Power Spectral Pythagorean Numbers

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Plimpton 322 is a cuneiform tablet dated to be from the Old Babylonian Period (19th–17th century AD) and displays a method of generating Pythagorean triples [4].

Abstract

The spectral basis of $\mathbf{Z}_n$, where $n = p_1^{e_1} \cdots p_k^{e_k}$ has at least two prime factors, implements the isomorphism $\mathbf{Z}_n \cong \mathbf{Z}_{p_1^{e_1}} \oplus \cdots \oplus \mathbf{Z}_{p_k^{e_k}}$. An interesting possibility is when the spectral basis consists entirely of primes or powers, and we call such a number *power spectral*. The spectral sum, the sum of all elements of the spectral basis, is of the form $an + 1$, and another interesting possibility is if $an + 1$ is also a power. A search yields only five examples, with two of them being $3^2 + 4^2 = 5^2$, where $\{9, 16\}$ is the spectral basis of 24, and $15^2 + 8^2 = 17^2$, where $\{15^2, 8^2\}$ is the spectral basis of 288. It is the purpose of this note to show that these two are the only power spectral Pythagorean triples. Some observations are made about the other three examples.

1 Statement of the problem

1.1 Pythagorean triples

Recall that a *Pythagorean triple* is a triple a, b, c of positive integers such that $a^2 + b^2 = c^2$. If x and y are positive integers of opposite parity, with $x > y$, then $a = x^2 - y^2$, $b = 2xy$, $c = x^2 + y^2$ is a primitive Pythagorean triple, and it can be shown that all primitive triples arise in this way. A few example are $3^2 + 4^2 = 5^2$, $5^2 + 12^2 = 13^2$, $8^2 + 15^2 = 17^2$. Here are several facts we will need about primitive Pythagorean triples, stated without proof.

Lemma 1.1 (Pythagorean triples)**.** *Let a, b, c be a primitive Pythagorean triple. Then*

1. *Exactly one of a, b is divisible by 3, but never c.*
2. *Exactly one of a, b is divisible by 4, but never c.*
3. *c is of the form $4k + 1$.*

Table 1: Power spectral Pythagorean numbers

n	Factorization	Spectral basis						
24	$(2)^3(3)$			3^2	$+$	4^2	$=$	5^2
288	$(2)^5(3)^2$			15^2	$+$	8^2	$=$	17^2
2400	$(2)^5(3)(5)^2$	15^2	$+$	40^2	$+$	24^2	$=$	49^2
4704	$(2)^5(3)(7)^2$	63^2	$+$	56^2	$+$	48^2	$=$	97^2
9408	$(2)^6(3)(7)^2$	63^2	$+$	56^2	$+$	48^2	$=$	97^2

1.2 The spectral basis

Let us recall the spectral basis of $\mathbf{Z}_n$, where n has at least two prime factors. The case when n is a prime power will not concern us, but if $n = p^e$, then any element x of $\mathbf{Z}_n$ has a unique decomposition $x = \sum_{i=0}^{e-1} a_i p^i$, where $0 \le a_i < p$, $0 \le i < e$. Suppose $n = p_1^{e_1} \cdots p_k^{e_k}$, where $k \ge 2$. Define

$$\begin{aligned} u_i &= n/p_i^{e_i}, \\ \bar{u}_i &= u_i^{-1} \mod p_i^{e_i}, \\ \pi_i &= \bar{u}_i u_i, \end{aligned}$$

then we have

$$\begin{aligned} \pi_i^2 &\equiv \pi_i \pmod{n}, \\ \pi_i \pi_j &\equiv 0 \pmod{n}, \quad i \ne j, \\ \pi_1 + \cdots + \pi_k &\equiv 1 \pmod{n}. \end{aligned}$$

Consequently, if x is any element of $\mathbf{Z}_n$, then we have the unique decomposition

$$\begin{aligned} x^r &= \bar{x}_1^r \pi_1 + \cdots + \bar{x}_k^r \pi_k, \\ \bar{x}_i^r &= x^r \mod p_i^{e_i}, \end{aligned}$$

where r is any nonnegative integer. If x is relatively prime to n, then r can also be negative. The spectral basis implements the isomorphism

$$\mathbf{Z}_n \cong \mathbf{Z}_{p_1^{e_1}} \oplus \cdots \oplus \mathbf{Z}_{p_k^{e_k}}.$$

We shall often say "the spectral basis of n" rather than "the spectral basis of $\mathbf{Z}_n$."

