

Stochastic Transitions Between Neutral and Bimodal Affective States Under Contradictory Stimulation

Renato Vieira dos Santos ^{*}

*Instituto de Ciência, Tecnologia e Inovação – ICTIN,
Universidade Federal de Lavras – UFLA, Campus Paraíso, MG 37950-000, Brazil*

Contemporary information environments expose individuals to streams of messages that alternate rapidly between positive and negative affective valences, a condition we term contradictory stimulation. What are the long-term consequences of such structured inconsistency on an individual’s core affective state? The difficulty lies in representing the pattern of contradiction, rather than the content of any single message, within a mathematically tractable framework. We approach this problem by treating the strength of external influence as a stochastic parameter, thereby converting a deterministic homeostatic model of affect into a stochastic differential equation with multiplicative noise. The resulting stationary distribution reveals two distinct regimes separated by a critical boundary in parameter space: a unimodal regime centered at affective neutrality, emerging when the noise intensity exceeds a threshold, and a bimodal regime corresponding to affective instability, where the distribution exhibits two well-separated peaks. We map the phase diagram in the parameter space and identify an island of bimodality for intermediate regulatory capacity, as well as a lock-in region for near-neutral default bias. These findings offer a minimal, analytically transparent mechanism through which the statistical architecture of an information environment, independent of any specific content, can shape the qualitative form of affective experience.

Keywords: stochastic dynamics; affective states; noise-induced transitions; sociophysics; Fokker-Planck equation

I. INTRODUCTION

The influence of mass media on individual attitudes and emotional states has been studied extensively across the social sciences. Classical models of opinion dynamics typically treat external information as a coherent field that pulls a person’s belief or attitude toward a fixed position [1–3]. A substantial body of empirical work demonstrates that consistent messaging can shift attitudes in predictable directions [4, 5]. Yet contemporary media environments rarely present a single coherent narrative. Instead, they deliver a rapid alternation of messages with opposing affective valences: one moment encouraging optimism, the next provoking outrage, often within the same session of exposure. This condition, which we call contradictory stimulation, raises a question that remains underexplored from a formal modeling perspective. How does the statistical structure of an information stream, independent of its specific content, shape the long-term distribution of an individual’s emotional state? In the language of statistical physics, we ask whether the spectral properties of an external noise source can induce a qualitative change in the stationary distribution of a homeostatic system, and under what conditions this transition occurs.

The difficulty in addressing this question lies in representing the pattern of contradiction within a tractable dynamical framework. Most opinion dynamics models encode the content of messages as an attractive force in an abstract opinion space [6, 7, 20, 21]. In such mod-

els, contradiction would require multiple competing attractive forces, which rapidly becomes analytically intractable when the alternation is fast and irregular. An alternative approach, drawn from the theory of stochastic processes, is to absorb the inconsistency into a random variable rather than into multiple deterministic forces. We adopt this strategy by treating the strength of external influence as a stochastic parameter, effectively converting the deterministic dynamics into a stochastic differential equation (SDE) with multiplicative noise. The intensity of this noise then quantifies the degree of contradictory stimulation, while the underlying deterministic drift captures the homeostatic tendencies of the affective system.

The object of our analysis is not an opinion or a belief, but the core affective state: the continuous dimension of valence that underlies all emotional experience [8, 9]. Psychological research has established that this state behaves as a homeostatically regulated variable, returning to an individual-specific baseline after perturbation [10, 11]. By modeling core affect rather than discrete emotions or cognitive attitudes, we isolate the most elementary affective signal that external information can modulate. This choice yields a dynamical system that is psychologically grounded yet mathematically transparent, allowing us to derive exact results for the stationary distribution without resorting to numerical simulation as the primary tool.

Our main contribution is the identification of two qualitatively distinct regimes separated by a critical boundary in parameter space. When the noise intensity exceeds a threshold, the stationary distribution collapses to a single peak centered at affective neutrality, a state in which the individual experiences neither positive nor negative

^{*} renato.santos@ufla.br

valence with any persistence. We interpret this as a noise-induced transition to emotional disengagement, consistent with the observation that chronic exposure to contradictory information can produce apathy rather than heightened engagement. When the noise intensity lies below this threshold and the deterministic influence is sufficiently strong, the stationary distribution becomes bimodal, with peaks separated by a shallow minimum. In this regime, the individual's affective state oscillates randomly between two distinct levels, a phenomenon we term affective instability. We emphasize that this bimodality is an intra-individual phenomenon, describing a single person's oscillation between two affective states, and is distinct from social polarization, which refers to divergence between groups. The transition between these regimes is controlled by the relative magnitudes of the homeostatic parameters, and its location in parameter space exhibits non-trivial structure that we map through a systematic sensitivity analysis.

Before proceeding, we emphasize that the contribution of this paper is conceptual rather than technical. The mathematical tools we employ are standard; the novelty lies in recognizing that contradictory stimulation can be represented as multiplicative noise and in showing that this representation yields a rich phase structure with testable implications. The most striking of these implications is anti-intuitive: increasing the intensity of contradictory messages does not simply heighten emotional engagement; above a critical threshold, it suppresses sustained affect altogether, producing a state of indifference. This prediction offers a candidate mechanism for the co-existence of widespread disengagement and periodic affective volatility in contemporary media environments.

Specifically, we make the following contributions. First, we operationalize contradictory stimulation as multiplicative noise in the influence parameter, converting a deterministic homeostatic model into an SDE whose noise intensity quantifies the degree of inconsistency in the information stream. Second, we derive the exact stationary distribution of the resulting process and identify the condition under which its number of modes changes. Third, we map the phase diagram in the space of the homeostatic parameters, revealing an island of bimodality for intermediate regulatory capacity and a lock-in region for near-neutral default bias. Fourth, we extend the analysis to the case of multiple correlated noise sources, showing that the stationary distribution can become trimodal. These results establish a direct link between the statistical architecture of an information environment and the qualitative form of individual affective dynamics.

