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Geometric decomposition of the orbital energy into components Ω_A and Ω_R.

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Abstract.

This article **identifies separate energy channels**, with a **focus on the Coriolis force energy channel as their integrator**. The author demonstrates that the total kinetic energy of a launched satellite in the unsteady state results from the **geometric sum of momentum and angular momentum, as two vectorial and orthogonal kinetic energy carriers coupled together in spacetime by a dynamic phase. This phase results from energy transfer through the Coriolis effect energy channel, which mixes the components without introducing additional energy**. The article identifies a scientific interpretation gap and proposes a new, enriched perspective in the field of cosmology.Two main energy components were distinguished:

- Ω_A — **(active)** identified with linear momentum (translational energy opposing the gravitational force) integrated along the ascension radius, ultimately converted into the satellite's potential energy with a mass of m .

- Ω_R — **(reactive)** identified with angular momentum (rotational energy equivalent to the opposing Coriolis force) force projected onto the tangent to the ascent spiral, then integrated along the spiral, finally transformed into rotational orbital energy (encoded into the angular momentum of the mass m) .

Each of them is a power integral associated with the orthogonal motion components (radial and tangential). In standard orbital mechanics, angular momentum energy is implicitly included in the orbital kinetic energy along the ascent spiral, but it is not treated as an independent, orthogonal component in the energy balance in the dynamic

ascent state. In this paper, the rotational energies Ω_R and Ω_A are separated into separate physical quantities, showing for a circular orbit (as in Fig. 1):

$$\Omega_A = \frac{p_r^2}{2m} = \frac{GMm}{r}$$

$$\Omega_R = \frac{L^2}{2I} = \frac{GMm}{2r}$$

where L is the angular momentum of the mass m in orbit and p_r built up over time is the radial momentum. The key conclusion is, that while in an inertial frame of reference the Coriolis force is treated as fictitious, in a non-inertial frame it generates a real (equivalent) energy contribution associated with providing angular momentum to mass m during this process. Module of total lifting energy is:

$$\boxed{\Omega_\Lambda = \sqrt{\Omega_A^2 + \Omega_R^2}}$$

The above notation, which also represents mechanical power in the orbital domain, is an analogous formula commonly used in electrical engineering, which is analogous to the representation of apparent power in AC circuits.

