A Model for a Machian Cosmology Reproducing Dark Matter, Dark Energy, and the Fine Structure Constant

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Abstract

Motivated by the suggestion made by R. H. Dicke in 1957 that the speed of light may be correlated with the gravitational potential of the entire universe, we develop a model for a Machian cosmology in which matter determines the speed of light through a scalar field which has a mathematical form similar to, but distinct from, the gravitational potential. We show that this leads naturally to a cosmology in which the speed of light was higher in the early universe and is decreasing in cosmological time, providing an explanation for the isotropy of the cosmic microwave background (CMB) without the need for inflation. This cosmology results in an apparent amplification of the baryonic mass density (dark matter) and predicts a critical acceleration reproducing that of modified Newtonian dynamics (MOND). We also fit the model to high-redshift supernova data from the Supernova Cosmology Project, showing that an excellent fit is obtained with only baryonic matter. Finally, we derive a geometric relationship between cosmological parameters and the fine structure constant of quantum electrodynamics.

Keywords: general relativity, dark matter, dark energy, cosmology, MOND

1. Introduction

In 1957 R. H. Dicke published "Gravitation Without a Principle of Equivalence" in which he made the observation that twice the gravitational potential of the observable universe is approximately equal to the speed of light squared [1]. Dicke was considering the fact that the deflection of light around a gravitating body can be modeled as a local change in the refractive index of the form

$$n = \frac{c_0}{c} = 1 + \frac{2GM}{rc_0{}^2}. \quad (1)$$

He suggested that the number 1 on the right-hand-side of this equation has its origin in the gravitational potential of all matter in the observable universe. Upon performing the computation using the currently estimated mass of the observable universe and the particle horizon as its radius he did, in fact, find it to be a numerical coincidence. An obvious question is whether this is simply a numerical coincidence that happens to hold at our present cosmological time, or if it represents a deeper connection in physics between gravitation, electromagnetism, and causality.

In what follows we will develop a Machian cosmology inspired by Dicke's observation in which the speed of light is connected to all masses in the universe, although not precisely by the gravitational potential itself. This cosmology will be founded on three postulates: 1) A slightly modified form of the Einstein equation of general relativity differing only in the cosmological constant term, 2) a scalar field which resembles the Newtonian gravitational potential but differs in some respects, and 3) a Friedmann-Robertson-Walker (FRW) metric for the universe without dark energy or dark matter but with intrinsic negative spatial curvature. We will now breifly outline the motivation for these postulates.

1.1. A Modified Einstein Equation

The Einstein equation is given by

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \quad (2)$$

where $G_{\mu\nu}$ is the Einstein tensor constructed from the metric tensor $g_{\mu\nu}$, $T_{\mu\nu}$ is the stress-energy tensor of matter, and Λ is the cosmological constant. The cosmological constant term is often moved to the right hand side, and since $g_{\mu\nu}$ can be made equal to the Minkowski metric $\eta_{\mu\nu}$ locally through a coordinate transformation, the cosmological constant behaves as a source of stress-energy with positive energy density and negative pressure.

Writing the metric as the Minkowski metric plus a perturbation $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, we suggest slightly modifying the cosmological constant term to

$$G_{\mu\nu} + \Lambda h_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \quad (3)$$

This form of the Einstein equation remains generally covariant. Since $h_{\mu\nu}$ may be transformed away to be identically zero locally (but not everywhere), the cosmological constant no longer represents an effective stress-energy tensor but rather a screening factor for the metric.

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[1]This work was not financially supported by NIST and the views presented are solely those of the author and do not represent those of NIST or the U.S. Department of Commerce.

To make this clear, we will represent the components of $h_{\mu\nu}$ as

$$\begin{aligned} h_{00} &= -2\Phi \\ h_{0i} &= w_i \\ h_{ij} &= 2s_{ij} - 2\Psi\delta_{ij} \end{aligned} \tag{4}$$

where Ψ encodes the trace of h_{ij} and s_{ij} is traceless so that we have

$$\begin{aligned} \Psi &= -\frac{1}{6}\delta^{ij}h_{ij} \\ s_{ij} &= \frac{1}{2}\left(h_{ij} - \frac{1}{3}\delta^{kl}h_{kl}\delta_{ij}\right). \end{aligned} \tag{5}$$

We will assume $\left|h_{\mu\nu}\right| \ll 1$. In the transverse gauge the components of the Einstein equation in units where $c = 1$ can be written as follows [2]

$$G_{00} = 2\nabla^2\Psi - 2\Lambda\Phi = 8\pi G T_{00} \tag{6}$$

$$G_{0j} = -\frac{1}{2}\nabla^2 w_j + 2\partial_0\partial_j\psi + \Lambda w_j = 8\pi G T_{0j} \tag{7}$$

$$\begin{aligned} G_{ij} = \left(\delta_{ij}\nabla^2 - \partial_i\partial_j\right)(\Phi - \Psi) - \partial_0\partial_{(i}w_{j)} + 2\delta_{ij}{\partial_0}^2\Psi \\ -\Box s_{ij} + \Lambda\left(2s_{ij} - 2\Psi\delta_{ij}\right) = 8\pi G T_{ij}. \end{aligned} \tag{8}$$

The (0,0) equation in (6) reduces to

$$\nabla^2\Psi - \Lambda\Phi = 4\pi G\rho. \tag{9}$$

In the case of dust where $T_{\mu\nu} = \rho u_\mu u_\nu$, from (8) we have $\Psi = \Phi$ so that (9) reduces to a screened Poisson equation

$$\nabla^2\Phi - \Lambda\Phi = 4\pi G\rho. \tag{10}$$

Equation (9) is simply the Newtonian equation for the gravitational potential with characteristic screening length $\sqrt{\Lambda}$ with solution given by

$$\Phi(r) = -\frac{GM}{r}e^{-r\sqrt{\Lambda}}. \tag{11}$$

We will come back to this solution when discussing the nature of the proposed scalar field in Section 2.