The remainder of this paper is organized as follows. In Section II we define the model, including the deterministic homeostatic dynamics and the saturating influence function. Section III introduces the stochastic extension and derives the stationary distribution. The main results, including the phase diagrams and the identification of the critical boundaries, are presented in Section IV. Section V discusses the implications of our findings for so-

ciophysics and outlines directions for future work. Technical derivations and numerical methods are collected in the appendices.

II. MODEL OF AFFECTIVE DYNAMICS

We model the core affective state of an individual as a scalar variable $x(t)$ evolving in continuous time. Positive values of x correspond to pleasant valence, negative values to unpleasant valence, and $x = 0$ to affective neutrality. The variable x is dimensionless, so that all parameters governing its evolution carry dimensions of inverse time. This normalization reflects the interpretation of x as an affective intensity bounded by the finite capacity of the emotional system. The parameter m plays the role of a restoring force constant, b is a constant bias, and c modulates a saturating feedback term. All three have dimensions of inverse time, so the dynamics are overdamped by construction. This choice reflects the observation that core affect is not oscillatory but relaxes monotonically toward its homeostatic baseline in the absence of sustained external input.

A. Homeostatic regulation and default bias

In the absence of external influence, the dynamics of x are governed by two parameters with clear psychological interpretation. The regulatory capacity m , with dimensions $[m] = T^{-1}$, quantifies how quickly the individual returns to emotional equilibrium after a perturbation. Large values of m describe rapid recovery, whereas small values describe prolonged emotional responses. The default bias b , also with dimensions $[b] = T^{-1}$, sets the baseline emotional state in the absence of stimulation. A positive b indicates a disposition toward positive affect, while a negative b indicates a disposition toward negative affect.

The simplest dynamical equation incorporating these two parameters is the linear relaxation

$$\frac{dx}{dt} = -mx + b. \quad (1)$$

This equation has a single stable fixed point at $x = b/m$, toward which the system relaxes with characteristic time $1/m$. Equation (1) captures the homeostatic tendency of affective states documented in psychological research [10, 11]. It is, however, insufficient to describe the effect of external information, which we introduce next.

B. Influence function and affective resonance

External stimuli, such as media messages, modify the affective state by adding a driving term to the right-hand side of Eq. (1). We denote this term by $I(x)$ and require that it satisfy three conditions. First, the influence

must saturate at extreme values of x , reflecting the observation that individuals already in a strongly positive or negative state are less susceptible to further shifts in the same direction. Second, the influence must be sign-flexible, meaning that the same functional form can describe both positive and negative messages depending on the sign of a control parameter. Third, the influence must vanish at $x = 0$, since a perfectly neutral individual has no preferred emotional direction to amplify.

A function that satisfies all three conditions is

$$I(x) = c \tanh(x), \quad (2)$$

where c is the influence strength with dimensions $[c] = T^{-1}$. The hyperbolic tangent is approximately linear near $x = 0$, so that small deviations from neutrality are amplified by the external message, while its saturation for large $|x|$ prevents unbounded growth.

The interpretation of Eq. (2) requires care. A naive reading would suggest that a neutral individual, with $x = 0$, receives no influence at all, since $\tanh(0) = 0$. This observation misses the dynamical role of the influence term. It is the *derivative* of $I(x)$ at $x = 0$, not the value, that determines the local response. For $c > 0$, a small positive fluctuation is amplified, while a small negative fluctuation is damped. This property is precisely what is meant by *affective resonance*: the influence is strongest when the individual's current state is already congruent with the message's valence. A positive message, represented by $c > 0$, reinforces positive affect and suppresses negative affect, in agreement with the well-documented phenomenon of mood-congruent processing [12, 13]. A negative message, represented by $c < 0$, has the opposite effect.[25]

C. Bistability in the deterministic limit

Combining Eq. (1) with Eq. (2), the full deterministic dynamics are

$$\frac{dx}{dt} = -mx + b + c \tanh(x) \equiv -\frac{dV(x)}{dx}, \quad (3)$$

where the effective potential is

$$V(x) = \frac{m}{2}x^2 - bx - c \log[\cosh(x)]. \quad (4)$$

Equation (4) is defined up to an additive constant, which does not affect the dynamics.

Equilibrium points of Eq. (3) satisfy

$$c \tanh(x) = mx - b. \quad (5)$$

The left-hand side is a bounded function with maximum magnitude $|c|$, while the right-hand side is a straight line with slope m and intercept $-b$. Depending on the values of m , b , and c , Eq. (5) may have one or three solutions, as illustrated in Fig. 1.

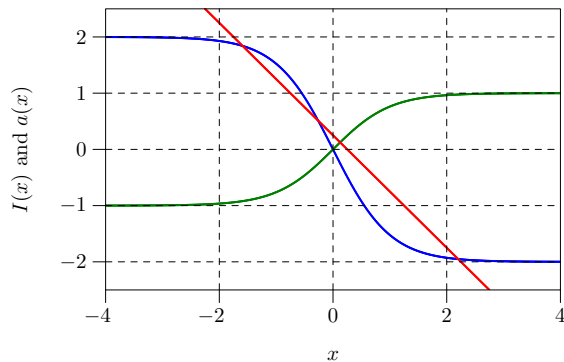


FIG. 1. Graphical solution of Eq. (5) for $m = 1$, $b = 1/4$, and two values of c . The red line is $mx - b$. The green and blue curves are $c \tanh(x)$ for $c = -1$ and $c = 2$, respectively. The number of intersections determines the number of equilibria. For $c = -1$, there is a single stable fixed point near $x = 0$. For $c = 2$, there are three fixed points, two stable and one unstable.

When three solutions exist, two are stable minima of the potential $V(x)$, and one is an unstable maximum. This is the bistable regime, in which the individual's affective state may settle into one of two distinct equilibria depending on initial conditions. Figure 2 shows the potential $V(x)$ for fixed m and c and varying b . As b increases, the left minimum disappears, leaving only the right minimum. This corresponds to a situation in which a sufficiently positive default bias prevents the individual from remaining in a negative affective state.

