De Broglie's Internal Clock of Electrons in Hydrogen Atoms as Cause of the Quantization of Orbital Angular Momentum

Martin Kraus (kraus.martin@gmail.com)

January 27, 2026

Abstract

De Broglie showed that a hypothetical internal clock of electrons could cause the quantization of energy levels of closed electronic orbits in the Bohr model of hydrogen-like atoms. Inspired by this insight, this work presents a model for the quantization of orbital angular momentum of electronic Bohr-Sommerfeld orbits with apsidal precession. The predictive power of the presented model is very limited since a parameter of the model is calculated based on a previously published quantization of the electron's orbital angular momentum.

1 Introduction and Previous Work

De Broglie's doctoral thesis [dB25] explained experimentally observed wave-like features of electrons with a hypothetical internal clock, which is locally in phase with hypothetical "phase waves." In particular, de Broglie used the concept of phase waves to derive Bohr's quantization condition for the energy levels of closed electronic orbits in the non-relativistic Bohr model of hydrogen-like atoms. As is well known, de Broglie's work motivated Schrödinger's early work on non-relativistic quantum wave mechanics [Sch26].

It is less well known that de Broglie's quantitative description of phase waves relied strongly on Einstein's special theory of relativity. Thus, core features of *non-relativistic* quantum mechanics (e.g. Bohr's quantization condition) can be derived from the *relativistic* mechanics of clocks ticking at the Compton frequency, which depends on Planck's constant h. This is in contrast to the common belief that quantum mechanics is fundamentally different from and more general than classical mechanics, which supposedly could only be derived from quantum mechanics in the limit of $h \to 0$. How could one resolve this apparent contradiction? The popular head-in-the-sand solution is to ignore de Broglie's concept of an internal clock.

This work takes a different approach. As in previous work [Kra25a], the internal clock of an orbiting electron is assumed to drive a forced vibration in the space around the nucleus, and the electron's internal clock is (at least approximately) locally in phase with this forced vibration. If the electron is in a steady state on a closed Bohr-Sommerfeld orbit, previous work [Kra25a] concluded that the phase relation between the electron's internal clock and the forced vibration after one period of the closed orbit implies Einstein's (coordinate-independent) quantization condition [Ein17].

However, elliptical Bohr-Sommerfeld orbits are not closed but show a small amount of apsidal precession (i.e. a rotation of the line of apsides), which is a relativistic effect as described by Sommerfeld [Som23, pages 467–470]. The small apsidal rotation angle $\Delta\phi$ per orbital period of the corresponding closed orbit depends on the orbital angular momentum p_ϕ of the electron. Based on Einstein's quantization condition, a continuum of values of p_ϕ is possible for any value of the primary quantum number n. In general, the resulting apsidal rotation angle advances the endpoint of an electronic orbit to a position where the electron's internal clock is not in phase with the forced vibration in the space around the nucleus, which violates the basic requirement for steady states in our model. For a few, discrete values of p_ϕ, however, the resulting apsidal rotation angles advance the endpoint of the orbit to positions where the electron's internal clock is (approximately) in phase with the forced vibration. (If one period of the orbit starts at its apocenter or if the electron's speed is constant on the orbit, one could imagine that the apsidal precession advances the endpoint of the orbit after one period by an integer number of wavelengths of the electron's phase wave corresponding to the same number of ticks (i.e. periods) of the electron's internal clock.)

Realistically, the quantization condition on p_ϕ is not a local condition at any single point but more similar to a minimization condition on an integral of (weighted) local phase differences between the electron's internal clock and the forced vibration at all points of the rotated orbit. In this work, however, the condition is reduced to a local condition at single points with a parameter that is adjusted based on the $\sqrt{l(l+1)}$ correction described by Bucher [Buc08]. This local condition is the topic of the next section.

