The Universe and Large Numbers : Cosmic Web, Cosmic Microwave Background and Cosmological Constant.

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Abstract

This paper presents a purely illustrative exploration of some numerical hierarchies that emerge when comparing different energy and time scales in the present-day Universe. A dimensionless parameter is introduced, constructed from the ratio between the energy density associated with the nonlinear gravitational structure and that of the cosmic microwave background, weighted by the ratio between the so-called Planck's time and atomic unit of time. The resulting extremely small value reflects the enormous separation of scales between the physics of the early Universe, the relict background radiation, and the late cosmological structure. This analysis does not aim to establish new dynamic relationships or fundamental magnitudes, but rather to offer a numerical curiosity in the spirit of traditional discussions about *large numbers* in cosmology.

Keywords. *Cosmic web,Cosmological constant,Cosmic microwave background,Large numbers,Arithmetic.*

Introduction

From the earliest developments of modern cosmology, various authors such as Eddington, Dirac, and Weyl (Dirac, 1937; Eddington, 1931; Weyl, 1919) [1] [2][3]drew attention to the emergence of enormous numerical *hierarchies* when comparing fundamental physical quantities. In particular, Dirac observed that certain ratios between cosmological, atomic, and gravitational constants resulted in extraordinarily large or small numbers, which motivated the well-known *Large Numbers Hypothesis*. Although these coincidences did not always lead to verifiable dynamical theories, they did establish a tradition of conceptual exploration based on the comparison of very different physical scales.

In this spirit, the present work proposes a purely illustrative comparison between some characteristic quantities of the present-day Universe: the energy associated with the nonlinear gravitational structure, the energy of the cosmic microwave background, and the timescales linked to *primordial gravitational waves*. The aim is not to derive new physical laws, but rather to highlight the enormous separation of scales that emerges when juxtaposing phenomena from the early Universe, the relict background radiation, and the late cosmological structure. These hierarchies are presented here as a *numerical curiosity*, in continuity with the tradition of "large numbers" in cosmology.
We must remark on the fact that a numerical analysis of physical constants only makes sense using *dimensionless* numbers .

Method and results.

The parameters introduced in this work are not intended to represent fundamental magnitudes, direct observables or dynamic relationships, but only to *illustrate* numerical hierarchies between different cosmological scales.
Let's start defining the first dimensionless number that works as a true large number and that will be useful to analyze relationships among different large numbers . Such definition involves two length dimensions : *Bohr radius* and *Planck's length*.Besides ,our definition includes a mathematical constant, *Euler's number* : 2.7182818...
Planck's length $\approx 1.6162\, x\, 10^{-35} m$ is a base unit in the system of Planck units, also known as natural units. Most physicists state that Planck length defines the smallest scale's length where space breaks down.
Bohr radius [4] is a physical constant that refers to the unit of length in atomic units. In the hydrogen atom an electron '*orbits'* a proton.The ground state of one electron in the hydrogen atom defines the smallest possible orbit. This *orbital radius* is almost equal to the Bohr radius $a_0 \approx 5.2918\, x\, 10^{-11} m$, which gives the maximum probability density of the electron in its ground state energy.

Therefore we have two small length dimensions ; on one hand Planck's length is associated with the physics of the space itself , and on the other hand Bohr radius is associated with the scale of the most simple atom , the atom of hydrogen. Since both physical constants are given in *meters* , the ratio of Bohr radius over Planck's length gives a dimensionless number. From a theoretical point of view we must consider the gap between both length dimensions , i.e. the orders of magnitude that separates Planck's length from Bohr radius .

Let’s define a dimensionless parameter:

$$(N...) = \frac{a_0}{e\, l_P} = 1.2045 \qquad (1)$$

$a_0 = 5.2918\, x\, 10^{-11} m$

$l_P = 1.6162\, x\, 10^{-35} m$

$e = 2.71828...$

Let's get down to business with the *big numbers*. It can be said that most of the matter that stars and interstellar space (not taking into account what is known as *dark matter*) are made of hydrogen atoms. According to estimates, the observable universe contains about 10^{80} Hydrogen atoms. In addition to *baryonic* matter, the universe contains *radiation*, which we know as the cosmic microwave background.

