De Broglie's Phase Waves of Electrons on Cyclotron Orbits

Martin Kraus (kraus.martin@gmail.com)

February 5, 2026

Abstract

The discrete energy levels of electronic orbits in Bohr's model of hydrogen atoms may be derived from the requirement that de Broglie's electronic phase waves are continuous on closed orbits. This short work applies the same reasoning to electronic cyclotron orbits. The resulting discrete energy levels include the non-relativistic Landau levels and additional (half-integer) levels. It is hypothesized that transitions to these additional levels are usually strongly suppressed.

1 Introduction and Previous Work

In his doctoral thesis [dB25], de Broglie employed a hypothetical internal clock of electrons and associated "phase waves" to explain experimentally observed wave-like features of electrons. In particular, de Broglie derived the discrete energy levels in Bohr's model of hydrogen-like atoms by requiring that phase waves on circular electronic orbits are continuous. More recent previous work [Kra25] derived Einstein's quantization condition [Ein17] for the case of closed Bohr-Sommerfeld orbits. In the present work, the same approach is applied to circular electronic cyclotron orbits resulting in the non-relativistic Landau levels [LL65] and additional (half-integer) levels.

The notation in this work follows previous work [Kra25, dB25] (including speed of light c, Planck constant h, mass of electron m_0, speed of electron as fraction β of speed of light, and the Lorentz factor $\gamma \stackrel{\text{def}}{=} 1/\sqrt{1-\beta^2}$) but uses $|\mathbf{v}|$ for the speed of an electron and $|e|$ for the elementary electric charge. Furthermore, the reduced Planck constant $\hbar \stackrel{\text{def}}{=} h/(2\pi)$ and the orbital angular frequency $\omega \stackrel{\text{def}}{=} 2\pi/\tau$ (with orbital period τ) are employed in order to provide the standard form of some equations.

In this notation, the orbital angular frequency ω of electronic cyclotron orbits in a uniform magnetic field B is given [CM91] by

$$\omega = \frac{|e|B}{\gamma m_0}. \tag{1}$$

With these prerequisites in mind, the following sections discuss electronic phase waves on circular orbits and their quantization.

2 Phase Waves on Circular Orbits

De Broglie proposed that each electron features a periodic process (i.e. an internal clock) of the Compton frequency $\nu_0 \stackrel{\text{def}}{=} m_0c^2/h$ in its rest frame [dB25]. If the electron is moving with constant speed $|\mathbf{v}| = \beta c$, the time-dilated frequency of this moving clock in the rest frame of a fixed observer is $\nu_1 \stackrel{\text{def}}{=} m_0c^2\sqrt{1-\beta^2}/h = m_0c^2/(\gamma h)$. Furthermore, according to de Broglie, this internal clock "appears constantly in phase with a wave" [dB25, page 9] (i.e. the electron's "phase wave") of frequency $\nu \stackrel{\text{def}}{=} m_0c^2/(h\sqrt{1-\beta^2}) = m_0c^2\gamma/h$.

If an electron moves on a periodic circular orbit with constant speed $|\mathbf{v}|$, its phase wave may be considered part of a steady-state forced vibration of an unspecified oscillating field, which is driven by the electron's internal clock [Kra25]. Thus, each fixed point of this field oscillates at the same uniform frequency ν as the electron's phase wave.

3 Quantization Condition

As mentioned, the electron's internal clock and the phase wave are always in phase. For a steady-state forced vibration, this results in the quantization condition that after one orbital period τ, the internal clock and the oscillating field at a fixed point have to return to the exact same phase *difference*. In other words, the quantization condition requires that the difference between the number of cycles n_{of} of the oscillating field during period τ and the number of cycles n_{ic} of the electron's internal clock during period τ is an integer [Kra25].

In the case of a circular cyclotron orbit, the orbital period is $\tau = 2\pi/\omega$ with $\omega = |e|B/(\gamma m_0)$. Since the forced vibration is characterized by frequency ν, the number of cycles n_{of} of the oscillating field during period τ is

$$n_{\text{of}} \stackrel{\text{def}}{=} \tau\nu = \frac{2\pi}{\omega} \cdot \frac{m_0 c^2 \gamma}{h}. \tag{2}$$

The number of cycles n_{ic} of the internal clock (with time-dilated frequency ν_1) during period τ is

$$n_{\text{ic}} \stackrel{\text{def}}{=} \tau\nu_1 = \frac{2\pi}{\omega} \cdot \frac{m_0 c^2}{h\gamma}. \tag{3}$$