1.2. An FRW Metric with Negative Spatial Curvature

In order to motivate our postulate that the universe has negative spatial curvature, we begin with the Friedmann equations which govern the expansion of the universe in general relativity

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{\kappa c^2}{a^2} + \frac{\Lambda c^2}{3} \tag{12}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \tag{13}$$

where $\kappa = k/{R_0}^2$ is the spatial curvature, R_0 is the radius of curvature at the present time, Λ is the cosmological constant, and $\dot{a}$ and $\ddot{a}$ represent first and second time derivatives of the scale factor [2]. The parameter k in the spatial curvature may take the values $+1, -1$, or 0 corresponding to closed, open, and flat universes.

With the modified form of the Einstein equation proposed above, we may eliminate the cosmological constant terms from these equations since this constant no longer represents an effective stress-energy tensor of the vacuum but rather a screening factor for the metric tensor. In a universe dominated by baryonic matter the pressure terms in the stress-energy tensor are also zero. Taking the curvature term to be negative we have $\kappa = -1/{R_0}^2$. The Friedmann equations therefore simplify to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{c^2}{a^2{R_0}^2} \tag{14}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho. \tag{15}$$

The Machian aspect of our framework is that the speed of light squared is somehow determined by all masses in the universe. Therefore we might expect to obtain an expression for the speed of light squared which is proportional to the average density of baryonic matter. We will show later that the speed of light also depends on the scale factor $a(t)$ and takes the following general form

$$c^2 = \delta\rho a^2 \tag{16}$$

where δ is a constant of proportionality to be determined, ρ is the average baryonic mass density and a is the scale factor. Using (16) in (14) yields

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi G\rho}{3} + \frac{\delta\rho}{{R_0}^2} \\ &= \frac{8\pi G\rho}{3}\left[1 + \frac{3\delta}{8\pi G{R_0}^2}\right]. \end{aligned} \tag{17}$$

Although we began with a universe having negative spatial curvature, (17) looks effectively like a flat universe having a baryonic mass density artificially amplified by a factor of $1 + 3\delta/(8\pi G{R_0}^2)$. The artificial amplification of the baryonic mass density from the perspective of the expansion rate is suggestive that the origin of dark matter is simply negative spatial curvature combined with a speed of light having the form in (16). We should note that this does not explain the apparent accelerated expansion rate of the scale factor, since (15) suggests that the universe should be decelerating. We will show in the section on cosmology that the apparent accelerated expansion rate which was deduced based on observations of distant supernovae is also an illusion having its origin in negative spatial curvature.

This brief introduction, however, highlights and motivates why negative curvature is one of the three postulates of the proposed cosmology which we will continue to expand upon in the sections that follow.

2. The Scalar Field

We now outline the properties of the scalar field which gives rise to a relationship of the form in (16). In the introduction we found that the modified Einstein equation leads to a screened Poisson equation for the Newtonian potential of the form

$$\nabla^2\Phi - \Lambda\Phi = 4\pi G\rho \tag{18}$$

with solution given by

$$\Phi(r) = -\frac{GM}{r}e^{-r\sqrt{\Lambda}}. \tag{19}$$

However, this is the solution in flat space. In a universe with negative spatial curvature the Laplacian in (18) is

$$\nabla^2\Phi = \frac{1}{\sinh^2(r/R)}\frac{d}{dr}\left[\sinh^2(r/R)\frac{d\Phi}{dr}\right] \tag{20}$$

where R is the radius of negative curvature. The general solution to (18) for a point mass M in this negatively curved space is then

$$\Phi(r) = -\frac{GM}{R}\frac{e^{-r/\lambda}}{\sinh(r/R)} \tag{21}$$

with

$$\lambda = 1/\sqrt{\Lambda + 1/R^2}. \tag{22}$$

Without a cosmological constant screening term we have $\lambda = R$ and the Newtonian potential decays exponentially over a length scale corresponding to the radius of negative curvature. The addition of the cosmological constant enhances the screening effect resulting in a faster exponential decay. Note that for $r << R$, $R\sinh(r/R) \approx r$, and $e^{-r/\lambda} \approx 1$ so that we recover the correct Newtonian potential $\Phi(r) = -GM/r$ in the limit of small r.

R. H. Dicke suggested that the scalar field we are searching for which relates mass to the speed of light squared is simply twice the gravitational potential itself. However, this approach is problematic when viewed from the perspective of Newtonian gravitational forces. To understand why, consider the Schwarzschild metric around a spherically symmetric mass M

$$ds^2 = -\left({c_0}^2 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r{c_0}^2}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{23}$$

The Newtonian gravitational acceleration is given by $1/2\nabla g_{00}$ which is simply the expected Newtonian result $-GM/r^2$. Dicke suggested

$$\begin{aligned} {c_0}^2 &= \sum_i \frac{2GM_i}{r_i} \\ &= \frac{2GM_U}{R_U} + \frac{2GM}{r}. \end{aligned} \tag{24}$$

where the subscript U signifies the contribution from all masses in the universe other than the mass M of the body of interest. Note that g_{00} now becomes

$$\begin{aligned} g_{00} &= {c_0}^2 - \frac{2GM}{r} \\ &= \frac{2GM_U}{R_U} + \frac{2GM}{r} - \frac{2GM}{r} \\ &= \frac{2GM_U}{R_U}. \end{aligned} \tag{25}$$

The g_{00} component of the metric now has no dependence on the mass of the gravitating body or the distance from it. It is simply a constant dependent upon the properties of the rest of the universe. In other words, in the universe Dicke proposed, *there are no Newtonian gravitational forces*. What this implies is that if the speed of light squared does in fact depend on mass according to some scalar function of M, that function must effectively be a constant when close to that mass instead of having a $1/r$ dependence. Only then do we have a restoration of Newtonian gravitational forces.