Cosmic microwave background

In the observable universe, radiation is dominated by the *cosmic microwave background* photons.
According to the standard model of cosmology[5] (called the ΛCDM model), the cosmic microwave background (CMB) is the *echo* or *fossil radiation* resulting from the *Big Bang*. It represents the oldest light we can detect in the universe.With the formation of neutral atoms, free electrons almost completely disappeared. Without electrons for photons to collide with, they could finally travel freely through space without being absorbed or deflected.The universe continued to expand.This expansion *stretched* the wavelength of the photons (redshift effect) until they reached the microwave region of the electromagnetic spectrum.

The well known Planck-Einstein equation [6] reads:

$$E = N_{\gamma} \hbar \nu_{0} \tag{2}$$

where $\hbar$ is the reduced Planck constant: $1.05457 \, x \, 10^{-34} Js$, N_{γ} is the number of photons $\sim 10^{90}$ and $\nu_{0} = 1.603 \, x \, 10^{11} s^{-1}$ refers to the spectral radiance *peak* of the cosmic microwave background (CMB) that occurs at this frequency mode of vibration[7].There is a consensus that the ratio between the number of photons and barions is about 10^{10},therefore $N_{\gamma} \sim 10^{90}$

Equation (2) in numbers:

$$E = N_{\gamma} \hbar \nu_{0} = 10^{90} x \, 1.05457 \, x \, 10^{-34} Js \, x \, 1.603 \, x \, 10^{11} s^{-1}$$

resulting

$$E = 1.69 \, x \, 10^{67} J \tag{3}$$

We are more interested in *energy density* than energy itself :

$$\rho_{CMB,0} \equiv \frac{E}{V} \tag{4}$$

V: volume of observable universe [8].Later we will do the same with the gravitational binding energy density of the filaments of the cosmic web, so the volume itself *cancels out* when calculating the ratio between both densities.

Gravitational binding energy

The gravitational binding energy of a system is the total energy required to completely *dismantle* the system, separating all its material to infinite distances. It is a measure of how strongly the body or system is held together by its own gravity.
The analysis of gravitational binding energy (Carroll & Ostlie, 2017)[9] using the expression

$$\Omega = -\, f \,\frac{GM^2}{L} \tag{5}$$

allows us to characterize the morphology and dynamic state of a massive system.
A sphere of uniform density has a *form factor* f = 0.6 and highly centralized structures like the Sun exhibit f values of approximately 1.75 due to their concentration of mass in the core.
According to cosmological simulations (Aragón-Calvo et al., 2010; Cautun et al., 2014)[10], filaments exhibit weak gravitational potentials compared to *virialized systems*.
A system defined by $f \simeq 0.001$ describes a configuration where self-gravity is practically negligible relative to its spatial extent. Physically, this value is consistent with *large-scale filamentary structures or extremely diffuse gas clouds*, where the effective gravitational radius indicates a diffuse, filamentous, or *expanding system*.
What comes to mind when we talk about diffuse, filamentous, *expanding system*? Perhaps the *cosmic web?*
In cosmological simulations, the *filaments* of the cosmic web exhibit characteristic gravitational potentials,far inferior to those of virialized systems, so that an effective geometric factor $f \sim 10^{-3}$ is obtained.

We define an effective gravitational energy density as the total gravitational binding energy associated with the filaments of the cosmic web, averaged over a comoving cosmological volume

$$\rho_{grav,0} \equiv \frac{1}{V}\sum_{i} |\Omega, i| \qquad (6)$$

where Ω, i is the gravitational binding energy of the filament i estimated to a first approximation as

$$\Omega, i \sim \frac{GM_i^2}{L_i} \qquad (7)$$

where M_i, L_i represents the characteristic mass and filament length, respectively.