2 Parameterized Local Quantization Condition

2.1 Notation

This work uses SI units and several well-known physical constants, including Planck's constant h, reduced Planck's constant $\hbar \stackrel{\text{def}}{=} h/(2\pi)$, speed of light c, vacuum permittivity ε_0, (positive) elementary electric charge e, mass of electron m_e, (Sommerfeld's) fine-structure constant $\alpha \stackrel{\text{def}}{=} e^2/(4\pi\varepsilon_0\hbar c)$, Bohr radius $r_\text{B} \stackrel{\text{def}}{=} \hbar/(m_\text{e}c\alpha) = 4\pi\varepsilon_0\hbar^2/(m_\text{e}e^2)$, and Compton frequency of electrons $f_\text{e} \stackrel{\text{def}}{=} m_\text{e}c^2/h = m_\text{e}c^2/(2\pi\hbar)$.

As mentioned, n denotes the (integer) primary quantum number, and l denotes the (integer) orbital angular momentum quantum number with $0 \leq l < n$ as suggested by Bucher for the Bohr-Sommerfeld model [Buc08]. However, cases with $l = 0$ are not discussed in this work since they require special treatment [Buc08, Kra25b]. ϕ and r are polar coordinates of the electron's (two-dimensional) position relative to the (fixed) position of the nucleus with ϕ being the angle around the nucleus between the position of the electron and the pericenter of an elliptical orbit, and r being the electron's distance from the nucleus. $p_\phi = p_\phi(n, l)$ is the orbital angular momentum and depends on n and l, which is the quantization that this work is concerned with.

2.2 Derivation of Quantization Condition at Pericenter

According to Sommerfeld [Som23, pages 467–468], the apsidal rotation angle $\Delta\phi$ per orbital period of the corresponding closed orbit is (in good approximation as long as $\Delta\phi$ is small)

$$\Delta\phi \approx \frac{\pi\alpha^2\hbar^2}{p_\phi^2}. \tag{1}$$

The apsidal precession characterized by $\Delta\phi$ is the only relativistic effect that we need to consider in this section; thus, the following relations are non-relativistic approximations.

In order to relate $\Delta\phi$ to the ticking of the electron's internal clock, we compute the angle that the electron is advancing during one tick (i.e. period) of the internal clock. Since the frequency of the internal clock is f_e, the period Δt of the internal clock is (neglecting relativistic effects):

$$\Delta t \approx \frac{1}{f_\text{e}} = \frac{h}{m_\text{e}c^2} = \frac{2\pi\hbar}{m_\text{e}c^2}. \tag{2}$$

Given the tangential speed $v_t(\phi)$ of the electron and its distance $r(\phi)$ from the nucleus, the electron moves by the angle $v_t(\phi)\Delta t/r(\phi)$ in the ϕ-coordinate during one tick of the internal clock, and by l times this angle during l ticks, i.e. $l\, v_t(\phi)\Delta t/r(\phi)$. If the value of the latter expression at the pericenter (i.e. for $\phi = 0$) is equal to $\Delta\phi$, then the electron's internal clock after one full orbit is (approximately) in phase with its former self on the previous orbit (and, therefore, with the forced vibration) near the pericenter. For all other points (with $\phi \neq 0$) of an elliptical orbit, the tangential speed $v_t(\phi \neq 0)$ is smaller and the radial distance $r(\phi \neq 0)$ is larger than at the pericenter; thus, the angle $l\, v_t(\phi = 0)\Delta t/r(\phi = 0)$ is in general too large for all other points to be in phase with the corresponding (i.e. rotated by $\Delta\phi$) points of the previous orbit.

Thus, in order to minimize the phase differences on the whole orbit, the angle has to be less than $l\, v_t(\phi = 0)\Delta t/r(\phi = 0)$, which we take into account by a real parameter κ_{nl}^peri with $0 < \kappa_{nl}^\text{peri} < 1$. Hypothetically, it is possible to compute κ_{nl}^peri by a minimization of an integral of (weighted) phase differences on the whole orbit. In the present work, however, κ_{nl}^peri is computed (in Section 2.4) based on Bucher's $\sqrt{l(l+1)}$ correction.