To gain some insight into the form of the scalar function which has this property of being approximately constant for small r, we return to the solution for the gravitational potential in (21)

$$\Phi(r) = -\frac{GM}{R}\frac{e^{-r/\lambda}}{\sinh(r/R)} \tag{26}$$

with

$$\lambda = 1/\sqrt{\Lambda + 1/R^2}. \tag{27}$$

Our choice for the scalar function Θ should look similar but without the $1/r$ dependence. A reasonable guess might be

$$\Theta(r) = \frac{2GM}{R}e^{-r/\lambda}. \tag{28}$$

where R again is the radius of negative curvature, and r the distance from the body of mass M. Note that for $r << R$ this reduces approximately to $\Theta(r) = 2GM/R$, which is a constant. Whereas Dicke proposed a relationship of the form

$$c^2 = \sum_i \frac{2GM_i}{r_i} \tag{29}$$

we are now proposing

$$c^2 = \sum_i \frac{2GM_i}{R}e^{-r_i/\lambda}. \tag{30}$$

It is important to point out that in order for all inertial observers to agree on the same speed of light, by convention the M_i in (30) must be the rest mass and not the relativistic mass. In addition, it should be noted that Equation (30) is a non-propagating constraint, meaning the scalar field is instantaneous, much like the gravitational potential in Newtonian gravity. If the speed of causality has an origin (in this case the scalar field), it is only logical that the origin of causality cannot itself be causal.

We may express (30) in integral form by assuming an average density of baryonic matter ρ and integrating to infinity over mass shells of area $4\pi R^2\sinh^2(r/R)$ and thickness dr

$$\begin{aligned} c^2 &= \frac{2G\rho}{R}\int_0^\infty 4\pi R^2\sinh^2(r/R)\,e^{-r/\lambda}dr \\ &= 8\pi G\rho R\int_0^\infty \sinh^2(r/R)\,e^{-r/\lambda}dr \\ &= \frac{16\pi G\rho R\lambda}{(R/\lambda)^2 - 4} \\ &= \frac{16\pi G\rho\gamma\lambda^2}{\gamma^2 - 4}. \end{aligned} \tag{31}$$

where we have defined the dimensionless parameter $\gamma = R/\lambda$. This equation allows us to interpret the speed of light in terms of cosmological parameters as one unit of cosmological length λ divided by one unit of cosmological time $1/\sqrt{G\rho}$ multiplied by a geometric prefactor $\sqrt{16\pi\gamma/(\gamma^2-4)}$.

We now have one equation with two unknowns γ and λ. We can obtain a second equation relating these unknowns to the Hubble parameter $H = \dot{a}/a$ using the first Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{c^2}{a^2 {R_0}^2}. \tag{32}$$

We will assume that the screening length λ is increasing along with the scale factor $a(t)$ so that we may express λ^2 in (31) as $\lambda^2 = {\lambda_0}^2 a^2$ where λ_0 is the screening distance of the scalar field at the present cosmological time. If $\lambda = 1/\sqrt{\Lambda + 1/R^2}$ is proportional to the scale factor $a(t)$, this implies that the cosmological constant which has units of $1/m^2$ is also evolving with the scale factor as $\Lambda \propto 1/a^2$.

Substituting the expression for c^2 given in (31) into (32) and replacing $\dot{a}/a$ with the Hubble parameter H we obtain

$$\begin{aligned} H^2 &= \frac{8\pi G\rho}{3} + \frac{16\pi G\rho\gamma{\lambda_0}^2}{(\gamma^2-4)\,{R_0}^2} \\ &= \frac{8\pi G\rho}{3} + \frac{16\pi G\rho\gamma}{(\gamma^2-4)\,(R_0/\lambda_0)^2} \\ &= \frac{8\pi G\rho}{3} + \frac{16\pi G\rho}{\gamma\,(\gamma^2-4)} \\ &= \frac{8\pi G\rho}{3}\left[1 + \frac{6}{\gamma\,(\gamma^2-4)}\right] \end{aligned} \tag{33}$$

Since G and c are known and H and ρ are approximately known based on cosmological observations we now have two equations with two unknowns γ and λ which may be solved for uniquely. We may use (33) to solve numerically for γ and then use this value for γ in (31) to solve for λ.

Using a value for the Hubble parameter of $H = 70km/s/Mpc$ and an average density of baryonic matter of $\rho = 4.08 \times 10^{-28} kg/m^3$ we obtain $\gamma = 2.0339$ and $\lambda = 6.648 \times 10^{25} m$. It is important to emphasize that we are only including baryonic matter in our model and not dark matter or energy. Since $\gamma = R/\lambda$ we may solve for R to obtain $R = 1.352 \times 10^{26} m$. Thus, the radius of negative spatial curvature of the universe lies just beyond the radius of the Hubble sphere which is $R_H = c/H = 1.321 \times 10^{26} m$. Using $\lambda = 1/\sqrt{\Lambda + 1/R^2}$ we may solve for the present value of the cosmological constant which in our model is not a constant but changes along with the scale factor like $a^{-2}(t)$ to obtain $\Lambda = 1/\lambda^2 - 1/R^2 = 1.716 \times 10^{-52} m^{-2}$.