1. Introduction:

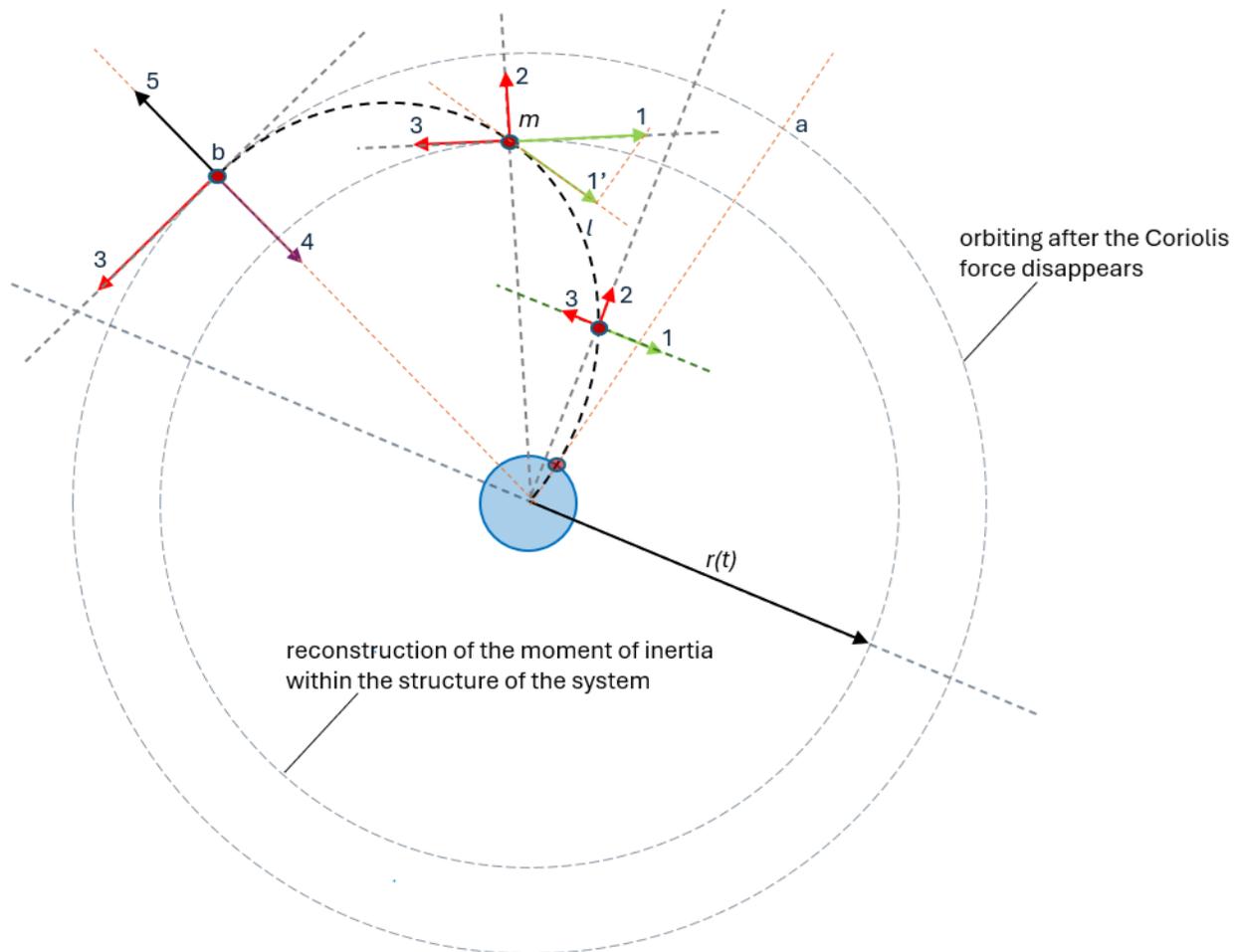Figure 1 below shows the satellite's ascent trajectory into a circular orbit.

Fig.1

Figure description:
1 – Coriolis force, $F_{cor}(t)$
1' – projection of the Coriolis force onto the tangent to the arc l
2 – radial velocity $v_r(t)$
3 – tangential speed $V_\theta(t)$
4 – centripetal force
5 – centrifugal force
a,b – points determining the projection on arc l of spiral

Typical orbital mechanics textbooks present the energy balance as follows:

- Change in potential energy: $\Delta E_p = GMm(\frac{1}{R} - \frac{1}{r})$

- Final kinetic energy: $E_k = \frac{GMm}{2r}$

- Total energy consumption: $E_{total} = \Delta E_p + E_k$

- The rocket's ascent velocity is treated as one vector (thrust) with a variable inclination over time, and the energies are treated as "before" and "after" states.

Although mathematically speaking this approach is correct, it hides the physical fact that a satellite entering orbit absorbs two orthogonal energy components that are phase shifted in space and phase shifted in time:

- kinetic energy of translational motion (upward radial momentum), ultimately converted into potential energy upon entering orbit,

- rotational energy around the planet, encoded in the angular momentum $L = mv_{orb}r$. This rotational energy is different from the kinetic energy of translational motion – it is a separate physical quantity that must be supplied against the Coriolis force during the dynamic process of changing the satellite's moment of inertia.

