Physical Characteristics of Geodesics For A Yilmaz Point Mass Metric.

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Abstract:
Since Schwarzschild's first solution of the Einstein field equations, the simple model of a single, point mass gravitating source has encompassed an impressive array of phenomena that have provided confirmation for Einstein's theory of General Relativity. These include gravitational time dilation and spectral redshifts, gravitational refraction of light, perihelion precession of planetary orbits, innermost stable orbits of accretion disks and, recently, the shadows of the photon spheres of extremely compact masses. These phenomena are associated with the geodesic motions of material particles or photons in the immediate vicinity of large masses that can be regarded as point sources of gravity.

The limited purposes of this article are to present the underlying physics of the exponential metric of Yilmaz and to demonstrate that it correctly encompasses the observed phenomena. As an isotropic metric, It may be the only one also in accord with the observed isotropy of inertia.

1 Introduction

Although the first solution of the Einstein field equations was provided by Schwarzschild, the mathematical expression known as the "Schwarzschild metric" is not Schwarzschild's result. It arose from the work of Droste and Hilbert (Hilbert 1917). Schwarzschild's solution had no event horizon and no black holes - see https://arxiv.org/pdf/physics/9905030. Nevertheless, the use of the "Schwarzschild metric" with its event horizon continues without any observational justification. Schwarzschild's original solution metric is regular all the way in to the location of the central point particle. As discussed below, this could be a boundary condition requirement for the metric of a point mass source.

The Yilmaz exponential metric for a point mass has a Newtonian potential as an integral embedded part. It is also regular all the way in to the

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location of the central point particle. The geodesic motions of test particles in this metric satisfy all of the requirements that are normally considered to be confirmation of the theory of General Relativity. Gravitational time dilation and the existence of a photon sphere are correctly confirmed in this metric. Models of compact masses based on this metric permit black hole candidate objects to possess intrinsic magnetic moments. As previously shown, e.g. (Robertson & Leiter 2006), the interactions of magnetic fields and accretion disks provide for a common mechanism for the spectral state switches of neutron stars and galactic black hole candidates in low mass x-ray binary systems and the radio loud/quiet transitions of active galactic nuclei.

2 Geodesic motions of test particles

A spacetime metric is a relationship that tells us how much space to add to an amount of time to make a "spacetime". In special relativity the metric is the invariant spacetime interval $ds^2 = c^2dt^2 - dl^2$, where c is the speed of light in free space, t is time and $dl = \sqrt{dx^2 + dy^2 + dz^2}$ an interval of distance traversed by a particle or photon in time interval dt. When sufficient mass or energy is present in the vicinity, the intervals of space and time are altered by gravitation. Our measures of space and time are distorted. The metric of space and time and time external to a point mass, M, with no angular momentum is a diagonal matrix in the coordinates. The effect of gravity is encapsulated in the metric coefficients, g_{ii}, in the invariant interval

$$ds^2 = g_{00}c^2dt^2 + g_{11}(dx^1)^2 + g_{22}(dx^2)^2 + g_{33}(dx^3)^2 \tag{1}$$

Here x^i are coordinates such that $x^0 = ct$, is the time coordinate and x^1, x^2, x^3 are spatial coordinates. In the static metrics considered here, the metric coefficients, g_{ii}, are functions of coordinates, x^i, excepting time, such that $g_{00} \geq 0, \;\; g_{11} \leq 0, \;\; g_{22} \leq 0, \;\; g_{33} \leq 0.$ [2]

Hilbert's choice of conventional spherical coordinates centered on M, along with $g_{22} = g_{33} = -1$ leads to the misnamed Schwarzschild metric with metric coefficients

$$g_{00} = -1/g_{11} = 1 - 2R_g/r \tag{2}$$

[2]Although Eq. 1 could be written more compactly and more generally as $ds^2 = g_{ij}dx^idx^j$, the intent here is to deliberately restrict the discussion to the most simple circumstances and observations of geodesics in ordinary space and time.

where $R_g = \kappa M/c^2$ is known as the "gravitational radius" and κ is the Newtonian gravitational force constant.

The vanishing of g_{00} for the "**Schwarzschild radius",** $r = \mathbf{2R_g}$, inappropriately attributes the occurrence of an event horizon to Schwarzschild. Nevertheless $r = 2R_g$ marks the occurrence of the event horizon; a hypothetical place from which not even light can escape and time stands still with $g_{00} = 0$.

Since many astronomical observations have been interpreted in terms of the geodesic motions of test particles in a Schwarzschild metric, we continue with the form of Eq. 1, but with metric coefficients that will differ from those of the Schwarzschild metric. Additionally, we note that in a frame of reference coincident with and moving with a test particle, $ds^2 = c^2 d\tau^2$, where τ is the proper time kept by a clock moving with the test particle (Chow 1994). Since ds^2 is an invariant, we can set Eq. 1 equal to $c^2 d\tau^2$ for the description of geodesic motion of a test particle moving in the vicinity of M.

Since the mass of a test particle at rest depends upon its position in a gravitational field, we define m_0 to be the mass of a test particle at rest, far away from any gravitational sources at a location where the gravitational potential would be zero. This use of subscript zero to denote rest mass is no longer common, but in the circumstances to follow this is a convenient way to provide a necessary distinction between the moving inertial mass of a test particle and its rest mass in a gravitational field.

Multiplying $ds^2 = g_{00}c^2dt^2 + g_{11}(dx^1)^2 + g_{22}(dx^2)^2 + g_{33}(dx^3)^2 = c^2 d\tau^2$ by m_0^2 and dividing by $d\tau^2$, converts Eq. 1 to a useful energy-momentum equation for test particles in geodesic motion.

$g_{00}(m_0 c dt/d\tau)^2 + \Sigma_{i=1}^3 g_{ii}(m_0 dx^i/d\tau)^2 = m_0^2 c^2$ or, better yet

$$\Sigma_{i=0}^3 \; g_{ii}(p^i)^2 = m_0^2 c^2 = \Sigma_{i=0}^3 \; g^{ii}(p_i)^2 \qquad (3)$$

where $g^{ii} = 1/g_{ii}$, contravariant energy-momentum components are $p^i = m_0 dx^i/d\tau$ and covariant components are $p_k = g_{kj}p^j$. In a static metric such that g_{ij} does not vary with time, $p_0 = E/c$ where $E = mc^2$ is the conserved energy of a test particle of mass m in geodesic motion. Clothing Eq. 1 in the language of momentum of Eq. 3 does not change the implications of the geodesic equations. They still identify the constants of geodesic motions and rename them as energy and angular momentum when Eq. 3 is independent of time or angular coordinates.

