Quantum Amplitude Estimation for Expected Loss Computation
under a Discretized Latent-Factor Model

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Abstract—We present a quantum algorithm for estimating the expected loss of a credit portfolio driven by a latent risk factor. The method discretizes a continuous latent variable, encodes its probability distribution into quantum amplitudes, embeds the loss function via controlled rotations on an ancilla qubit, and applies Grover-style amplitude amplification. The resulting state admits a two-subspace decomposition enabling estimation of the expected loss using Quantum Amplitude Estimation (QAE) or a maximum-likelihood estimator (MLE) based on Grover power measurements. The approach achieves a quadratic speedup over classical Monte Carlo methods.

Index Terms—Quantum amplitude estimation, Grover operator, credit risk, expected loss, latent factor models.

I. INTRODUCTION

Estimating expected loss (EL) is a central problem in credit risk management and financial engineering. Classical Monte Carlo methods approximate expectations with sampling complexity $O(\varepsilon^{-2})$. Quantum Amplitude Estimation (QAE), introduced by Brassard et al. [1], enables a quadratic speedup, reducing complexity to $O(\varepsilon^{-1})$. In this paper, we formulate expected loss computation under a latent-factor model as an amplitude estimation problem using Grover-style operators.

II. PROBLEM SETTING

Let $Z \sim N(\mu, \sigma^2)$ be a latent systematic risk factor. Define the loss

$$L(Z) = EAD \cdot LGD \cdot PD(Z), \qquad (1)$$

and the target quantity

$$E[L] = \int L(z)\,dP(z). \qquad (2)$$

III. DISCRETIZATION OF THE LATENT VARIABLE

Using n qubits, we define $N = 2^n$ grid points over

$$[\mu - 3\sigma, \mu + 3\sigma]. \qquad (3)$$

Let

$$\Delta = \frac{6\sigma}{N - 1}, \qquad i = 0, \ldots, N-1.$$
$$z_i = \mu - 3\sigma + i\Delta, \qquad (4)$$

Discrete probabilities p_i approximate $P(Z = z_i)$ and satisfy $\sum_i p_i = 1$.

IV. UNCERTAINTY STATE PREPARATION

Define the uncertainty state

$$|\psi_Z\rangle = \sum_{i=0}^{N-1} \sqrt{p_i}\,|i\rangle. \qquad (5)$$

A unitary U prepares $|\psi_Z\rangle$ from $|0\rangle^{\otimes n}$:

$$U|0\rangle^{\otimes n} = |\psi_Z\rangle. \qquad (6)$$

V. LOSS ENCODING VIA CONTROLLED ROTATIONS

Let

$$L(i) = L(z_i), \qquad f(i) = \frac{L(i)}{L_{\max}} \in [0, 1]. \qquad (7)$$

Introduce an ancilla qubit and define

$$U_f(|i\rangle|0\rangle) = |i\rangle\left(\sqrt{1 - f(i)}\,|0\rangle + \sqrt{f(i)}\,|1\rangle\right). \qquad (8)$$

This is implemented as

$$U_f = \sum_{i=0}^{N-1} |i\rangle\langle i| \otimes R_y(\theta_i), \qquad \theta_i = 2\arcsin\left(\sqrt{f(i)}\right). \qquad (9)$$

Define

$$A = U_f(U \otimes I). \qquad (10)$$

VI. AMPLITUDE INTERPRETATION

Applying A to $|0\rangle^{\otimes n}|0\rangle$ yields

$$|\Psi\rangle = \sum_{i=0}^{N-1} \sqrt{p_i}\,|i\rangle\left(\sqrt{1 - f(i)}\,|0\rangle + \sqrt{f(i)}\,|1\rangle\right). \qquad (11)$$

Define

$$a = \sum_{i=0}^{N-1} p_i f(i) = \frac{\mathbb{E}[L]}{L_{\max}}. \qquad (12)$$

Then

$$|\Psi\rangle = \sqrt{1-a}\,|\Psi_0\rangle + \sqrt{a}\,|\Psi_1\rangle, \qquad (13)$$

where $|\Psi_1\rangle$ corresponds to ancilla = 1.