2.Introduction of components: Ω_A and Ω_R

The author defines two basic energy components of the orbital system:

- **Ω_A — (active)** translational kinetic energy: work opposing gravity when lifting a mass from radius R to r , associated with the radial component of motion:

$$\Omega_A = \int_R^r -F_{grav}\,dr = -\frac{GMm}{r}$$

- **Ω_R — (reactive)** rotational kinetic energy: energy stored in orbital angular momentum, associated with the component of tangential motion against the Coriolis force due to the change in the moment of inertia of the subsystem:

$$\Omega_R = \int_{a(spiral)}^b -F_c'\,dl' \quad = \Omega_R = -\frac{GMm}{2r}$$

For a circular orbit of radius r these quantities are in the ratio:

$$\Omega_R = \frac{1}{2}\Omega_A = \frac{GMm}{2r} = \frac{1}{2}mv_{orb}^2$$

The use of Ω *(omega)* highlights the well-established scientific connection with motion (kinetic energy) and draws an analogy to the classification of energy categories .

The indices A (active) and R (reactive) refer to the nomenclature used in the field of electrical energy (active power/reactive power) where these quantities are added geometrically, which constitutes a similarity.

This sets the stage for broader applications in cosmology, where momentum and angular momentum, as two distinct yet phase-conjugated vector quantities carrying kinetic energies, are fundamental to the large-scale structure of the Universe.

3.The role of the Coriolis force as an indicator of Ω_R

In an inertial rotating frame, the Coriolis force:

$$F_c = -2m(\omega \times v_r)$$

is a fictitious force and formally does not perform work in this frame of reference. However, in a non-inertial (co-rotating) frame, the engines must perform work against this force to place the satellite in the target orbit because the subsystem changes its moment of inertia I relative to the axis of rotation. This work constitutes the actual energy input and is equal to:

$$\Omega_R = -\frac{GMm}{2r}$$

The author cautions the reader not to make the cognitive error of associating the perpendicularity of the Coriolis force to the radial displacement with velocity, v_r as this textbook assumption leads to the mental shortcut that the work is zero. The correct approach is to consider the torsional action of the Coriolis force torque relative to the axis of rotation as a result of the change in angular momentum L:

$$\frac{dL}{dt} = \tau_C \neq 0$$

$$W_c(t) = \int_R^r \tau_C(t) \cdot \omega(t) \cdot t = \int_R^r (-2\omega(t) \times v_r(t)) \cdot r(t) \cdot \omega(t) \cdot t$$

This work as Ω_R after the dynamic process is quantitatively equal to half the value of the active translational energy Ω_A. In an inertial frame, the same energy Ω_R manifests itself as the need to provide tangential velocity v_θ, which creates (gradually builds up) orbital angular momentum. **The Coriolis force therefore acts as a physical indicator of the need to provide reactive energy Ω_R**, even if formally it is an artifact of non-inertial frames of reference.

4. The novelty of this work

The author emphasizes the key role of the Coriolis force as an integrator, the possibility of representing a power triangle and that the process of launching a satellite into orbit is the physical equivalent of achieving energy matching, at the expense of the energy introduced in which:

1. Two energy components: Ω_A (active, opposite to the gravitational force)and Ω_R (reactive, rotational, equivalent work opposite to the Coriolis force) — are coupled in space with a constant phase of 90°, while in time the coupling is variable (see Fig. 2).
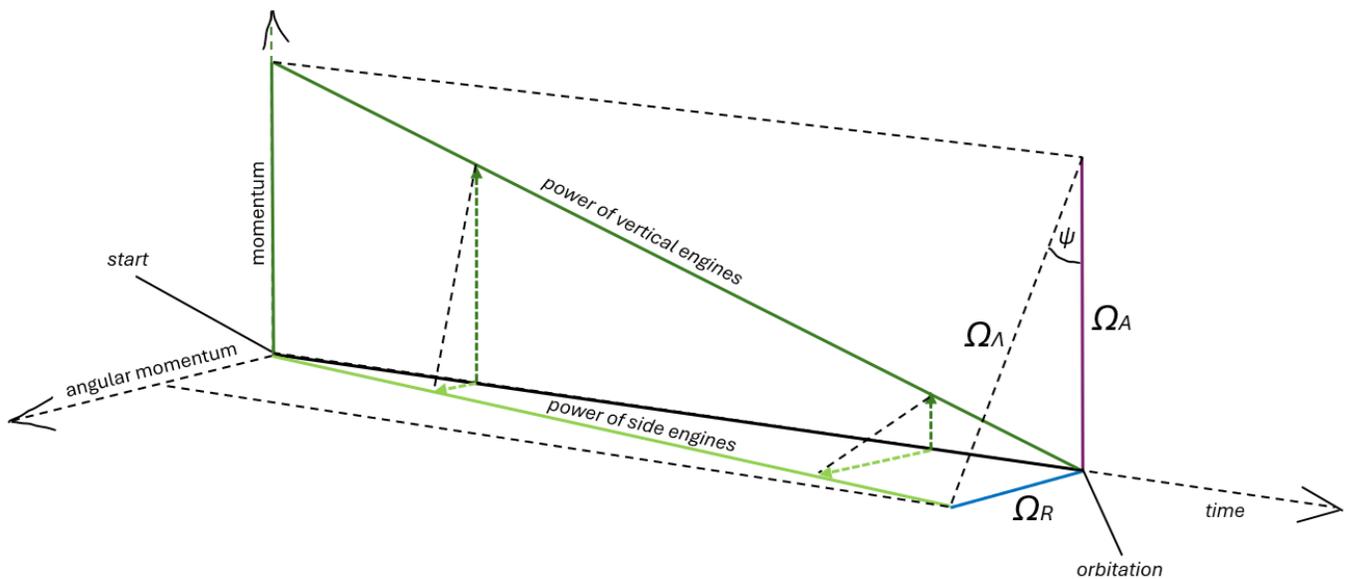

Fig. 2 (illustrative drawing)

2. The total energy is a geometric sum resulting from the vector and complex nature of momentum and angular momentum:

$$\Omega_\Lambda = \sqrt{\Omega_A^2 + \Omega_R^2}$$

Figure 2 aims to illustrate the orthogonality of the two energy fluxes during a gravity maneuver – they do not change the energy balance but illustrate its physical origin.
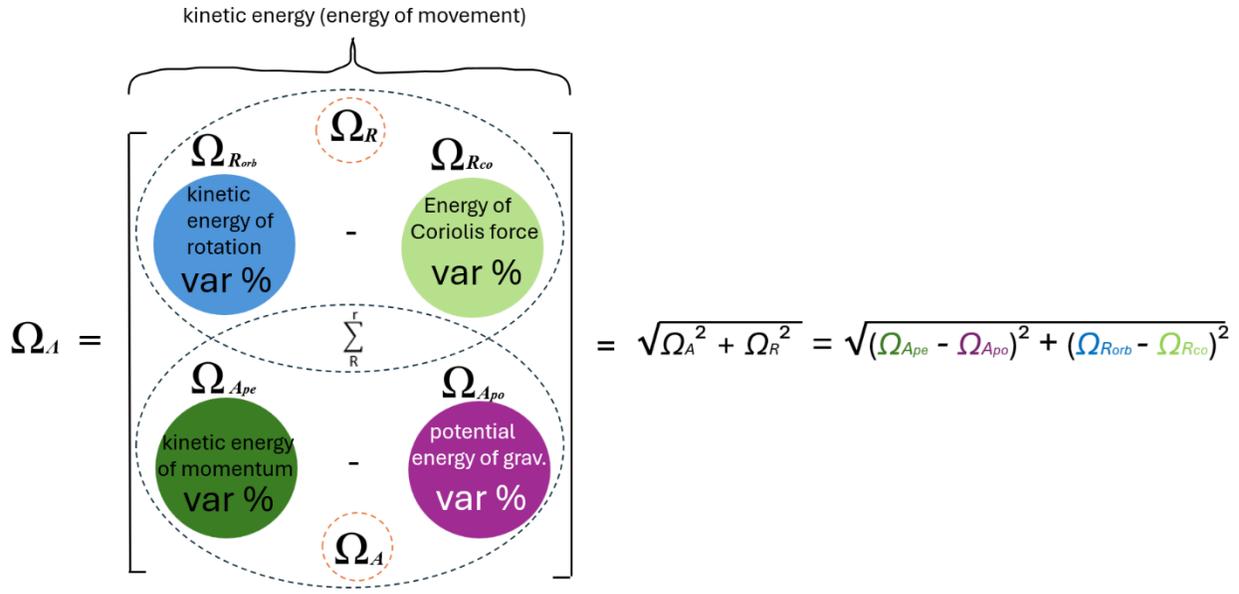
kinetic energy (energy of movement)