There is an interesting relationship contained within these numbers. If we take the ratio of the radius of negative curvature R to the radius of the Hubble sphere R_H we obtain $R/R_H = 1.0229$. If we then subtract 1 and divide by π we obtain $0.00729 = 1/137$ which is precisely the fine structure constant. We therefore have the empirical relationship

$$\frac{R}{R_H} - 1 = \pi\alpha \tag{34}$$

with α the fine structure constant. For some, this may appear to be numerology instead of physics. However, we should not completely exclude the possibility that the fine structure constant has cosmological origins. Dicke's observation has also been criticized as numerology, but as we have shown, his observation could potentially have deeper physical meaning. The relationship in (34) may ultimately give further insight into the connection between gravitation, the geometry of the universe, and quantum electrodynamics.

Finally, we point out that in equation (33), the baryonic mass density is amplified by a factor of $1 + 6/(\gamma(\gamma^2-4))$. Using $\gamma = 2.0339$ as calculated above, we find that only about 4.4% of the universe appears to be baryonic matter. Therefore, in our model, dark matter is not a special type of matter but rather has its origin in negative spatial curvature.

3. The Critical Acceleration in Modified Newtonian Dynamics

Modified Newtonian dynamics (MOND) is a modified theory of Newtonian gravity which was proposed to explain the flat velocity versus radial distance curves for stars orbiting most galaxies including the Milky Way Galaxy [3, 4, 5]. Stars in the outer regions of these galaxies are orbiting at tangential velocities which are faster than expected, suggesting the existence of a lower limit for acceleration a_0 [6, 7, 8, 9]. In MOND this acceleration is a fitting parameter which is still fundamentally unexplained, but seems to accurately fit the experimental data. Its empirically determined value is $a_0 = 1.2 \pm 0.3 m/s^2$. In the following section we will show that the numerical value for this critical acceleration can be obtained from our proposed scalar field.

The FRW metric with negative spatial curvature is given by

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[dr^2 + R^2 \sinh^2(r/R) d\Omega^2\right]. \tag{35}$$

As we showed above with the Schwarzschild metric

$$ds^2 = -\left({c_0}^2 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r{c_0}^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{36}$$

the Newtonian gravitational acceleration is obtained from the metric as $1/2\nabla g_{00}$. Applying this to the Schwarzschild metric we obtain $-GM/r^2$ which is the expected Newtonian result. However, so far we have overlooked the fact that c^2 is determined by the scalar field and should be included when taking the gradient of g_{00}.

For the FRW metric in (35) we have

$$\begin{aligned} \frac{1}{2}\nabla g_{00} &= -\frac{1}{2}\nabla c^2 \\ &= -\frac{1}{2}\nabla \sum_i \frac{2GM_i}{R} e^{-r_i/\lambda} \\ &= \frac{G}{R\lambda} \sum_i M_i e^{-r_i/\lambda} \cos\theta. \end{aligned} \tag{37}$$

The factor of $\cos\theta$ takes into account that the acceleration is a vector. We will be interested in determining the acceleration along the z-axis corresponding in spherical coordinates to $\theta =$

0. The projection of the acceleration vector onto the z-axis for non-zero θ is simply $\cos\theta$. We may now convert the sum into an integral over hemispherical shells

$$\begin{aligned}\frac{1}{2}\nabla g_{00} &= \frac{G}{R\lambda}\sum_i M_i e^{-r_i/\lambda}\cos\theta \\ &= \frac{G}{R\lambda}\int_0^{\infty}\int_0^{\pi/2}\int_0^{2\pi}\rho R^2\sinh^2(r/R)e^{-r/\lambda}\sin\theta\cos\theta dr d\theta d\phi \\ &= \frac{\pi G R\rho}{\lambda}\int_0^{\infty}\sinh^2(r/R)e^{-r/\lambda}dr \\ &= \frac{\pi G R\rho}{\lambda}\left[\frac{2\lambda}{(R/\lambda)^2-4}\right] \\ &= \frac{2\pi G\rho\gamma\lambda}{\gamma^2-4}\end{aligned} \tag{38}$$

where we have used $\gamma = R/\lambda$. Using $\gamma = 2.0339$ and $\lambda = 6.648\times10^{25}m$ which we obtained above, we obtain a critical acceleration of $a_0 = 1.69\times10^{-10}m/s^2$. This is remarkably close to the empirically observed value of $a_0 = 1.2\pm0.3\times10^{-10}m/s^2$. The critical acceleration therefore appears to have its origin in the cumulative repulsive gravitational effects from the scalar fields of all matter in the universe.

Note that since $\lambda\propto a(t)$ and $G\rho\propto a(t)^{-3}$, the critical acceleration scales with the scale factor like $a_0\propto a(t)^{-2}$. This means that at a redshift of 9 the critical acceleration would have been 100 times larger, roughly $1.2\times10^{-8}\ m/s^2$. It has been argued that MOND - especially a version of MOND in which the critical acceleration is scale-dependent - may be capable of explaining recent observations made by the James Webb Space Telescope (JWST) showing much faster structure formation than previously thought possible [10, 11, 12, 13, 14, 15, 16]. The scale-dependence of the critical acceleration in our theoretical framework would seem to support this hypothesis.