Putting everything together, the local quantization condition for p_ϕ at the pericenter becomes:

$$\Delta\phi \approx \kappa_{nl}^{\text{peri}} l \frac{v_t(\phi=0)\Delta t}{r(\phi=0)} \qquad \text{with} \quad 0 < \kappa_{nl}^{\text{peri}} < 1. \tag{3}$$

In order to evaluate this condition for an elliptical orbit specified by n and l, note that $v_t(\phi=0) = \sqrt{\alpha\hbar c/(m_\text{e}\Lambda_{nl})}(1+\epsilon_{nl})$ with the semi-latus rectum $\Lambda_{nl} = a_{nl}(1-\epsilon_{nl}^2)$, the major semi-axis $a_{nl} = n^2 r_\text{B}$, and the eccentricity $\epsilon_{nl} = \sqrt{1 - p_\phi^2/(n^2\hbar^2)}$ [Wik26] with $p_\phi = p_\phi(n,l)$, which has to be determined by the quantization condition. Also note that $r(\phi=0) = a_{nl}(1-\epsilon_{nl})$ [Wik26].

The next objective is to solve Eq. (3) for $\kappa_{nl}^{\text{peri}}$ in order to express $\kappa_{nl}^{\text{peri}}$ as a function of $p_\phi = p_\phi(n,l)$, or, equivalently, as a function of ϵ_{nl}:

$$\frac{\pi\alpha^2\hbar^2}{p_\phi^2} \approx \kappa_{nl}^{\text{peri}} l \sqrt{\frac{\alpha\hbar c(1+\epsilon_{nl})^2}{m_\text{e}\Lambda_{nl}}} \frac{2\pi\hbar}{m_\text{e}c^2} \frac{1}{a_{nl}(1-\epsilon_{nl})} \tag{4}$$

$$\frac{\pi\alpha^2\hbar^2}{n^2\hbar^2(1-\epsilon_{nl}^2)} \approx \kappa_{nl}^{\text{peri}} l \sqrt{\frac{\alpha\hbar c(1+\epsilon_{nl})^2}{m_\text{e}a_{nl}(1-\epsilon_{nl}^2)}} \frac{2\pi\hbar}{m_\text{e}c^2} \frac{1}{a_{nl}(1-\epsilon_{nl})} \tag{5}$$

$$\frac{\pi\alpha^2}{n^2(1-\epsilon_{nl}^2)} \approx \kappa_{nl}^{\text{peri}} l \sqrt{\frac{\alpha\hbar c(1+\epsilon_{nl})^2}{m_\text{e}n^2 r_\text{B}(1-\epsilon_{nl}^2)}} \frac{2\pi\hbar}{m_\text{e}c^2} \frac{1}{n^2 r_\text{B}(1-\epsilon_{nl})} \tag{6}$$

$$\frac{\pi\alpha^2}{(1-\epsilon_{nl}^2)} \approx \frac{\kappa_{nl}^{\text{peri}} l}{n} \sqrt{\frac{\alpha\hbar c(1+\epsilon_{nl})^2}{m_\text{e}r_\text{B}(1-\epsilon_{nl}^2)}} \frac{2\pi\hbar}{m_\text{e}c^2} \frac{1}{r_\text{B}(1-\epsilon_{nl})} \tag{7}$$

$$\frac{\pi\alpha^2}{(1-\epsilon_{nl}^2)} \approx \frac{\kappa_{nl}^{\text{peri}} l}{n} \sqrt{\frac{\alpha^2\hbar c^2(1+\epsilon_{nl})^2 m_\text{e}}{m_\text{e}\hbar(1-\epsilon_{nl}^2)}} \frac{2\pi\hbar}{m_\text{e}c^2} \frac{m_\text{e}c\alpha}{\hbar(1-\epsilon_{nl})} \tag{8}$$

$$1 \approx \frac{2\kappa_{nl}^{\text{peri}} l}{n} \sqrt{\frac{(1+\epsilon_{nl})^2(1-\epsilon_{nl}^2)}{(1-\epsilon_{nl})^2}} \tag{9}$$