$$\Omega_\Lambda = \begin{bmatrix} \Omega_{R_{orb}} & - & \Omega_{R_{co}} \\ & \sum_{R}^{r} & \\ \Omega_{Ape} & - & \Omega_{Apo} \end{bmatrix} = \sqrt{\Omega_A^2 + \Omega_R^2} = \sqrt{(\Omega_{Ape} - \Omega_{Apo})^2 + (\Omega_{Rorb} - \Omega_{Rco})^2}$$

Ω_R

Ω_A

kinetic energy of rotation — var %

Energy of Coriolis force — var %

kinetic energy of momentum — var %

potential energy of grav. — var %

Fig.3

Figure description:

Ω_Λ – total energy of the satellite

Ω_A – activ energy of the satellite (momentum)

Ω_R – reactive energy of the satellite (rotation)

$\Omega_{\Lambda pe}$ – kinetic energy of momentum

$\Omega_{\Lambda ro}$ – kinetic energy of rotation (angular momentum)

$\Omega_{\Lambda co}$ – kinetic energy against torque of the Coriolis force

$\Omega_{M\Lambda po}$ – potencial energy (potencial of gravity)

3. The natural consequence of this orthogonality is the full analogy to the apparent power in an RLC circuit with a power factor cos (ψ) of 0.89 in steady state

4. Fig. 3 above shows the energy dependencies in the dynamic state of launching into orbit and the method of summing the energy depending on the assigned group (algebraic/geometric).

5. Orbital launch is treated here as a dynamic process of changing the moment of inertia of a rotating, closed Earth-satellite system, whose interior is subject to dynamic changes (mass shifts) under the influence of external energy. This shift in perspective allows us to treat this system as rotating around a common center of gravity (a point close to Earth's center of gravity), which in turn explains the role of the Coriolis force as a component whose influence must be included in the energy balance. **This shift in the perception of the system leads to the emphasis on the fact that the subsystem (the satellite) changes its angular momentum L(t) in a dynamic process**, according to the following relationship:

$$I(t) = m\big(r(t)\big)^2$$

$$\omega(t) = \frac{v_\theta(t)}{r(t)}$$

$$L(t) = I(t) \cdot \omega(t) \neq const.$$

$$\frac{dL}{dt} = \tau_C \neq 0$$

$$W_c(t) = \int_R^r F_c(t) \cdot r(t) \cdot \omega(t) \cdot t = \int_R^r \tau_C(t) \cdot \omega(t) \cdot t = \int_R^r \big(-2\omega(t) \times v_r(t)\big) \cdot r(t) \cdot \omega(t) \cdot t$$

This interpretation does not change the mathematics, but **it gives physical meaning to the separation of rotational energy Ω_R as an independent component of the balance associated with the Coriolis force. Although this force is apparent in the inertial frame, for a non-inertial frame it reflects the need to transfer angular momentum to the satellite. The work $Wc\ (t)$ performed against the inertial Coriolis force constitutes the final rotational energy. After integrating this force along the ascent trajectory, we obtain the final component Ω_R of energy in the frame striving to achieve energy matching in orbit. Orbitation can therefore be considered a state of gravitational matching where energy equilibrium occurs, ending the dynamic process of change in momentum and angular momentum of the body reaching the (ψ) orbital trajectory with cos = 0.89**

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5. Calculating integrals along an ascending spiral.