2.1 The form of g_{00}

With no more information than this we can deduce a major feature of the metric. For a test particle at rest on a geodesic path, as might occur at the apex of flight of a particle thrown straight upward against a gravitational field, the right member of Eq. 3 and $p_0 = E/c$ would reveal that

$$g^{00}(E/c)^2 = g^{00}(mc)^2 = m_0^2 c^2 \rightarrow m = m_0\sqrt{g_{00}} \tag{4}$$

If the throw be repeated after giving the particle additional energy dE, it would reach a more distant apex. This same position could be achieved by holding the particle at the first apex by a non-geodesic force in opposition to the attraction of M, and then displacing it quasi-statically by distance $d\vec{r}$. The change of particle energy associated with the displacement would be equal to the increment of work done against gravity. By the definition of a potential, this would also be the change of potential energy in the change from first apex to its new apex position. Thus $dE = md\tilde{U}$, where $\tilde{U}$ is the conventional potential function associated with the source, M, that interacts with m. If we render $\tilde{U}$ dimensionless by dividing by c^2 such that $U = \tilde{U}/c^2$, then $dE = mc^2 dU = EdU$.

While the last terms of Eq. 4 show how the rest mass depends on position via the metric element g_{00}, it is of more interest to differentiate it and show how g_{00}, which is a static feature of the spacetime of the source, M, is related to U.

$$dE = m_0 c^2 d(\sqrt{g_{00}}) = mc^2 dU = m_0 c^2\sqrt{g_{00}}dU \tag{5}$$

Subject to the condition that $g_{00} = 1$ where $U = 0$, this integrates to

$$g_{00} = e^{2U} \tag{6}$$

This is a necessary result that follows from just a metric description of geodesic motion, the definition of a potential, and the most famous relationship of special relativity, $E = mc^2$. Although Eq. 6 is not a part of any generally accepted solution of the Einstein field equations, it has also been shown to be an EXACT requirement of special relativity and the principle of equivalence (Alley 1994, p 131). Note that it is not limited to weak fields, which occur for $|U| << 1$, nor is it necessary that U arise from only a single gravitating source. Whether it is of any importance or utility remains to be seen. At the moment, it merely ties the geodesic motions of test particles to the definition of a potential via the metric coefficient g_{00}. If taken as a definition of the potential, we can see via Eq. 6 that the potential associated with a metric would be $U = ln(g_{00})/2$.[3] [4]

Although it is possible that the potential might be something other than Newtonian, the Newtonian potential

$$U = U_N = -(\kappa M/(c^2 r) = -R_g/r \qquad (7)$$

will be used except where otherwise noted and

$$g_{00} = e^{-2R_g/r} \approx 1 - 2R_g/r + 2(R_g/r)^2 \ . \ . \ . \ . \qquad (8)$$

Eq. 6 is the way the Newtonian potential becomes embedded in the Yilmaz metric. We know that in the weak field limit, Newtonian gravity is applicable. The question remaining is whether it would fail somewhere near a sufficiently compact mass. In this regard, $U_N \approx 2 \times 10^{-6}$ at the surface of the sun and is much smaller elsewhere in the solar system. The solar system tests that have been used to establish the validity of general relativity have been conducted with circumstances for which second and higher powers of U are negligible. Perhaps they are not always negligible in strong fields. In the remainder of this article we will examine some interesting results for some very strong fields.

Before completing the spatial parts of the metric, an important feature of the the temporal part of the metric deserves comment. $\sqrt{g_{00}}dt$ is the rate of passage of proper time. We can think of g_{00} as the control for the tick rate of a standard clock at a point in spacetime. The nearer you get to the location of the central mass, M, the slower the tick rate. Eventually, as $r \to 0$, with $U \to -\infty$, we expect the clock rate to slow to near zero. Thus it seems that $\lim g_{00}(r \to 0) = 0$ should be taken as a boundary condition on the metric.[5]

2.2 A metric based on physical principles

H*ü*seyin Yilmaz envisioned a metric theory in which gravitational potentials are embedded in the metric tensor in a way that might allow them to be

[3]For the "Schwarzschild" metric $U = (1/2)ln(1 - 2R_g r) = U_N - U_N^2 + 4U_N^3/3 + \ldots$, where U_N is the Newtonian potential. U is the same to this order in the original solution of Schwarzschild for which $g_{00} = 1 - 2R_g/(r^3 + (2R_g)^3)^{1/3}$, but it differs in higher order terms that are more important as $r \to 0$.

[4]A problem with the "Schwarzschild" metric is that the gravitational force on a test particle would diverge at the event horizon $F = -mc^2 \nabla U = \frac{-mc^2}{\sqrt{|g_{11}|}} \frac{\partial U_{Sch}}{\partial r} = -\frac{\kappa M m/r^2}{\sqrt{(1-2R_g/r))}}$.

[5]The misnamed "Schwarzschild" metric fails this necessary condition, but the metric of Schwarzschild's original solution passes with $g_{00} = 1 - 2R_g/(r^3 + (2R_g)^3)^{1/3}$.

reconciled with a field theory. Eq. 6 achieves part of this intent. For the remaining parts of a metric that is capable of encompassing astrophysics in strong gravitational fields, we adopt the static isotropic metric form

$$ds^2 = e^{2U} c^2 dt^2 - e^{\lambda}(dx^2 + dy^2 + dz^2) \tag{9}$$

where λ is a function of spatial coordinates.[6]

In the matter free space exterior to mass or energy concentrations, Eq. 9, with all spatial dimensions affected by gravity in the same way, would seem to be generally necessary for consistency with the Hughes-Drever experiments (Hughes, Robinson & Beltran-Lopez 1960, Drever 1961) that have revealed the isotropy of inertia. This point has been emphasized by Yilmaz (1977). But it is not clear that this is a necessity inside continuous distributions of mass or energy. For the remainder of this article, we will continue with forms similar to Eq. 9, but restrict our consideration to a theory of particles and fields rather than a hydrodynamic continuum. This is quite consistent with much of astrophysics in which the gravitating matter is granular. The same situation occurs in electromagnetism, where the particulate nature of the charged matter in plasmas is not a major problem.