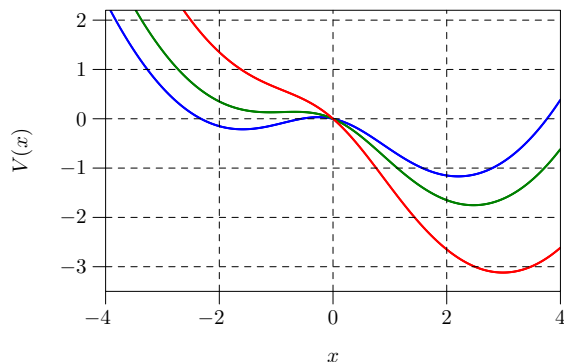


FIG. 2. Effective potential $V(x)$ from Eq. (4) for $m = 1$, $c = 2$, and three values of b . Increasing b shifts the potential upward and to the right, eventually removing the left minimum and leaving only a single stable state at positive x .

Figure 3 shows the effect of varying the regulatory capacity m . As m increases, the non-trivial minima become shallower and eventually disappear, leaving a single minimum near $x = 0$. This is the phlegmatic regime, in which strong homeostatic regulation prevents the persistence of any strongly positive or negative state.

The deterministic model defined by Eq. (3) provides the backbone for the stochastic extension developed in Section III. The key feature that will prove crucial is the

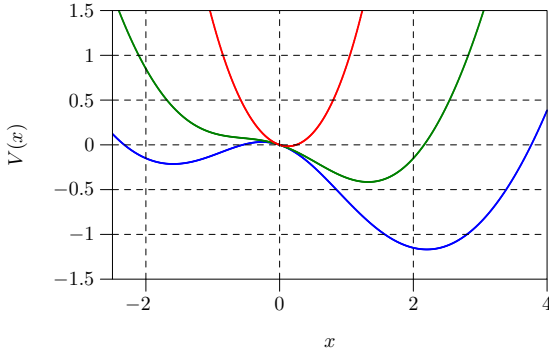


FIG. 3. Effective potential $V(x)$ from Eq. (4) for $b = 1/4$, $c = 2$, and three values of m . Increasing m flattens the potential and eventually removes the off-center minima, leaving a single stable state near neutrality.

existence of a bistable regime, which, as we will show, is transformed into a bimodal stationary distribution when the influence strength is allowed to fluctuate.

III. CONTRADICTION STIMULATION AS STOCHASTIC INFLUENCE

We now introduce the central mechanism of this work: the operationalization of contradictory stimulation as stochastic noise in the influence parameter c . This section defines the stochastic differential equation, derives the stationary distribution, and performs a bifurcation analysis that reveals the transition between the neutral and bimodal regimes. The notation introduced here will be used consistently throughout the remainder of the paper.

A. Stochastic influence parameter

In an environment where messages alternate rapidly between positive and negative valences, the effective influence strength c is no longer a fixed constant but a fluctuating quantity. We model this by replacing c with $c + \sqrt{2\sigma_c}\xi(t)$, where $\xi(t)$ is Gaussian white noise with zero mean and unit intensity:

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t)\xi(t') \rangle = \delta(t - t'). \quad (6)$$

The parameter σ_c , with dimensions $[\sigma_c] = T^{-1}$, quantifies the intensity of the contradictory stimulation. Substituting this into Eq. (3) yields the Itô stochastic differential equation

$$dx_t = [-mx_t + b + c \tanh(x_t)]dt + \sqrt{2\sigma_c} \tanh(x_t) dW_t, \quad (7)$$

where dW_t is the increment of a standard Wiener process with $\langle dW_t \rangle = 0$ and $\langle dW_t^2 \rangle = dt$. The multiplicative nature of the noise, through the factor $\tanh(x)$, reflects the fact that contradictory messages have no effect at

exact neutrality but become increasingly influential as the individual moves away from neutrality. This property is consistent with the interpretation of affective resonance introduced in Section II B.

B. Stationary distribution and effective potential

For a one-dimensional Itô SDE of the form

$$dx_t = f(x_t)dt + g(x_t)dW_t, \quad (8)$$

with $\langle dW_t^2 \rangle = dt$, the stationary probability density $P_{\text{st}}(x)$ is given by

$$P_{\text{st}}(x) = \mathcal{N} \exp[-\mathcal{V}(x)], \quad (9)$$

where $\mathcal{N}$ is a normalization constant and the effective stochastic potential is

$$\mathcal{V}(x) = - \int \frac{2f(x')}{g(x')^2} dx' + \ln g(x)^2. \quad (10)$$

For our specific case, $g(x) = \sqrt{2\sigma_c} \tanh(x)$, so that $g(x)^2 = 2\sigma_c \tanh^2(x)$.

Substituting $f(x) = -mx + b + c \tanh(x)$ into Eq. (10) yields a closed-form expression for $\mathcal{V}(x)$, which is lengthy and therefore omitted; the extrema condition derived below is sufficient for the analysis. The extrema of $P_{\text{st}}(x)$ are located at the points x_m where $d\mathcal{V}/dx = 0$, which is equivalent to the condition

$$0 = f(x_m) - g(x_m)g'(x_m), \quad (11)$$

or explicitly,

$$mx_m - b = c \tanh(x_m) - 2\sigma_c \tanh(x_m) \text{sech}^2(x_m). \quad (12)$$

Equation (12) is the central equation for the analysis of the stationary regimes. It generalizes the deterministic equilibrium condition Eq. (5) by the addition of a noise-induced term proportional to σ_c .

C. Noise-induced transition to neutrality

The effective potential $\mathcal{V}(x)$ defined by Eq. (10) determines the qualitative shape of the stationary distribution. For small σ_c , the potential retains two minima corresponding to positive and negative affective states, separated by a barrier near $x = 0$. As σ_c increases, the barrier is gradually suppressed, and the two minima merge into a single minimum at $x = 0$. This is a noise-induced transition from a bimodal to a unimodal distribution, as illustrated in Fig. 4.