5.1. Vertical work against gravity – energy Ω_A

Considering the optimal spiral trajectory, part of the work input by the engines is converted into vertical momentum kinetic energy and will be:

$$\Omega_A = \int_R^r -F_{grav}\, dr = \int_R^r \left(-\frac{GMm}{r^2}\right) dr$$

5.2 . Work against the Coriolis force — energy Ω_R

In a rotating frame of reference, the Coriolis force is purely tangential. The engine's work to overcome this force:

$$\Omega_{cor}(t) = \Omega_R = W_C(t) = \int\limits_{a(\text{spiral})}^{b} -F_C' \, dl' = \int\limits_{R}^{r} \left(-2\omega(t) \times v_r(t)\right) \cdot r(t) \cdot \omega(t) \cdot t$$

For an optimal gravitational spiral leading to a circular orbit, the following integral gives:

$$W_c(t) = \frac{1}{2} m v_{orb}^2 = \Omega_R = \frac{GMm}{2r}$$

Although $F_c \perp dr$, the moment of force $\tau_c = r \times F_c$ about the Earth's rotation axis gives mathematically the reactive mechanical power that must be supplied in the plane perpendicular to the rotation axis during the satellite's path bending maneuver:

$$P_c(t) = \omega(t) \cdot \tau_C(t)$$

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6. Phase relationship - power (energy) triangles.

In the optimal gravitational spiral, the phase parameterization ψ (t):

- $v_r(\psi) = v_{spiral} \cos(\psi)$
- $v_\theta(\psi) = v_{spiral} \sin(\psi)$

The momentum vectors (radial and tangential) are shifted in phase in space by exactly 90°. The energy portioning Ω_A (radial motion) and Ω_R (tangential motion) evolves in phase with time during the ascent as shown in Fig. 4. The final steady-state state is shown in Fig. 5.
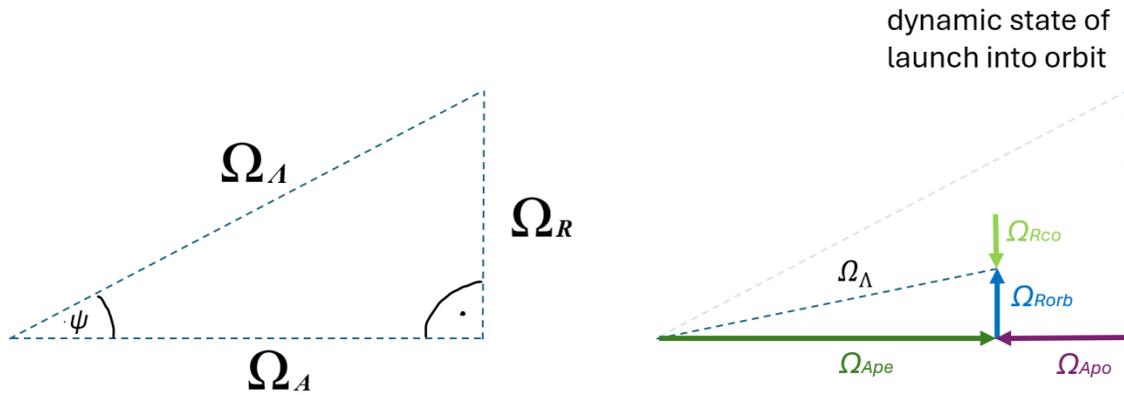
Consequently, the total mechanical power distribution over time is analogous to the way power is summed in RLC electrical systems and can be represented as a triangle of mechanical power supplied to the system with a phase shift:

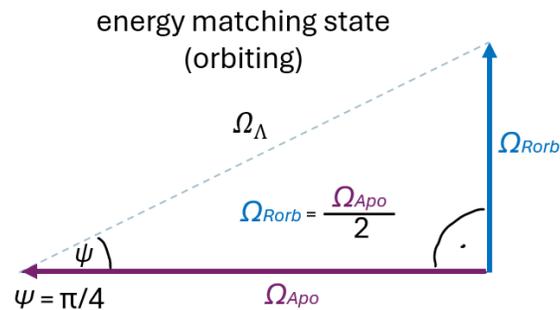$$\cos(\psi) = \frac{\Omega_A}{\Omega_\Lambda}$$

Where :

$$\Omega_\Lambda = \sqrt{\Omega_A^2 + \Omega_R^2}$$

The essence of this physical analogy transferred from the electrical domain to the mechanical (cosmological) domain is shown in Fig. 4 and Fig. 5 below:

$$\Omega_\Lambda = \sqrt{\Omega_A{}^2 + \Omega_R{}^2} = \sqrt{(\Omega_{Ape} - \Omega_{Apo})^2 + (\Omega_{Rorb} - \Omega_{Rco})^2} \neq const.$$

Fig.4

$$\Omega_\Lambda = \sqrt{\Omega_A{}^2 + \Omega_R{}^2} = \sqrt{(-\Omega_{Apo})^2 + (\Omega_{Rorb})^2} = const.$$

Fig.5

7. Theorem: *total orbital lift energy*

In the case of a satellite launch into a circular orbit in an ideal flat configuration, the total energy Ω_Λ consists of two energy components absorbed during the maneuver Ω_A i Ω_R:

$$\Omega_\Lambda = \sqrt{\Omega_A^2 + \Omega_R^2}$$

$$\Omega_\Lambda = \sqrt{\left(\int_R^r -\frac{GMm}{r^2}\,dr\right)^2 + \left(\int_R^r \left(-2\omega(t) \times v_r(t)\right) \cdot r(t) \cdot \omega(t) \cdot t\right)^2}$$

The geometric sum results from the complex nature of momentum and angular momentum, which, being vector carriers of kinetic energy (motion), are coupled in time and space, where the phase coupling in space is constant and equal to $\pi/2$, and the phase coupling in time evolves from the value 0 at the starting point to the value $\pi/4$ at the point of entering the orbit.

8. Difference from the standard virial theorem

virial theorem for $U \propto 1/r$ directly gives $E_k = -E_p/2$, so for a circular orbit:

$$E_{total} = (\Omega_\Lambda) = E_k + E_p$$

virial theorem treats the system as a whole "before" and "after" and gives the relationship between **the average** kinetic and potential energy, but:

- o Neglects the role of the Coriolis force as an integrator between the radial linear momentum and the tangential angular momentum of the ascent

trajectory when the moment of inertia of the system under consideration changes

- o It does not extract the components Ω_A and Ω_R explicitly
- o It does not identify rotational energy as an independent physical quantity that is summed geometrically.
- o There are no considerations related to the change in the moment of inertia I and the change in the angular momentum L of the closed Earth-satellite system and the launch dynamics.

9.What is different about the new approach?

The new approach results from the orthogonality of two vector quantities (momentum and angular momentum) as separate complex entities carrying energy fluxes in time (Ω_A and Ω_R), which evolve in time and influence each other in space-time using **the "clutch" in the form of the Coriolis force.**

This explanation is:

- Physically intuitive (phase geometry, Fig. 2)
- Directly related to the actual trajectory of the rocket (Fig. 1)
- It highlights the phase dependencies in time and space and gives a new interpretation of the dynamic matching factor of a rotating system as $\cos(\psi)$,(Fig.4,Fig.5)
- It can provide a new conceptual basis for cosmological applications, in which the decomposition of energy into components Ω_A and Ω_R and **the emphasis on the role of Coriolis forces** can better describe the fundamental processes of evolution of the large-scale structures of the Universe.

