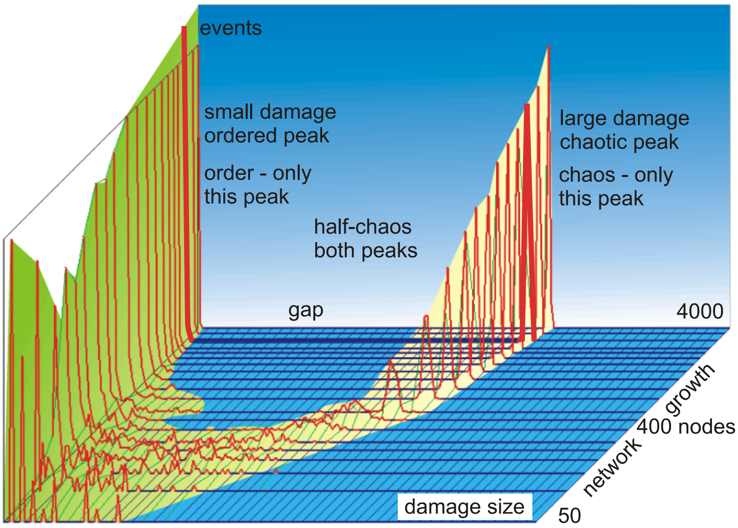
Half-chaos in complex networks challenge for mathematical modelling

Andrzej Gecow[1], Laszlo Barna Iantovics[2]

Abstract
Half-chaos was detected using simulations of complex, autonomous, fixed-size Kauffman networks, but it also occurs in open, growing networks. Mathematicians dealing with deterministic chaos expect its description in mathematical terms. This article (read at least to ch.3.5) can significantly help to create such a description. Kauffman, studying the statistical properties of a set of completely random systems, identifies two states of the system: either it is ordered, where a small disturbance practically always dies out, or it is chaotic, where a small disturbance practically always causes significant damage (a change in functioning). In between, there is a narrow (in the system parameters) phase transition. The assumption of a small attractor causes the system to cease to be fully random, and a third state is revealed - half-chaotic, in which small and large damage have a similar share, despite the system parameters are chaotic. If this system were fully random, it would almost always give strong chaos. A mechanism for increasing system stability following a permanent disturbance has been identified. It involves limiting secondary initiations by shortening the attractor. Leaving in a half-chaotic system only changes that give small damage do not lead out of half-chaos. In the distribution of the damage size, there is a large gap between small and large damages, which naturally defines 'small changes'. Leaving one change which causes large damage leads to a practically irreversible transition to ordinary chaos. Unique stability of half-chaos extends the range of allowed parameters for models of human-created and living systems, previously limited to the edge of chaos by the famous Kauffman hypothesis. Half-chaos explains the essence of the life process.

Keywords: deterministic chaos; complex networks; Kauffman networks; system stability; life on the edge of chaos; Darwinian mechanism.

Visual abstract:



1 Introduction

This paper is an appeal to mathematicians to describe of half-chaos in language of mathematics. The main observed features of order, half-chaos and chaos are depicted on the visual abstract, however, it is for open growing network (h 4,3), which is the most complicated version descibed in this article in ch.4. Half-chaos in an autonomous network with fixed number of nodes should be descibed by mathematicians in the first stage. Distribution of damage size in such case is shown in bold in the graphical abstract, but it is an effect, not an assumption. The assumption are summarised in ch.3.5 and can be comprehensible after ch.2 reminding Kauffman model, and ch.3.1-4 where modification to this model are introduced and mechanism of half-chaos based on short attractor and secondary initiotions are explained.

1.1 This is not a republication of the discovery of half-chaos

Half-chaos was first published in a paper (Gecow 2019/21) five years after the first attempt at its publication (Gecow 2016a,b). The main obstacle was the lack of its description in the language of mathematics. Providing an

[1] Independent researcher, Warsaw, Poland andrzej.gecow@gmail.com
https://sites.google.com/site/andrzejgecow/home
[2] Corresponding author, barna.iantovics@umfst.ro George Emil Palade University of Medicine, Pharmacy, Science, and Technology of Targu Mures, Marinescu, 38, Târgu Mureș, Mureș, 540142, Romania

unambiguous algorithm and computer simulation results did not satisfy the publishers. This remains a fundamental problem in communicating with the community studying deterministic chaos and complex networks. The report went virtually unnoticed, and conferences and simplified descriptions in conference papers were of no help (Gecow 2020; Gecow, Nowostawski 2021). This paper is an appeal to mathematicians fluent in the desired language of mathematics to attempt such a description of half-chaos. Unfortunately, the description in terms of continuum power sets is inadequate here; it must be limited to the range of finite integers. The article contains all the basic data necessary to translate the two basic types (especially the first, simplest) of half-chaos into the mathematical language and also shows the prospect of further, more complex types for growing networks (Gecow, Iantovics 2022, Gecow 2017, 2023). Half-chaos is particularly important for understanding the essence of the life process and is suitable for modeling the majority of complex, functioning entities of human civilization.

It is difficult to expect that a reference, only pointing out the aforementioned articles containing many aspects unnecessary for this task, forcing the reader to read longer descriptions and select relevant information, would be a sufficiently encouraging form. This is why it is necessary to repeat much of the data already published. **It should be emphasized that this article does not present any new discoveries in the field of half-chaos**; it merely organizes the already published knowledge so that it is necessary and sufficient to undertake the postulated task. Recently, a paper (Gecow 2025) was published that also describes half-chaos, but its goal was to identify the essence of the life process, which required a different set of half-chaos characteristics. This paper, on the other hand, gathers the data to translate the description of simulation effects into the mathematical language.

Although this text has much overlap with (Gecow 2025), **it cannot be considered a republication**. Both articles are aimed at different audiences and have different goals. In both cases, the characteristics of half-chaos described have been published previously and are merely a necessary, selective reminder, without which both articles would be insufficiently readable.

1.2 Deterministic chaos in complex networks

Kauffman (1969) proposed a logical network (also called Boolean or Kauffman's), which functions and is suitable for describing complex systems, including living organisms. With its help, he tried to describe the multitude of tissues despite having the same genome. It was only a decade later that physicists noticed this network; biologists have not been interested in it so far. Intensive research on this network led to the creation of the field of complex networks.

The first part of the studies concerned autonomous networks of a fixed size. **One of the basic issues is the stability of such networks**, i.e., their resistance to small disturbances. The results of studies of deterministic Kauffman's networks indicated the existence of two of their states - ordered (resistant to disturbances) and chaotic, in which a small disturbance results in a large change in functioning. The theory of deterministic chaos concerns variables from the sets of the continuum cardinality, its application to networks that are finite and mainly operate on discrete signals requires risky approximations. Despite this, the chaotic state of a deterministic network should be considered deterministic chaos, but it requires the development of mathematical tools for its description.

1.3 Conflict with Kauffman's hypothesis: life on the edge of chaos, discovery of half-chaos

Conclusions from general research on chaos in network's functioning, based mainly on simulations, provided the basis for formulating the Kauffman's famous hypothesis: 'life on the edge of chaos'. It, however, led to many doubts, including a conflict with the assumptions of my (Andrzej Gecow - AG) model explaining certain regularities of biological evolution, which was being developed in parallel since 1974. My (AG) research from 2011 to 2016 on autonomous networks with a fixed number of nodes solved this problem by discovering half-chaos. This research and its foundations are described by (AG) using the first person form. Laszlo Barna Iantovics (BI) participated in later studies of growing and open networks, which are more suitable for practical modeling. The existence of half-chaos was also identified there, which is important for assessing the usefulness of the half-chaotic state and its mathematical description.

Based on many premises shared by other researchers, **I assumed network parameters that should produce strong chaos**. However, the problem is broader, not only biological. Networks describing large systems created by humans and their stability have a practical dimension. The need to defend my model forced me to **look for sources of a radical increase in the stability of networks** describing living and technical systems. In accordance with common belief, I first investigated the sufficiency of regulatory mechanisms, i.e., negative feedback loops, but in the long term, such loops turned out to be insufficient (Gecow 2019/21, 2016a). Secondly, I investigated modularity, which also turned out to be insufficient. A detailed analysis of the mechanisms of instability to small random disturbances led to the indication of the role of short attractors.