4. Dark Energy and Cosmology

So far we have shown that our proposed model is capable of explaining the origin of dark matter, but we have not yet addressed the fact that it predicts a decelerating universe which is at odds with dark energy. We will now develop a more complete cosmological model so that we can fit the model to high-redshift supernova data from the Supernova Cosmology Project [17, 18], demonstrating that we can obtain an excellent fit with only baryonic matter.

To summarize our results thus far, our proposed universe has an FRW metric with negative spatial curvature given by

$$ds^2 = -c^2dt^2 + a(t)^2\left[dr^2 + R^2\sinh^2(r/R)d\Omega^2\right]. \tag{39}$$

The first and second Friedmann equations take the form

$$\left(\frac{\dot a}{a}\right)^2 = H^2 = \frac{8\pi G\rho}{3}\left[1+\frac{6}{\gamma(\gamma^2-4)}\right] \tag{40}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\rho. \tag{41}$$

As we mentioned previously, although our model has contributions from both matter and curvature, from a mathematical standpoint it appears effectively to consist of only matter but with an amplified average mass density. In a matter-only universe, the scale factor evolves in time according to

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3} \tag{42}$$

where t_0 is the age of the universe which is related to H_0 by $t_0 = 2/(3H_0)$. The proper distance to an astronomical object is given by

$$d_p(t_0) = \int_{t_e}^{t_0}\frac{c}{a(t)}dt. \tag{43}$$

where t_e is the time light from the source was emitted [19]. In our model the speed of light varies in cosmological time. From (31) we have

$$c^2 = \frac{16\pi G\rho\gamma\lambda^2}{\gamma^2-4}. \tag{44}$$

Since $\lambda\propto a(t)$ and $\rho\propto a(t)^{-3}$, we have $c^2\propto a(t)^{-1}$ and $c\propto a(t)^{-1/2}$. Therefore we may express c as $c = c_0 a(t)^{-1/2}$ where c_0 is the present value for the speed of light. Equation (43) then becomes

$$\begin{aligned}d_p(t_0) &= c_0\int_{t_e}^{t_0}\frac{1}{a(t)^{3/2}} = c_0\int_{t_e}^{t_0}\frac{1}{t/t_0} = c_0t_0\ln\left[\frac{t}{t_0}\right]_{t_e}^{t_0} \\ &= c_0t_0\left\{\ln[1]-\ln\left[\frac{t_e}{t_0}\right]\right\} \\ &= c_0t_0\ln\left[\frac{t_0}{t_e}\right]\end{aligned} \tag{45}$$

where we have used (42) for the time-dependence of the scale factor.

The redshift is related to the scale factor by $1+z = 1/a(t)$

$$1+z = \frac{1}{a(t)} = \left(\frac{t_0}{t}\right)^{2/3} \tag{46}$$

which we can solve for t_0/t to obtain

$$\left(\frac{t_0}{t}\right) = (1+z)^{3/2}. \tag{47}$$

Our expression in (45) for the proper distance then becomes

$$\begin{aligned}d_p(t_0) &= c_0t_0\ln(1+z)^{3/2} = \frac{2c_0}{3H_0}\ln(1+z)^{3/2} \\ &= \frac{c_0}{H_0}\ln(1+z).\end{aligned} \tag{48}$$

where we have used the fact that in a matter-only universe $t_0 = 2/(3H_0)$.

Although we have so far neglected curvature in regard to all equations related to the expansion of the universe, with regard

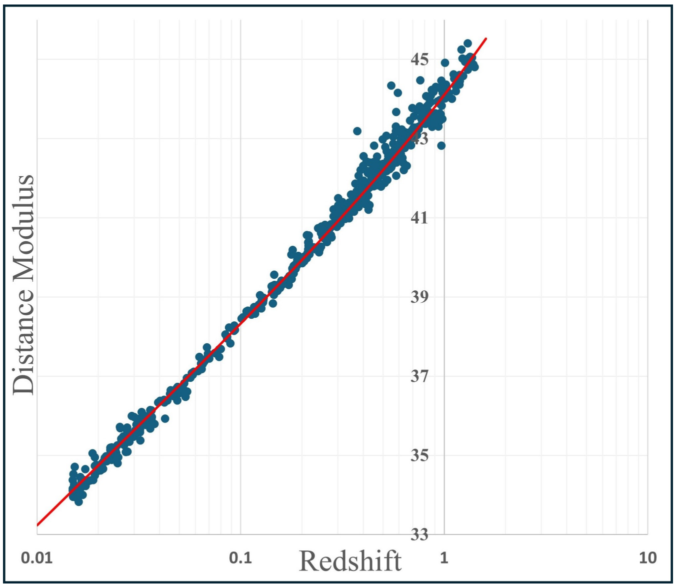

Figure 1: Plot of distance modulus vs. redshift using data from the Supernova Cosmology Project. The solid red line shows the model prediction.

to luminosity distance we must take into account the negative curvature of the universe. In this case luminosity distance is related to the proper distance by [19]

$$\begin{aligned} d_L &= R_0 (1+z) \sinh(r/R_0) \\ &= R_0 (1+z) \sinh\left(d_p(t_0)/R_0\right). \end{aligned} \tag{49}$$

Using the expression in (48) for the proper distance in (49) and taking $R_0 \approx c_0/H_0$ equal to the radius of the Hubble sphere which we showed is approximately true in Section 2, we then obtain

$$\begin{aligned} d_L &= \frac{c_0}{H_0}(1+z)\sinh(\ln(1+z)) \\ &= \frac{c_0}{2H_0}\left[(1+z)^2 - 1\right]. \end{aligned} \tag{50}$$

The luminosity distance is in turn related to the distance modulus defined as the difference between the apparent magnitude and absolute magnitude of a light source by

$$m - M = 5\log_{10}\left(\frac{d_L}{1Mpc}\right) + 25. \tag{51}$$

Figure 1 shows a plot of distance modulus as a function of redshift based on publicly available data from the Supernova Cosmology Project Union 2.1 Compilation [17, 18]. The red line shows our model fit based on a luminosity distance given by (50). Note that the model provides an excellent fit to the data even at high redshift where the ΛCDM model fails without dark matter and dark energy. We are only considering ordinary baryonic matter in our model.