This transition can be understood as a form of P-bifurcation, in which the number of modes of the stationary distribution changes as a control parameter is varied [16]. The critical value σ_c^* at which the bimodal structure disappears can be estimated by solving Eq. (12) together with the condition that the two non-trivial extrema coalesce, i.e., $d^2\mathcal{V}/dx^2 = 0$. A detailed analysis is deferred to Appendix B.

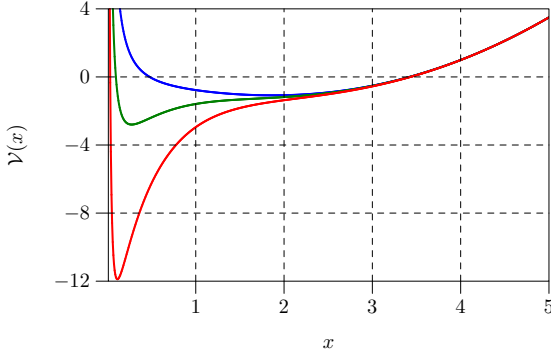


FIG. 4. Effective stochastic potential $\mathcal{V}(x)$ computed from Eq. (10) for $b = 1$, $c = 1$, $m = 1$, and three values of σ_c . As σ_c increases, the two wells merge into a single well at $x = 0$.

D. Extensions with multiple stochastic parameters

Having established the single-noise case, we now consider a more realistic scenario in which both the baseline b and the influence strength c are allowed to fluctuate. This extension allows us to explore richer stationary distributions, including trimodality, and to assess how correlations between different sources of variability affect the phase structure.

In realistic scenarios, more than one psychological parameter may fluctuate. We consider the general case in which both the baseline b and the influence strength c are allowed to fluctuate independently, with intensities σ_b and σ_c , and correlation λ . The SDE becomes

$$dx_t = [-mx_t + b + c \tanh(x_t)]dt + \sqrt{2\sigma_c} \tanh(x_t) dW_t^{(c)} + \sqrt{2\sigma_b} dW_t^{(b)}, \quad (13)$$

where $\langle dW_t^{(c)2} \rangle = \langle dW_t^{(b)2} \rangle = dt$, and $\langle dW_t^{(c)} dW_t^{(b)} \rangle = \lambda dt$.

The stationary distribution for this two-noise case is derived in Appendix A. The extrema of $P_{\text{st}}(x)$ satisfy

$$mx_m - b = c \tanh(x_m) - 2 \operatorname{sech}^2(x_m) [\lambda \sqrt{\sigma_c \sigma_b} + \sigma_c \tanh(x_m)]. \quad (14)$$

This equation reduces to Eq. (12) when $\sigma_b = 0$ and $\lambda = 0$. The additional terms shift the location and number of extrema, potentially giving rise to trimodal distributions under appropriate parameter combinations, as we explore in Section IV.

IV. RESULTS

We now present a systematic analysis of the stationary regimes emerging from the model defined in Section III. The focus is on the dependence of the number of modes of the stationary distribution on the parameters

(σ_c, c, m, b) , and on the mapping of the critical boundaries separating unimodal and bimodal regimes. All numerical results are obtained using the Euler-Maruyama method described in Appendix C, with the same convention for the noise intensity established in Section III A.

A. Neutralization under intense contradiction

Figure 4 illustrates the basic mechanism: as the contradiction intensity σ_c increases, the two off-center minima of the effective potential are suppressed, and the stationary distribution becomes sharply peaked at $x = 0$. This is the regime of affective neutralization, in which the individual experiences neither positive nor negative affect with any persistence. This is the central anti-intuitive result of our analysis: stronger contradiction does not amplify affective response but extinguishes it.

The critical value σ_c^* at which the bimodal structure disappears depends on the remaining parameters. This dependence is explored in detail in Section IV C.

B. Bimodal regime and affective instability

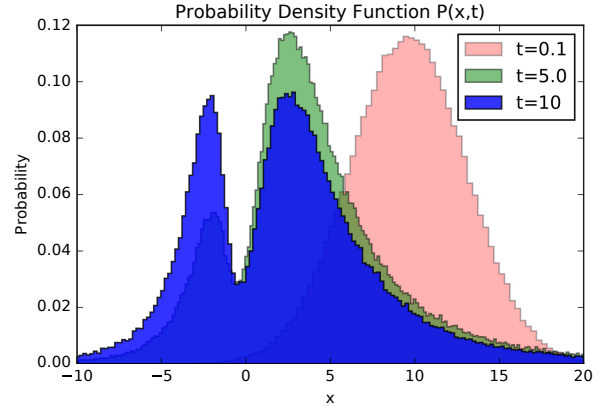


FIG. 5. Time evolution of the probability density $P(x, t)$ for $b = 1$, $c = 5$, $m = 1$, $\sigma_c = 0.845$, $\sigma_b = 0.5$, and $\lambda = 0.6$, starting from an initially positive state $x(0) = 10$. The distribution, sharply peaked at $t = 0.1$ (red), broadens and eventually splits into two well-separated peaks at $t = 10$ (blue), one near neutrality and one at distinctly positive values. This spontaneous emergence of bimodality from the interplay of homeostatic regulation and stochastic contradictory stimulation provides the dynamical mechanism for affective instability.

Figure 5 demonstrates that this bimodal structure is not an artifact of the stationary approximation, but emerges dynamically from an initially localized state as the system evolves under the influence of the stochastic forcing.

For $\sigma_c < \sigma_c^*$ and sufficiently large c , the stationary distribution is bimodal, with two distinct peaks separated

by a shallow minimum. In this regime, the individual's affective state oscillates randomly between the two peaks, a phenomenon we term affective instability. This is illustrated in Fig. 6, which shows the effective potential for fixed σ_c and increasing c .