In examining a system with an extremely short attractor equal to one step, which is a point attractor, I obtained particularly high stability. Obtaining more sensible small attractors was possible through the evolution of such a system by leaving changes that increased the attractor. Systems created in this way still had a strongly increased permanent stability, attractors had a tendency to decrease, and long evolution, when leaving only disturbances giving small changes in functioning, did not lead to typical chaos. The state of the system with a short attractor with parameters that should give strong chaos, turned out to be other than ordered or chaotic, I called it half-chaotic.

There are two basic types of half-chaos. Starting with these simulations from the point attractor state could have introduced assumptions not included in the definition of half-chaos (only short attractor). I checked this by generating a short attractor in different ways, and it turned out that indeed both types of half-chaos differ slightly, although not fundamentally. **'Half-chaos_1' has a weaker assumption - only a sufficiently short attractor**, but its left peak (of small changes) on the distribution of the size of the change in functioning contains only extremely small changes (Fig. 2). **'Half-chaos_2' arises from a point attractor**, which is an additional assumption. It has a left-hand peak falling exponentially, contains small changes, but not only extremely small ones, which is clearly more adequate to the described reality. It also creates a specific modularity encountered in the percolation process, which allows for large global attractors of the entire network, resulting from the composition of small attractors in many of these modules.

The identification of the mechanism of half-chaos_2 was verified by building such a network of many small active modules separated by inactive 'ice', without starting from the point attractor.

The disturbance was a change in the function of one node for the initial state of the inputs of that node. **This article aims to show the essence of half-chaos_1, i.e., a mechanism leading to a radical increase in the permanent stability of the system to a small random disturbance caused by an exceptionally short attractor. This is to enable the construction of a mathematical description of this phenomenon. Such attempts should begin with the simplest example** (see ch.3.5**)**. However, it is good to be immediately aware of what the later expansion of this description is to concern, especially since half-chaos_2 is more useful.

The definition of half-chaos is based on short attractor, however, observing a system we can suspect the state of half-chaos based on its properties: it is the state of a particular system that is subject to various disturbances and it turns out that despite the parameters obliging to expect chaos, a significant part of these disturbances does not lead to a loss of stability. The damage size distribution has two widely spaced peaks separated by a distinct gap. For changes from the left peak (of small changes of functioning), the system functions similarly, remains itself, and has not lost its stability.

The next phenomenon is the '**evolutionary stability of half-chaos**'. In this case, the **disturbing changes leading to a small change in functioning are left, which causes the evolution of the system.** The functioning of the system should be distinguished from its evolution. It turns out that even a very long such evolution does not lead out of a state of half-chaos. A small change in functioning has a natural limitation of the range of the left peak of the size distribution of such changes, behind which lies a large gap, and behind it, the right peak of large chaotic changes. Leaving one change from the right peak gives an irreversible entry into ordinary chaos, which, in the case of living objects, models elimination (death). Natural selection, therefore, corresponds to leaving only small changes, maintaining identity, and this requires reproduction. Together, this is a Darwinian mechanism.

The above studies concerned autonomous networks of a fixed size. **Growing networks, both autonomous and open**, were further studied (Gecow 2017, 2023; Gecow, Iantovics 2022), since we deal with such in practical applications. They also have half-chaos, but there are many additional, very complex phenomena, some of which will only be mentioned here. In these studies, the disturbance was the addition or subtraction of a node, which is clearly a larger and more complex disturbance.

The significance of half-chaos results from its applications to the description of human-created systems and living objects. It indicates features that have not been given much attention so far and allows the use of a significantly larger and more adequate range of network parameters modeling such systems than Kauffman's research suggested. When applied to the description of living objects, it provides a new, deeper interpretation of quite basic concepts such as the natural criterion of identity (survival) and elimination, necessary to describe the Darwinian mechanism of natural selection and understand the essence of the life process (Gecow 2025).

2 Kauffman model

This chapter recalls Kauffman's model, in which half-chaos is found in the later stages. This description is sufficient to understand the subsequent text; it is not necessary to study Kauffman's works. A similar reminder is found in (Gecow 2025).

2.1 Kauffman network

The Kauffman network consists of nodes connected by links. Input links provide signals that are transformed by the function contained in the node into its result, called the node state. This state is then transmitted as a signal through output links to other nodes, for which they are input links. The states of all nodes at a given moment are the state of the network.

Kauffman's research concerned Boolean networks, where the number of possible signal states[3] **s**=2. They can have different probabilities **p**, but in this simplified article, let's assume that they are equally probable.

Kauffman (1969, 1971, 1986, 1990, 1993) considered autonomous networks, i.e., without links introducing signals from the environment and outputting them to the environment. In simulation studies, it is convenient to determine a constant for all nodes of a given network, the number **K** of input links to the node, while the number **k** of outputs[4] remains varied.

These are directed networks, because signals flow in one direction through the links; dynamic, because they function – new states of all nodes are calculated[5] every time step; deterministic, because the functions are unambiguous.

2.2 Chaos and order in Kauffman's random network

Kauffman studied the statistical properties of a set of any systems[6], generating systems completely randomly. This model indicates two states of the system – either it is ordered, and a small disturbance practically always dies out, or it is chaotic, and a small disturbance practically always causes a great change in functioning. In between, there is a narrow (in the system parameters[7]) phase transition.

The main feature of the chaotic behavior of dynamic systems is great sensitivity to initial conditions, leading to maximally different effects for very similar initial conditions. Such a definition of chaos is primary, and theories using Lyapunov exponents are secondary; they show this phenomenon more conveniently, but in narrower circumstances. The term "deterministic chaos" is not reserved for one of these possible circumstances and is undoubtedly well defined for large but finite, dynamic, discrete deterministic networks. In this respect, we use it in the same sense as Kauffman and many others do, e.g., (Gros 2012; Cappelletti et al. 2022).

Let's randomly generate a network of **N** nodes, where N is large, e.g., 400 or 4000. Functions assigned to nodes, the initial state of the network, and the connections of nodes by links are random. The latter has several ways of random generation, resulting in different variants[8] of the network, but half-chaos occurs in each of the variants studied.

We have two identical networks, in one of which we make a small disturbance. Later, we studied various disturbances, e.g., adding or subtracting a node. Such a disturbance is not small, but half-chaos also occurred. **We**

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[3] In the following, however, we will assume that the number of signal states can be larger than 2 (s≥2), because statistical studies for s=2 are clearly less general and less adequate to reality, but we propose to leave the name of the Kauffman network for such a network, which will distinguish it from the Boolean network.

[4] This is a synchronous method of calculating the functioning of the network, there are also others, but we have not used them.

[5] The names K and k are customary, used by many researchers. The number k is also called the node degree,

[6] He used RBN networks (Random Boolean Networks). This type of network cannot grow.

[7] Typically, K was considered. Derrida and Pomeau (1986) found, that K=2 is critical for p=1/2.

[8] For a long time, networks of the Erdős-Rényi (1960) type called Random Boolean Network (RBN) have been studied. I use the name "**er**" here. Their creation was determined by the rule: fixed N nodes connected randomly. The functions and states of the nodes are drawn separately. Due to the later use of other types of random networks, Serra et al. (2004a) propose the name Classic Random Boolean Network (CRBN) for classical RBNs. They have a characteristic "bell-shaped" distribution P(k) of node degrees k. Until 1999, they had practically no competition. Barabási and Albert (1999) showed that in nature and technology, including the Internet, networks with a different formula, called scale-free (often - BA, here - "**sf**"), usually occur. They have a growth rule in which a new node joins those already present in the network with a probability proportional to the degree of node k. The distribution P(k) here is a straight line on a log-log graph. These networks therefore have few "hubs", i.e. nodes with a particularly large k, but in this type of network they are the largest. Networks with a less extreme formula are also considered - here the hubs do not reach such large k, but there are more of them, for it P(k) is a straight line on a logarithmic graph. Such a network (Albert, Barabási 2002) is called single scale (here - "ss"), and a new node is added with equal probability to any already present in the network. The sf and ss networks have an important property - they can grow, which the er network cannot. Iguchi et al. (2007) used the names SFRBN for sf and Exponential-Fluctuation Networks (EFRBN) for ss. All three of these networks (er, sf, ss) are originally not directed, so for the purposes of the Kauffman network studies they had to be adapted accordingly (Gecow 2019/21, 2016b, 2024) which requires separate treatment of input and output links. Usually, K was set.

will first limit ourselves to a permanent[9] change in the value of the function in one node for the initial state of the inputs of this node. Now we start both networks to function. They are deterministic, then if it were not of this small disturbance, both networks would always have identical states, but the disturbance is there, and the states of the network at specific moments may differ. This difference is measured by the number **A** (from Avalanche introduced in (Serra et al. 2004b)), differing in the state of nodes or **d**=A/N (from damage, this term was used already in 1994).