To constrain the function λ we can require that the metric spacetime permit the passage of realistic physical waves of light or gravity. For light, $ds^2 = 0$ and the coordinate speed for photons in Eq. 9 is $\sqrt{(dx^2 + dy^2 + dz^2)}/dt = ce^{U-\lambda/2}$, which clearly varies from place to place in space. Assuming that photons can also be represented as wave packet superpositions of generic plane waves of the form $\psi = e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})}$ with angular frequency ω and wave vector $\mathbf{k}$, we might expect ψ to obey a generalized d'Alembertian wave equation

$$\Box\psi = \frac{1}{\sqrt{-g}}\partial_j(\sqrt{-g}g^{jp}\partial_p\psi) = 0 \tag{10}$$

where (g) is the determinant of the metric tensor. Using Eq. 9 for the metric this produces

$$e^{-2U}\omega^2/c^2 - e^{-\lambda}k^2 + ie^{-\lambda}\mathbf{k}\cdot\nabla(U + \lambda/2) = 0 \tag{11}$$

According to this equation, the phase speed of the wave, $(\omega/k) = ce^{(U-\lambda/2)}\sqrt{1 + (i/k^2)\mathbf{k}\cdot\nabla(U + \lambda/2)}$ will depend not only on position in space, but also on the direction of travel of the wave via the $\mathbf{k}$ in the

[6] For the isotropic form of Eq. 9, an exact solution of the Einstein field equations yields $g_{00} = (1 - R_g/(2r))^2/(1 + R_g/(2r))^2 \approx 1 - 2R_g/r + 2(R_g/r)^2 + \ldots$, which agrees with Eq. 6 through second order and $\lambda = (1 + R_g/2r)^4$. Its event horizon is at $r = R_g/2$, where the gravitational force on a test particle is divergent.

right member. Worse, yet, the group velocity $d\omega/dk$ and phase speed ω/k will differ unless $\nabla(U+\lambda/2)=0$. We impose this condition to ensure that photon wave packets can move in this spacetime without particle and wave separating or dispersing.[7]. Thus we require $\lambda=-2U$ as a condition that satisfies boundary conditions, leaves the speed of light at a point independent of its direction of travel and unifies phase and group speeds of photons, considered as wave packets. An appropriately physically constrained metric for matter free space now takes the form [8]

$$ds^2 = e^{2U}c^2dt^2 - e^{-2U}(dx^2+dy^2+dz^2) \tag{12}$$

This exponential form of the metric was first proposed and then elaborated by H*ü*seyin Yilmaz (Yilmaz 1958, 1971, 1992) but it was not derived with this emphasis on elementary physical requirements. Additional theoretical work on the Yilmaz theory is needed to incorporate rotating frames and radiation fields and extend it for applications to cosmology, but Eq. 12 is adequate for the present purposes.

The exponential metric of Eq. 12 can be changed to spherical coordinates by the substitutions, $x=rsin\theta cos\phi,\ y=rsin\theta sin\phi,\ z=rcos\theta$ yielding

$$ds^2 = e^{2U}c^2dt^2 - e^{-2U}(dr^2+r^2d\theta^2+r^2sin^2\theta d\phi^2) \tag{13}$$

Which can be rewritten as before, in Eq. 3, as an energy-momentum equation for geodesic test particles with momenta $p^0 = m_0cdt/d\tau,\quad p^r = m_0dr/d\tau,\quad p^\theta = m_0d\theta/d\tau,\quad p^\phi = m_0d\phi/d\tau,\quad p^0 = g^{00}p_0$ and $p_0 = g_{00}p^0 = E/c$ again. The metric coefficients for Eq. 3 are $g_{00}=e^{2U},\quad g_{11} = -e^{-2U},\quad g_{22} = -e^{-2U}r^2,\quad g_{33} = -e^{-2U}r^2sin^2\theta$. For the special case of a geodesic trajectory restricted to the equatorial plane $\theta=\pi/2,\ d\theta=0$, the right member of Eq. 3 with this metric becomes

$$e^{-2U}E^2/c^2 - e^{2U}(p_r^2+p_\phi^2/r^2) = m_0^2c^2 \tag{14}$$

In this static metric, E is the conserved energy of a test particle in geodesic motion and p_ϕ is its conserved angular momentum. For photons, it must be remembered that $m_0=0$ and geodesics are null.

[7] This last term arises from parts of the d'Alembertian, $[\partial_i(\sqrt{-g}\ g^{ij})]\partial_j\psi$. The choice $\partial_i(\sqrt{-g}\ g^{ij})=0$ is known as the harmonic coordinate condition (Weinberg 1972, p163). By dropping this constraint and allowing $\lambda\neq-2U$, Eq. 9 can be extended to describe a fluid continuum.

[8] Restricted to the matter free space external to M excepting test particles of negligible mass.

With U as the Newtonian potential for solar mass M, solutions of Eq. 13 or Eq. 14 correctly account for all of the solar system tests of relativistic gravity theory that have been taken as confirmation of General Relativity. These include the perihelion shift of the orbit of planet Mercury, the Shapiro time delay of radar echoes from Venus that pass the limb of the sun ($m_0 = 0$), and the bending of light rays from stars as they pass near the sun on the way to earth.

2.3 Closed orbits and relativistic accretion disks:

Beyond the solar system, several important applications of relativistic gravity can be understood in terms of orbits of particles attracted by a single mass source. Using the Newtonian potential for central mass M for these, $U = -R_g/r$, Eq. 14 can be rewritten as (Robertson 1999)

$$(e^{2U} p_r/(m_0 c))^2 = (\frac{dr}{cd\tau})^2 = (E/(m_0 c^2))^2 - (e^{2U} + a^2 U^2 e^{4U}) \qquad (15)$$

where $a = p_\phi c/(\kappa M m_0)$ is a dimensionless constant angular momentum parameter. The last two terms are an effective potential for radial motion, $V(r) = e^{2U} + a^2 U^2 e^{4U}$. Turning points of the orbit occur for $p_r = 0$. Circular orbits occur where the turning points coincide, with the first derivative of the effective potential vanishing. Finally, the circular orbit is at least marginally stable if the second derivative of the effective potential is not negative. Turning points occur for

$$a^2 = -\frac{e^{-2U}}{U + 2U^2} \qquad (16)$$

and for circular orbits

$$E = m_0 c^2 e^U \sqrt{\frac{1+U}{1+2U}} \qquad (17)$$

Where both first and second derivatives of the effective potential vanish, there is an innermost marginally stable orbit. This occurs for $U = -R_g/r = (-3 \pm \sqrt{5})/4$. Using the positive sign and inverting gives $r = 5.24GM/c^2$ and $a^2 = 12.413$. Although Eq. 17 would yield infinite energy and angular momentum for $U \to -1/2$, this does not occur for a circular orbit of a particle with $m_0 > 0$.