1.3 Power spectral Pythagorean triples

Definition 1.1 (Power spectral number). *A power spectral number is a positive integer whose spectral basis consists entirely of primes or powers.*

Examples 1. Observe that the spectral basis of $\mathbf{Z}_{12}$ is $\{3^2, 2^2\}$, the spectral basis of $\mathbf{Z}_{24}$ is $\{3^2, 4^2\}$, and the spectral basis of $\mathbf{Z}_{288}$ is $\{15^2, 8^2\}$.

If one investigates whether or not the spectral sum $\sum \pi_i$ of a power spectral number can also be a power, then one finds only the five examples listed in Table 1. Observe that the first two entries are Pythagorean triples and that last two entries in Table 1 are an example of an *isospectral pair*, namely numbers n_1 and n_2 such that $n_1 = 2n_2$ with the same spectral basis.

Definition 1.2 (Power spectral Pythagorean triple). *A* ***power spectral Pythagorean triple*** *is a primitive Pythagorean triple a, b, c such that $\langle a^2, b^2 \rangle$ is the spectral basis of $n = c^2 - 1$. Note that this implies that n has only two primary parts.*

We shall prove the following theorem.

Theorem 1.1. *The only two power spectral Pythagorean triples are 3,4,5 and 8,15,17, corresponding to the spectral basis* $\{3^2, 4^2\}$ *of* $n = 24$*, and the spectral basis* $\{15^2, 8^2\}$ *of* $n = 288$*, respectively.*

As for power spectral Pythagorean numbers with three factors, we will find useful the following two theorems [3]. See Section 4.

Theorem 1.2 (Mersenne)**.** *Let* M_p *be a Mersenne prime with Mersenne exponent* $p > 2$*. Then*

1. $2^{2p-1} \cdot 3 \cdot M_p^2$ *has power spectral basis*

$$\left\{M_p^2(M_p+2)^2, M_p^2(M_p+1)^2, (M_p^2-1)^2\right\}$$

with index 2.

2. $2^{2p} \cdot 3 \cdot M_p^2$ *has power spectral basis*

$$\left\{M_p^2(M_p+2)^2, M_p^2(M_p+1)^2, (M_p^2-1)^2\right\}.$$

3. $2^{2p+1} \cdot 3 \cdot M_p^2$ *has power spectral basis*

$$\left\{M_p^2(M_p+2)^2, 4M_p^2(M_p+1)^2, (M_p^2-1)^2\right\}.$$

Note that numbers 1 and 2 comprise an isospectral pair, that is, numbers n_1 *and* n_2 *such that* $n_1 = 2n_2$ *that have the same spectral basis.*

Remark 1*.* The fourth and fifth entries in Table 1 are $M_3 = 7$ in entries 1 and 2 of Theorem 1.2, respectively

Theorem 1.3 (Fermat)**.** *Let* F_i *be a Fermat prime with exponent* $f_i = 2^i$*,* $i > 0$*. Then*

1. $2^{2f_i-1} \cdot 3 \cdot F_i^2$ *has power spectral basis*

$$\left\{(F_i-2)^2F_i^2, (F_i-1)^2F_i^2, (F_i^2-1)^2\right\}$$

with index 2.

2. $2^{2f_i} \cdot 3 \cdot F_i^2$ *has power spectral basis*

$$\left\{(F_i-2)^2F_i^2, (F_i-1)^2F_i^2, (F_i^2-1)^2\right\}.$$

3. $2^{2f_i+1} \cdot 3 \cdot F_i^2$ *has power spectral basis*

$$\left\{(F_i-2)^2F_i^2, 4(F_i-1)^2F_i^2, (F_i^2-1)^2\right\}.$$

Note that numbers 1 and 2 comprise an isospectral pair, that is, numbers n_1 *and* n_2 *such that* $n_1 = 2n_2$ *that have the same spectral basis.*

Remark 2*.* The third entry in Table 1 is $F_1 = 5$ in entry 3 of Theorem 1.3.