This supernova data was previously interpreted as implying that the rate of expansion of the universe is accelerating [20, 21]. This conclusion was drawn based on the use of the ΛCDM model. In our model the scale factor increases as if the universe is geometrically flat and contains only baryonic matter but with an amplification factor for the baryonic mass density. In a flat universe with only baryonic matter the deceleration parameter is $q = +0.5$, meaning the expansion of the universe is decelerating, not accelerating. The universe only appears to be accelerating due to the fact that the speed of light is decreasing in cosmological time according to $c = c_0 a(t)^{-1/2}$ and the negative geometric curvature affects the calculation of luminosity distance.

There is another important point that must be made about our model before concluding this section. The fact that the speed of light varies in cosmological time naturally implies other physical constants must also vary in time. The idea of time-varying constants in cosmology is not new, as it was first proposed by Paul Dirac in 1937 and has since been expounded upon by many others [22, 23, 24, 25, 26]. For example, the speed of light is related to the permittivity and permeability of free space, meaning that if the speed of light is varying in time either one or both of those constants must also vary in time. We must check that the variability of other constants which are connected to the speed of light is consistent with experiment.

The fine structure constant is given by

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}. \tag{52}$$

Experiments utilizing electronic transitions in atomic clocks as well as cosmological observations have constrained the time variation of α to $\dot{\alpha}/\alpha \sim 10^{-17} yr^{-1}$, which strongly suggests that it is in fact a constant [27, 28, 29, 30]. We have already shown that in our model we have $c \propto a(t)^{-1/2}$. Since $c = 1/\sqrt{\mu_0 \varepsilon_0}$ we could take $\mu_0 \propto a(t)^{1/2}$ and $\varepsilon_0 \propto a(t)^{1/2}$ which would give the correct dependence of the speed of light on $a(t)$. Note from the expression for α in (52) if $c \propto a(t)^{-1/2}$ and $\varepsilon_0 \propto a(t)^{1/2}$ then α does not depend on $a(t)$ as long as $\hbar$ and e are true constants.

In our cosmological model we also assumed that the wavelength of atomic transitions does not depend on $a(t)$. In other words, we assumed that a hydrogen atom emitted light at the same wavelengths in the past as it does now. The redshift we observe is due solely to the expansion of the universe so that $1 + z = 1/a(t)$, and not due to any intrinsic variation in the properties of the hydrogen atom that would cause it to emit at different wavelengths in the past. The wavelength at which hydrogen emits is given by the expression

$$\lambda_{n,m} = \frac{4\pi\hbar}{m_e c \alpha^2}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)^{-1}. \tag{53}$$

If α and $\hbar$ are true constants, and $c \propto a(t)^{-1/2}$, then in order for hydrogen to emit at the same wavelength independent of the scale factor we must also have $m_e \propto a(t)^{1/2}$. It would seem reasonable to propose that all particle masses are varying with the scale factor, not only particles interacting with the Higgs field, such that the proton-electron mass ratio μ remains constant which is strongly supported by observational evidence [31, 32, 33]. This implies that the energy scale for quantum chromodynamics scales like $\Lambda_{QCD} \propto a(t)^{-1/2}$ so that the proton mass m_p then scales like $m_p \propto \Lambda_{QCD}/c^2 \propto a(t)^{-1/2}/a(t)^{-1} =$

Table 1: Dependence of various natural constants on scale factor

Parameter	Symbol	Units	Variation
Gravitational Constant	G	$m^3/(kg \cdot s^2)$	$a(t)^{-1/2}$
Particle Mass	m	kg	$a(t)^{1/2}$
Speed of Light	c	m/s	$a(t)^{-1/2}$
Permittivity of Free Space	ϵ_0	F/m	$a(t)^{1/2}$
Permeability of Free Space	μ_0	H/m	$a(t)^{1/2}$
Energy	E	$kg \cdot m^2/s^2$	$a(t)^{-1/2}$
QCD Energy Scale	Λ_{QCD}	$kg \cdot m^2/s^2$	$a(t)^{-1/2}$
Radius of Curvature	R	m	$a(t)$
Radius of Hubble Sphere	R_H	m	$a(t)$
Hubble Parameter	H	$1/s$	$a(t)^{-3/2}$
Cosmological Constant	Λ	m^{-2}	$a(t)^{-2}$
MOND Critical Acceleration	a_0	m/s^2	$a(t)^{-2}$
Electric Charge	e	C	*constant*
Fine Structure Constant	α		*constant*
Planck Constant	h	$kg \cdot m^2/s$	*constant*
Proton-Electron Mass Ratio	μ		*constant*

$a(t)^{1/2}$. Note that the Bohr radius (the effective size of an atom) which is given by

$$a_0 = \frac{\hbar}{m_e c \alpha} \tag{54}$$

remains constant in this framework.