The occurrence of a smallest stable orbit with a radius of $5.24R_g$ is an important result for the physics of astrophysical accretion disks. Accreting particles in these disks begin with energy $E \approx m_0 c^2$ far away from the central mass, M, in the outer disk. Viscous transport of angular momentum along with heating and radiative energy losses allow them to slowly swirl into the innermost marginally stable circular orbit. At this location, according to Eq. 17, with $U = -(3 - \sqrt{5})/4 = -.190983$, the particle energy has been reduced by 5.5 %. For a given amount of accreting mass, this provides a great deal more luminosity for an accretion disk than would be provided by the kinds of nuclear reactions that provide the luminosities of stars. For comparison, the Schwarzschild metric has an innermost orbit radius of $6R_g$ and its energy dissipation is negligibly different at 5.7% of the accreted rest mass energy.

2.4 Shadow Images of the photon sphere:

Eq. 14 can be applied to photons by setting $m_0 = 0$ with some cautions. It needs to be remembered that their geodesics are null and $d\tau = 0$. Although photons have energy and momentum, p^i must be related to dx^i via an affine parameter that is not a time increment. $E/c = p_0$ is still a conserved quantity in photon geodesic motion, but E is the usual photon energy only in places for which $g_{00} = 1$. This will be discussed further in the consideration of gravitational redshift. Eq. 14, the spherical coordinate form of Eq. 3, can be applied for photon orbits and, using $r = -R_g/U$, can be rearranged as

$$(rp_r/p_\phi)^2 = (\frac{R_g E}{p_\phi c})^2 \frac{1}{(Ue^{2U})^2} - 1 \tag{18}$$

A circular geodesic with $p_r = 0$ occurs if $(\frac{R_g E}{p_\phi c})^2 \frac{1}{(Ue^{2U})^2} = 1$. For $U < 0$, this has a minimum radius for $U = -1/2$, for $r = 2R_g$. This is the location of an unstable circular photon orbit and $r = 2R_g$ is the radius of this "photon sphere". For $0 > U > -1/2$ there are turning points outside the photon sphere for geodesics that are open to large radii. For $U < -1/2$ there are turning points for geodesics trapped inside the photon sphere. For $R_g E/(cp_\phi) > 1/(2e)$ photons can travel in either direction without turning points.

For an object that is small enough to reside within its photon sphere and illuminated from its surroundings, photons that pass inside the photon sphere would eventually reach the central object and be captured. To get past the central object, their paths must come no closer than the photon sphere radius from the center. After these photon sphere grazing photons have spiraled out to a large distance, they can be observed as a series of parallel rays that will appear to have come from an object of radius b such that $p_\phi = bp$. At this distance, $U = 0$ and the conserved E can be recognized as the photon energy $E = pc$. From Eq. 18, with $U = -1/2$ for photon geodesics that graze the photon sphere we then find, for a Newtonian potential, that

$$p_\phi c/E = pb/p = b = e^1 2R_g \tag{19}$$

Thus the apparent radius of the image of the photon sphere is $b = e^1\,(2R_g) \approx 5.436R_g$ for the metric of Eq. 13.

A similar calculation for the Schwarzschild metric yields a photon sphere with $p_\phi c/E = \sqrt{27}R_g \approx 5.196R_g$ and a photon sphere radius of $3R_g$. This apparent radius for the shadow of the photon sphere given by Eq. 19 is larger than that of the Schwarzschild metric by only 4.6%. The observed values of R_g for the black hole candidates are not known with sufficient accuracy to allow measurements of these shadows to distinguish between the metric of Eq. 13 and the Schwarzschild metric. Lastly, it should be noted that the practice of calling these the shadows of event horizons is grossly misleading. They are nothing more than the shadows of a photon sphere. Obtaining them has been a great achievement and photon spheres are exotic enough in their own right for the recognition of it.

The exponential metric with a Newtonian potential is arguably correct in the weak-field limit, where second and higher order terms in U are negligible, but we have seen that it can be retained down to an innermost marginally stable circular orbit for material particles, where it is capable of correctly accounting for the luminosities of accretion disks. Now we see that it even extends to a photon sphere with an image size appropriate for comparison with the images that have been obtained by the Event Horizon Telescope collaboration for M87 (Akiyama 2019). If the concept of a Newtonian potential is to fail at some location, it would seem that it would need to be somewhere inside the photon sphere. $|U| = 1/2$ is large enough to no longer qualify as a "weak-field" case.

2.5 Inside the photon sphere and $r \leq 2R_g$:

In both Schwarzschild and exponential-Newtonian metrics (hereafter Yilmaz metric), particles that pass inside the photon sphere can become trapped. As shown by Eq. 16, there are no particle trajectory turning points for test particles with real mass and real angular momentum for $U < -1/2$ inside the photon sphere. For suitably low angular momentum and sufficient energy, outbound particles can escape from within the photon sphere, but these are constrained to have smaller angular momentum with increasing depth below the photon sphere. For the Schwarzschild metric neither massive particles nor photons can escape from an event horizon at $2R_g$, but for the Yilmaz metric there is no event horizon at $2R_g$. In particular, outbound photons can always escape from within $2R_g$ for the Yilmaz metric for sufficiently small angular momentum.

In the isotropic Yilmaz metric, the left side of Eq. 18, rp_r/p_ϕ, is the cotangent of the angle of a photon geodesic relative to a straight radial line outward from the center at the photon's location. Denoting this angle as α, it is apparent from Eq. 18 that $1 + cot^2\alpha = (sin\ \alpha)^{-2}$, from which we find that the photons are on paths such that

$$sin^2\alpha = (p_\phi c/(R_g E))^2 (Ue^{2U})^2 \tag{20}$$

At the photon sphere, with $U = -1/2$, all outbound photons with $\alpha < \pi/2$ can escape. For those with $\alpha \to \pi/2$, Eq. 20 shows that $p_\phi c/(R_g E) = 2e$. For a surface inside the photon sphere and emitting photons isotropically there is a smaller escape cone. Only those photons with suitably small angular momentum as they leave the emitting surface can escape. Those that spiral out and reach the photon sphere with $\alpha < \pi/2$ can escape. In the limit as $\alpha \to \pi/2$ and $U = -1/2$, the fraction, f, that can escape is given by

$$f = [2Ue^{(1+2U)}]^2 \tag{21}$$

For an emitting surface at the photon sphere, with $U = -1/2$, all outbound photons escape, some even tangentially, with $f = 1$, but the escape fraction would be less than 1% for $r \sim 0.4R_g$ and ten times less at $r = 0.2R_g$. This emitting surface would be well inside $2R_g$. Eq. 21 is of interest when considering the properties of objects that might result from matter collapsing into extremely compact states. If there is no event horizon to swallow the mass, there is no reason to suppose that photons emitted by collapsing matter cannot be seen from some photon sources in locations where $U << -1/2$.