VII. Grover Operator

Define reflections

$$S_\chi = I - 2(I \otimes |1\rangle\langle 1|), \qquad (14)$$

$$S_0 = I - 2|0\cdots 0\rangle\langle 0\cdots 0|. \qquad (15)$$

The Grover operator is

$$Q = AS_0 A^\dagger S_\chi. \qquad (16)$$

Let $a = \sin^2\theta$. Then after m Grover iterations,

$$\Pr(\text{ancilla} = 1 \mid Q^m) = \sin^2\big((2m+1)\theta\big). \qquad (17)$$

VIII. Estimation Methods

A. Quantum Amplitude Estimation

Quantum phase estimation on Q yields eigen phases $\pm 2\theta$ and thus $a = \sin^2\theta$ with complexity $O(\varepsilon^{-1})$ [1].

B. Maximum Likelihood Estimation

Repeated measurements after Q^m yield binomial samples

$$k_m \sim \text{Binomial}\big(N_m,\ \sin^2((2m+1)\theta)\big). \qquad (18)$$

Let

$$p_m(a) = \sin^2\big((2m+1)\arcsin(\sqrt{a})\big). \qquad (19)$$

The MLE is

$$\hat{a} = \arg \max_{a\in(0,1)} \sum_m \Big[k_m \log p_m(a) + (N_m - k_m)\log(1 - p_m(a)) \Big]. \qquad (20)$$

IX. Complexity and Resources

Classical Monte Carlo: Sampling complexity $O(\varepsilon^{-2})$.

Quantum method: QAE achieves $O(\varepsilon^{-1})$ queries to the oracle A [1], [3]. The total number of qubits is $n + 1$ plus auxiliary work qubits. Circuit depth is dominated by controlled rotations and Grover iterations.

X. Numerical Results (Synthetic Study)

A. Experimental Setup

We consider a synthetic latent-factor model

$$Z \sim N(0,1), \qquad (21)$$

with probability of default

$$PD(Z) = \Phi(\alpha Z + \beta), \qquad (22)$$

where Φ is the standard normal CDF, $\alpha = 0.8$, and $\beta = -0.3$. We set EAD = LGD = 1, so $L(Z) = PD(Z)$. The true expected loss is computed via high-precision numerical quadrature.

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TABLE I

RMSE comparison between classical Monte Carlo and quantum amplitude estimation for synthetic expected loss estimation.

Samples / Queries (N)	MC RMSE	QAE RMSE
10^2	2.545×10^{-2}	9.047×10^{-3}
3×10^2	1.389×10^{-2}	3.042×10^{-3}
10^3	7.295×10^{-3}	8.917×10^{-4}
3×10^3	4.548×10^{-3}	2.933×10^{-4}
10^4	2.380×10^{-3}	8.882×10^{-5}
3×10^4	1.535×10^{-3}	2.865×10^{-5}

B. Error Scaling

Classical Monte Carlo estimation is performed using N independent samples. Quantum amplitude estimation is simulated by ideal Grover queries and exact amplitude extraction. Figure 1 reports the RMSE as a function of the sample size or oracle queries on a log–log scale.
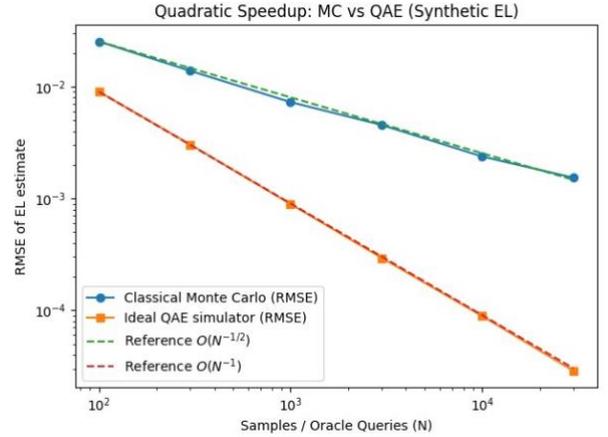

Fig. 1. Root-mean-square error (RMSE) of expected loss estimation versus number of samples (classical Monte Carlo) and oracle queries (quantum amplitude estimation) on a log–log scale. Classical Monte Carlo exhibits $O(N^{-1/2})$ convergence, while quantum amplitude estimation achieves $O(N^{-1})$ scaling, demonstrating a quadratic speedup.

XI. Conclusion

We formulated expected loss estimation under a discretized latent-factor model as a quantum amplitude estimation problem. Synthetic numerical experiments confirm the theoretically predicted quadratic speedup over classical Monte Carlo methods. This framework provides a principled foundation for quantum credit risk applications as quantum hardware matures.

References

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