Lunar laser ranging experiments have also constrained time variation in the gravitational constant to $\dot{G}/G \sim 10^{-13} yr^{-1}$ [34]. However, this constraint is based on the assumption that mass is not varying which is not correct in our model. These laser ranging experiments are not actually measuring variations in G directly but variations in GM. Since masses are varying like $M \propto a(t)^{1/2}$, we must have $G \propto a(t)^{-1/2}$ in order for the product GM to be conserved. The dependence of various physical constants and their dependence on scale factor $a(t)$ is summarized in Table 1.

Finally, we note that in a cosmology with $c \propto a(t)^{-1/2}$ the speed of light would have been significantly higher in the early universe. If the universe truly began as a singularity then it would have been infinite. Variable speed of light theories have been investigated in the past as an alternative to inflation to explain the isotropy of the CMB [35, 36, 37, 38, 39, 40]. This could be considered another potential advantage of the cosmological framework we have presented.

5. Conclusion

Motivated by the observation made by R. H. Dicke in 1957 that the speed of light has its origin in the gravitational potential of the entire universe, we developed a cosmological model based on a scalar field which couples matter to the speed of light. We showed how this scalar field, when combined with negative spatial curvature and a slightly modified form of the Einstein equation, leads naturally to an explanation of dark matter. The model is consistent with cosmological observations such as the flat rotation curves of galaxies and provides a means to numerically predict the critical acceleration in MOND from first-principles. We also fit the model to luminosity and redshift data from the Supernova Cosmology Project, showing that an excellent fit is obtained without dark matter and dark energy. Finally, recent observations from JWST of galaxies at very high redshift show much faster structure formation than previously thought possible. This may be explained by a drastically larger MOND critical acceleration in the early universe predicted by the proposed model.

Since the speed of light scales inversely with the square root of the scale factor, we derived a cosmology in which many natural "constants" are also varying in cosmological time with the scale factor. While this model appears promising as a path to explaining dark matter and energy, further work is certainly needed to understand if this model is empirically consistent with observations of the CMB, baryon acoustic oscillations (BAOs), and big bang nucleosynthesis (BBN). These observations are often used as evidence to rule out models which include time-varying natural constants. However, in our model many different constants instead of just one are varying with the scale factor, and this must be taken into account when being put into the context of observations of the CMB, BAOs, and BBN. Clearly such studies will need to be done before this model can be seriously considered as a possible alternative to ΛCDM.

References

[1] R. H. Dicke, Gravitation without a principle of equivalence, Reviews of Modern Physics 29 (1957) 363–376. doi:https://doi.org/10.1103/RevModPhys.29.363.

[2] S. M. Carroll, Spacetime and Geometry: An Introduction to General Relativity, Cambridge University Press, 2019.

[3] M. Milgrom, A modification of the newtonian dynamics as a possible alternative to the hidden mass hypothesis 1, The Astrophysical Journal 270 (1983) 365–370. doi:10.1086/161130.

[4] M. Milgrom, The mond paradigm, arXiv preprint arXiv:0801.3133 (3 2008).

[5] M. Milgrom, Mond theory, Canadian Journal of Physics 93 (2015) 107–118. doi:10.1139/cjp-2014-0211.

[6] K. G. Begeman, A. H. Broeils, R. H. Sanders, Extended rotation curves of spiral galaxies: dark haloes and modified dynamics, Monthly Notices of the Royal Astronomical Society 249 (3) (1991) 523–537. doi:10.1093/mnras/249.3.523.

[7] G. Gentile, B. Famaey, W. J. G. de Blok, Things about mond, A&A 527 (2011) A76. doi:10.1051/0004-6361/201015283.

[8] F. Lelli, S. S. McGaugh, J. M. Schombert, Sparc: Mass models for 175 disk galaxies with spitzer photometry and accurate rotation curves, The Astronomical Journal 152 (6) (2016) 157. doi:10.3847/0004-6256/152/6/157.

[9] P. Li, F. Lelli, S. McGaugh, J. Schombert, Fitting the radial acceleration relation to individual sparc galaxies, A&A 615 (2018) A3. doi:10.1051/0004-6361/201732547.

[10] S. S. McGaugh, J. Schombert, F. Lelli, J. Franck, J. R. Franck, Accelerated structure formation: The early emergence of massive galaxies and clusters of galaxies, Astrophysical Journal 976 (1) (2024) 13. doi:10.3847/1538-4357/ad834d.

[11] R. Eappen, P. Kroupa, N. Wittenburg, M. Haslbauer, B. Famaey, The formation of early-type galaxies through monolithic collapse of gas clouds in milgromian gravity, Monthly Notices of the Royal Astronomical Society 515 (2022) 1541–1553. doi:10.1093/mnras/stac2229.

[12] R. Eappen, P. Kroupa, The formation of compact massive relic galaxies in mond, Monthly Notices of the Royal Astronomical Society 528 (3) (2024) 4264–4271. doi:10.1093/mnras/stae286.

[13] R. Eappen, P. Kroupa, Scaling relations of early-type galaxies in mond, Galaxies 13 (2) (2025) 22. doi:10.3390/galaxies13020022.

[14] M. Milgrom, A modification of the newtonian dynamics as a possible alternative to the hidden mass hypothesis, Astrophysical Journal 270 (1983) 365–370. doi:10.1086/161130.

[15] R. H. Sanders, Structure formation in a mond cosmology, Monthly Notices of the Royal Astronomical Society 296 (1998) 1009–1018. doi:10.1046/j.1365-8711.1998.01483.x.

[16] A. Nusser, Structure formation in a modified gravity framework, Monthly Notices of the Royal Astronomical Society 331 (2002) 909–916. doi:10.1046/j.1365-8711.2002.05373.x.

