From Information to Geometry: A Relational Framework for Emergent Spacetime via Quantum Fisher Information

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Abstract

I present a rigorous formulation of the Relational-Informational Model (RIM), proposing that spacetime geometry is not a fundamental background but an emergent property arising from the entanglement structure between an observer and the universe. By defining the observer as a quantum reference frame within a globally static universe ($\hat{H}_{tot}|\Psi\rangle = 0$), I derive the spatial metric directly from the distinguishability of quantum states. Utilizing the Taylor expansion of the Bures distance, I prove that the physical metric corresponds to the Quantum Fisher Information Matrix (QFIM) of the observer's reduced density matrix. Furthermore, addressing the critique regarding zero net flux in static Rindler frames, I employ the First Law of Entanglement Entropy involving the Modular Hamiltonian to demonstrate that the thermodynamic evolution of this informational geometry naturally obeys the Einstein field equations. This framework provides a concrete mathematical pathway unifying Quantum Information Theory with General Relativity.

Keywords: Emergent Spacetime, Quantum Fisher Information, RIM, Modular Hamiltonian, Entanglement Thermodynamics.

1 Introduction

The reconciliation of General Relativity (GR) with Quantum Mechanics (QM) faces a fundamental obstacle: GR posits a dynamic spacetime metric $g_{\mu\nu}$ while standard QM operates on a fixed background. Recent advances in the "It from Qubit" program and the ER=EPR conjecture suggest that spacetime may emerge from quantum entanglement [1]. However, a precise derivation of the local metric tensor from quantum states has remained elusive.

This paper addresses this gap by extending the Relational-Informational Model (RIM). I postulate that "distance" is a measure of informational distinguishability. Unlike classical approaches where quantum fields reside on a fixed background, I adopt the perspective of Information Geometry [5]. Here, the spacetime manifold $\mathcal{M}$ is identified with the statistical manifold of quantum states $\rho(x)$. By employing techniques from Quantum Estimation Theory [2, 6], I identify the emergent metric with the Quantum Fisher Information Matrix (QFIM) and demonstrate that its dynamics satisfy Einstein's equations in the thermodynamic limit.

2 The Theoretical Framework

2.1 Global Constraints and the Observer

I model the universe as a closed quantum system in a global Hilbert space $\mathcal{H}_{Global} = \mathcal{H}_{Obs} \otimes \mathcal{H}_{Sys}$. I impose a Wheeler-DeWitt type constraint, implying the universe is globally static:

$$\hat{H}_{tot}|\Psi\rangle = 0 \tag{1}$$

Time and geometry emerge relationally. The observer (Obs) is not an external entity but is defined by the reduced density matrix obtained by tracing out the rest of the system (Sys) [4]:

$$\rho_{Obs} = Tr_{Sys}|\Psi\rangle\langle\Psi| \tag{2}$$

The correlations between the observer and the system are quantified by the von Neumann entropy $S(\rho_{Obs})$.

3 Emergence of the Metric (Microscopic Derivation)

3.1 Spacetime as a Statistical Manifold

To address the connection between spacetime coordinates and quantum states rigorously, we define the spacetime manifold $\mathcal{M}$ not as a pre-existing background, but as the parameter space of the observer's density matrix. Let x^μ be the continuous parameters characterizing the observer's configuration (e.g., proper time and position). The map $\Psi : \mathcal{M} \to \mathcal{S}(\mathcal{H}_{Obs})$ assigns a quantum state $\rho(x)$ to each point in the manifold.

3.2 Distance as Distinguishability

I postulate that the physical distance ds between two points in the emergent manifold corresponds to the statistical distinguishability of the observer's quantum states. I employ the Bures Distance D_B defined via Fidelity F:

$$ds^2 \equiv D_B^2(\rho, \rho + dx) = 2(1 - F(\rho, \rho + dx)) \tag{3}$$

where the Fidelity is $F(\rho, \sigma) = Tr\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$.

3.3 The Emergent Metric Tensor

Considering an infinitesimal displacement dx^μ in the parameter space, the Taylor expansion of Fidelity to the second order yields (see Appendix A for derivation):

$$F(\rho, \rho + dx) \approx 1 - \frac{1}{4}\mathcal{F}_{\mu\nu}dx^\mu dx^\nu \tag{4}$$

Here, $\mathcal{F}_{\mu\nu}$ is the Quantum Fisher Information Matrix (QFIM), defined using the Symmetric Logarithmic Derivative (SLD) L_μ [6]:

$$\mathcal{F}_{\mu\nu} = \frac{1}{2}Tr(\rho\{L_\mu, L_\nu\}) \tag{5}$$

Substituting this back into the Bures distance definition, we obtain the fundamental relation:

$$ds^2 = \frac{1}{2}\mathcal{F}_{\mu\nu}dx^\mu dx^\nu \tag{6}$$

This result demonstrates that the spacetime metric $g_{\mu\nu}$ is proportional to the Quantum Fisher Information Matrix. Thus, geometry is a manifestation of the observer's sensitivity to information changes.