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10. Cosmological Implications and Further Developments

Introduction of new coexisting quantities Ω_A and Ω_R as independent components of the energy balance may open up new interpretation perspectives in cosmology:

- **Translation and rotation in large-scale structures:** if a satellite's orbit clearly requires energy Ω_R associated with angular momentum, analogous processes may occur in galactic and cosmological systems where rotation plays a fundamental role. Symbol Ω_A (Greek lambda) indicates a connection with the currently described cosmological effects of motion and variability of space.
- **Phase shift in the evolution of structures:** the orthogonality **and coupling in spacetime of the vector components (momentum and angular**

momentum) carrying the components Ω_A and Ω_R suggests that radial and rotational processes in astrophysical systems can be naturally phase-separated, and their current degree of energetic matching cos(ψ) can be calculated. It also means that identical energetic components belonging to "active" and "reactive" can be distinguished, which can be summed, but this time algebraically within a given group .

- **Thermodynamic Analogy:** Like apparent power in AC circuits, the distribution of Ω_A and Ω_R can be useful in describing the thermodynamics of cosmological processes. It should be emphasized that the analogy to an AC circuit does not imply a physical identity, but rather provides an auxiliary geometric scheme that organizes the distribution of power (energy) over time. The orthogonality of the energy inputs Ω_A and Ω_R during the satellite's ascent results from the dynamical structure of the gravitational spin maneuver, not from transferring electrical engineering terms by analogy.

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11. Final conclusions

The above detailed development of these ideas is beyond the scope of this work and will be the subject of further publications, especially those concerning the role of the Coriolis force . The present study aims to **establish a conceptual framework** and demonstrate that Ω_A and Ω_R are independent and measurable components of the energy balance, which are not analyzed separately in standard orbital mechanics, thus neglecting the Coriolis force as a term integrating both quantities. The Coriolis effect represents the energy flux that travels in a dynamic state from an "active" channel to a "reactive" channel or vice versa, depending on the case under consideration (orbital launch/deorbital deorbit). The total energy of orbital entry:

$$\Omega_\Lambda = \sqrt{\Omega_A^2 + \Omega_R^2}$$

naturally corresponds to energetically balanced (gravitationally tuned) systems and provides a bridge between classical orbital mechanics and cosmology based on spin and angular momentum.

The complex nature of momentum and angular momentum can also be represented in the form of complex numbers - which will be the subject of subsequent publications:

$$Z = p + iL$$

$$Z_{\Omega_\Lambda} = \Omega_A + i\Omega_R$$

$$\left| Z_{\Omega_\Lambda} \right| = \sqrt{\Omega_A^2 + \Omega_R^2} = \Omega_\Lambda$$

$$\Omega_\Lambda(t) = |\Omega_\Lambda| \cdot e^{i\psi}$$

12. References

[1] Bate , R., Mueller, D., and White, J. (1971). Fundamentals of Astrodynamics . Dover.

[2] Vallado , D. (2013). Fundamentals of astrodynamics and its applications (4th ed.).

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13. Notation

- Ω_A : active energy (momentum, radial)
- Ω_R : reactive energy (angular momentum, tangent)
- Ω_A : total energy (kinetic motion)
- r : radius orbits
- R : Radius Earth
- m : mass of the satellite
- M : mass of the Earth
- in_r : radial velocity (for the Coriolis force)
- G : constant gravitational
- E_p : potential energy (conventional notation)
- E_k : kinetic energy (conventional notation)
- E_{total} : total energy (conventional notation)
- v_θ : speed tangent
- F_c : force Coriolis
- F_{grav} : force gravitational
- v_{orb} : speed orbital
- l : arc length (orbit segment)
- τ_C : Coriolis moment of force
- *Toilets* : energy in the Coriolis "energy channel"
- p_r – radial momentum
- L – angular momentum
- z – complex momentum