2.6 Gravitational redshifts

As mentioned previously, photons travel on null geodesics, have no proper time, and their momentum and energy are not always easily related to coordinates, velocities and other kinematic quantities. This inconvenience can be partially compensated by considering that they have useful qualities of "proper energy" and "proper momentum". Using $m_0 = 0$ and the Yilmaz metric with rectangular coordinates, the right member of Eq. 3 is

$$e^{-2U}E^2/c^2 - e^{2U}(p_x^2 + p_y^2 + p_z^2) = 0 \tag{22}$$

If we define proper momentum as $p' = e^U \sqrt{(p_x^2 + p_y^2 + p_z^2)}$ and proper energy $E' = e^{-U}E$, then Eq. 22 becomes $E' = cp'$, which holds anywhere on photon geodesics. We can also use $E' = h\nu$ and $p' = h/\lambda$ where h is Planck's constant and ν and λ, the frequency and wavelength of the wave associated with a photon at its location. Further, since E is conserved for geodesics in a static metric, we find that $\nu = e^{-U(x^i)}E/h$ gives the photon frequencies at locations on the geodesic. Then if frequencies ν_1 and ν_2 correspond to locations with coordinates (x_1^i) and (x_2^i), on a photon geodesic, they are related by

$$\nu_2 = \nu_1 e^{-(U_2 - U_1)} \tag{23}$$

from which it can be seen that if $U_2 > U_1$), the frequency at position 2 will be lower than when the photon is at position 1. The wavelength at position 2 is correspondingly increased. The conventional redshift, z of photons at position 2 relative to position 1 is defined to be

$$z = (\lambda_2 - \lambda_1)/\lambda_1 = e^{(U_2 - U_1)} - 1 \tag{24}$$

Eq. 23 was stated by Einstein in a 1907 paper (translation by Schwartz, 1977). Although Einstein had first arrived at a first order approximation for gravitational redshift, he stated that "**in all strictness**" this first order result must be replaced by the exponential form of Eq. 23. In 1907 Einstein clearly maintained that the metric coefficients must be strictly exponential functions in order to conform to the requirements of special relativity and the principle of equivalence (Alley 1994, p 131), but for reasons unknown, his final development of General Relativity satisfied the requirement only to terms of first order.

So far we have seen that a Newtonian potential can be retained in the exponential metric from the weak field limit all the way to the interior of the photon sphere while remaining in accord with observations. There is only one consequential difference between this Yilmaz metric and the Schwarzschild metric. The latter has an event horizon boundary with infinite gravitational force and redshift while the former does not. While neither massive particles nor photons can escape from within an event horizon, radially directed photons can always escape from the central object of the exponential metric. Eq. 20 is trivially satisfied with $\alpha = 0$, $p_\phi = 0$ in this case.

2.7 Radial geodesics inside $2R_g$

Lacking turning points inside $r = 2R_g$, the most interesting parts of geodesic trajectories of test particles can be examined for purely radial motions. These can be described by rearranging Eq. 14 with $p_\phi = 0$ as follows:

$$v^2 = (dr/dt)^2 = c^2 e^{-4R_g/r}(1 - (m_0c^2/E)^2 e^{-2R_g/r}) \tag{25}$$

Since $g_{00} = e^{-2R_g/r} \leq 1$ for $r \geq 0$ and $(m_0c^2/E)^2 \geq 0$, it is apparent that radial geodesic motions of test particles will be limited to speeds less than c in this metric.

Eq. 25 can be differentiated to obtain the acceleration of the test particle as

$$a = vdv/dr = (GM/r^2)(2e^{-4R_g/r} - 3(m_0c^2/E)^2 e^{-6R_g/r}) \tag{26}$$

The leading multiplicative term in this equation is the free-fall acceleration of Newtonian gravity. A test particle freely falling from rest from very large r has $E = m_0c^2$ and Eq. 26 yields an initial acceleration of $-GM/r^2$, as expected. But contrary to our expectation of gravitation always being attractive, the acceleration of this particle becomes zero for $r = 4.93R_g$ and positive for smaller r. It reaches a peak of about $0.037c^2/R_g$ at $r = 1.42R_g$ before approaching zero speed and acceleration for $r \to 0$. [9]

Who knew that gravity could become repulsive? In fact, since $e^{-R_g/r} \leq 1$, Eq. 25 shows that the acceleration of a test particle would always remain positive for $E \geq \sqrt{3/2}m_0c^2$. Neglecting the effect of the gravitational potential, a particle would have this energy if moving at a speed of $c/\sqrt{3}$.

A test particle launched inward toward M from large r would experience an outward directed acceleration for $E > \sqrt{3/2}m_0c^2$. It would have a peak outward directed acceleration somewhere inside $r = 2R_g$ before approaching zero speed and acceleration for $r \to 0$. In this case, gravity opposes its motion toward M all the way, but it continues inward.

As previously mentioned, radially outgoing particles with sufficient energy can always escape from points outside $r = 0$. This is illustrated in Figure 1 for a test particle started outward from near $r = 0$ with energy $E = 10m_0c^2$. It accelerates outward. A peak acceleration of $0.067c^2/R_g$ is obtained at $2R_g$. It eventually obtains a terminal speed of $0.995c$. After the strong acceleration phase, the elapsed travel time increases fairly uniformly

[9] In both the original Schwarzschild metric and the Schwarzschild (Hilbert) metric, free fall from rest produces repulsion. In the latter, the acceleration becomes positive outward at $r = 6R_g$ before reaching a peak of $0.47c^2/R_g$ at $r = 2.46R_g$. The free fall speed goes to zero at $r = 2R_g$.

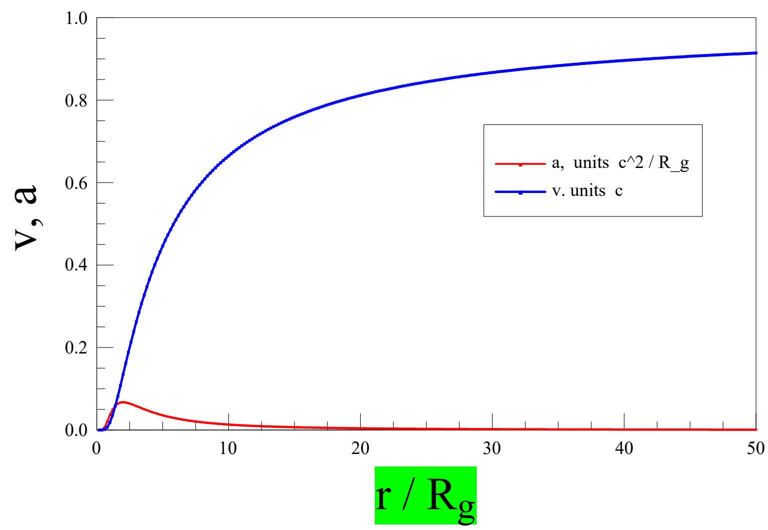

Figure 1: Speed and acceleration along an outgoing radial geodesic trajectory for a test particle with energy $E = 10 m_o c^2$.

with distance or conversely, distance increases nearly uniformly with time elapsed.