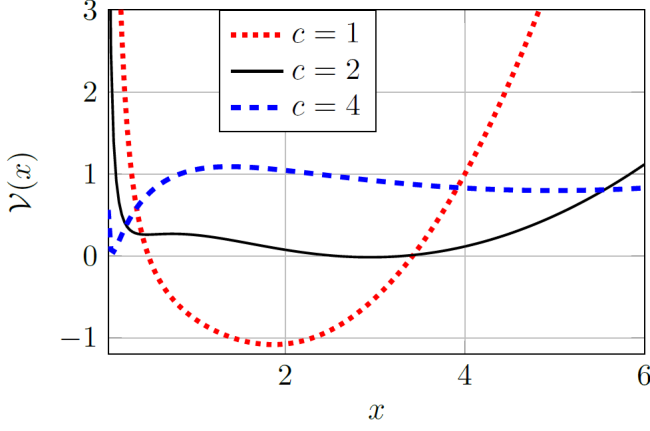


FIG. 6. Effective stochastic potential $\mathcal{V}(x)$ for $m = 1$, $b = 1$, $\sigma_c = 0.5$, and increasing c . For large c , a secondary well emerges, corresponding to a distinct positive affective state.

We emphasize that the bimodal distribution is not a signature of polarization in the social sense, but rather of intra-individual instability. The same individual samples both positive and negative states over time, with no sustained commitment to either. This distinction is important for the interpretation of the model's implications, as we discuss in Section V.

C. Phase diagrams and critical boundaries

To map the regions of parameter space corresponding to unimodal and bimodal regimes, we solve Eq. (14) numerically and track the number of real solutions as a function of the control parameters. The results are summarized in the phase diagrams of Fig. 7.

The phase diagrams reveal several non-trivial features. First, the critical value σ_c^* is not monotonic in c : it increases for small c and decreases for large c , so that the bimodal region is bounded in the (σ_c, c) plane. Second, the critical boundary in the (σ_c, m) plane forms an island, indicating that the bimodal regime exists only for intermediate regulatory capacities. Third, the (σ_c, b) plane shows a lock-in region near $b = 0$, where the system remains bimodal for all σ_c , regardless of the noise intensity. These boundaries can be read as a map of the parameter regimes in which an individual's affective state is either robust to the statistical properties of the information stream or sensitive to them. Below the critical curve, the homeostatic system is strong enough to maintain two distinct affective basins despite the contradictory input; above it, the noise overwhelms the deterministic structure and drives the system toward the single

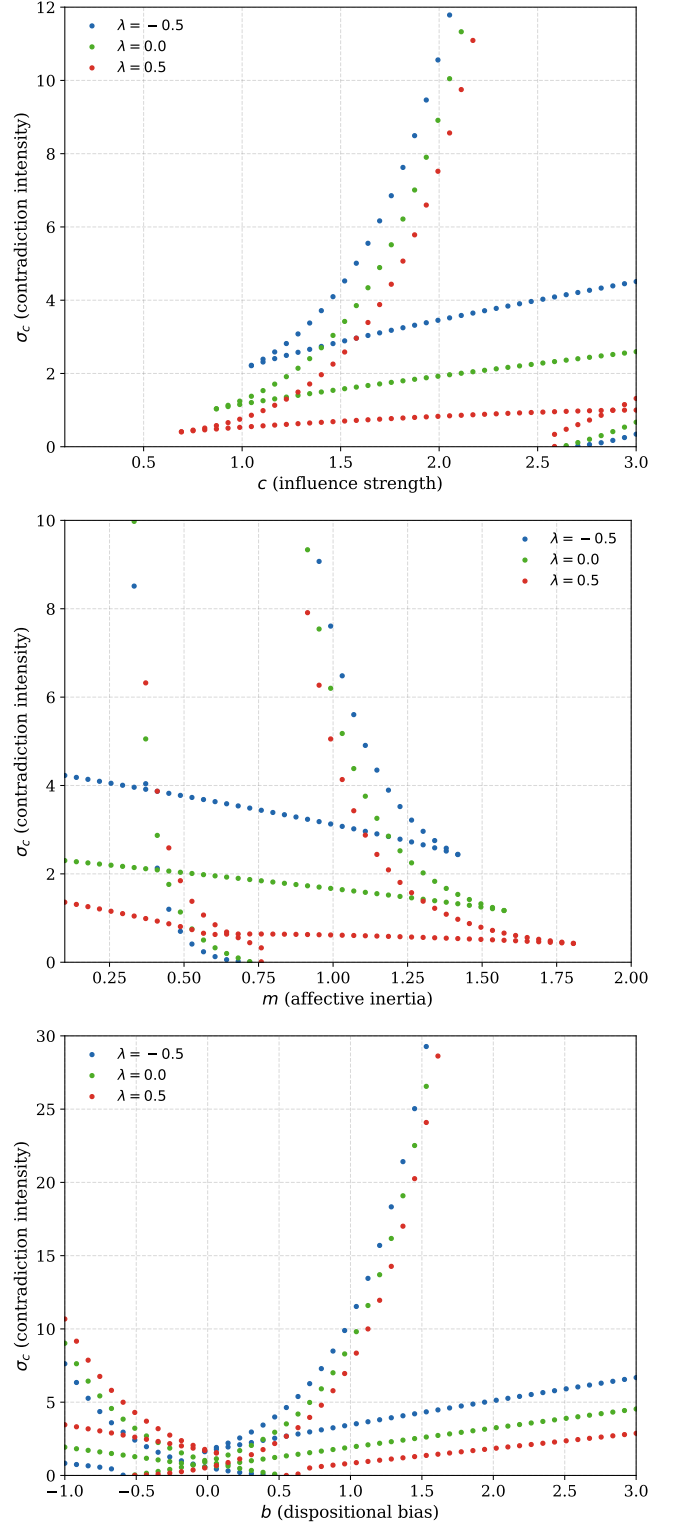


FIG. 7. Critical values of the contradiction intensity, σ_c^* , delimiting the transition between bimodal and unimodal regimes. For each parameter set and correlation coefficient λ , the stationary distribution is bimodal for $\sigma_c < \sigma_c^*$ and unimodal for $\sigma_c > \sigma_c^*$. Boundaries obtained from Eq. (14). (a) $\sigma_c^*(c)$; (b) $\sigma_c^*(m)$; (c) $\sigma_c^*(b)$.

neutral state.

