theory, known as the EPR-paradox. In contrast to Bohr, Einstein believed that particles have definite properties at any time and that therefore Quantum Theory was not complete.

Reasoning

The reasoning in this paper is as follows:

1. An object observed or detected from different viewpoints usually produces different outcomes.
2. Conclusions cannot be drawn from the comparison of these outcomes.
3. To draw conclusions from outcomes of observations or detections from different viewpoints, the observations or detections have to be made equivalent. This means: as if from one viewpoint.
4. This can be achieved by taking into account all rotations of the observers or detectors in respect of each other and in respect of the object.

All of this will be explained in this paper.

Experiment

David Bohm thought of an experiment to solve this paradox. He proposed to measure the spin directions of entangled particles in different directions and calculate correlations for the combinations of outcomes. Spin of a particle can be measured in only one direction. Spin directions of a pair of entangled particles are opposite and can be measured separately in different directions. So experiments must show correlations between probabilities for certain combinations of outcomes (equal spin outcomes or opposite spin outcomes for a certain pair) and the relative settings of detectors. Quantum Mechanics predicted specific probabilities and correlations. Experiments showed exactly the correlations, predicted by Quantum Mechanics, between probabilities for the combinations of outcomes and the relative settings of the detectors. Somehow it was concluded that Bohr was right and Einstein was wrong. This conclusion was premature.

The experiments are called: Bell-test experiments, after John S. Bell [1], who established probabilities for the combinations of outcomes occurring in those experiments. His probabilities did not correspond to the probabilities predicted by QM. He therefore stated that the QM probabilities could not be explained in classical physics: Bell's theorem. This theorem led to the inexplicable notion of quantum entanglement. The theorem is a 'no-go theorem' which cannot be proven.

Explanations

Although the correlations are correctly predicted by QM, they were not understood. Until now an understandable explanation has not been given. Many attempts have been made. Some advocate super-determinism. Someone else proposes a multiverse. Actually this one is correct as far as the multiverse is conceived as a Di-verse: the Universe observed from two viewpoints. Many try to prove that Bell's inequalities are wrong. They are not, they only are not applicable to the experiments (they don't describe them correctly, as we shall see, because they in fact describe the experiments from one viewpoint). Most assert that the difficulty to explain the outcomes arises from the execution of the experiments and in fact that is correct, but not in the way they believe it is. It is because something is missing in the explanation: the fact that the particles of an entangled pair are being detected from different viewpoints and the consequence of this.

An explanation is a statement about what is there, what it does, and how and why. Good explanations enhance science and the understanding of the world. [2]

A prediction by a theory is not an understandable physical explanation. To explain the correlations that occur in experiments we have to find out which entangled pairs show combinations of equal spin and which show combinations of opposite spin and the probability with which they occur. This is not an easy task since we cannot see the particles. Yet it is possible to deduce the conditions to explain the correlations. To accomplish this a simple model is described in an ideal situation.

To understand this explanation for the seemingly incomprehensible results in Bell-test experiments, it should be acknowledged that two essential different kinds of movement exist: translation (shifting) and rotation (spinning). In translation no directions change and in rotation all directions change except for the direction of the axis of the rotation. (Rotation transforms to translation only when the radius of rotation approaches infinity or when the angle of rotation approaches zero (becomes infinitely small)). This difference in movement is important to understand the behaviour of vectors in vector spaces on which this explanation rests.