Of course, the change in functioning, called damage, can die out or remain at a low level, it can also transform into an avalanche and reach a saturation[10] state called Derrida's chaotic equilibrium (Derrida, Weisbuch 1986) (Fig. 1a, b variable dmx).

For Boolean networks (s=2 signal states, we assumed that they are equally probable) and for the number of entries to the node ‹K›<2 (here ‹K› is the average for networks with diverse K), damage usually dies out or remains at a very low level. Such a network is called ordered; it has only a left peak on the damage size graph (as in Fig.2). However, for ‹K›>2, an avalanche almost always[11] occurs (Fig.1a,d) leading to Derrida equilibrium (dmx), which is near the half of all node states (Fig.1b), i.e., near a state almost completely unrelated to the state of the undisturbed network. Such network is called chaotic; it has only a right peak on the damage size graph as in Fig.2.

2.3 Chaos-order phase transition

Passing through ‹K›=2 in the space of different[12] networks, the order-chaos phase transition occurs - a radical change in the statistical dynamic properties associated with stability. In this Kauffman model, a random network can be either chaotic or ordered, it practically cannot be intermediate. Its place is determined by parameters such as ‹K›, s, and omitted[13] in this article, p.

Since in the development of large systems created by humans and in biological evolution, changes are necessary, but small, and such changes occur only near this phase transition, Kauffman put forward the famous hypothesis that life is on the edge of chaos and order. This conclusion is correct, but only for fully random networks. However, neither systems created by humans nor living organisms are fully random; they are usually the result of deliberate selection of changes or natural selection, and in this aspect, this model is too simplified.

I have thoroughly investigated this simplification, which led to the detection of half-chaos (Gecow 2019/21, 2016b, 2020; Gecow, Nowostawski 2021), in which, despite the '**chaotic parameters**' – indicating a highly chaotic network if it were from the set completely random, the behavior of the network after a small disturbance can be chaotic or ordered in a similar proportion, which is contrary to Kauffman's conclusions.

2.4 Attractors

To define and understand half-chaos, we will need the concept of an attractor. It is a cyclic trajectory of the state of the system. Since the number of nodes N and states of nodes s are finite, the number of possible states of the network is also finite, although huge, then after a finite time of the network's functioning, some state must repeat itself. From this point, it will repeat cyclically, because the network is deterministic. Starting from any state, the network must always fall into some attractor (of the very many[14] possible in this network) and this happens after a finite, usually small number of steps. For large networks (e.g., of the order of N=400), attractors are usually very long, but finite - they have a finite number of steps.

In the theory of deterministic chaos, it is slightly different. This theory is built for functions with arguments and values from a set with cardinality of the continuum. The probability of encountering the same input state to a

[9] This network change involves a single computational process. Later, when we study evolution, this change is assessed and sometimes left in the network in time subsequent initializations are tested.

[10] Note that A≤N, so the difference in the states of the undisturbed and disturbed network cannot diverge to infinity. Derrida equilibrium is the maximum loss of information about the similarity of the network based on functioning.

[11] In the tested set of any networks, i.e. for a randomly generated network.

[12] First of all, they differ in the randomly selected structure, functions and initial state, but we can also add the network type (er, sf, ss) and the modifications of sf and ss (sh and si, respectively) resulting from the introduction of node subtraction.

[13] Because we have established p=1/s, i.e. for Boolean networks p=1/2.

[14] One system (network) typically has lot of attractors. Most of the network states are not elements of any attractor, but starting from any state the network always reaches some attractor. The set of states leading to a particular attractor is called `**basin of attraction**'. Kauffman (1969) has proposed that different types of cells in the animal body are an effect of different attractors of its common gene regulatory network (see also (Serra et al. 2010)). Therefore, number and length of attractors for a network with a particular set of parameters is one of the main investigated themes.

function or of the extinction of a disturbance is almost zero. Here, trajectories approach attractors asymptotically, i.e., they reach them at infinity. This theory offers convenient tools that are sometimes[15] used to estimate conclusions in complex networks, but this is a risky approximation.

