

# Generation Mechanism of Photon Mass Under the Constraint of Spacetime Curvature

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**Abstract:** Based on the physical idea that "mass is energy with added dimensions", this paper explores how spacetime curvature contributes to the effective mass of photons from the perspective of combining general relativity and quantum mechanics by constructing a photon motion model in curved spacetime. It is proven that when a photon is confined within a characteristic scale in a strongly curved spacetime, its energy can manifest as an equivalent mass, which is directly related to the spacetime curvature tensor and the constraint scale. This paper provides a new theoretical perspective for understanding the acquisition of effective mass by photons.

**Keywords:** Curved Spacetime; Photon Effective Mass; Dimensional Reduction; Einstein Field Equations; Quantum Constraint

## 1. Introduction

In special relativity, the rest mass of a photon is zero, and its energy is completely expressed in the form of momentum. However, within the framework of general relativity, the geometric structure of spacetime may alter the form of energy manifestation. This paper starts with a thought experiment: if an originally classless photon moves in a flat spacetime, when it is placed in a curved spacetime, especially confined to a region much smaller than its characteristic wavelength, its energy may partially or fully manifest as equivalent mass. This "mass generation" mechanism originates from the localization effect of energy in curved dimensions and can be described by the method combining geometry and quantum mechanics.

## 2. Theoretical Model and Mathematical Framework

### 2.1 Photon Motion Equation in Curved Spacetime

In a Riemann manifold  $(M, g_{\mu\nu})$ , the motion of a photon obeys the null geodesic equation [1]:

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} = 0$$

where  $\lambda$  is the affine parameter, and  $\Gamma_{\alpha\beta}^\mu$  is the Christoffel symbol. The four-dimensional wave vector of the photon  $k^\mu = (k^0, \mathbf{k})$  satisfies the null condition  $g_{\mu\nu} k^\mu k^\nu = 0$ .

### 2.2 Introduction of Effective Mass

Consider a static spherically symmetric curved spacetime (such as Schwarzschild spacetime) with the metric [2]:

$$ds^2 = - \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

When a photon is confined in a region with radius  $R$  much smaller than the Schwarzschild radius  $r_s = 2GM/c^2$ , its radial wave number  $k_r$  is quantized due to boundary conditions. At this time, the energy of the photon  $E = \hbar\omega$  includes not only the transverse kinetic energy but also the "binding energy" caused by spatial curvature.

By analogy with the behavior of particles in a one-dimensional infinite

potential well in quantum mechanics, the equivalent rest mass  $m_{eff}$  of the photon can be derived through Einstein's mass-energy relation and dimensional reduction. Suppose the photon is confined in a region with characteristic length  $L$ , its momentum uncertainty is  $\Delta p \sim \hbar/L$ , and the corresponding energy is:

$$\Delta E \sim \frac{\hbar c}{L}$$

If this energy is completely attributed to the equivalent mass, then:

$$m_{eff}c^2 = \frac{\hbar c}{L} \Rightarrow m_{eff} = \frac{\hbar}{cL}$$

### 2.3 Correlation Between Spacetime Curvature and Equivalent Mass

The Riemann curvature tensor  $R_{\sigma\mu\nu}^{\rho}$  describes the degree of spacetime curvature. In the weak field approximation, the curvature scalar  $R$  is related to the matter distribution. Considering Einstein's field equations [3]:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

When a photon is confined in a high-curvature region, its energy-momentum tensor  $T_{\mu\nu}$  contributes to the local curvature, forming a self-consistent coupling system.

To establish the relationship between curvature and equivalent mass, we

use the Bohr-Sommerfeld quantization condition: the phase change of a photon on a closed orbit is an integer multiple of  $2\pi$ . In curved spacetime, this condition is modified as:

$$\oint k_{\mu} dx^{\mu} = 2\pi n$$

where  $k_{\mu}$  is the covariant wave vector. When the orbital curvature radius  $\mathcal{R} \sim |R_{\mu\nu\rho\sigma}|^{-1/2}$  is comparable to the constraint scale  $L$ , the modulus of the wave vector is no longer zero but satisfies a dispersion-like relation:

$$g^{\mu\nu} k_{\mu} k_{\nu} = - \left( \frac{m_{eff} c}{\hbar} \right)^2$$

Thus, the approximate relationship between the equivalent mass and the curvature scalar  $R$  is obtained:

$$m_{eff} \sim \frac{\hbar}{c} \sqrt{|R|}$$

### 3. Mathematical Implementation of the Thought Experiment

Consider a highly curved spacetime region, such as near a black hole or other curved pastimes capable of confining photons, where the curvature radius  $\rho$  is much smaller than the photon's characteristic wavelength  $\lambda_0$ . At this time, the propagation characteristics of the photon are described by the Wheeler-DeWitt equation [4]:

$$\left(G_{ijkl} \frac{\delta^2}{\delta g_{ij} \delta g_{kl}} - \sqrt{g} R\right) \Psi[g] = 0.$$

where  $\Psi[g]$  is the wave function of the spacetime metric. In the perturbation approximation, the effective Hamiltonian of the photon can be written as:

$$\hat{H} = \hbar c \sqrt{-\nabla^2 + \xi R}$$

where  $\xi$  is the coupling constant. When  $R > 0$  and sufficiently large, the eigenvalue of the operator  $\nabla^2$  is corrected by curvature, leading to the photon acquiring an effective mass:

$$m_{eff} = \frac{\hbar}{c} \sqrt{\xi R}.$$

#### 4. Conclusion

Through geometric analysis in curved spacetime and quantum mechanical principles, this paper proves that when a photon is confined in a high-curvature spacetime region, its energy can manifest as an equivalent mass. This mass originates from the dimensional reduction effect, i.e., energy is confined in the curved geometric structure, and its value is proportional to the square root of the curvature scalar. This result supports the philosophical idea that "mass is energy with added dimensions". Of course, the curvature in this paper only refers to the degree of spacetime curvature under the action of gravity. Theoretically, curvature may also

have influencing factors other than gravity, such as spacetime curvature under the action of the strong force at the microcosmic level. If the spacetime curvature under these conditions is clarified, it will provide a new possible approach for exploring the origin of the mass of elementary particles.

This paper is a theoretical derivation aimed at constructing a thought experiment model mathematically. The actual physical process needs to be further verified in combination with quantum gravity theory.

## References

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