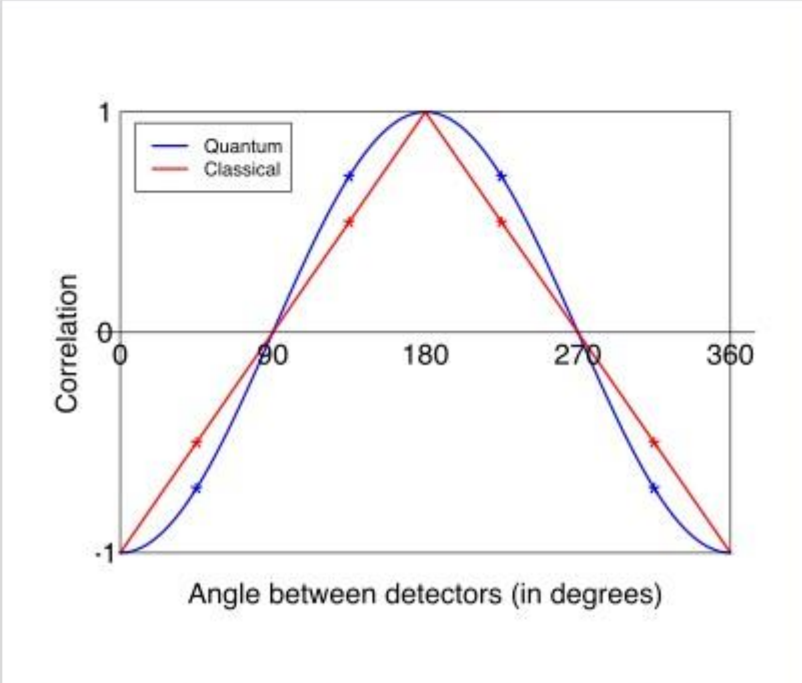
The explanation hasn't been found for about halve a century by now probably because the process of observation / detection is perfectly natural. From a different viewpoint, or in another direction, we see the Universe differently, a different Universe. We see and we immediately adjust because we think it is the same Universe we see. But although the Universe doesn't change, we change our viewpoint or direction. From another viewpoint, or in another direction, we see the ‘Uni-verse’ as a ‘Di-verse’: a different Universe. This is the real problem and it takes Bell-test experiments to make us realize.

The Model

Pairs of entangled particles are produced in a source. The particles of a pair have opposite properties: they move in opposite directions, along a line of motion, and they have opposite spin. Spin is considered to be rotation of a particle around an axis. The direction of that axis, together with the direction of rotation, can be represented by a vector, applying the right hand rule: put the fingers of your right hand around the particle in the rotating direction and then your thumb gives the direction of the vector. The direction of the spin vector of a particle is random in space. So the spin directions of a pair of entangled particles is represented by opposite vectors, randomly, but oppositely, pointing in space. The detectors to detect the spin of the particles are placed at either side of the source, perpendicular on the line of motion of the particles. The setting of a detector is a direction (it might be a field direction) in a plane perpendicular on the line of motion. The setting of a detector can be varied in that plane. A setting also is represented by a vector, or actually a one-directional line.

Measurement of the spin of a particle boils down to compare the component of the spin vector to the setting of the detector: if the component is in the direction of the setting, then the outcome is spin up (+) and if the component is in the opposite direction in respect of the setting, then the outcome is spin down (–). In case of opposite spin and equal settings of the detectors, spin outcomes of one pair of entangled particles always are opposite. In case the settings of the detectors are different, then combinations of equal spin outcomes can occur. The probability for a certain combination of outcomes to occur at a certain combination of settings of the detectors, is exactly given by QM. If the angle between the settings is φ, then the probability for a combination of equal spin outcomes is $\sin^2(\varphi/2)$ and the probability for a combination of opposite spin outcomes is $\cos^2(\varphi/2)$. Together the probabilities add up to 1, as the sum of probabilities should do, and their subtraction makes a negative cosine, defining the correlation. (See diagram below, blue line).

Diagram 1



The best possible local realist imitation (red) for the quantum correlation of two spins in the singlet state (blue), insisting on perfect anti-correlation at 0°, perfect correlation at 180°. Many other possibilities exist for the classical correlation subject to these side conditions, but all are characterized by sharp peaks (and valleys) at 0°, 180°, and 360°, and none has more extreme values (±0.5) at 45°, 135°, 225°, and 315°. These values are marked by stars in the graph, and are the values measured in a standard Bell-CHSH type experiment: QM allows $\pm 1/\sqrt{2} = \pm 0.7071\ldots$, local realism predicts ±0.5 or less.