3 Correction of the Kauffman model, half-chaos

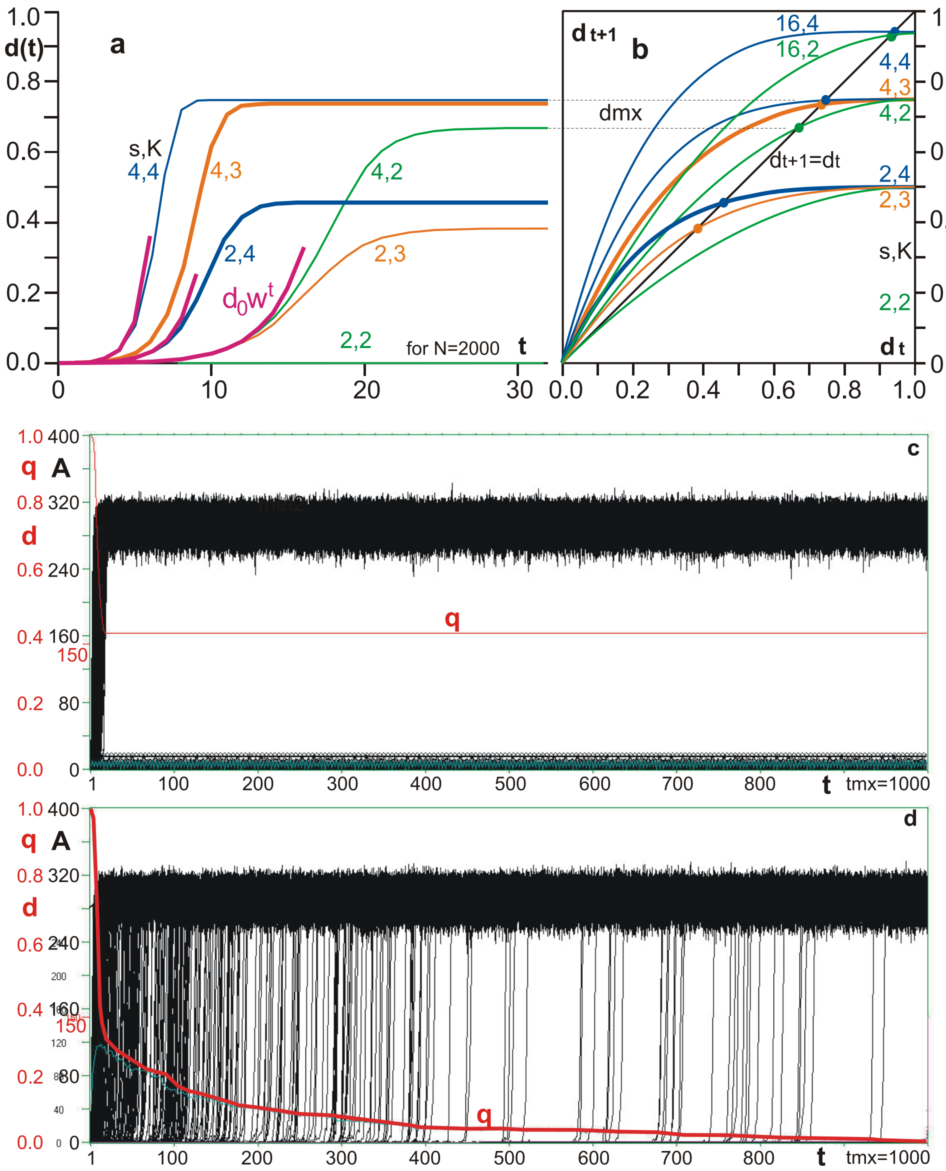

Fig. 1. Damage size in dependency of t – number of functioning calculation steps; tmx is arbitrarily defined maximum of t. Radical difference in q level for half-chaotic and chaotic states.
b- dmx – maximum of damage as 'Derrida equilibrium level' in the so-called 'Derrida plot' from theoretical 'annealed approximation model' built for fully random complex networks (Derrida, Weisbuch 1986), extended for s=4 and 16 (Gecow 2011). Notes – s,K=2,2 as phase transition has no dmx>0.
a- damage size d(t) after small disturbance during the first 30 steps, calculated using 'Derrida annealed approximation model'. In addition, an approximation of the first stage of the explosion to chaos using the 'coefficient w of change reproduction (for signal change on one input link)' (d_0w^t). In later stages, more input links are typically disturbed.
c,d- Below two sets of such trajectories d(t) as in **a** from simulation experiments for half-chaotic **(c)** and chaotic **(d)** networks sf for s,K=4,3, N=400, and all possible (s-1)×N = 1200 initiations (disturbances). Damage size in scale A(t) (back) and d(t)=A(t)/N (red). In addition, the level of order q(t) – part of 1200 processes, still not exploded to chaos, is shown in red. The measure q is connected to stability. As can be seen, the half-chaotic network ends to explode to chaos at the very beginning

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[15] In the case of large finite, discrete, and fully random networks, continuity and the transition to infinity are used to apply the already mastered, typical tools used for the analysis of chaotic phenomena (Aldana, Coppersmith, Kadanoff 2003a; Aldana 2003b).

due to its attractor being short (here it is equal to 20 for the starting state of disturbances), and q remains stable at a high level near 0.4, while q for the chaotic network drops to 0 at tmx. Here, the threshold of small damage is set on A=150 (see gap in Fig.2).

3.1 More than 2 signal values

The Kauffman model allowed us to enter the new world of complex networks and look around. This is a big step that had to start with the simplest possible models. However, some of these simplifications are too large for statistical studies, including primarily the limitation to two-state – logical signals (Gecow 2011). A logical network can describe any complex relationship, but some of the possible states of such a network are usually inadmissible, which does not interfere when its functioning is monitored, but they are taken into account in statistical studies, which introduces a significant error.

Even $\langle K\rangle<2$ is an extreme and somewhat specific situation (it must contain nodes with one input, i.e., K=1). The number of different, equally probable signal states s=2 is also extreme, especially for K=2 (Fig.1.b). In the remaining range of s and K, random networks must (statistically) be chaotic, so the proximity of the phase transition is extremely exceptional. Formally, logical, Boolean, and Kauffman networks are synonyms. By introducing $s\geq 2$, we propose (as in Gecow 2011) to maintain the name Kauffman network for such networks.

3.2 Short attractor, half-chaos, not a fully random system

There are many arguments that living objects have chaotic parameters (Aldana et al. 2003a; Lugue, Ballesteros 2004; Sole, Luque, Kauffman 2000; Turnbull et al. 2018; Gecow 2011), but significantly higher stability than chaotic systems. It is commonly believed that this increase in stability is only the result of the increased presence of negative feedbacks, i.e., regulatory loops, relative to the random state. I also thought so at the beginning of my research, but it turned out that negative feedbacks are only a major auxiliary factor (Gecow 2016a,b, 2019/21, 2024), and the basis for the long-term stability of the system under conditions of random variability is a short attractor, which has become a defining feature of half-chaos. For larger networks, this is a very exceptional feature – half-chaotic networks are therefore selected for this feature, i.e., they are not fully random.

The importance of negative feedbacks was also appreciated by Kauffman, but he introduced this relationship incorrectly (Gecow 2011), which is why he did not take into account that networks with an increased presence of them cease[16] to be fully random. In the face of the importance of half-chaos, this shortcoming ceases to be important, but it led to the not very accurate 'order for free' hypothesis (Kauffman 1996) and attempts to explain increased stability by 'canalyzing' (Kauffman, Peterson, Samuelsson 2004).

3.3 Half-chaos mechanism maintains stability

For chaotic parameters, a disturbance entering a node with one link on average comes out of it with a larger number of links, so the avalanche should grow. This is described by the 'change reproduction coefficient' $w=\langle k\rangle(s-1)/s$ (where $\langle k\rangle$ is the average k). It shows how many output links will be disturbed on average if one input link is disturbed. Its meaning and interpretation correspond to Lyapunov exponents in continuous and infinite spaces. I introduced it already in (Gecow 1975, 1986, 2005; Gecow, Hoffman 1983) for aa[17] networks by analogy to the nuclear reaction and R_0 in biology. When w>1, the damage increases, so the system is chaotic, and when w<1, it decreases, which is a feature of an ordered system. This coefficient describes very well and simply the initial, critical stage of the explosion into chaos (d_0w^t), as shown in Fig. 1.2. After this critical period, the avalanche is already so great that a return to the area of the left peak with a small damage is practically improbable. In the next stages, the probability of more than one damaged input grows and grows, then the explosion ends at Derrida equilibrium.

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[16] Kauffman, describing negative feedback loops, uses after Ashby(1960) the "essential variable". "In the context of Boolean networks, keeping the essential variables in bounds corresponds most simply to holding them fixed" (Kauffman 1993 page 211). In order to model the stability mechanism, all variables (including the essential ones) must be flexible. They cannot be "frozen", because this is only a substitute for regulation by their effects. In the set of networks studied by Kauffman, there are therefore no networks whose increased stability results from the increased (relative to random) presence of negative feedback loops in them, which can give stability even in the range of K>>2, i.e. the parameter considered by Kauffman giving chaos. Such modeling of negative feedback loops creates a tautology: it assumes that due to their effectiveness the system is in the 'ordered' range, which groundlessly excludes great stability in the range of parameters giving chaos in the case of random networks. This error results from identifying the effect (stability) with its cause (system parameter K).

[17] I introduced the aa network in 1974 and studied only this type of network until 2005, when I also started studying the er and sf networks. It has fixed K=k and the result of the function is k-dimensional. Each signal of this result is output by a separate link. In this article we omit this specific network, although half-chaos occurs in it the strongest.

Since Kauffman and I study networks with fixed K, and ‹k›=K for such autonomous networks, it is convenient to see this coefficient in the form: w=K(s-1)/s. From this perspective, it is immediately clear that for s=2, and w=1 there is a "critical connectivity" K_c =2. It indicates a phase transition between order and chaos. K<2 seems out of the scope of interest. In this situation, w≤1 occurs only for s=2 and K=2, which is an extreme. In principle, it is therefore difficult to expect that the real network will be ordered. This can happen for a logical network, where p≠0.5, and small K, as Derrida and Pomeau (1986) indicated with the formula $2K_c$ p(1-p) = 1, seeking the critical K_c for logical networks. The value 2K p(1-p), with a similar interpretation to the coefficient w, has been noticed and used by many[18] researchers.

In the first steps of calculating the functioning of the network after a disturbance, the law of large numbers does not yet apply (there are few nodes with a changed state), so despite such an average (w>1), the disturbance has the right to die out, and this is not unlikely. In further functioning, the initial state of the node's inputs for which its function was changed may re-occur, which will cause a secondary initiation of damage (disturbance of functioning). This happens on average every s^K steps, (for simulations as in Fig. 1 and 2 s,K=4,3 it is 64 steps), which can be enough for the path to the attractor and most of the short attractor. The initial state in which the first disturbance occurred is rarely already on the attractor; the second disturbance, if the attractor has not yet rotated, is therefore usually in new circumstances and may also die out, but it does not have to. If it already happens on the attractor and dies out, then after the attractor rotates, it will also die out. For short attractors, it may not happen at all. Since in the typical situation the attractor is large, so many secondary initiations will occur, the chance of extinction in all secondary initiations is practically zero, and the network practically always reaches the chaotic state of Derrida. In a half-chaotic network, there are very few or no secondary initiations in new circumstances, which allows for a large share of initiations with the final extinction of the damage. We write here the ‘extinction’ of damage, but it includes the remaining damage at a very small level.