Therefore, the difference $\Delta n_{\text{of,ic}}$ (which is required to be an integer by the quantization condition) is

$$\Delta n_{\text{of,ic}} \stackrel{\text{def}}{=} n_{\text{of}} - n_{\text{ic}} = \frac{2\pi}{\omega} \cdot \frac{m_0 c^2 \gamma}{h} - \frac{2\pi}{\omega} \cdot \frac{m_0 c^2}{h\gamma} = \frac{2\pi m_0 c^2}{\omega h} \left(\gamma - \frac{1}{\gamma}\right) \tag{4}$$

$$= \frac{2\pi m_0 c^2}{\omega h} \cdot \frac{\gamma^2 - 1}{\gamma} = \frac{2\pi m_0 c^2}{\omega h} \cdot \frac{1 - (1-\beta^2)}{(1-\beta^2)\gamma} = \frac{2\pi m_0}{\omega h} \cdot \gamma |\mathbf{v}|^2. \tag{5}$$

Thus:

$$\omega\hbar \Delta n_{\text{of,ic}} = \gamma m_0 |\mathbf{v}|^2 \qquad \text{with} \quad \Delta n_{\text{of,ic}} > 0. \tag{6}$$

The odd values of $\Delta n_{\text{of,ic}}$ (i.e. $1, 3, 5, \ldots$) represent the (integer) Landau levels. They may be enumerated by an integer $n \geq 0$ with $\Delta n_{\text{of,ic}} = 2n+1$. Inserting this relation into Eq. (6) and assuming $\gamma \approx 1$ results in the well-known equation for non-relativistic Landau levels E_n [LL65, page 425]:

$$E_n \stackrel{\text{def}}{=} \omega\hbar \left(n + \tfrac{1}{2}\right) = \tfrac{1}{2} m_0 |\mathbf{v}|^2 \qquad \text{with} \quad n \geq 0. \tag{7}$$

4 Discussion

While the successful calculation of non-relativistic Landau levels supports the validity of the presented approach, the quantization condition allows for additional energy levels for even values of $\Delta n_{\text{of,ic}}$ (i.e. $2, 4, 6, \ldots$) and, therefore, half-integer values of n (i.e. $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$). Half-integer Landau levels are an active research area, but experimental observations are still rare [YJY+16]. Hypothetically, transitions from integer energy levels to half-integer energy levels (and vice versa) are strongly suppressed such that half-integer levels are usually inaccessible to electrons (since the ground-state energy level is E_0). Potential factors contributing to this suppression include the similarity of phase waves for neighboring integer values of n, the proximity of cyclotron orbits for neighboring integer values of n (compared to, for example, corresponding electronic orbits in the Bohr model of hydrogen), and destabilizing effects of other electrons occupying orbits for neighboring integer values of $\Delta n_{\text{of,ic}}$.

5 Conclusion and Future Work

This work calculates the non-relativistic Landau energy levels of electronic cyclotron orbits based on de Broglie's requirement that an electron's phase wave has to be continuous on a periodic electronic orbit [dB25], which was interpreted in previous work [Kra25] as a requirement for steady-state forced vibrations driven by an electron's internal clock. Thus, discrete Landau levels appear to be a consequence of a relativistically moving clock instead of being an inherently quantum mechanical phenomenon.

Future work includes research on the existence of half-integer Landau levels, research on the degeneracy of Landau levels in terms of different modes of forced vibrations, and a derivation of relativistic Landau levels.

References

[CM91] Mario Conte and William W. MacKay. *An Introduction to the Physics of Particle Accelerators*. World Scientific Publishing, Singapore, 1991.

[dB25] Louis de Broglie. Recherches sur la théorie des Quanta. *Annales de Physique*, 10(3):22–128, January 1925. English translation by A. F. Kracklauer.

[Ein17] Albert Einstein. Zum Quantensatz von Sommerfeld und Epstein. *Verhandlungen der Deutschen Physikalischen Gesellschaft*, 19(9–10):82–92, May 1917.

[Kra25] Martin Kraus. De Broglie's phase waves of electrons in hydrogen-like atoms as parts of steady-state forced vibrations. viXra:2503.0008, 2025.

[LL65] Lev Davidovich Landau and Evgenii Mikhailovich Lifshitz. *Quantum Mechanics: Non-Relativistic Theory*, volume 3 of *Course of Theoretical Physics*. Pergamon Press, 2nd edition, 1965.

[YJY+16] W. Yu, Y. Jiang, J. Yang, Z. L. Dun, H. D. Zhou, Z. Jiang, P. Lu, and W. Pan. Quantum oscillations at integer and fractional Landau level indices in single-crystalline $ZrTe_5$. *Scientific Reports*, 6(35357), 2016.

A Revisions

- Original version submitted to vixra.org on February 5, 2026.