2 n is powerful

In this section we show that if n is powerful and power spectral Pythagorean, then $n = 288$. Recall that a positive integer is *powerful* if and only if all the exponents in its prime factorization are greater than one. We shall say that a positive integer is *nonpowerful* if and only if it is not powerful.

Suppose $a^2 + b^2 = c^2 = n + 1$ is the spectral basis of $n = p^\alpha q^\beta$, $\alpha, \beta > 1$. Define $u = n/p^\alpha = q^\beta$ and $v = n/q^\beta = p^\alpha$. Further, define $\bar{u} = u^{-1} \mod p^\alpha$ and $\bar{v} = v^{-1} \mod q^\beta$ so that we have $\bar{u}u + \bar{v}v = n+1$ as the spectral basis for n. The conditions are

$$\bar{u}q^\beta + \bar{v}p^\alpha = p^\alpha q^\beta + 1, \tag{1}$$
$$a^2 = \bar{u}q^\beta, \tag{2}$$
$$b^2 = \bar{v}p^\alpha, \tag{3}$$
$$c^2 = p^\alpha q^\beta + 1. \tag{4}$$

Since c is of the form $4k + 1$, we have

$$\begin{aligned}(4k+1)^2 &= p^\alpha q^\beta + 1\\ 16k^2 + 8k + 1 &= p^\alpha q^\beta + 1\\ 16k^2 + 8k &= p^\alpha q^\beta\\ 8k(2k+1) &= p^\alpha q^\beta.\end{aligned}$$

We may now assume that $p = 2$ and that q is an odd prime. Let $k = 2^\kappa$ so that we have

$$2^{\kappa+3}(2^{\kappa+1} + 1) = 2^\alpha q^\beta.$$

Thus, $\alpha = \kappa + 3$. If $\kappa + 1$ has an odd factor, then $2^{\kappa+1} + 1$ would have more than one prime factor, so $\kappa + 1$ must be a power of 2, that is, $\kappa = 2^\lambda - 1$ and, consequently, $\alpha = 2^\lambda + 2$. Thus,

$$2^{\kappa+1} + 1 = q^\beta$$
$$q^\beta - 2^{\kappa+1} = 1. \tag{5}$$

Let us now recall Catalan's conjecture.

Catalan's conjecture. *Let x, y be positive integers and let a, b be positive integers greater than one. Then the only solution to $x^a - y^b = 1$ occurs for $3^2 - 2^3 = 1$.*

Remark 3. *Catalan's conjecture* was conjectured by Eugene Charles Catalan in 1844 and proven by Preda Mihailescu in 2002. Consequently, the conjecture is also referred to as *Mihailescu's theorem*.

The only solution to (5) is $q = 3$, $\beta = 2$, and $\kappa + 1 = 3$. Thus, $\alpha = 2 + 3 = 5$ and $n = 2^5 \cdot 3^2 = 288$.

Corollary 2.1. *The only powerful power spectral Pythagorean number with two prime factors is $n = 288 = 2^5 \cdot 3^2$, with spectral basis $\{15^2, 8^2\}$ and spectral sum $15^2 + 8^2 = 17^2$. See Table 1.*

3 n is nonpowerful

In this section we show that if n is nonpowerful, then $n = 24$. Either $\beta \le 1$ or $\kappa + 1 \le 1$. If $\beta = 1$, then $q = 2^{\kappa+1} + 1$, and q must be a Fermat prime with $\kappa = 2^\lambda - 1$, $\alpha = 2^\lambda + 2$. If $\kappa + 1 = 1$, then $\kappa = 0$, $q^\beta = 3$, so $\beta = 1$, $q = 3$, $\alpha = 3$, $n = 24$. From now on, we assume that $\beta = 1$ and $\kappa > 0$.