$$\kappa_{nl}^{\text{peri}} \approx \frac{n}{2l} \sqrt{\frac{(1-\epsilon_{nl})^2}{(1+\epsilon_{nl})^2(1-\epsilon_{nl})(1+\epsilon_{nl})}} \sqrt{\frac{(1+\epsilon_{nl})}{(1+\epsilon_{nl})}} \tag{10}$$

$$\kappa_{nl}^{\text{peri}} \approx \frac{n}{2l} \frac{\sqrt{1-\epsilon_{nl}^2}}{(1+\epsilon_{nl})^2} \tag{11}$$

$$\kappa_{nl}^{\text{peri}} \approx \frac{p_\phi(n,l)}{2l\hbar\left(1+\sqrt{1-(p_\phi(n,l)/(n\hbar))^2}\right)^2} \tag{12}$$

As mentioned, this approximation and Bucher's correction $p_\phi(n,l) = \sqrt{l(l+1)}\hbar$ are used in Section 2.4 to compute numerical values of $\kappa_{nl}^{\text{peri}}$.

2.3 Derivation of Quantization Condition at Apocenter

This section repeats the calculation of the previous section, but for the apocenter ($\phi = \pi$) instead of the pericenter of the elliptical orbit. The resulting parameter κ_{nl}^{apo} times l specifies the number of "wavelengths" of the internal clock that the apsidal rotation advances the apocenter compared to the apocenter of the previous orbit. If the two apocenters are in phase, the expectation is that $\kappa_{nl}^{\text{apo}} l$ is close to an integer greater than or equal to 1; thus, the local quantization condition at the apocenter becomes:

$$\Delta\phi \approx \kappa_{nl}^{\text{apo}} l \frac{v_t(\phi=\pi)\Delta t}{r(\phi=\pi)} \qquad \text{with} \quad 1 \le \kappa_{nl}^{\text{apo}} l. \tag{13}$$

For the evaluation at the apocenter, note that $v_t(\phi=\pi) = \sqrt{\alpha\hbar c/(m_\text{e}\Lambda_{nl})}(1-\epsilon_{nl})$ and $r(\phi=\pi) = a_{nl}(1+\epsilon_{nl})$ [Wik26]. Thus, the quantization condition at the apocenter evaluates to:

$$\frac{\pi\alpha^2\hbar^2}{p_\phi^2} \approx \kappa_{nl}^{\text{apo}} l \sqrt{\frac{\alpha\hbar c(1-\epsilon_{nl})^2}{m_\text{e}\Lambda_{nl}}} \frac{2\pi\hbar}{m_\text{e}c^2} \frac{1}{a_{nl}(1+\epsilon_{nl})} \tag{14}$$

$$\frac{\pi\alpha^2\hbar^2}{n^2\hbar^2(1-\epsilon_{nl}^2)} \approx \kappa_{nl}^{\text{apo}} l \sqrt{\frac{\alpha\hbar c(1-\epsilon_{nl})^2}{m_\text{e}a_{nl}(1-\epsilon_{nl}^2)}} \frac{2\pi\hbar}{m_\text{e}c^2} \frac{1}{a_{nl}(1+\epsilon_{nl})} \tag{15}$$

$$\frac{\pi\alpha^2}{n^2(1-\epsilon_{nl}^2)} \approx \kappa_{nl}^{\text{apo}} l \sqrt{\frac{\alpha\hbar c(1-\epsilon_{nl})^2}{m_\text{e}n^2 r_\text{B}(1-\epsilon_{nl}^2)}} \frac{2\pi\hbar}{m_\text{e}c^2} \frac{1}{n^2 r_\text{B}(1+\epsilon_{nl})} \tag{16}$$

$$\frac{\pi\alpha^2}{(1-\epsilon_{nl}^2)} \approx \frac{\kappa_{nl}^{\text{apo}} l}{n} \sqrt{\frac{\alpha\hbar c(1-\epsilon_{nl})^2}{m_\text{e}r_\text{B}(1-\epsilon_{nl}^2)}} \frac{2\pi\hbar}{m_\text{e}c^2} \frac{1}{r_\text{B}(1+\epsilon_{nl})} \tag{17}$$