[17] N. Suzuki, et al., The hubble space telescope cluster supernova survey. v. improving the dark-energy constraints above z > 1 and building an early-type-hosted supernova sample*, The Astrophysical Journal 746 (1) (2012) 85. doi:10.1088/0004-637X/746/1/85.

[18] V. Ruhlmann-Kleider, et al., The union2.1 supernova catalog, The Astrophysical Journal Letters 843 (2) (2017) 97.

[19] B. Ryden, Introduction to Cosmology, 2nd Edition, Cambridge University Press, Cambridge, UK, 2017.

[20] A. G. Riess, et al., Observational evidence from supernovae for an acceleration universe and a cosmological constant, The Astronomical Journal 116 (1998) 1009–1038. doi:10.1086/300499.

[21] S. Perlmutter, Supernovae, dark energy, and the accelerating universe, Physics Today 56 (4) (2003) 53–60. doi:10.1063/1.1580050.

[22] P. A. M. Dirac, Cosmological models and the large numbers hypothesis, Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences 338 (1615) (1974) 439–446. doi:10.1098/rspa.1974.0095.

[23] C. J. A. P. Martins, The status of varying constants: a review of the physics, searches and implications, Reports on Progress in Physics 80 (12) (2017) 126902. doi:10.1088/1361-6633/aa860e.

[24] C. J. A. P. Martins, Primordial nucleosynthesis with varying fundamental constants: Degeneracies with cosmological parameters, Astronomy & Astrophysics 646 (2021) A47. doi:10.1051/0004-6361/202039605.

[25] M. Deal, C. J. A. P. Martins, Primordial nucleosynthesis with varying fundamental constants: Solutions to the lithium problem and the deuterium discrepancy, Astronomy & Astrophysics 653 (2021) A48. doi:10.1051/0004-6361/202140725.

[26] J.-P. Uzan, The fundamental constants and their variation: observational and theoretical status, Rev. Mod. Phys. 75 (2003) 403–455. doi:10.1103/RevModPhys.75.403.

[27] N. Leefer, C. T. M. Weber, A. Cingöz, J. R. Torgerson, D. Budker, New limits on variation of the fine-structure constant using atomic dysprosium, Phys. Rev. Lett. 111 (2013) 060801. doi:10.1103/PhysRevLett.111.060801.

[28] Planck Collaboration, P. A. R. Ade, N. Aghanim, et al., Planck intermediate results. XXIV. Constraints on variations in fundamental constants, Astronomy & Astrophysics 580 (2015) A22. doi:10.1051/0004-6361/201424496.

[29] G. Li, L. Sun, X. Chen, H. Zhou, Time variation of fine-structure constant constrained by [O III] emission-lines at $1.1 < z < 3.7$, Monthly Notices of the Royal Astronomical Society 527 (3) (2024) 4913–4935. doi:10.1093/mnras/stad3240.

[30] S. M. Kotuš, M. T. Murphy, R. F. Carswell, High-precision limit on variation in the fine-structure constant from a single quasar absorption system, Monthly Notices of the Royal Astronomical Society 464 (3) (2017) 3679–3703. arXiv:1609.03860, doi:10.1093/mnras/stw2543.

[31] S. A. Levshakov, M. Dessauges-Zavadsky, S. D'Odorico, P. Molaro, A new constraint on cosmological variability of the proton-to-electron mass ratio, Monthly Notices of the Royal Astronomical Society 333 (2) (2002) 373–377. doi:10.1046/j.1365-8711.2002.05408.x.

[32] J. A. King, J. K. Webb, M. T. Murphy, R. F. Carswell, Stringent null constraint on cosmological evolution of the proton-to-electron mass ratio, Physical Review Letters 101 (2008) 251304. doi:10.1103/PhysRevLett.101.251304.

[33] J. Bagdonaite, W. M. G. Ubachs, M. T. Murphy, J. B. Whitmore, Constraint on a varying proton-electron mass ratio 1.5 billion years after the big bang, Physical Review Letters 114 (7) (2015) 071301. doi:10.1103/PhysRevLett.114.071301.

[34] J. G. Williams, S. G. Turyshev, D. H. Boggs, Progress in lunar laser ranging tests of relativistic gravity, Phys. Rev. Lett. 93 (2004) 261101. doi:10.1103/PhysRevLett.93.261101.

[35] A. Albrecht, J. Magueijo, Time varying speed of light as a solution to cosmological puzzles, Phys. Rev. D 59 (1999) 043516. doi:10.1103/PhysRevD.59.043516.

[36] J. W. Moffat, Variable speed of light cosmology, primordial fluctuations and gravitational waves, The European Physical Journal C 76 (130) (2016). doi:10.1140/epjc/s10052-016-3971-6.

[37] J. W. Moffat, Nonlinear perturbations in a variable speed of light cosmology, arXiv:1501.01872 [astro-ph.CO] (2015). arXiv:1501.01872.

[38] M. A. Clayton, J. W. Moffat, Dynamical mechanism for varying light velocity as a solution to cosmological problems, Physics Letters B 460 (1999) 263–269. doi:10.1016/S0370-2693(99)00774-1.

[39] S. Bhattacharjee, P. K. Sahoo, Temporally varying universal gravitational "constant" and speed of light in energy momentum squared gravity, arXiv:2001.06569 [gr-qc] (2020). arXiv:2001.06569.

[40] D. Youm, Brane world cosmologies with varying speed of light, arXiv:hep-th/0101228 (2001). arXiv:hep-th/0101228.