To confirm the consistency between the analytical phase diagrams and direct numerical simulations, we performed Euler-Maruyama simulations for selected parameter combinations spanning the three phase diagrams. The number of modes observed in the numerically estimated stationary distribution matched the prediction of the phase diagram in each case. This agreement confirms that the analytical boundaries are not artifacts of the stationary approximation, but are reproduced by the full stochastic dynamics. A representative subset of these validation points is listed in Table I.

These findings are the main quantitative results of this paper. They demonstrate that the interplay between homeostatic regulation and contradictory stimulation produces a rich landscape of affective regimes, which cannot be reduced to a simple threshold condition.

It is instructive to compare these findings with other sociophysical models that incorporate external influence or non-conformist behavior. In bounded-confidence models, agents update opinions only when the difference between their state and a neighbor's state falls below a threshold, leading to fragmentation or consensus [7]. In models with contrarian agents, a fraction of the population systematically opposes the prevailing trend, introducing a noise-like effect that can stabilize coexistence [3]. Our mechanism differs in that the noise is not intrinsic to the agent's decision rule but arises from the statistical structure of the external information stream itself. This distinction offers a complementary pathway for generating bimodality and neutralization, one that does not require heterogeneity in agent rules or thresholds. Recent topical reviews [24] have highlighted the diversity of such mechanisms in sociophysical systems, underscoring the value of parsimonious, analytically tractable models like the one presented here.

D. Trimodal distributions under combined noise

When both b and c fluctuate with appropriate correlation, the stationary distribution can exhibit three peaks. An example is shown in Fig. 8, obtained by solving Eq. (14) and verifying the numerical solution of the full SDE.

The emergence of trimodality indicates that the combination of fluctuating baseline and fluctuating influence can fragment the affective landscape into three distinct basins. This regime is more fragile than the bimodal one and requires fine-tuning of parameters, but it illustrates the potential complexity of the dynamics when multiple sources of stochasticity interact. The graphical solution of the extrema condition for this case is shown in Fig. 9.

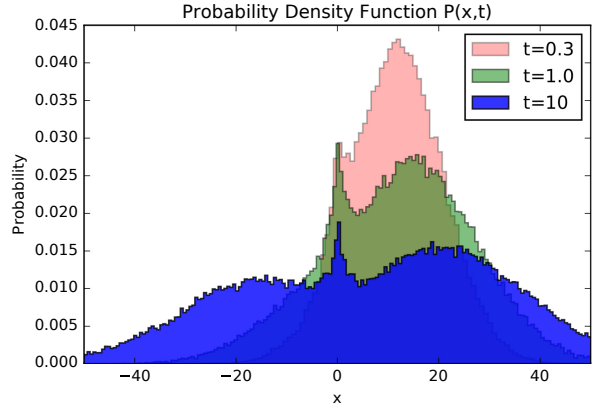


FIG. 8. Stationary distribution $P_{\text{st}}(x)$ for $b = 0.8$, $c = 7$, $m = 0.4$, $\sigma_c = 0.5$, $\sigma_b = 112.5$, and $\lambda = 0.3$, showing three distinct modes.

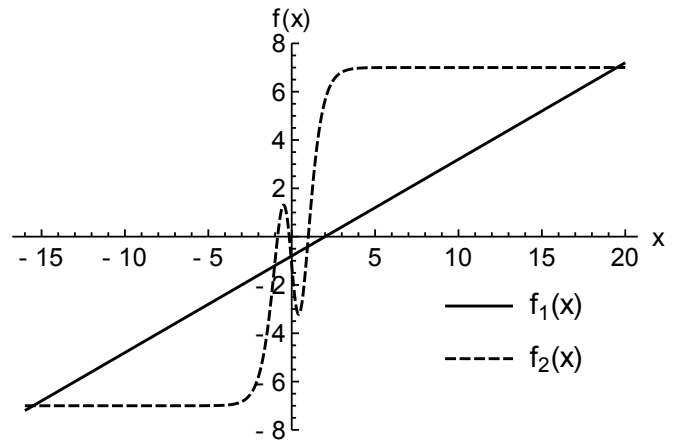


FIG. 9. Graphical solution of Eq. (14) for the parameters of Fig. 8, showing three intersection points corresponding to the three modes.

V. DISCUSSION

The results presented in Section IV demonstrate that contradictory stimulation, modeled as multiplicative noise in the influence parameter, can drive a homeostatic affective system into one of two qualitatively distinct regimes. When the noise is weak relative to the deterministic influence, the system exhibits bimodal affective instability, with the individual sampling two distinct emotional states over time. When the noise is strong, the system collapses to a single neutral state, a phenomenon we interpret as affective disengagement. The transition between these regimes is controlled by the interplay between the homeostatic parameters m and b , the influence strength c , and the noise intensity σ_c .

A. Interpretation of the regimes

The unimodal regime centered at $x = 0$ admits a direct psychological reading. When the external environment delivers messages that cancel each other too rapidly for the homeostatic system to track, the most probable state becomes affective neutrality. This is not a peaceful calm, but rather an emotional flatness in which the individual ceases to respond strongly to either positive or negative stimuli. This counterintuitive suppression of affect by excessive contradiction is the signature prediction of the model. The mathematical mechanism is transparent: the noise-induced term in Eq. (12) grows with σ_c , eventually dominating the deterministic bistability and collapsing the two potential wells into one. This result aligns with the broader literature on noise-induced transitions, in which multiplicative noise is known to suppress multistability and stabilize the trivial state [16, 17, 19].