Suppose $a^2 + b^2 = c^2 = n + 1$ is the spectral basis of $n = 2^\alpha q$, where $q = 2^{2^\lambda} + 1$ is a Fermat prime, $\alpha = 2^\lambda + 2$. Define $u = n/2^\alpha = q$ and $v = n/q = 2^\alpha$. Further, define $\bar{u} = q^{-1} \mod 2^\alpha$ and $\bar{v} = 2^{-\alpha} \mod q$ so that we have $\bar{u}u + \bar{v}v = n + 1$ as the spectral basis for n.

Lemma 3.1. *Suppose $q = 2^{2^\lambda} + 1$ is a Fermat prime, and let $\bar{u}$ and $\bar{v}$ be defined as above. Then*

1. *If $\lambda = 0$, then $\bar{u} = 3$ and $\bar{v} = 2$.*
2. *If $\lambda > 0$, then $\bar{u} = 3q - 2$ and $\bar{v} = 2^\omega$, $\omega = 2^\lambda - 2$.*

Proof. The case $\lambda = 0$ is easy to compute. Assume $\lambda > 0$. Let us show that $\bar{v} = 2^\omega$, where $\omega = 2^\lambda - 2$. Observe that

$$\begin{aligned} 2^{\alpha+\omega} &= 2^{(2^\lambda+2)+(2^\lambda-2)} \\ &= 2^{2\cdot 2^\lambda} \\ &= (2^{2^\lambda})^2 \\ &= (q-1)^2 \\ &= q^2 - 2q + 1 \quad (q > 3). \end{aligned}$$

Consequently,

$$2^{\alpha+\omega} \equiv 1 \pmod{q},$$

and, therefore, $\bar{v} = 2^\omega$. Furthermore, we have

$$\begin{aligned} \bar{u}q + (q-1)^2 &= 2^\alpha q + 1 \\ \bar{u}q + q^2 - 2q + 1 &= 2^\alpha q + 1 \\ \bar{u}q + q^2 - 2q &= 2^\alpha q \\ \bar{u} + q - 2 &= 2^\alpha \\ \bar{u} &= 2^\alpha - q + 2 \\ &= 2^{2^\lambda+2} - q + 2 \\ &= 4 \cdot 2^{2^\lambda} - q + 2 \\ &= 4(q-1) - q + 2 \\ \bar{u} &= 3q - 2 \quad (q > 3). \end{aligned}$$

□

Remark 4. Observe that

$$\begin{aligned} (3q-2)q + (q-1)^2 &= 2^\alpha q + 1 \\ 3q^2 - 2q + q^2 - 2q + 1 &= 2^\alpha q + 1 \\ 4q^2 - 4q &= 2^\alpha q \\ q - 1 &= 2^{\alpha-2} \\ q &= 2^{\alpha-2} + 1, \end{aligned}$$

so q must be a Fermat prime, with $\alpha = 2^\lambda + 2$, $\lambda > 0$.

Remark 5. Observe that $2^\alpha q = 4(q-1)q$ so that we have in fact the polynomial identity

$$(3q-2)q + (q-1)^2 = 4(q-1)q + 1 = (2q-1)^2.$$

Theorem 3.1. *The spectral basis for $n = 2^\alpha q$, where $q = 2^{2^\lambda} + 1$ is a Fermat prime, $\alpha = 2^\lambda + 2$, is given by*

$$\begin{aligned} (3q-2)q + (q-1)^2 &= 2^\alpha q + 1 \quad (q > 3), \\ 3^2 + 4^2 &= 2^3 \cdot 3 + 1 \quad (q = 3). \end{aligned}$$

Corollary 3.1. *The only nonpowerful power spectral Pythagorean number with two prime factors is $n = 24 = 2^3 \cdot 3$, with spectral basis $\{3^2, 4^2\}$ and spectral sum $3^2 + 4^2 = 5^2$.*