Obtaining a short attractor is not easy. An exception is an extremely short attractor (one step), i.e., a **point attractor**. Here, it is enough to change the values of the random functions of all nodes for their current input state to the current state of the node after drawing the network. This does not disturb the statistics of randomness of the network parameters, although we know that the functions cease to be fully random. Starting the above-described random variability of the network from the point attractor, we get a shape of the left peak more adequate[19] (than using only the short attractor assumption) to reality and a strong tendency to shorten the attractor. In order to keep the attractor within interesting limits (not too small), it was necessary to limit the admissibility of smaller values than, for example, 7. It turned out that there is a specific type of dynamic modularity[20], which was also observed by Kauffman near the phase transition. This mechanism does not occur when starting from a forced short attractor (more detailed description in the article (Gecow 2019/21) and documentation (Gecow 2016a)).

The above description of half-chaos is precise, and the descriptions of simulation programs (Gecow 2019/21, 2016a, 2017, 2023) are unambiguous and allow for obtaining the same results, but these are not descriptions in the language of mathematics that mathematicians expect. For them, simulations are not sufficient proofs. In the case of networks, the description concerns discrete quantities and finite networks, while the theory of deterministic chaos operates on continuum cardinality sets, which loses important aspects of half-chaos, primarily the chance for fade out in the few first steps and existence and role of secondary initiations. The extension of the mathematical theory is therefore desirable and awaits a brave mathematician.

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[18] In (Aldana et al. 2003a), a similar equation (6.2) is given: $\mathbf{K_c}$ (**s** -1)/**s** =1, for condition **w=**1. They state: "The critical connectivity $\mathbf{K_c}$ decreases monotonically when s > 2, approaching 1 as s → ∞. The moral is that for this kind of multi-state networks to be in the ordered phase, the connectivity must be very small, contrary to what is observed in real genetic networks.". However, as I show indicating half-chaos, the assumption that such networks should be in the ordered area is false. Derrida and Pomeau (1986) searched for $\mathbf{K_c}$ and found that 2$\mathbf{K_c}$ **p** (1 - **p**) = 1. See also (Aldana 2003b; Fronczak et al. 2008). Shmulevich et al. (2005) used "expected network sensitivity" defined as 2**Kp** (1 - **p**), which Rämö et al. (2006) called "order parameter" or "the so-called Derrida parameter, the discrete analogue of the Lyapunov exponent of continuous dynamical systems" (Cappelletti et al. 2022). Serra et al. (2007 eq. 4.9) used **‹k›q** where **q** is the probability that a node will change its state if one of its inputs changes. It is therefore exactly my **w**. This value coincides with the "Derrida exponent," which has often been used to characterize RBN dynamics."

[19] The left peak contains small changes, but they reach (of course - rarely) the area of A=30. In the case of starting from a forced small attractor (length 21 was used), the left peak practically ends at A=2.

[20] These are small areas of activity with a short attractor inside, usually there are several of them in the network, but they do not contact each other. The remaining network area is 'inactive', i.e. its functioning does not change the states of the nodes, as in the case of a point attractor. Since the attractors in these excited areas are usually of different lengths, the attractor of the entire network can even be quite large - it is the near product of the lengths of these different attractors.

3.4 In-ice-modularity, half-chaos variants

As was mentioned above, we have observed two variants of half-chaos; they differ in the shape of the left peak on the damage size graph. This is visible in Fig.2, more exactly in the book (Gecow 2024) and research documentation (Gecow 2016a). The **condition of a small global attractor alone** gives a large share of zero damage (A=0) and at most small amounts for A=1 and A=2. Damage A=3 was not observed at all. This is variant '**half-chaos_1**'. A short global attractor for the entire network is an adequate description of the essence of the half-chaos phenomenon.

The second variant (from point attractor) - '**half-chaos_2**' is more important, it seems to better describe real living and human-built systems - gives a smooth exponential decline to about A=10, and even[21] to A=20 or more, i.e., it contains small changes, but not: almost only zero. This variant is easily obtained by starting the evolution from a point attractor; its functioning is invisible - the state of the network remains unchanged, so the states of all nodes do not change. The method to obtain a point-attractor network is described above. Such a network is difficult to distinguish from a random one.

Nodes that do not change their state are called ice; they typically occur in the ordered state of random networks, but here the network has chaotic parameters. After perturbations, about 99% of cases of small changes in functioning also give a point attractor. If we limit the attractor to not being too small, then after the disturbance, 'small lakes of activity' (originally: "unfrozen islands" (Kauffman 1990)) are created; usually there are several of them, but they are separated by the ice.

Kauffman observed such a picture near the phase transition in random networks. These are not classical modules, which are defined as more strongly connected by links. These modules can disappear (freeze) during evolution and reappear in a similar composition. In such 'in-ice-modules', the attractors are small because these modules contain a small number of nodes. The combination of these independent local attractors into the observed global attractor can even give a large global attractor. However, **not the global attractor is important in the mechanism creating half-chaos, but these local attractors**.

During the growth of the network (see ch.4.2, Fig.4), when the disturbances were the addition and subtraction of nodes, classical modules defined by different link densities were created. When the network and module were already large, typical chaos could appear in such a classical module. If the admissible "small change", the threshold of which for the entire network also increases, was still greater than the Derrida chaotic equilibrium level of such a module, then this disturbance was accepted. This was the only observed mechanism of exiting half-chaos to chaos despite meeting the condition of a small change (Gecow, Iantovics 2022; Gecow, Iantovics, Tez 2022; Gecow 2017, 2023). Here, the model should probably be expanded.

3.5 Summary of assumption for Mathematicians

Chaos theory has been practiced in mathematics since the 1990s. The starting point was Devaney's book (1989), which considers discrete-time dynamical systems. There are many mathematical approaches to the phenomenon of "deterministic chaos." There is no single, universally accepted definition of it. Many mathematicians adopt Devaney's definition, which identifies three features of chaos: 1. Topological transitivity; 2. Density of periodic points; and 3. Sensitive dependence on initial data. Feature [3] is generally considered the quintessence of chaos, but it has been shown (Banks 1992) that [1] and [2] yield [3], but not vice versa. In this theory, the global attractor is the limit of a function as t approaches infinity; it is the set to which trajectories deriving from its complement asymptotically approach.

A discrete-time process should be described. This process is a sequence of recursive function values, which are the successive states of a deterministic Kauffman network. Initially, let's limit our interest to autonomous networks. This function defines the entire network composed of N nodes. We are interested in the range 400<N<4000 – the networks shouldn't be too small, but they don't have to be very large. Each of these nodes contains a function that transforms the current signals at the node's inputs to its later state, which in the next time step will be transmitted via output links to other nodes, constituting their input signals. Signals are natural numbers in the range 0 to s-1. This means that the signal has s variants, assumed to be equally probable. (Originally, in the Kauffman network, s=2, meaning these are logical signals, but we also use s larger ones, say up to 16, however, we postulate retaining the name Kauffman network.) The functions in the nodes are unique, also randomly generated, with equal probability for each variant. Signals flow through the links in one direction, meaning the network is directed. The number of outputs from a node is conventionally described by the number k and can be any number. This is sometimes called the node's degree. It plays a significant role in defining the network type (e.g., scale-free (Barabási et al. 1999), single scale (Albert, Barabási 2002)). Generally, the number K of inputs to

[21] This is for the initialization of a change of function in one node for one input state. In the case of perturbations in the form of adding and subtracting nodes (see ch.4.2, Fig.4), which are not so small, the threshold of a small change grows with the size of the network, and the gap is gradually 'filled' as a result of the formation of classical modules.

a node can also be any number; we will leave this option for a later stage. It is convenient to assume a constant K for a given network, e.g., K=2, 3, or 4. Only for K=2 and s=2 can such a network be "ordered," while other combinations should result in "chaotic" networks. However, this statement is only statistically true. The network state at a given time step consists of all the states of the network's nodes. The initial state of the network should be random.