$\rho_{grav,0}$ should be understood as an effective quantity associated with the nonlinear structure, *not* as a global gravitational energy in the strict sense of general relativity.

Incidentally, since the visible mass of the cosmic web is mostly made of hydrogen, we will use the mass of the hydrogen atom as the reference value in the equation. A hydrogen atom has a mass of $1.673\ x\ 10^{-27} kg$ and *there are* about 10^{80} hydrogen atoms in the observable universe.Applying numerical values:

G: Universal Gravitational Constant = $6.674\ x\ 10^{-11} m^3 kg^{-1} s^{-2}$

m_H: hydrogen mass =$1.673\ x\ 10^{-27} kg$

N_H: number of hydrogen atoms in the observable universe $\sim 10^{80}$

L ~ radius of the observable universe $\sim 10^{26} m$

$$\Omega = - f \frac{GM^2}{L} = - 0.001 \frac{G (N_H m_H)^2}{L} = - 1.868 \, x \, 10^{67} J \qquad (8)$$

Just as we did with the CMB energy, we write the gravitational binding energy *density* using the absolute value to avoid problems with the minus sign

$$\rho_{grav,0} \equiv \frac{|\Omega|}{V} \qquad (9)$$

We see the dimensionless parameter R_0:

$$R_0 \equiv \frac{\rho_{grav,0}}{\rho_{CMB,0}} \sim 1.105 \qquad (10)$$

which quantifies the relative importance of the energy density associated with the filaments of the *cosmic web* versus the energy density of the cosmic microwave background *at present*.In the present cosmological time, the energy density associated with the gravitational structure is of the same order of magnitude as the energy density of the cosmic microwave background, although the two quantities are *not dynamically coupled*.
Although there is no direct dynamic coupling between the two, this comparison illustrates the change in cosmological regime from a *radiation-dominated* state to a *structure-dominated* one.
The regime in which only the CMB is relevant and the gravitational contribution is negligible. In other words

$$\frac{\rho_{CMB}}{\rho_{CMB}} \equiv 1$$

we define the dimensionless parameter Θ

$$\Theta \equiv 1 + R_0 \rightarrow 1 + 1.105 = 2.105 \tag{11}$$

where the unit represents the purely radiative regime *normalized* to the cosmic microwave background. In this sense, the constant term fixes the early boundary of the Universe.
$\Theta \rightarrow 1$ when the gravitational contribution associated with the structure is negligible, while the growth of R_0 quantifies the emergence of nonlinear structure in late epochs.
Briefly, R_0 measures structure *vs* radiation; $\frac{\rho_{CMB}}{\rho_{CMB}}$ radiation *vs* radiation. As for Θ It measures how far we move away from the radiative regime.

Planck units & atomic units

When we defined the parameter (*N*...), we took into account two constants: the Planck length and the Bohr radius. Now we are going to define the parameter τ using those two hierarchies, namely the *Planck time* and the *atomic time*.
Planck units are a set of physical units based on fundamental constants of nature [11]. These units are especially useful in theoretical physics, as they allow us to describe phenomena at extremely small scales, such as those found in quantum theory and gravity. The *Planck time*, denoted as t_P It is the smallest unit of time that has physical meaning and is defined as:

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \tag{12}$$

where $\hbar$ is the reduced Planck constant, G is the universal gravitational constant and c is the speed of light in vacuum.The Planck time is approximately $5.4\ x\ 10^{-44} s$, which indicates that it is the time it takes light to travel a distance equivalent to the Planck length.