Proof. The spectral basis of $n = 2^\alpha q$, where $q = 2^{2^\lambda} + 1$ is a Fermat prime, $\alpha = 2^\lambda + 2$, $\lambda > 0$, is given by

$$(3q - 2)q + 2^{2 \cdot 2^\lambda} = 2^\alpha q + 1, \quad (q > 3).$$

Clearly, 3 does not divide the second term on the left, so it must divide the first term. Thus, either $3|q$ or $3|(3q - 2)$, but both are impossible. Therefore, the only Pythagorean power spectral number occurs for $\lambda = 0$, that is, when $q = 3$, $\alpha = 3$, so $n = 2^3 \cdot 3 = 24$. □

Combining Corollaries 2.1 and 3.1 we have

Theorem 3.2. *The only power spectral Pythagorean numbers with two prime factors are* 24 *and* 288*.*

4 Three prime factors

Let d be a nonsquare positive integer, $d > 1$, and consider the Diophantine equation $x^2 - dy^2 = 1$, called *Pell's equation*. Joseph Louis Lagrange proved that, as long as d is not a perfect square, Pell's equation has infinitely many distinct integer solutions [8]. Recursive solutions to Pell's equation for specific d can be obtained from Dario Alpern's web page [9].

4.1 Mersenne

The power spectral numbers $2^{2p-1} \cdot 3 \cdot M_p^2$ and $2^{2p} \cdot 3 \cdot M_p^2$, where M_p is a Mersenne prime, have the spectral basis [3]

$$\left\{M_p^2(M_p + 2)^2, M_p^2(M_p + 1)^2, (M_p^2 - 1)^2\right\}.$$

If we let $q = M_p$, then the spectral sum of the spectral basis is equivalent to the polynomial identity

$$q^2(q + 2)^2 + q^2(q + 1)^2 + (q^2 - 1)^2 = 3q^2(q + 1)^2 + 1.$$

If we let $y = q(q+1)$ and $3y^2 + 1 = x^2$, then we have the Pell equation $x^2 - 3y^2 = 1$. A pair of recursive solutions is given by [9]:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

Using the first recursive solution, we obtain

$$[[1,0,1],[2,1,2],[3,4,7],[4,15,26],[5,56,97],[6,209,362]].$$

It just so happens that $y = 56 = 7 \cdot 8$, $x = 97$ is a solution to the Pell equation $x^2 - 3y^2 = 1$, and $q = 7 = 2^3 - 1$ is a Mersenne prime.

4.2 Fermat

The power spectral number $2^{2f_i+1} \cdot 3 \cdot F_i^2$, where F_i is a Fermat prime, $f_i = 2^i$, has power spectral basis [3]

$$\left\{(F_i - 2)^2 F_i^2, 4(F_i - 1)^2 F_i^2, (F_i^2 - 1)^2\right\}.$$

If we let $q = F_i$, then the spectral sum of the spectral basis is equivalent to the polynomial identity

$$q^2(q - 2)^2 + 4q^2(q - 1)^2 + (q^2 - 1)^2 = 6q^2(q - 1)^2 + 1.$$

If we let $y = q(q-1)$ and $6y^2+1 = x^2$, then we have the Pell equation $x^2 - 6y^2 = 1$. A pair of recursive solutions is given by [9]:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 5 & 12 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 5 & -12 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}.$$

Using the first recursive solution, we obtain

$$[[1,0,1],[2,2,5],[3,20,49],[4,198,485],[5,1960,4801],[6,19402,47525]].$$

It just so happens that $y = 20 = 5 \cdot 4$, $x = 49$ is a solution to the Pell equation $x^2 - 6y^2 = 1$, and $q = 5 = 2^2 + 1$ is a Fermat prime.

Only G*d knows what's going on here!

Let's summarize our results as a theorem.