$$\frac{\pi\alpha^2}{(1-\epsilon_{nl}^2)} \approx \frac{\kappa_{nl}^{\text{apo}} l}{n} \sqrt{\frac{\alpha^2\hbar c^2(1-\epsilon_{nl})^2 m_\text{e}}{m_\text{e}\hbar(1-\epsilon_{nl}^2)}} \frac{2\pi\hbar}{m_\text{e}c^2} \frac{m_\text{e}c\alpha}{\hbar(1+\epsilon_{nl})} \tag{18}$$

$$1 \approx \frac{2\kappa_{nl}^{\text{apo}} l}{n} \sqrt{\frac{(1-\epsilon_{nl})^2(1-\epsilon_{nl}^2)}{(1+\epsilon_{nl})^2}} \tag{19}$$

$$\kappa_{nl}^{\text{apo}} \approx \frac{n}{2l} \sqrt{\frac{(1+\epsilon_{nl})^2}{(1-\epsilon_{nl})^2(1-\epsilon_{nl})(1+\epsilon_{nl})}} \sqrt{\frac{(1-\epsilon_{nl})}{(1-\epsilon_{nl})}} \tag{20}$$

$$\kappa_{nl}^{\text{apo}} \approx \frac{n}{2l} \frac{\sqrt{1-\epsilon_{nl}^2}}{(1-\epsilon_{nl})^2} \tag{21}$$

$$\kappa_{nl}^{\text{apo}} \approx \frac{p_\phi(n,l)}{2l\hbar\left(1-\sqrt{1-(p_\phi(n,l)/(n\hbar))^2}\right)^2} \tag{22}$$

2.4 Parameter Values Based on Bucher's Correction

Inserting Bucher's $\sqrt{l(l+1)}$ correction, i.e. $p_\phi(n,l) = \sqrt{l(l+1)}\hbar$, into Eq. (12) and Eq. (22) allows us to compute numerical values of $\kappa_{nl}^{\text{peri}}$ and κ_{nl}^{apo} for pairs of n and l (excluding cases with $l = 0$ as mentioned in Section 2.1). For the results, see Tables 1 and 2.

Table 1: Numerical values of $\kappa_{nl}^{\text{peri}}$ for some pairs of n and l based on $\sqrt{l(l+1)}$ correction

	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$
$n=2$	$\kappa_{2\,1}^{\text{peri}} \approx 0.2426$				
$n=3$	$\kappa_{3\,1}^{\text{peri}} \approx 0.1997$	$\kappa_{3\,2}^{\text{peri}} \approx 0.2461$			
$n=4$	$\kappa_{4\,1}^{\text{peri}} \approx 0.1888$	$\kappa_{4\,2}^{\text{peri}} \approx 0.1910$	$\kappa_{4\,3}^{\text{peri}} \approx 0.2566$		
$n=5$	$\kappa_{5\,1}^{\text{peri}} \approx 0.1842$	$\kappa_{5\,2}^{\text{peri}} \approx 0.1748$	$\kappa_{5\,3}^{\text{peri}} \approx 0.1949$	$\kappa_{5\,4}^{\text{peri}} \approx 0.2669$	
$n=6$	$\kappa_{6\,1}^{\text{peri}} \approx 0.1819$	$\kappa_{6\,2}^{\text{peri}} \approx 0.1674$	$\kappa_{6\,3}^{\text{peri}} \approx 0.1750$	$\kappa_{6\,4}^{\text{peri}} \approx 0.2012$	$\kappa_{6\,5}^{\text{peri}} \approx 0.2762$

Table 2: Numerical values of κ_{nl}^{apo} for some pairs of n and l based on $\sqrt{l(l+1)}$ correction