Source: Wikipedia

Vector space

To explain the results of Bell-test experiments a vector space is established. The vector space is a spherical space that contains all vectors that participate in the experiment. All vectors start in the centre of the sphere. The length of the vectors is not important, only their direction is. There are three kinds of vectors participating in an experiment:

1. Two opposite vectors, representing the line of motion. All detected pairs of entangled particles in an experiment move along this line in opposite directions. The line of motion defines the position of an experiment in space because it is the particles moving along this line that are being detected and measured.
2. Two vectors, representing the setting of the detectors. These vectors can rotate in a plane at the centre, perpendicular on the line of motion. (Actually the settings are no vectors, but one-directional lines intersecting in the centre of the vector sphere).
3. Opposite vector pairs, representing the opposite spin directions of the pairs of entangled particles that are being detected in an experiment. These opposite vector pairs are randomly and proportionally distributed in the sphere.

All vectors that participate in an experiment are allowed to start in the centre of the sphere because they can all be translated (shifted) to the centre without change of direction. The centre of the sphere can considered to represent the source where the entangled pairs are created. Of course the detectors themselves are not at the source, but their settings, represented by one-directional lines, can imagined to be translated to the centre of the sphere. So in our imagination we have to switch between the situation in a 'real' experiment and the vector sphere.

Viewpoints

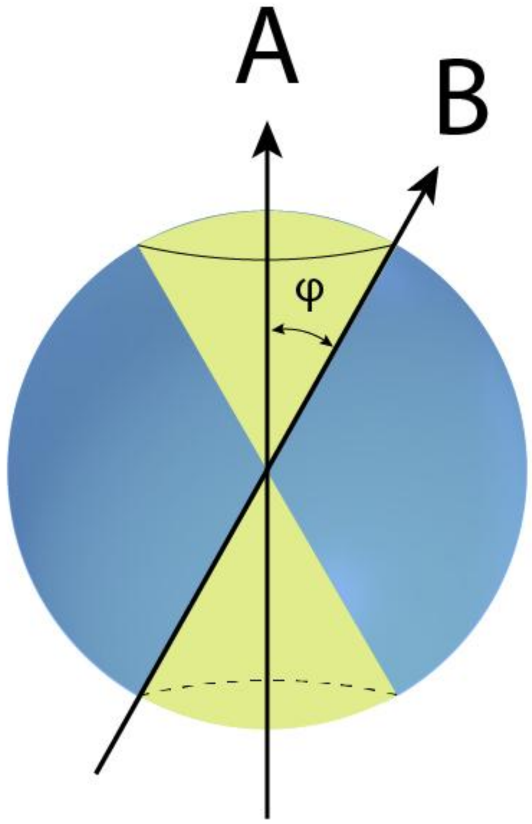
In a 'real' experiment the detectors (A and B) are positioned perpendicular on the line of motion at either side of the source, facing the source. Suppose the detectors to be observers: Alice and Bob. Imagine a pair of entangled particles (1 and 2) is created in the source and that their common spin axis coincidently coincides with the line of motion. Particle 1 moves towards Alice and particle 2 moves towards Bob. So if Alice sees particle 1 approaching, spinning for instance left way around, then Bob sees particle 2 approaching, also spinning left way around because 1 and 2 have opposite spin and they move in opposite directions. So when Alice and Bob compare their results, they see the particles spinning identically whereas in reality they have opposite spin. This means that results, obtained from different viewpoints, cannot be compared. Well, of course they can be put side by side and compared, but no meaningful conclusion may be drawn from that comparison. This is because observations from different viewpoints are not equivalent. An object observed from one direction usually looks differently when observed from another direction. This also goes for pairs of entangled particles.