Theorem 4.1. *1. Suppose a, b, c is a primitive Pythagorean triple such that $\{a^2, b^2\}$ is the spectral basis of $n = c^2 - 1$. Then $n = 24$ or $n = 288$, with spectral bases $\{3^2, 4^2\}$ and $\{15^2, 8^2\}$, respectively.*

2. *The power spectral numbers $2^{2p-1} \cdot 3 \cdot M_p^2$ and $2^{2p} \cdot 3 \cdot M_p^2$, where M_p is a Mersenne prime, $p > 2$, have the spectral basis*

$$\left\{M_p^2(M_p+2)^2, M_p^2(M_p+1)^2, (M_p^2-1)^2\right\}.$$

If we let $q = M_p$, then the spectral sum is equivalent to the polynomial identity

$$q^2(q+2)^2 + q^2(q+1)^2 + (q^2-1)^2 = 3q^2(q+1)^2 + 1.$$

If x and y solutions of the Pell equation $x^2 - 3y^2 = 1$, where $y = q(q+1)$, then the spectral sum is x^2. Observe that $y = 7 \cdot 8$, $x = 97$ is a solution, so we have

n	*Factorization*	*Spectral basis*						
4704	$(2)^5(3)(7)^2$	63^2	+	56^2	+	48^2	=	97^2
9408	$(2)^6(3)(7)^2$	63^2	+	56^2	+	48^2	=	97^2

Furthermore, 4704 *and* 9408 *comprise an isospectral pair.*

3. *The power spectral number $2^{2f_i+1} \cdot 3 \cdot F_i^2$, where F_i is a Fermat prime, $f_i = 2^i$, $i > 0$, has power spectral basis*

$$\left\{(F_i-2)^2F_i^2, 4(F_i-1)^2F_i^2, (F_i^2-1)^2\right\}.$$

If we let $q = F_i$, then the spectral sum is equivalent to the polynomial identity

$$q^2(q-2)^2 + 4q^2(q-1)^2 + (q^2-1)^2 = 6q^2(q-1)^2 + 1.$$

If x and y solutions of the Pell equation $x^2 - 6y^2 = 1$, where $y = q(q-1)$, then the spectral sum is x^2. Observe that $y = 5 \cdot 4$, $x = 49$ is a solution, so we have

n	*Factorization*	*Spectral basis*						
2400	$(2)^5(3)(5)^2$	15^2	+	40^2	+	24^2	=	49^2

4.3 Another Pell equation

Since the previous subsection had an application of the Pell equations $x^2 - 3y^2 = 1$ and $x^2 - 6y^2 = 1$, respectively, let us give honorable mention to the following result.

Theorem 4.2. *Let x and y be solutions to the Pell equation $x^2 - 2y^2 = 1$. Then*

$$\begin{aligned} a &= \frac{x-1}{2}, \\ b &= \frac{x+1}{2}, \\ c &= y, \\ d &= ab = \frac{x^2-1}{4}, \\ e &= d+1 = \frac{x^2+3}{4}, \end{aligned}$$

have the property

$$\begin{aligned} a^2 + b^2 &= c^2, \\ c^2 + d^2 &= e^2. \end{aligned}$$

Proof. Left to the reader. □

Corollary 4.1. *The Frenet frame of* $\mathbf{r}(t) = \langle e^{at}\cos bt, e^{at}\sin bt, be^{at}\rangle$*, an instructive assignment in Calc III, is computable without radicals.*

5 Sequences

The descriptions of these sequences have been edited to suit the needs of this document.

A000043: Mersenne exponents: primes p such that $2^p - 1$ is prime. Then $2^p - 1$ is called a Mersenne prime.

A000215: Fermat numbers: $a(n) = 2^{2^n} + 1$.

A000225: Mersenne numbers: $a(n) = 2^n - 1$.

A001075: Solutions x to the Pell equation $x^2 - 3y^2 = 1$.

A001078: Solutions y to the Pell equation $x^2 - 6y^2 = 1$. Numbers n such that $6n^2 + 1$ is a square.

A001079: Solutions x to the Pell equation $x^2 - 6y^2 = 1$.

A001348: Mersenne primes: $2^p - 1$, where p is prime.

A001353: Solutions y to the Pell equation $x^2 - 3y^2 = 1$. Numbers n such that $3n^2 + 1$ is a square.

A019434: Fermat primes: primes of the form $2^{2^k} + 1$, for some $k \geq 0$.

A103606: Primitive Pythagorean triples in nondecreasing order of perimeter, with each triple in increasing order, and if perimeters coincide then increasing order of the even members.

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