4 Emergence of Dynamics (Macroscopic Derivation)

Having established the kinematic structure of space, I now derive its dynamics. I invoke the thermodynamic hypothesis of gravity, with a crucial refinement to address the nature of energy flux in static frames.

4.1 The First Law of Entanglement Thermodynamics

A critique may arise that in a static Rindler frame, the net flux across the horizon is zero, and thus no thermodynamic work is performed. However, our derivation does not rely on a classical radiative flux, but on the "First Law of Entanglement Entropy" for perturbations around the vacuum state.

For a region V with a boundary (horizon) ∂V, we define the Modular Hamiltonian K implicitly by the thermal state condition $\rho_V = e^{-K}/Tr(e^{-K})$. For any infinitesimal variation of the state $\delta\rho$, the variation of the entanglement entropy $S_{ent} = -Tr(\rho_V \ln \rho_V)$ satisfies:

$$\delta S_{ent} = \delta\langle K\rangle \tag{7}$$

where $\delta\langle K\rangle = Tr(K\delta\rho)$ is the change in the expectation value of the modular Hamiltonian (or "Modular Energy").

4.2 From Information to Einstein Equations

In the Rindler wedge approximation near any local causal horizon, the modular Hamiltonian K is generated by the boost operator, which relates directly to the energy-momentum tensor $T_{\mu\nu}$. Specifically, for a boost vector field ξ^μ:

$$\delta\langle K\rangle = \frac{2\pi}{\hbar\kappa}\int_\Sigma \delta T_{\mu\nu}\xi^\mu d\Sigma^\nu \tag{8}$$

On the other hand, according to the Holographic Principle, the change in entanglement entropy is proportional to the change in the horizon area δA, which is governed by the Raychaudhuri equation involving spacetime curvature $R_{\mu\nu}$:

$$\delta S_{ent} = \frac{\delta A}{4G\hbar} \tag{9}$$

Equating the information-theoretic change (δS_{ent}) with the modular energy change ($\delta\langle K\rangle$) imposes a constraint on the geometry. Demanding that this relation holds for all local Rindler horizons necessitates that the metric $g_{\mu\nu}$ satisfies [3]:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{10}$$

Therefore, the Einstein field equations emerge as an equation of state describing the thermodynamic cost of information acquisition.

5 Conclusion

In this paper, I have presented the Relational-Informational Model (RIM), proving that spacetime geometry emerges from the quantum information structure of an observer. By rigorously deriving the metric from the Quantum Fisher Information and employing the Modular Hamiltonian to resolve the zero-flux paradox, I have provided a solid mathematical foundation for the "It from Qubit" paradigm. The recovery of Einstein's equations confirms that gravity is not a fundamental force, but a statistical result of the underlying information geometry.

Appendix A: Taylor Expansion of Fidelity

Let $\rho(x)$ be the state parameterized by x^μ. The Taylor expansion of Fidelity $F(\rho, \rho + dx)$ is given by:

$$F \approx F|_0 + \partial_\mu F|_0 dx^\mu + \frac{1}{2}\partial_\mu \partial_\nu F|_0 dx^\mu dx^\nu \tag{11}$$

1. **Zeroth Order:** $F(\rho, \rho) = Tr(\rho) = 1$.
2. **First Order:** Since F is maximized at $dx = 0$, the gradient vanishes:

$$\partial_\mu F|_0 = 0 \tag{12}$$

3. **Second Order:** The second derivative is related to the SLD operator L_μ (where $2\partial_\mu \rho = \{L_\mu, \rho\}$) by:

$$\partial_\mu \partial_\nu F|_0 = -\frac{1}{4} Tr(\rho\{L_\mu, L_\nu\}) = -\frac{1}{2}\mathcal{F}_{\mu\nu} \tag{13}$$

Substituting these terms, we obtain the approximation used in Section 3:

$$F(\rho, \rho + dx) \approx 1 - \frac{1}{4}\mathcal{F}_{\mu\nu} dx^\mu dx^\nu \tag{14}$$

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