Observations by Alice and Bob would be equivalent if they look from one viewpoint: they both would see one particle approach, spinning in one direction, and the other particle move away, spinning in the opposite direction. There would be no contradiction in their results. When an observer moves to another position and the observed object moves along in the same way, then the observation doesn't change and stays equivalent. So when Bob moves over from Alice's position to his position opposite of Alice, thereby rotating 180° around the source to keep facing the source, and particle 2 would move along with him, performing the same rotation, then Bob's observation of particle 2 would still be equivalent to Alice's observation of particle 1. Of course in reality particle 2 doesn't move along with Bob, but the idea gives the clue how to keep observations equivalent. It means that all rotations must be taken into account because rotations change directions, as is stated above. In Bell-test experiments these rotations are the rotations of the detectors in respect of each other and in respect of the line of motion. Spin of the particles doesn't change in respect of their movement along the line of motion. Since detectors are placed perpendicular onto the line of motion, also that rotation (90°) must be taken into account.

It are these relative rotations of the settings of the detectors in respect of each other and in respect of the line of motion that define the sub-space in the spherical vector space containing the opposite spin vectors of the pairs of entangled particles that yield combinations of equal spin outcomes.

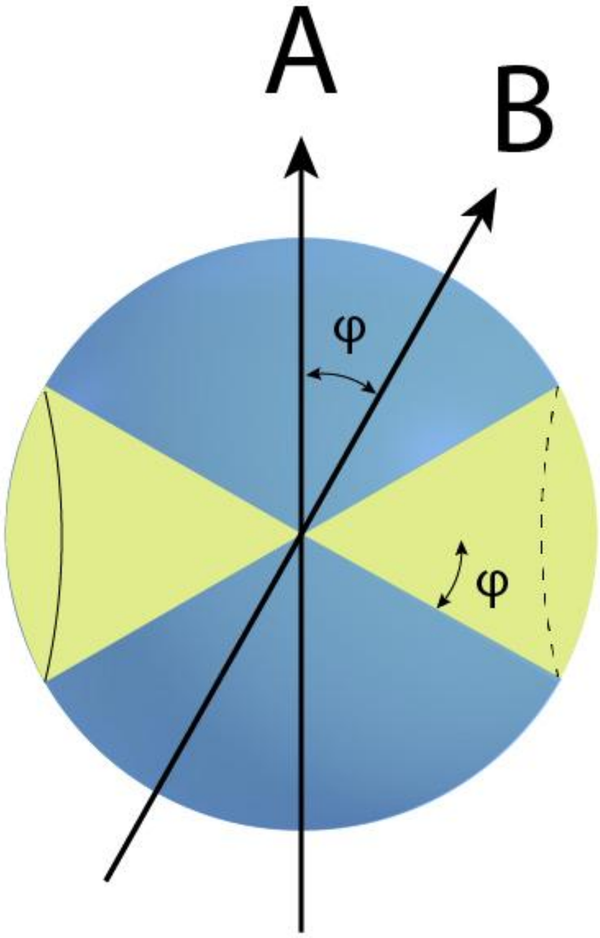
Switching over to the vector sphere, it becomes clear what we have to do. The movement of detector B from its position at A to its position opposite of A, keeping the relative angle between their settings constant while moving over, means in the vector sphere that the one-directional line, representing the setting of B, rotates around the one-directional line, representing the setting of A, keeping the relative angle (φ) constant. In doing so a vector sub-space is being cut out of the spherical vector space. This sub-space has the shape of a double-cone: two opposite cones with the setting of A as common axis, the centre of the vector sphere as their common top and top-angles of 2φ.

Fig.1 The double-cone produced by the rotation of B's setting around the setting of A



Taking also into account the 90° rotation that was needed to get detector A perpendicular onto the line of motion and reversing that rotation, the double-cone ends up having the line of motion as its axis. (See fig2.).

Fig.2: 90° rotated sub-space. The line of motion is the horizontal line through the centre. In this figure the plane containing A and B should be perpendicular onto the screen: from our point of view B should be behind or in front of A.