	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$
$n=2$	$\kappa_{2\,1}^{\text{apo}} \approx 8.243$				
$n=3$	$\kappa_{3\,1}^{\text{apo}} \approx 50.71$	$\kappa_{3\,2}^{\text{apo}} \approx 3.428$			
$n=4$	$\kappa_{4\,1}^{\text{apo}} \approx 169.5$	$\kappa_{4\,2}^{\text{apo}} \approx 13.96$	$\kappa_{4\,3}^{\text{apo}} \approx 2.309$		
$n=5$	$\kappa_{5\,1}^{\text{apo}} \approx 424.1$	$\kappa_{5\,2}^{\text{apo}} \approx 37.25$	$\kappa_{5\,3}^{\text{apo}} \approx 7.423$	$\kappa_{5\,4}^{\text{apo}} \approx 1.829$	
$n=6$	$\kappa_{6\,1}^{\text{apo}} \approx 890.8$	$\kappa_{6\,2}^{\text{apo}} \approx 80.67$	$\kappa_{6\,3}^{\text{apo}} \approx 17.15$	$\kappa_{6\,4}^{\text{apo}} \approx 5.031$	$\kappa_{6\,5}^{\text{apo}} \approx 1.564$

3 Discussion

As Table 2 suggests, $\kappa_{nl}^{\text{apo}} l$ based on the $\sqrt{l(l+1)}$ correction is always greater than 1.0, which supports the hypothesis that parts of the rotated orbit close to the apocenter are in phase with the previous orbit.

According to Table 1, this is not the case for $\kappa_{nl}^{\text{peri}}$ based on the $\sqrt{l(l+1)}$ correction. Instead, values of $\kappa_{nl}^{\text{peri}}$ in Table 1 are in a relatively consistent range between 0.15 and 0.3, which supports the hypothesis that the electron is usually out of phase near the pericenter but approximately in phase on the rest of the orbit.

Thus, the main conclusion to draw from the data in Tables 1 and 2 is that the data appears to be at least somewhat consistent with the presented model for the quantization of orbital angular momentum.

4 Conclusion and Future Work

This work presents a model of electronic orbits in hydrogen atoms that might explain a (coordinate-independent) quantization of orbital angular momentum of Bohr-Sommerfeld orbits with apsidal precession. The parametrized local quantization condition described in Section 2 allowed us to quantitatively express a given quantization (specifically the $\sqrt{l(l+1)}$ correction [Buc08]) in terms of this model. However, this is only a first step, which should be followed by future work on a way to calculate the parameter κ_{nl} of the local quantization condition without a given quantization, for example, by minimizing an integral of (weighted) phase differences on a full orbit.

Since the presented model in its current form is "only" a first step, some readers might criticize any publication of it as premature. *Maybe* they are right. But Sir Jonathan Paul Ive was *certainly* right when he said:

> *... while ideas ultimately can be so powerful, they begin as fragile, barely formed thoughts, so easily missed, so easily compromised, so easily just squished.*
>
> Sir Jonathan Paul Ive (October 19, 2011)

References

[Buc08] Manfred Bucher. Rise and fall of the old quantum theory. arXiv:0802.1366v1 [physics.hist-ph], 2008.

[dB25] Louis de Broglie. Recherches sur la théorie des Quanta. *Annales de Physique*, 10(3):22–128, January 1925. English translation by A. F. Kracklauer.

[Ein17] Albert Einstein. Zum Quantensatz von Sommerfeld und Epstein. *Verhandlungen der Deutschen Physikalischen Gesellschaft*, 19(9–10):82–92, May 1917.

[Kra25a] Martin Kraus. De Broglie's phase waves of electrons in hydrogen-like atoms as parts of steady-state forced vibrations. viXra:2503.0008, 2025.

[Kra25b] Martin Kraus. Spinning electrons on pendulum-paths in hydrogen atoms. viXra:2512.0020, 2025.

[Sch26] Erwin Schrödinger. Quantisierung als Eigenwertproblem (Erste Mitteilung). *Annalen der Physik*, 79:361–376, 1926.

[Som23] Arnold Sommerfeld. *Atomic structure and spectral lines*. E. P. Dutton and Company, New York, 1923. Translated from the 3rd German edition by Henry L. Brose.

[Wik26] Wikipedia contributors. Kepler orbit — Wikipedia, the free encyclopedia, 2026. [Online; accessed 26-January-2026].

A Revisions

- Original version submitted to vixra.org on January 27, 2026.