If a cloud of test particles near $r \sim R_g$ were all given such a burst of energy, the cloud expansion would be the reverse of an Oppenheimer-Snyder dust cloud collapse. From the viewpoint of a microbe on one of the interior particles of the cloud it would be in an expanding "universe" of dust particles. A simple cosmological model of this sort has been proposed (Dickau, Kauffmann & Robertson 2025) using the metric of Schwarzschild's original solution of the Einstein field equations for a point mass. This metric has no event horizon, has $g_{00}(r = 0) = 0$ and thus exhibits the same qualitative characteristics given by Eq. 26 and shown in Figure 1.

3 Gravitational Field Energy in the Yilmaz Metric

Einstein (1916) maintained that gravitational field energy was not localizable and should not be included as a source term in his field equations. To the contrary, the Yilmaz metric considered here demonstrates that gravitational field energy arises naturally as part of the total energy budget of a system of particles and fields. To examine this point, let T_i^j be the energy-momentum tensor right member of the Einstein field equations

$$G_i^j = -(8\pi\kappa/c^4)T_i^j \tag{27}$$

For the matter-free space (excepting test particles of negligible mass) external to a central point mass, M, Einstein said that we should have $T_i^j = 0$. For the G_0^0 component of the Einstein tensor, this equation for the static metric form of Eq. 9 is (Tolman 1934, 1987):

$$G_0^0 = e^{-\lambda}(\lambda'' + 2\lambda'/r + (\lambda')^2/4) = (-8\pi\kappa/c^4)T_0^0 \tag{28}$$

Where primes represent derivatives with respect to r. Substituting the exterior Newtonian potential of the Yilmaz metric, $\lambda = -2U = 2R_g/r$, into Eq. 28 (and also for Tolmans' other equations for other components of G_i^j) produces

$G_0^0 = e^{-\lambda}(\ \ 0 + \lambda'^2/4) = (-8\pi\kappa/c^4)T_0^0$
and
$T_0^0 = (-c^4/8\pi\kappa)(e^U U')^2 = -1/2\epsilon_g(c^2\nabla U)^2$

$$= (-1/2)\epsilon_g g^2 = T_2^2 = T_3^3 = -T_1^1 \tag{29}$$

where $\epsilon_g = 1/(4\pi\kappa)$. The Newtonian stress-energy tensor components are thus proportional to the square of gravitational field strength (defined as force per unit test mass) $\vec{g} = -c^2\vec{\nabla}U$, in complete analogy with the electric field energy density $(1/2)\epsilon_0 E^2$ for electric field $\vec{E}$ of a point charge source.

The presence of the term $\lambda'^2/4$ in Eq. 28 clearly represents a T_0^0 component of the gravitational field energy density, but in the Newtonian limit, its effect on spacetime curvature is negligible. To see when it is significant, consider the ratio $|(\lambda'^2/4)/(2\lambda'/r)| = |\lambda' r/8|$ of terms in Eq. 28. For the Newtonian potential $\lambda = 2\kappa M/r$, the ratio is $R_g/4r$. Not only is the field energy density term negligibly small in the solar system, it is only 1/8 as large as the preceding term in Eq. 28 at the photon sphere where $r = 2R_g$. This answers the question of where the Newtonian potential and exponential metric might become of questionable validity. It would seem to be somewhere well inside the photon sphere.

Yilmaz proposed that in matter free space, we should add the field energy density terms to the right member of the Einstein field equations. This would have the effect of canceling the same terms in the left member and leaving the terms that produce the Newtonian potential. One consequence of this is that if λ'^2 is not in Eq. 28, the event horizon at $r = R_g/2$ for the metric of Eq. 9 would not occur. There would be no black holes. In other circumstances, such as stellar interiors, where fluid density and pressure terms would be present in the right member of the field equations, it is only necessary to realize that they also represent the effects of the gravitational field energy that a fluid volume element would contain. The presence of static pressure owes it entire existence to the presence of the gravitational field.

4 Gravitational waves

As another example of how the inclusion of the field stress-energy terms as source terms in the right members of the field equations seems logical, consider a topic of considerable recent interest; viz, gravitational waves. We imagine these to be disturbances of spacetime that propagate essentially in a Minkowskian interstellar vacuum. In this case, the Einstein equations would have solutions equivalent to those obtained by setting the Ricci tensor components equal to zero. Consider a metric that might represent gravitational waves propagating along a z-axis with small perturbations of opposite amplitudes in the x and y directions, i.e.;

$$ds^2 = c^2dt^2 - e^{2U}dx^2 - e^{-2U}dy^2 - dz^2 \tag{30}$$

Where $U = U(ct \pm z)$ represents a wavelike distortion of what would otherwise be Minkowskian spacetime propagating along the z-axis. Denoting partial derivatives with respect to z with primes and using dots for derivatives with respect to (ct), the Ricci tensor components for the metric are:

$$R_{11} = -e^{2U}(\ddot{U} - U'') \qquad R_{22} = e^{-2U}(\ddot{U} - U'')$$
$$R_{00} = 2(\dot{U})^2, \qquad R_{33} = 2(U')^2, \qquad R_{30} = -2(\dot{U})(U') \tag{31}$$

If set equal to zero, solutions of the wave equations $R_{11} = 0$ and $R_{22} = 0$ represent waves of arbitrary amplitude propagating along the z-axis. Unfortunately, these waves are NOT solutions of the Einstein field equations because none of R_{00}, R_{30} or R_{33} are exactly zero, as demanded by Einstein. In fact, they look suspiciously like wave energy terms. If we continue and write the Einstein field equations

$$G_i^j = R_i^j - \delta_i^j R_k^k/2 = -(8\pi\kappa/c^4)T_i^j \tag{32}$$

we find that

$$G_0^0 = G_3^3 = \dot{U}^2 + U'^2 = (8\pi\kappa/c^4)T_0^0 \ \mathit{or}\ T_3^3 \tag{33}$$

For G_1^1 and G_2^2 we find that

$$G_1^1 = G_2^2 = \pm(\ddot{U} - U'') + \dot{U}^2 - U'^2 \tag{34}$$

Note that for plane waves, $\dot{U}^2 = U'^2$, therefore, where the wave equation $\ddot{U} - U'' = 0$ is satisfied, the metric remains, effectively Minkowskian despite the presence of a transverse, traceless wave. $T_0^0 = T_3^3 = (c^4/8\pi\kappa)(\dot{U}^2 + U'^2)$ would represent field stress-energy propagating with the wave, clearly localized along the z-axis.