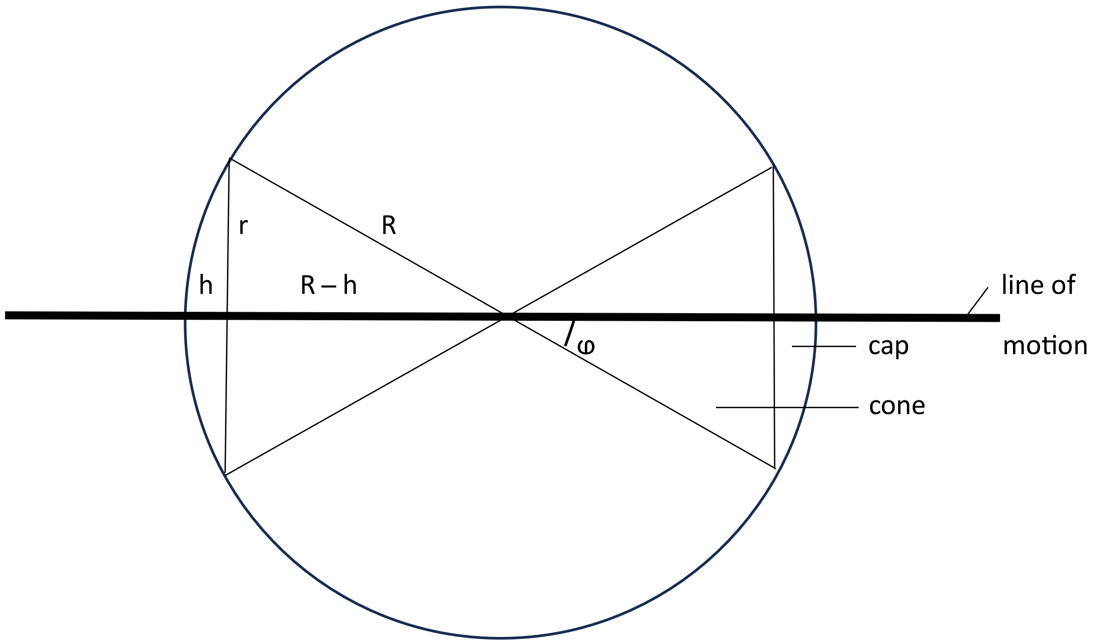


In this way the vector sub-space is discovered that contain the opposite spin vectors of the pairs of entangled particles that show combinations of equal outcomes. Notice that for spin vectors in this sub-space it makes no difference if they are observed or detected from one viewpoint or from opposite viewpoints, moving along or not: they always end up in that sub-space. Also notice that rotating the setting of A around the setting of B results in the same sub-space: the double-cone with the same top angles, having the line of motion as their common axis.

Since the opposite spin vectors of all pairs in an experiment are equally distributed in the sphere, the probability for an arbitrary pair to find its opposite spin vectors in the double-cone shaped vector sub-space is proportional to the volume of the sub-space. That probability is the volume of the sub-space divided by the volume of the total sphere. That probability is equal to the probability predicted by QM.

Proof

Fig.3: Double-cone in spherical vector space



I found the volumes and areas for caps and cones at Wikipedia.

Volume cap: $(1/3)\pi h^2(3R - h)$ Area cap: $2\pi Rh$

Volume cone: $(1/3)\pi r^2(R - h)$

The volume V of the double-cone is the volume of 2 cones plus 2 caps:

$$V = 2(1/3)\pi r^2(R - h) + 2(1/3)\pi h^2(3R - h)$$

$$=(2/3)\pi\{r^2(R - h) + h^2(2R +(R - h))\}$$

Because $r = R\sin\omega$, $(R - h) = R\cos\omega$ and $h = R - R\cos\omega$ it follows:

$$V = (2/3)\pi\{R^2\sin^2\omega(R\cos\omega) + (R - R\cos\omega)^2(2R + R\cos\omega)\}$$

$$= (2/3)\pi\{R^2\sin^2\omega(R\cos\omega) + (R^2 - 2R^2\cos\omega + R^2\cos^2\omega)(2R + R\cos\omega)\}$$

$$= (2/3)\pi\{R^2\sin^2\omega(R\cos\omega) + 2RR^2 - 4RR^2\cos\omega + 2RR^2\cos^2\omega + RR^2\cos\omega - 2RR^2\cos^2\omega + RR^2\cos\omega\cos^2\omega\}$$