The bimodal regime, by contrast, corresponds to affective instability. Here the individual does not settle into a single emotional baseline, but oscillates randomly between two distinct states. The presence of two well-separated peaks in the stationary distribution implies that the system spends a significant fraction of time in each state, with relatively rare transitions between them. This is the dynamical signature of a system with two competing attractors separated by a finite barrier, where the noise is strong enough to allow occasional transitions but weak enough to preserve the distinction between the basins.

It is important to emphasize that the bimodal regime is not a signature of polarization in the social sense. Polarization, as the term is used in political science, refers to the divergence of attitudes or affects between distinct groups within a population [18]. Our model describes a single individual, and the bimodality it exhibits is intra-individual: the same person samples both states. To connect this result to population-level phenomena, one would need to introduce heterogeneity in parameters or interactions between agents, which we discuss below.

The role of the multiplicative character of the noise deserves emphasis. In our model, the diffusion function is $g(x) = \sqrt{2\sigma_c} \tanh(x)$, which vanishes at $x = 0$ and saturates for large $|x|$. This structure is essential for the noise-induced transition described above. If the noise were additive, corresponding to $g(x) = \text{const}$, the effective potential would retain its deterministic form up to a trivial shift, and the number of minima would not change with σ_c . The multiplicative nature of the noise is therefore not a cosmetic choice but the mathematical feature responsible for the neutralization regime.

This mechanism is closely analogous to the suppression of potential barriers by multiplicative noise in reaction-diffusion systems, as discussed in the classic treatment by [17]. In such systems, noise that couples to the state variable can stabilize the trivial homogeneous state and destabilize non-trivial stationary states, even when the deterministic dynamics are bistable. Our model exhibits

the same phenomenology in a minimal setting: the contradictory stimulation acts as a state-dependent noise source that, above a critical intensity, renders the neutral state the only stable mode of the stationary distribution.

The correlation λ between the two noise sources acts as a control parameter that shifts the critical boundary without changing the qualitative structure. This follows from Eq. (14), where λ enters only through the cross term $\lambda\sqrt{\sigma_c\sigma_b}$. Its effect is therefore quantitative rather than topological, which explains why the phase diagrams for different values of λ exhibit similar shapes with shifted thresholds.

B. Implications for sociophysics

The model contributes to the growing literature on sociophysics, which seeks to understand social phenomena through the methods of statistical physics [2, 3, 20, 21, 23]. Within this tradition, opinion dynamics models typically treat external information as a coherent field that pulls agents toward a fixed position. Our work departs from this tradition by treating the statistical structure of the information stream, rather than its content, as the primary driver of individual-level dynamics.

The phase diagrams of Fig. 7 reveal that the neutral and bimodal regimes are not separated by a simple threshold in σ_c , but by a boundary whose shape depends non-trivially on the other parameters. The island of bimodality in the (σ_c, m) plane indicates that individuals with moderate regulatory capacity are most susceptible to the destabilizing effects of contradictory stimulation. Those with very low m remain locked into a bimodal state regardless of σ_c , while those with very high m are phlegmatic and never develop bimodality in the first place. The lock-in region near $b = 0$ in the (σ_c, b) plane suggests that individuals with near-neutral default bias are particularly resistant to noise-induced neutralization.

These predictions could be tested empirically using ecological momentary assessment methods, in which individuals report their affective state at regular intervals over several days [10, 22]. By exposing participants to controlled streams of contradictory messages and measuring the resulting affective time series, one could estimate the parameters m , b , c , and σ_c and test whether the predicted phase boundaries are reproduced.

A natural extension of this work is to embed the individual-level SDE in a network of interacting agents. Each agent would be subject to external contradictory stimulation and to social influence from neighbors, the latter modeled as an additional drift term proportional to the difference between the agent's state and the average state of its neighbors. Preliminary analysis suggests that the bimodal regime at the individual level can give rise to clustering at the population level, even in the absence of homophily or biased assimilation. This direction will be explored in a separate publication.

C. Limitations

The model has several limitations that should be acknowledged. First, the variable x is an abstraction of core affective valence, not a direct measurement of any observable quantity. The mapping from x to self-reported affect or to physiological indicators remains to be established. Second, the parameters m , b , and c are treated as constant in time, whereas in reality they may evolve under chronic stress or repeated exposure to contradictory information. A more complete model would allow these parameters to depend on the history of the system, creating feedback loops between environmental contradiction and psychological vulnerability. Third, the model does not represent cognitive processing, beliefs, or the content of messages. The predictions are therefore limited to the affective pathway, which is only one of several mechanisms through which contradictory information may exert psychological effects.

D. Directions for future work

Three directions for future work emerge from this study. The first is empirical validation, as outlined above. The second is the multi-agent extension, which would allow the model to address population-level phenomena such as the emergence of affective clustering or the diffusion of disengagement. The third is the introduction of parameter evolution, in which m and b adapt over time in response to the noise intensity. This would capture the gradual erosion of emotional resilience that may accompany chronic exposure to contradictory information, and could lead to a richer phase structure in which the system transitions from bimodal to unimodal over time even at fixed σ_c .

VI. CONCLUSION

We have shown that a minimal model of core affect, supplemented by a stochastic influence parameter, exhibits a noise-induced transition between bimodal and unimodal stationary distributions. The transition is controlled by the intensity of contradictory stimulation relative to the strength of homeostatic regulation. When the noise is weak, the system exhibits affective instability, with a single individual sampling two distinct emotional states over time. This intra-individual bimodality is distinct from social polarization, which involves divergence between groups. When the noise is strong, the system collapses to a single neutral state, corresponding to affective disengagement. This counterintuitive suppression of sustained affect by intense contradiction is the most distinctive consequence of the model.

The phase diagrams reveal non-trivial dependencies on the parameters, including an island of bimodality for intermediate regulatory capacity and a lock-in region near

neutral default bias. These results provide a formal mechanism through which the statistical architecture of an information environment, independent of its content, can shape the qualitative form of affective experience.