As in the previous section where we began with the known Newtonian gravity potential, when we begin with an assumed metric form, the Einstein field equations then tell us what the corresponding field stress-energy tensor must be. While it is possible that we might have assumed something that makes no sense physically, that does not appear to be the case here. The stress-energy of the point mass gravitational field that was found is exactly as we would expect in the Newtonian limit. The energy propagating through empty space in the form of the gravitational waves here is very plausibly similar in form to energy associated with other kinds of waves.

The thing that would make no physical sense here would be to dogmatically insist that $T_i^j \equiv 0$ must hold because we are in matter free space. In that case the waves considered here could be solutions only for waves of infinitesimal amplitude. It would seem to be more reasonable to say that there are energy densities that propagate with these waves and that they should be represented by nonzero T_i^j terms in the right members of the Einstein field equations. It is these fields that interact with the gravitational wave detector's test masses that respond to the waves. Although the generation of gravitational waves by accelerated masses has not been considered here, it has been shown (Lo 1995) that a self-consistent derivation of Einstein's quadrupole gravitational radiation formula requires the presence of a gravitational field stress-energy tensor t_i^j as a source term. But in order to prevent gravity fields from being self-generating it is necessary that the t_i^j for a field must affect spacetime curvature in a negative way compared to the effect of ordinary inertial/gravitational mass. If we say that ordinary mass produces a positive curvature of spacetime, then t_i^j must produce just the right amount of curvature with an opposite sense.

This gives us a way to understand why the misnamed Schwarzschild metric has an event horizon. If the right member of Eq. 28 had a $t_0^0 = e^{-\lambda}\lambda'^2/4$ to represent the field energy, the solution of the equation for the point mass becomes the Newtonian potential for which there is no event horizon. If this t_0^0 is missing from the right member the spacetime becomes curved too rapidly as we approach the location of the point mass and becomes singular at $r = R_g/2$. Including the t_0^0 term in the right member of the field equation cancels the $\lambda'^2/4$ term and this produces a reverse curvature effect that permits a regular metric all the way in to the central point mass.

5 Compact Objects in the Yilmaz metric

The Yilmaz metric may be applicable to masses of arbitrarily small radius. Even the extreme case of a point mass would not lead to a curvature singularity since the Kretschmann scalar is zero at $r = 0$ (Alley, Aschan & Yilmaz 1995, Yilmaz & Alley 1999). Lacking an event horizon to swallow mass, accretion processes or gravitational collapse in the Yilmaz metric would apparently continue until adiabatic heating and trapped photons could provide enough pressure to stabilize a very slow rate of collapse. Eventually an Eddington balanced state should be achieved with a very compact, hot radiating core, however, to reduce Eddington limit luminosities to those observed distantly for quiescent stellar mass galactic black hole candidates (GBHC), redshifts of $z \simeq 10^8$ would be needed. This is extraordinary, to say the least, but probably no more incredible than the $z = \infty$ of an event horizon. Achieving a redshift of 10^8 in the Yilmaz metric would require

$U = -R_g/R \simeq -20$ and $R \simeq R_g/20$. This would, indeed, be a very compact object, but if there is no event horizon to be avoided, then there is no reason to suppose that anything other than radiation pressure might eventually stop the collapse.

5.1 ECO-MECO Models

Models of gravitationally compact masses with extreme redshifts already exist within the scope of conventional General Relativity and the Schwarzschild metric. It has been shown (Herrera & Santos 2004, Herrera, DiPrisco & Barreto 2006, Mitra 2006a,b,c, Mitra & Glendenning 2010, Robertson & Leiter 2003) that it might be possible, in accord with Einstein's intuition, to achieve an Eddington balance before trapped surfaces or event horizons would occur. Such objects with radii $r \simeq 2R_g$ can have very large, but finite, redshifts, very low distantly observed luminosity and such extremely long radiative lifetimes that they have been called "eternally collapsing objects" or ECO (Mitra 1998, 2000, 2002).

Motivated by the similarities of spectral state switches of dwarf nova, and neutron star (NS) and galactic black hole candidates (GBHC) in low-mass x-ray binary systems, Robertson & Leiter (2002, 2003, 2004, 2006) suggested that the ECO must be strongly magnetic objects that they called MECO. From the ratios of luminosities in quiescence and at the spectral state switch, they estimated magnetic moments to be $\mu \sim 10^{27}\ G\ cm^3$ for atoll class NS and $\mu \sim 10^{29}\ G\ cm^3$ for GBHC. A GBHC that actually possessed an event horizon would not have a magnetic moment because the multipole moments of magnetic and gravitational fields are quenched in the process of gravitational collapse into an event horizon. Yet magnetic fields either too large to be generated by accretion flows (Gliozzi, Bodo & Ghisellini 1999, Gnedin et al. 2003, Robertson & Leiter 2002, Vadawale, Rao & Chakrabarti 2001) or too close to the Schwarzschild radius (Marti-Vidal et al. 2015, Johnson et al. 2015) have been observed. Future observations that clearly establish the characteristics of magnetic fields within or near the photon sphere might either confirm or reject the MECO model.

One of the attractive features of ECO-MECO models, whether in Yilmaz or Schwarzschild metrics, is that they provide a common mechanism for producing the spectral and timing properties of dwarf novae, neutron star, GBHC and AGN systems in terms of the interactions between the intrinsic magnetic moments of central objects and accretion flows (Robertson & Leiter 2003, 2004, 2006, 2010).

Nova outbursts are thought to occur when accretion disk instabilities permit disk plasma flows into the central object. For dwarf and NS systems, the accretion disks clearly must interact with a magnetic moment. In MECO models, GBHC and AGN interact in the same way, with a significant magnetic pressure exerted at the inner disk boundary. In the low/hard spectral state, plasma is ejected by a magnetic propeller effect (Ilarianov & Sunyaev 1975, Stella, White & Rosner 1986, Cui 1997, Zhang, Yu & Zhang 1997, Campana et al. 1998). Jets are sometimes observed for these states (Robertson & Leiter 2004). At sufficiently large accretion rates, the disk can push the magnetopause inside the corotation radius and produce much higher disk luminosities in softer x-rays from plasma that reaches the innermost stable orbit. For dwarf and NS systems additional luminosity arises when the flow reaches the star surface. In the case of GBHC systems, little luminosity arises from flow inside the innermost orbit, whether it would reach an event horizon or a MECO.