$$= (2/3)\pi R^3\{\sin^2\omega(\cos\omega) + 2 - 4\cos\omega + 2\cos^2\omega + \cos\omega - 2\cos^2\omega + \cos\omega(\cos^2\omega)$$

$$= (2/3\pi R^3\{\cos\omega(\sin^2\omega + \cos^2\omega) + 2 - 3\cos\omega\}$$

$$= (2/3\pi R^3\{\cos\omega + 2 - 3\cos\omega\}$$

$$= (4/3)\pi R^3\{1 - \cos\omega\}$$

To find the ratio of this volume in respect to the volume of the total sphere, we have to divide by the volume of the sphere ($(4/3)\pi R^3$) and then we find for the ratio: $(1 - \cos\omega)$. The value of this ratio is between 0 and 2 and that is not the probability for a random vector to end up in the double-cone. This is because the volume of the double-cone is defined by the tip angles of the cones, being 2ω. So when ω approaches 180° the volume of the double-cone takes the total volume of the sphere and when ω approaches 360° it takes 2 times the volume of the total sphere. So the real probability for a random vector to end up in the double-cone is the ratio divided by 2: $(1 – \cos\omega)/2$. This is a probability between 0 and 1.

This probability $(1 – \cos\omega)/2$ is identical to quantum mechanics probability $\sin^2(\omega/2)$ as the diagrams of their function show. Moreover, I found in a book from 1931 by R. Carmichael and E. Smith: 'Mathematical Tables and Formulas' at page 217, formula 31: $\cos2\alpha = 1 – 2\sin^2\alpha$, showing that the probabilities indeed are identical.

The same goes, of course, for the ratio of the total area of the caps in respect to the total area of the sphere.

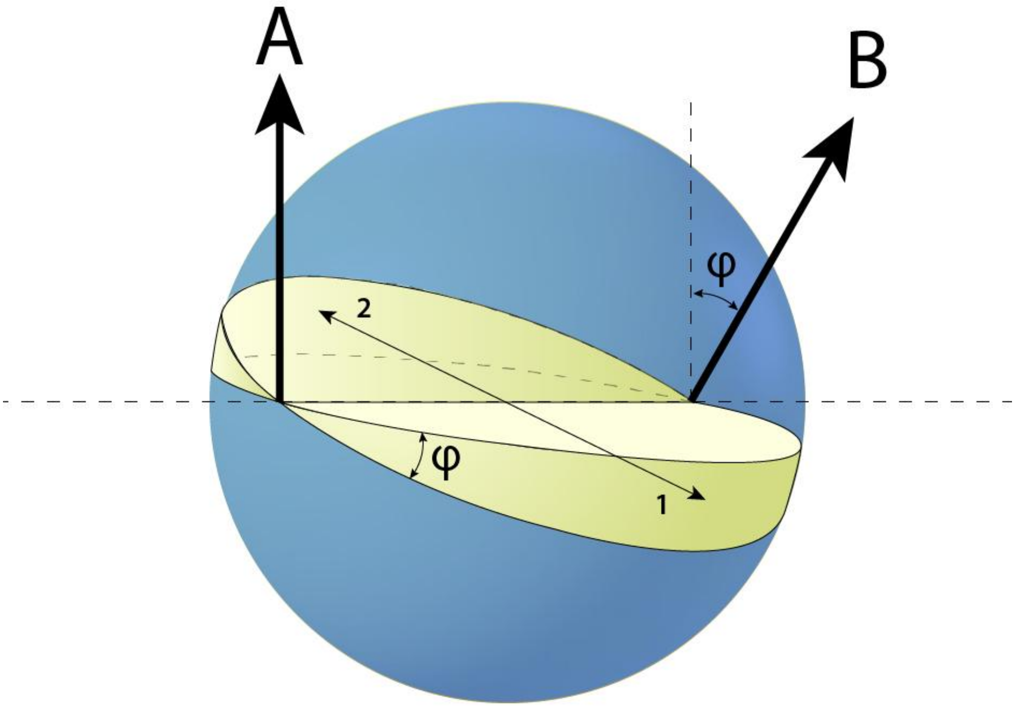
Area of the caps is: $2(2\pi Rh)$. This is $4\pi R^2(1 – \cos\omega)$. Dividing by the area of the spere ($4\pi R^2$) this gives the ratio: $(1 – \cos\omega)$ and thus the same probability $(1 – \cos\omega)/2$ for a random vector pair to end up in the double-cone and show a combination of equal spin outcomes.

