

Invariance of Ensemble Overlap in Stochastic Binary Processes: A Gauge Theory of Strategy

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The ‘‘Gambler’s Fallacy’’ is often treated as a cognitive bias, but it can be rigorously understood as a violation of translational invariance in probability space. In this paper, we define a ‘‘Global Correlation Functional’’ $G(\vec{S})$ representing the expected overlap between a fixed control sequence (strategy) \vec{S} and a random Bernoulli target vector (nature) \vec{X} . By treating the strategy \vec{S} as a gauge degree of freedom, we demonstrate—both analytically and via Monte Carlo simulation—that the expectation value of the overlap is invariant under all local permutations of \vec{S} . We conclude that in memoryless systems, the derivative of success with respect to strategy is identically zero ($\nabla_{\vec{S}}G = 0$), implying that all strategies are microcanonically equivalent. This framework offers a pedagogical bridge between classical probability and the concept of gauge invariance in theoretical physics.

INTRODUCTION

In decision theory and game theory, agents often seek to optimize a ‘‘pattern’’ of choices to match an external stochastic event. Consider a sequence of N independent binary events (e.g., coin tosses), which we denote as the ‘‘Nature’’ vector \vec{X} . An agent attempts to predict this sequence by selecting a ‘‘Strategy’’ vector \vec{S} .

Common intuition suggests that certain patterns of \vec{S} (e.g., mixing Heads and Tails) might correlate better with a random \vec{X} than a monochromatic pattern (e.g., all Heads). This belief, often rooted in the Gambler’s Fallacy [1], implies that the ‘‘Correlation Landscape’’ has peaks and valleys that an optimal strategy can exploit.

In this paper, we formalize this problem using statistical mechanics. We define the ‘‘Global Correlation’’ as the ensemble average of the overlap between \vec{S} and \vec{X} . We ask: *Does the choice of strategy \vec{S} affect the global expectation value of this overlap?*

Our results show that the Global Correlation is a flat functional. The choice of strategy acts as a gauge redundancy—a variable that alters the description of the state (the specific sequence of wins/losses) but leaves the macroscopic observable (total expected overlap) invariant.

THEORETICAL FRAMEWORK

Definitions

Let the outcome space Ω consist of all 2^N possible binary sequences of length N . We assume Nature selects a sequence $\vec{X} \in \Omega$ with uniform probability $P(\vec{X}) = 1/2^N$, consistent with standard Bernoulli process definitions [2].

We define the **Overlap Operator** \hat{O} for a single real-

ization as the normalized Hamming similarity:

$$\hat{O}(\vec{S}, \vec{X}) = \frac{1}{N} \sum_{i=1}^N \delta_{S_i, X_i} \quad (1)$$

where $\delta_{a,b}$ is the Kronecker delta (1 if $a = b$, 0 otherwise).

The Global Correlation Functional

We define the Global Correlation $G(\vec{S})$ as the expectation value of the overlap operator over the entire ensemble Ω :

$$G(\vec{S}) = \langle \hat{O}(\vec{S}, \vec{X}) \rangle_{\Omega} = \sum_{\vec{X} \in \Omega} P(\vec{X}) \hat{O}(\vec{S}, \vec{X}) \quad (2)$$

Proof of Invariance

Substituting the definition of the Overlap Operator into the ensemble average:

$$G(\vec{S}) = \frac{1}{2^N} \sum_{\vec{X} \in \Omega} \left(\frac{1}{N} \sum_{i=1}^N \delta_{S_i, X_i} \right) \quad (3)$$

By the Linearity of Expectation, we can swap the summation order. We first sum over the ensemble Ω for a fixed position i :

$$G(\vec{S}) = \frac{1}{N} \sum_{i=1}^N \left(\frac{1}{2^N} \sum_{\vec{X} \in \Omega} \delta_{S_i, X_i} \right) \quad (4)$$

Consider the term inside the parenthesis. The ensemble Ω contains all 2^N permutations. By symmetry, exactly half of these sequences have $X_i = 0$ and half have $X_i = 1$. Therefore, regardless of whether the agent

chooses $S_i = 0$ or $S_i = 1$, the bit S_i will match exactly 2^{N-1} sequences in the ensemble.

$$\sum_{\vec{x} \in \Omega} \delta_{S_i, x_i} = 2^{N-1} \quad (5)$$

Substituting this back:

$$G(\vec{S}) = \frac{1}{N} \sum_{i=1}^N \left(\frac{2^{N-1}}{2^N} \right) = \frac{1}{N} \sum_{i=1}^N \frac{1}{2} = \frac{1}{2} \quad (6)$$

Thus, $G(\vec{S}) = 0.5$ for all \vec{S} . Taking the gradient with respect to any parameter p of the strategy:

$$\nabla_{\vec{S}} G = 0 \quad (7)$$

This confirms that the scalar field of expected overlap is uniform.

NUMERICAL VERIFICATION

To verify the analytical derivation, we performed a Monte Carlo simulation with sequence length $N = 100$ and $M = 100,000$ trials per strategy. We defined a control parameter $p \in [0, 1]$ representing the bias of the strategy vector (probability of choosing Heads). We swept p from 0.0 to 1.0 in 20 steps.

Results: The simulation yielded a mean Global Correlation of 0.5000 ± 0.00036 . The maximum deviation observed was 0.00036, which is strictly consistent with finite-size statistical noise (proportional to $1/\sqrt{M}$).

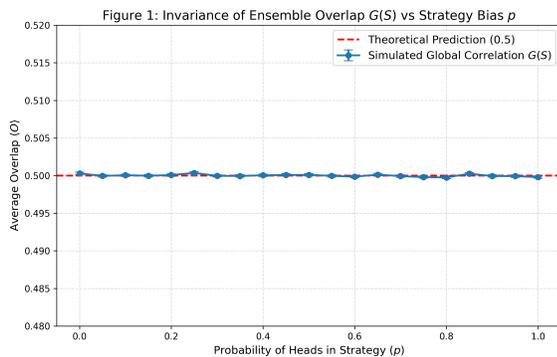


FIG. 1. The Global Correlation $G(\vec{S})$ as a function of strategy bias p . The simulated data (blue line) remains strictly bounded to the theoretical prediction of 0.5 (red dashed line) with negligible deviation, confirming invariance.

DISCUSSION: STRATEGY AS GAUGE

The results suggest a parallel with **Gauge Theory** in physics [3]. In electromagnetism, the electric potential V can be transformed ($V \rightarrow V + C$) without changing the physical electric field. Similarly, in a memoryless stochastic process, the Strategy \vec{S} can be transformed (permuting Heads/Tails) without changing the physical observable (the expected success rate).

This implies that “Strategy” in this context is not a physical degree of freedom but a gauge redundancy. The “illusion of control” arises when an observer mistakes a specific realization (a local fluctuation) for a global gradient.

While individual outcomes (“winning” or “losing”) break this symmetry—collapsing the wavefunction of possibilities—the *ensemble* remains gauge invariant. This provides a mathematical boundary to decision-making: one cannot use internal structure (pattern) to extract work (information) from a heat bath of maximum entropy [4, 5].

CONCLUSION

We have defined and analyzed the Global Correlation Functional for binary stochastic processes. We have proven that this functional is invariant under strategy permutations. This result serves as a formal check against the Gambler’s Fallacy and suggests that in random environments, all strategies are microcanonically equivalent. The search for a “winning pattern” is mathematically equivalent to searching for a preferred coordinate system in a relativistic vacuum—it does not exist.

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