With only small differences of numerical factors, such as e^1 and $\sqrt{27}/2$, that show up in many of the equations, quantitatively similar results are found for the Yilmaz and Schwarzschild metrics for luminosities, redshifts, photosphere temperatures, etc. MECO have an outer layer of baryons that are supported by a net outward flow of photons with a distantly observed luminosity of $1.26 \times 10^{38}(M/M_{\odot})f/(1+z_p) \;\; erg\ s^{-1}$, where z_p is the photosphere redshift and f is the escape fraction given by Eq. 21.

For the Schwarzschild metric Robertson & Leiter (2003) found the redshift of the baryon surface to be $z_s = 5.67 \times 10^7\sqrt{M/M_{\odot}}$. For that metric, the entire diminution of luminosity was attributed to redshift with $f = 1$ for escape from very near an event horizon. For the Yilmaz metric with a radiating surface well inside the photon sphere, $f << 1$ and the radiating surface is at a photosphere above an electron-pair atmosphere. The photosphere is a last scattering surface, with a temperature of $T_p \approx 2.5 \times 10^8\ K$ and redshift of $z_p = 10^3(M/M_{\odot})^{0.34}$. The distantly observed temperature is $T_\infty = T_p/(1+z_p)$. More details for both metrics are available for interested readers (Robertson & Leiter 2003, 2006, 2010, Robertson 2016).

6 Summary and Conclusions

Eq. 6 revealed a result that is required by special relativity, but incompatible with the accepted solutions of the Einstein field equations for static fixed mass gravitational sources. Applying this equation to the Schwarzschild metric revealed that a test particle in geodesic motion would experience an infinite gravitational force when it reached the event horizon. In addition to creating some curiosity about the reality of event horizons, this produced some motivation to look for either an alternative metric or, as Einstein wished, to find some way to avoid the possibility that matter might ever become so compact as to produce an event horizon. Both of these possibilities have been explored here. If an Eddington balance can be achieved in a Schwarzschild metric before the formation of a trapped surface or event horizon, it would provide a way to avoid infinite forces and redshifts at an event horizon. This was discussed above in the descriptions of ECO-MECO models of compact objects. These models are in good accord with an extensive set of observations.

Requiring a Newtonian potential limit for Eq. 6 and that light speed be independent of its direction produced a Yilmaz metric. When applied to geodesic motions in accretion disks or to photons, it clearly produces results in agreement with observations. There is an innermost marginally stable accretion disk orbit with acceptable luminosity and the image size of the photon sphere is essentially the same as that found for the Schwarzschild metric.

With the Newtonian potential, a point mass singularity remains as $r \rightarrow 0$, and $g_{00} \rightarrow 0$, but this is just a problem with the concept of a point mass and not a pathological feature of the metric as there is no curvature singularity for $r \rightarrow 0$. In this regard, we can see that the gravitational field energy density has a negative curvature effect on the spacetime metric. If sources of positive mass-energy densities produce positive curvature of spacetime, gravitational field stress-energy densities inherently produce the opposite effect on curvature. Without this required negative contribution (Lo 1995), spacetime becomes too warped too soon as we approach the point particle and an event horizon forms for $r > 0$. If field energy is a source of spacetime curvature, gravity would also be self-generating without the negative curvature requirement.

In the development of Einstein's field equations, the stress-energy tensors of all forms of matter and energy were considered as potential sources for the right members of the equations, with the exclusion of gravitational field energy (Einstein 1916). Einstein's fiat exclusion of gravitational field energy and requiring $T_i^j \equiv 0$ in the matter free space outside a point mass is what led to the Hilbert solution with an event horizon. As shown above, it can be eliminated simply by retaining the Newtonian gravitational potential and placing terms that represent the gravitational field energy density, t_i^j, in the right members of the Einstein field equations. A nonzero right member t_i^j is also needed in order to produce the correct gravitational binding energy of masses in the Newtonian limit. In addition, it was shown that gravitational waves of arbitrary amplitude can be solutions of the field equations if they are permitted to exist and propagate with a nonzero t_i^j.

Although the Yilmaz metric eliminates the event horizon and yields an obvious and intuitive form for $t_i^j \rightarrow g^2/(8\pi\kappa)$, in the weak-field Newtonian limit, it was left less than obvious what terms in the Einstein tensor correspond to an appropriate field stress-energy tensor t_i^j. They are generally the nonlinear terms within G_i^j that would be part of Einstein's stress-energy pseudotensor (Weinberg 1972, p165). Yilmaz (1992) discussed the problem with pseudotensors and concluded that they can be always be avoided by choosing rectangular coordinates. In various writings, Yilmaz offered expressions for t_i^j, but these often incorporated a harmonic coordinate condition, which might not hold in a matter continuum such as a stellar interior.

One last reason for choosing the Yilmaz metric to satisfy the requirement of Eq. 6 is that it may open a path to a quantum field theory of gravity. In the Schwarzschild metric, the metric tensor components must describe deviations from Minkowskian geometry and also serve as potentials. The vanishing of the covariant derivatives of the metric tensor creates problems for a quantum theory. In the Yilmaz metric, however, a quantum theory of its potentials might be possible (Yilmaz 1992). This might separate the quantum effects from the metric in a way that does not require spacetime quanta.

Although Einstein's exclusion of gravitational field energy density as a source of spacetime curvature was accomplished by fiat, it has allowed General Relativity to be a purely geometric theory of gravity. This simplicity has great appeal, but Weinberg (1972, Preface) notes that it has "**driven a wedge**" between the theory of elementary particles and General Relativity. What really matters is whether the theory correctly accounts for observations and experiments. A philosophical question is raised by the Yilmaz metric; given that it correctly describes relativistic gravitational phenomena, is it to be disregarded merely because it was not the first theory to do so?

At this point, the main intended points of this work have been completed. A physically well motivated metric has been shown to be capable of encompassing the main observational features of galactic black hole candidates all the way from Newtonian limit distances to deep inside the photon sphere. With no event horizon, there should be no central singularity. The central mass should settle into a MECO configuration with a substantial magnetic moment. This key feature of a MECO is open to observational tests that would distinguish a MECO from a true black hole.

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