This description uniquely determines the trajectory through the network states space. Since N and s are finite, the network state space is also finite, and after a finite time of such a deterministic process, some state of the network must repeat. From this point on, the network states form a closed loop called an attractor. Such a global attractor is therefore reached after a finite time. Typically, these attractors are large for such network parameters, but they can be short, for example, on the order of several dozen steps. The length of the attractor, along with the network's path to the attractor, is crucial for the network's stability.

We consider stability to a small perturbation. For the sake of clarity, assume that such a small initial perturbation is a change in the function value at one node for the initial state of the inputs to that node. Stability is measured by the number A of node states differing, at a given time, between the perturbed and unperturbed network. The perturbation, called damage d=A/N, under average conditions (in a set of random networks, not for K=2 and s=2), increases to a high level called "Derrida's chaotic equilibrium" (Fig.1.ab), but for a set of networks with a sufficiently short attractor, a significant portion of initializations do not cause such an increase in damage, which remains at a very low level. This results in behavior similar to ordered networks, despite the parameters of these networks practically always resulting in chaos in a set of random networks. Networks with a sufficiently short attractor are therefore neither ordered (with very little damage after many steps) nor chaotic (with the resulting damage at the Derrida equilibrium level). This third state, called half-chaotic, requires a mathematical description. It is fundamental to modeling real systems.

4 Simulation results and interpretations

4.1 Basic simulation results

The main result of the simulation studies is the recognition of the mechanism of a radical increase in the stability of a system in the set of systems with a short attractor. This mechanism has already been described above and is the main basis for building a mathematical description of half-chaos. It consists in limiting the number of secondary initiations after a permanent disturbance.

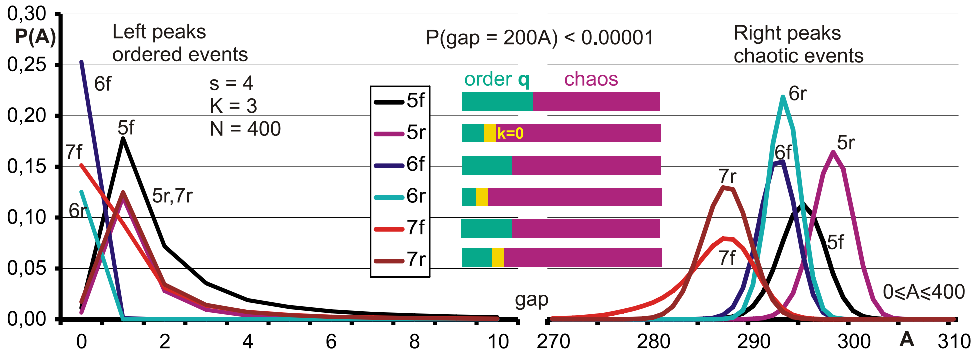

Fig. 2. The main result – distribution of damage size. A – Avalanche, number of different node states at maximal t (tmx=2000) in disturbed and undisturbed networks. The first character of the curve description (5, 6, 7) indicates the experiment as described in (Gecow 2019/21, 2016a), while the second shows the network *type* ('f' – scale-free, 'r' – Erdős-Rényi 'random'). In the networks, there are N = 400 nodes, and damage, d = A/N, s = 4, and K = 3. Fully random networks with such values of parameters s and K are chaotic (only the right peak exists), but here, the left peak exists, and its share is not negligible (order - *q* and chaos are shown in the middle). Such a picture, with two peaks, is for each particular network. They are neither ordered nor chaotic; thus, we call them 'half-chaotic'. The results from a few hundred networks for each experiment are summarized here. Errors are not calculated due to the presence of many types of rare causes, which make such a calculation inadequate—the smoothness of the curves is enough. The range of argument A (0≤A≤400) is divided into parts: left 0-10 for an exact view of left peaks, where experiment 6 exhibits an unusable feature; gap between A=10 and A=270, where probability is negligible, and area of right peaks of damage chaotic equilibrium. In the middle, over the gap, fractions of ordered (q) and chaotic (1 - q) events of avalanche after a small disturbance is shown. In the range of q in 'r'—an Erdős-Rényi 'random networks, an order resulting from the absence of output in some nodes (k = 0)' is shown in yellow. All results presented here concern only the effects of limiting global attractors. Experiment 5 – start from point-attractor, 6 – attractors are artificially reduced to 21 steps, 7 – networks are built using rules ('unfrozen lakes of activity') observed in experiment 5. Typical chaotic networks have q too small to be visible.

However, in physics, the basis and ultimate reference is the experiment. The mechanism is recognized on its basis, so it is a secondary explanation of observations. That is, the definition of half-chaos as a state of a system with parameters implying chaos for almost every system from a system set of fully random (we call such parameters ‘chaotic’), based on the assumption of a short attractor, is secondary. The original definition of half-chaos directly describing the experiment is the separation of systems with chaotic parameters, which, in the distribution of damage size (after a small disturbance), have two peaks with a similar share - not only the 'right' peak of large damage near Derrida's chaotic equilibrium, but also the 'left' one, containing tiny damage. Here, the similar share of both peaks despite the chaotic parameters is crucial.

For er networks, the left peak occurs in a noticeable size even in fully chaotic networks, but it results from the statistical presence of 'blind' nodes, i.e., those without output links, so the disturbance of these nodes cannot propagate. This is a reason for the increased stability, which we do not want to include in the half-chaos.

The simulation results presented in Fig. 1c, d, and 2 have a more psychological task, allowing us to see the effects of limiting to a short attractor, and thus to believe in its specificity. Fig. 1c, d show particularly clearly the difference between a half-chaotic and a chaotic system in terms of stability. The theoretical results shown in Fig. 1a, b are useful for understanding it. Fig. 2 shows, first of all, two peaks in the distribution of damage size occurring simultaneously for a single half-chaotic system, a large gap between them defining a ‘small change’ significant in the **evolutionary stability of half-chaos**[22], and the shares of cases of stability and instability after a random disturbance, despite the chaotic parameters. The left peak in Fig. 2 shows the difference between half-chaos_1 and _2, which is important for applications.

4.2 Autonomous growing networks, growing classic modules

So far, the results obtained from the simulation of autonomous networks with a constant number N of nodes have been discussed. Such networks were the main subject of research by Kauffman and many other researchers because they are much simpler. However, Barabási et al. (1999) noticed that the growth of the network and the rules of this growth have a huge impact on its properties. The main network studied until then, er (Erdős, Rényi 1960), cannot grow.