For a more visual description, including supporting computer programs, see: [3].

Violation

Bell calculated probabilities for combinations of outcomes that did not correspond to the predictions of Quantum Mechanics and derived inequalities from them. These inequalities were violated by Quantum Mechanics, and later on by the results of experiments. He was convinced that his probabilities applied to the experiments. The reason that they do not apply, is that Bell the probabilities calculated as observing the experiment from one viewpoint without realizing that outcomes from different viewpoints cannot be compared. The probabilities that Bell described were not the vector pairs in the double-cone, but they were the vector pairs in the sphere segments between the centre perpendicular planes of the settings. (See fig.3)

Fig 4. Sphere segments between the centre perpendicular planes of A and B (greenish) representing the sub-spaces containing the opposite vector pairs showing Bell's probabilities.



It can easily be deduced that opposite vector pairs (1 and 2) in the sub-space between the centre perpendicular planes of the settings yield combinations of equal spin outcomes when 1 is measured by A and 2 by B or vice versa.

The probability for a random vector pair to end up in those segments, is indeed Bell's probability for combinations of equal spin outcomes: (2ω / 2π). Together with the probability for combinations of opposite spin results: (1 – (2ω/2π) = (π – ω)/π) this gives a correlation as is represented by the red line in diagram 1.

(Bell's probabilities are valid for 0 < ω <180° because at 180° the centre perpendicular plane of B 'moves through' the centre perpendicular plane of A and because of that the opposite side of the planes are facing each other when ω >180°. This means that the sphere segments between the centre perpendicular planes contain not anymore the opposite spin vectors yielding combinations of equal spin outcomes, but they contain the opposite spin vectors of pairs yielding combinations of opposite spin outcomes when ω > 180°. This causes the symmetry in diagram 1.)

These probabilities seem correct, either observed from A, or observed from B. But if B would move over to A, in order to obtain equivalent detections (and comparable outcomes), keeping ω constant, then the sphere segments between the centre perpendicular planes, would move along and become totally different sub-spaces. (The segments would be mirrored in the centre perpendicular plane of A)

Therefore the opposite vector pairs in these sphere segments are not the spin vectors of pairs showing combinations of equal spin outcomes. For that reason Bell's inequalities are violated.

Quantum Computer

From this explanation it can be deduced that the QM probabilities apply on pairs of particles and that those probabilities depend on the relative angle (φ) between the settings of the detectors that detect the particles and measure them. These probabilities therefore do not apply to qubits measured by only one detector. These QM probabilities are represented on the Bloch sphere, so the probabilities of the Bloch sphere do not apply to single qubits. If the working of quantum computers is based on the Quantum Mechanics notion of entanglement, and the probabilities of the Bloch sphere, then they probably will at most function as analogue computers without the advantages of digital computers. They are expensive, technically complicated and have a high risk for errors. In respect to digital computers they will not be faster, better or able to solve problems that are more complicated. Quantum computers will not be able to produce the miracles that are promised.

Conclusion

This explanation shows that results in Bell-test experiments, predicted by QM, can very well be comprehended classically. It shows that entanglement is not an incomprehensible Quantum Mechanic phenomenon, but only the result of opposite properties that entangled particles acquire because of conservation laws (resonance).

References:

[1] J.S. Bell; Bertlmann's socks and the nature of reality.

https://cds.cern.ch/record/142461/files/198009299.pdf

[2] David Deutsch: The Beginning of Infinity. (book).

[3] Gerard van der Ham: The Principle of Perspective

https://bell-game-challenge.vercel.app/