The model is deliberately minimal, and its value lies in the transparency of the mechanism it exposes. We do not claim mathematical novelty in the tools employed; rather, the contribution lies in recognizing that contradictory stimulation can be represented as multiplicative noise and in demonstrating that this representation yields a rich, testable phase structure. By treating contradiction as a statistical property rather than a semantic one, we isolate the role of the information stream's structure from the role of its content. This separation is the primary contribution of the work, and it opens a path toward more elaborate models in which the individual-level dynamics derived here serve as the building block for population-level simulations.

Several questions remain open. How does the transition between regimes depend on the temporal correlations of the noise, which we have idealized as white? What happens when the homeostatic parameters themselves evolve in response to chronic contradiction? And how do the individual-level regimes interact when agents are coupled in a social network? These questions define the agenda for future work, and the framework presented here provides the foundation for addressing them.

Appendix A: Derivation of the extrema condition for multiple noises

We derive the condition for the extrema of the stationary distribution when two correlated noise sources are present, as defined by Eq. (13). For an Itô SDE with multiple Wiener processes,

$$dx_t = f(x_t)dt + \sum_k g_k(x_t)dW_t^{(k)}, \quad (\text{A1})$$

the extrema of $P_{\text{st}}(x)$ satisfy

$$0 = f(x_m) - \sum_{k,l} \lambda_{kl} g_k(x_m) g'_l(x_m), \quad (\text{A.1})$$

where $\lambda_{kk} = 1$ and λ_{kl} is the correlation between noises k and l . For our specific case, we have $g_1(x) = \sqrt{2\sigma_c} \tanh(x)$, $g_2(x) = \sqrt{2\sigma_b}$, and $\lambda_{12} = \lambda$.

Substituting these functions into Eq. (A.1), we obtain the diagonal term for $k = l = 1$, which is $g_1 g'_1 = 2\sigma_c \tanh(x_m) \text{sech}^2(x_m)$. The diagonal term for $k = l = 2$ vanishes because $g'_2 = 0$. The two cross terms, $k = 1, l = 2$ and $k = 2, l = 1$, together give $2\lambda\sqrt{\sigma_c\sigma_b} \text{sech}^2(x_m)$. Summing all terms yields

$$0 = -mx_m + b + c \tanh(x_m) - 2 \text{sech}^2(x_m) [\lambda\sqrt{\sigma_c\sigma_b} + \sigma_c \tanh(x_m)], \quad (\text{A.2})$$

which, after rearranging, is exactly Eq. (14).

Appendix B: Analysis of the critical boundary

The transition from a bimodal to a unimodal stationary distribution occurs when the two off-center extrema of the effective potential coalesce with the central extremum. To locate this boundary in parameter space, we impose two conditions simultaneously. The first is the stationarity condition for the extrema, given by Eq. (12), which we rewrite as

$$F(x) \equiv mx - b - c \tanh(x) + 2\sigma_c \tanh(x) \operatorname{sech}^2(x) = 0. \quad (\text{B.1})$$

The second is the condition that the extremum is degenerate, meaning that the second derivative of the effective potential also vanishes. This is equivalent to the derivative of $F(x)$ with respect to x being zero,

$$F'(x) \equiv m - c \operatorname{sech}^2(x) + 2\sigma_c [\operatorname{sech}^4(x) - 2 \tanh^2(x) \operatorname{sech}^2(x)] = 0. \quad (\text{B.2})$$

The critical boundary σ_c^* in the (σ_c, c) plane, for fixed m and b , is obtained by solving Eqs. (B.1) and (B.2) simultaneously for σ_c and x . Eliminating x yields a relation between σ_c , c , m , and b that defines the boundary. The same procedure applies to the multi-noise case by replacing Eq. (12) with Eq. (14).

The system of equations admits a simple solution in the symmetric case $b = 0$. Here $x = 0$ is always a solution of Eq. (B.1). The degeneracy condition at $x = 0$ reduces to

$$m - c + 2\sigma_c = 0, \quad (\text{B.3})$$

which yields the linear critical line $\sigma_c = \frac{c-m}{2}$. For $b \neq 0$, the solution is obtained numerically, and the resulting boundaries are those shown in the phase diagrams of Fig. 7.

Appendix C: Numerical methods

All numerical simulations were performed using the Euler-Maruyama method for Itô stochastic differential equations. For a general SDE of the form

$$dx_t = f(x_t)dt + \sum_k g_k(x_t)dW_t^{(k)}, \quad (\text{C.1})$$

the method discretizes time into steps of size Δt and updates the state according to

$$x_{t+\Delta t} = x_t + f(x_t)\Delta t + \sum_k g_k(x_t)\Delta W_t^{(k)}, \quad (\text{C.2})$$

where $\Delta W_t^{(k)}$ are independent Gaussian random variables with zero mean and variance Δt , consistent with the convention adopted in Eq. (6). For correlated noises, the increments are constructed via Cholesky decomposition of the covariance matrix.

The time step was set to $\Delta t = 0.01$, and 10^4 independent trajectories were generated for each parameter combination. The stationary distribution $P_{\text{st}}(x)$ was estimated by histogramming the final state of each trajectory after discarding a transient of duration $t_{\text{trans}} = 50$. Convergence was verified by halving Δt to 0.005 and doubling the number of trajectories to 2×10^4 , with no significant change in the estimated distributions or in the number and location of the modes.

σ_c	c	m	b	Predicted vs. observed modes
0.5	2.0	1.0	0.0	2 vs. 2
1.0	2.0	1.0	0.0	1 vs. 1
0.8	7.0	0.4	0.8	3 vs. 3
1.5	1.0	1.0	0.5	1 vs. 1

TABLE I. Comparison between the number of modes predicted by the analytical phase diagrams and those observed in Euler-Maruyama simulations for selected parameter combinations.

The complete Python implementation used to generate all numerical results is available at <https://github.com/econofisico/affective-dynamics-contradictory-stimulation/tree/main>. The repository includes scripts for reproducing the phase diagrams, the stationary distributions, and the time evolution shown in the figures.

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