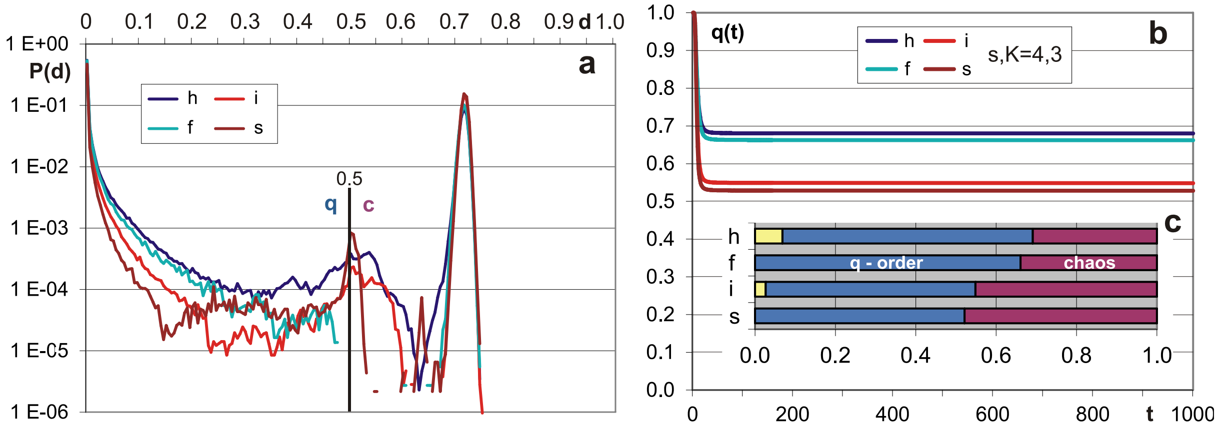

Fig.3. The main results of checking half-chaos existence in the autonomous growing networks.
a - Specific for half-chaos two peaks of P(d) with a deep gap between exist.
b - High level q of ordered reaction on disturbance despite s=4 and K=3 stabilizes during the time of calculation.
c - Fraction q of ordered reaction (below threshold) and chaotic (at Derrida chaotic equilibrium).
Damage size distribution P(d) in **a** during growth from N=50 to N=550. Damage d scaled to the current N. The threshold of small change is 0.5 of the current N. For each network type (only the second letter of the network name is used in figures) of h, i, f, s 600 networks' growth are simulated. Note, that it is a logarithmic scale and the gap between peaks, however not empty, is deep, on linear scale (as in Fig.2) capacity of the gap are invisible. The capacity of gap will be later discussed, it is created by modules, which for particular networks give much clearer additional peaks, but in a different place. These additional peaks appear most intensively for the i and h networks in which the nodes are removed; much weaker in f and s, where there is no removal. They are intensified by the increase of the threshold level.

The growth of the network is primarily the addition of new nodes. Such networks were proposed by Barabási et al. (1999) - the scale free (sf) network and Albert and Barabási (2002) - the single scale (ss) network. Their rules did not include the removal of nodes, which also has significant practical significance, so in the research presented here, these two networks were supplemented with subtraction, which gave the networks sh and si, respectively. The exact rules of these operations are not simple; they require a long description, which is not important in this article - they are described in detail in (Gecow 2011, 2024).

[22] The evolutionary stability of half-chaos was already defined in ch.1.3.

Adding and subtracting nodes is no longer a very small disturbance; moreover, it introduces many additional phenomena that spoil the purity of the obtained results, making it difficult to assign causes to the observed effects. It also strongly affects the technical problems of the simulation, mainly the comparison determines the amount of damage. In the subject of half-chaos, the basic question was whether the presence of half-chaos is still observed in such circumstances. Fig. 3 shows the basic results in this area. Especially, Fig.3c clearly shows that half-chaos is strongly present in the growing networks.

The main phenomenon affecting the readability of the results and indeed dangerous for the evolutionary stability of half-chaos is the presence of growing classical modules. In a relatively small share, these are particularly large modules growing faster than the small change threshold. This phenomenon is the only one found so far that can lead to the end of half-chaos despite accepting only changes below the small change threshold.

An example of the evolution of an autonomous network si 4,3 with such a large growing module is shown in Fig. 4. Here, the networks grow from N=50 to N=550 by randomly adding and subtracting nodes. In the subsequent d(t) graphs (compare Fig. 1), trajectories are collected after each accepted initiation until the network grows by the next 50 nodes. Fig. 4 shows the initial segments of these dependencies from three such stages: 100-150, 350-400, 500-550. The small change threshold is 0.5N, i.e., it grows proportionally to N. The large module at the beginning (Fig.4a) is below the small change threshold for the entire network, so even when initiation within its range leads to chaos - it reaches the Derrida equilibrium level within the module, but does not release damage beyond its limits, it is accepted. Almost every change within the module is accepted, so the module grows quickly, much faster than the entire network. In phase b, it is already on the border of a small change, but the trajectory often returns below this threshold, and the average in the last section turns out to be below the threshold; the change is accepted. The red line of level q is determined based on the number of processes below the small change threshold, so in phase b, it is very oscillatory and much lower than in phases a and c, where it is smooth and almost horizontal. In phase c, the module has already exceeded the small change threshold, and the changes initiated in it are usually eliminated, but the level of q remains high.

If after passing the threshold, the trajectory does not go under the threshold even once in the next 70 steps, it is considered to have reached Derrida equilibrium, and it is not worth counting it any further. The basics of these rules and their detailed descriptions can be found in (Gecow 2017) and (Gecow, Iantovics 2022). As you can see in stage b, it happens that damage leaves the module and reaches Derrida's chaotic equilibrium for the entire network.

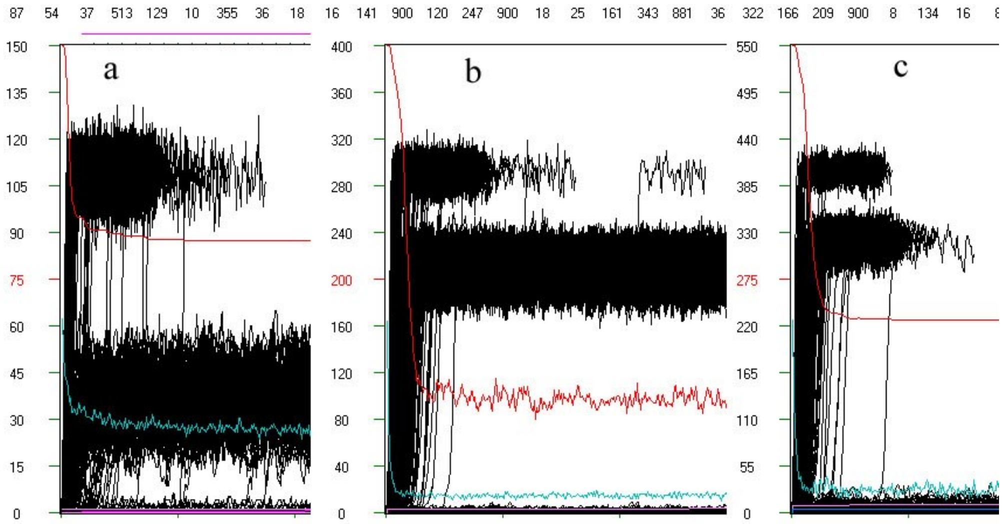

Fig. 4. Initial slices of the d(t) dependence from three stages of N growth of si 4,3: a 100-150, b 350-400, c 500-550. The small change threshold is 0.5N, q is marked in red as in Fig. 1c,d. Above the lengths of subsequent attractors after changes accepted from the beginning of the growth stage a, 900 means no attractor detected at tmx=1000. More detailed description in the text.

4.3 Open growing networks

In practice, we have not only growing networks, but also open ones, which further complicate both simulations and interpretations of their results. Documentation of research in this area can be found in (Gecow 2023) and a published short description in (Gecow, Iantovics 2022). Laszlo Barna Iantovics participated in modifying the simulation program, which had to be radically rebuilt. The networks always had 64 inputs and 64 outputs connecting them to a constant ‘environment’. The amount of damage was measured as before by the number A of changed states, but also at the output of the network by the number L of changed signals. Both of these quantities were averaged over the final, stable section just before tmx. The dependencies for L, although interesting for applications, will be omitted here. The networks were seen to tmx=1000 and grew to N=4000.

The presence of system outputs meant that the ‘complexity limit’ of the network (Gecow 2009), above which the presence of half-chaos had to be studied, was of the order of N=500. Here, too, the image was strongly influenced by classical modules already known from growing autonomous networks. The occurrence of these modules was not uniform – in some networks there were almost none (Fig. 5a), in others there were a few (Fig. 5b), in still others – many. Sometimes they disappeared or appeared near the end of growth. The presence of modules shifts the right peak of Derrida equilibrium to lower values, causing the formation of many such ‘right’ peaks associated with specific modules.

Half-chaos clearly occurs in such networks, and is particularly visible when the modules interfere little, as in Fig. 5a. Despite the complexity of the model and long simulation times, much more should be put into this model – in particular, the control of half-chaos in modules should be expanded. This theme is important for cancer (Gecow, Iantovics, Tez 2022). The starting from the point attractor turned out to be unnecessary for the observed half-chaos to be of type 2 (with a large amount of ice, modules in the ice, and an extended left peak). For this paper, it is important to show that in the range of practically interesting systems - growing and open- the basic properties of half-chaos are superimposed on many very complex phenomena. It is therefore difficult to expect that the mathematical description of half-chaos obtained will be immediately useful in practice.