Atomic units, also known as Hartree units [12], are a system of units that simplifies the equations of quantum mechanics by using fundamental constants as a basis.
The *atomic time* unit is a measure used in the context of quantum mechanics and is defined as the time it takes an electron to travel a distance comparable to its own wavelength. This unit is related to the Planck constant and is expressed in terms of the Planck time, although it is more commonly associated with the Hartree energy E_H. Atomic time is defined as:

$$t_a = \frac{\hbar}{E_H} \tag{13}$$

$$E_H = 4.36\ x\ 10^{-18} J \tag{14}$$

This unit of time, t_a is approximately $2.42\ x\ 10^{-17} s$ .Let's calculate the ratio between both units denoted by greek letter τ

$$\tau = \frac{t_P}{t_a} = 2.23\ x\ 10^{-27} \tag{15}$$

Discreteness

Spacetime doesn't necessarily have to be a *fundamental* aspect of the universe; it could emerge collectively, much like temperature in a gas. A single molecule *has no* temperature; temperature only arises when many molecules interact. Similarly, the continuous spacetime described by relativity could emerge from smaller, discrete elements. On a large scale, space and time are perceived as continuous, like a fabric viewed from afar. However, upon closer inspection, this fabric would be composed of elementary "threads," comparable to *atoms of space*[13].

Let's write down an equation that incorporates the parameters explored above plus a parameter that could quantify those hypothetical *atoms of space* that could quantify those hypothetical atoms of space. Such a parameter consists, no more and no less, than the parameter (N...) raised to the fourth power:

$$D = \frac{1}{(N...)^4} = 4.75\, x\, 10^{-97} \tag{16}$$

therefore:

$$\left[\left(1 + \frac{|\rho_{grav,0}|}{\rho_{CMB,0}}\right)\left(\frac{t_P}{t_a}\right)\right]D \approx \Lambda_{observed} \tag{17}$$

in numbers:

$$4.75\, x\, 10^{-97}\left[(2.23\, x\, 10^{-27}) + (2.23\, x\, 10^{-27} x\, 1.105)\right] = 2.2\, x\, 10^{-123}$$

$\Lambda_{observed}$ denote *observed* cosmological constant.Although in cosmology the value of the cosmological constant is usually expressed in the form $\sim 10^{-52} m^{-2}$, since the rest of the parameters are dimensionless, we express the value of $\Lambda_{obs.}$ in dimensionless units or Planck units. It is worth noting that the observed value of the cosmological constant in Planck units is on the order of an extreme discrepancy with theoretical predictions (Smolin, 2007) [14]

Writing the equation (17) in a more compact form:

$$D\left[\tau + \tau R_0\right] \approx \Lambda_{obs.} \qquad (18)$$

Discussion

The calculations described in the previous section involve both observational numbers, such as the cosmic microwave background and the cosmological constant, and quantities derived from fundamental concepts, such as gravitational binding energy density. There is also a comparative scale or hierarchy parameter, the (Tau) τ parameter, Planck's time versus atomic time. And finally, the *D* parameter, which is not observational but rather a large number that is being applied under the hypothetical concept of *discrete or granular* spacetime.

Since our analysis is developed within the *popular science field* of the curiosities of large numbers, we could also replace the parameter τ (tau) with another in the frequency space. Namely, the frequency of gravitational waves on the one hand, and the frequency of the cosmic microwave background radiation on the other.

Hypothetical gravitational waves from the early universe [15], generated during primordial cosmic events such as inflation, are characterized by frequencies of approximately $10^{-16} s^{-1}$.
These waves, which are disturbances in the fabric of spacetime, could provide valuable information about the conditions of the universe in its earliest moments.Let’s write the ratio

$$\omega = \frac{W_{GB}}{W_{CMB}} \approx \frac{10^{-16} s^{-1}}{10^{11} s^{-1}} \approx 10^{-27} \tag{19}$$

so that equation (18) is slightly modified, with an equivalent result:

$$D\left[\omega + \omega R_0\right] \approx \Lambda_{obs.} \tag{20}$$

Conclusion

Amateur exploration of numerical curiosities, inspired by the large number hypothesis, has yielded some noteworthy results. The purely informative purpose of this article does not imply predictions or new theories. We have carefully selected the various hierarchies and simplified the arithmetic relationships between physical scales.

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