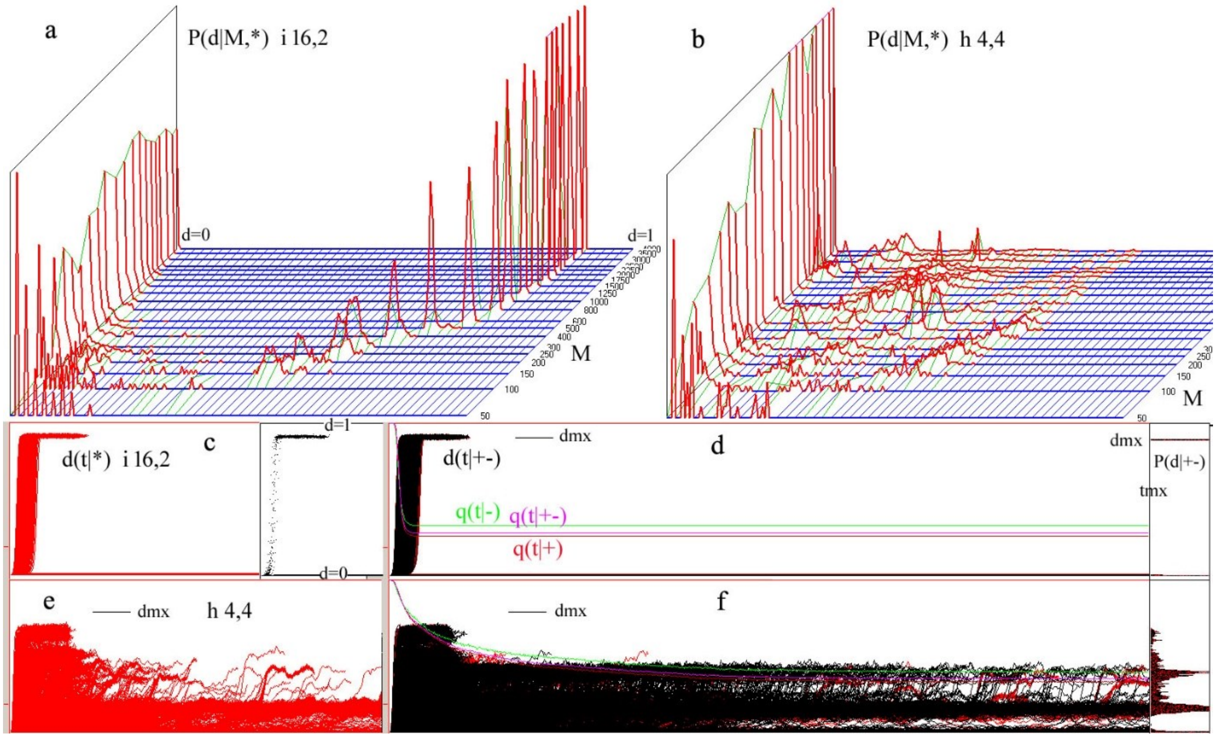

Fig.5. Example of simulation results of open, growing networks si 16.2 and sh 4.4.
a,b - P(d|M,*). M are successive states of network growth; the increments between them are not similar, rather they are presented on a logarithmic scale, and the numbers indicate N. The symbol ‘*’ denotes the measurement of the network state by a small disturbance as was used for autonomous networks with constant N (change in the value of the function of one node for its initial state of inputs). While a clear break separates the left and right peaks in **a,** then in **b,** several right peaks of systematically growing modules can be seen.
c,e – Dependence d(t) for the measurement as in **a,b** for N=4000. Only the initial part is presented, because the rest to tmx was practically identical. Here, too, as in the case of autonomous networks in Fig. 4, counting was stopped after 70 steps after passing the small change boundary.
d,f – Measurement of d(t) during the growth between N=3500 and 4000. Here, q is marked for the whole (node additions and subtractions) and separately.
In graphs **c** and **d,** the level of dmx is exactly as calculated in Fig.1.b, but in **e** and **f,** the achieved Derrida equilibrium level is lower, because the part of the network accessible to the chaotic avalanche does not contain modules into which damage has not entered.

4.4 Natural criterion of identity, Darwinian mechanism

Let us return to the interpretation of disruptive changes and damage initiated by them – changes in functioning. If damage has died out or remained at a small level, i.e., the change in the functioning of the system is small, then such a system remains 'itself' – it functions very similarly. We call such a change ordered.

However, if damage has developed into an avalanche, so it has achieved Derrida's equilibrium as in the case of chaos, then the system functions completely differently, it ceases to be the same system, even though the links and functions in the nodes (apart from one for one input state of one node) have remained unchanged. If this system described a living organism before initiation, then after that it undoubtedly describes something completely different – such an event is a model of death, in other words – elimination. The probability that the assumed goals of a system built by man will still be achieved after a small, unforeseen change in the system, which will result in a large, chaotic change in functioning, can be assumed to be practically zero. In this situation, we say that the system 'has stopped working' (implicitly – effectively/correctly).

When we leave only disruptive changes that initiate small changes in functioning, we get the evolution of a system that preserves its identity. What is important – it also preserves the state of half-chaos. I called this important feature 'evolutionary stability of half-chaos'. It is quite surprising, so I made many attempts to help in achieving chaos, but chaos was not achieved in this way (Gecow 2016a).

Leaving one large change (large damage after a small disruptive change) causes a practically irreversible transition to normal chaos. The decision not to leave such an initiating change requires a return to the state before this change (if it is still possible), and for systems not controlled by intentional beings, this means the necessity of reproduction.

We get such a full mechanism of Darwinian natural selection. This way, we obtain a deep view of what the life process is (Gecow 2025). The question remains: what does it mean that the change in functioning is 'small'?

The distribution of damage size is specific: it contains 2 peaks (Fig. 2): the left one – small 'orderly' changes, and the right one – large changes close to Derrida's chaotic equilibrium. There is a large gap between them, where changes of such an intermediate size practically do not occur (Gecow 2016a,b, 2020). The indication of a small change is therefore determined in a natural, objective way; it is not an arbitrary decision of the observer. This creates a natural criterion of the identity[23] of the evolving system, which we have not had so far.

5 Summary

Half-chaos is an intermediate state in terms of stability between chaos and order in the scope of complex dynamic networks with a finite number of signal variants and chaotic parameters. Its basic feature is a short attractor.

Half-chaos was detected and demonstrated by computer simulation methods. These are rigorous methods, also described sufficiently unambiguously, allowing for the repetition of experiments. The analyses carried out allowed us to understand the mechanism in which half-chaotic systems have significantly increased the stability. This provides a sufficient basis for mathematicians to create a description in the language of mathematics and to extend the theory of deterministic chaos to finite, discrete complex networks.

Humans and the systems they build are part of the life process and are governed by the same laws. Therefore, half-chaos is important both for understanding the life process and for describing and predicting stability in systems engineering – this is a practical application of half-chaos. Such predictions are typically made using mathematical methods, which is why developing a mathematical description of half-chaos is so important.

The process of life is maintaining a system in a state of half-chaos despite random variability. This model, described in more detail in the first chapter of the book "Draft of the Deductive Theory of Life" (Gecow 2024), provides a definition of life (Gecow 2025), indicating its essence while defining as precisely as possible the basic concepts in which we describe this process. Mainly, they are half-chaos and the natural criterion of the identity of the evolving system, which allows us to strictly consider the evolution of anything at all. We have not had this natural criterion so far. The famous Kauffman hypothesis, "life on the edge of chaos" is significantly corrected to "life evolves in the half-chaos of not fully random systems", which allows modeling living objects using networks with chaotic parameters. This is a much larger, more adequate area. So far, such a possibility has been blocked by the Kauffman hypothesis.

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[23] In another aspect, the identity criterion for living objects was given in (Nowostawski, Gecow 2011).

Author Contributions:
The concept of half-chaos research was based on the experience of A.G., while L.B.I. did the main work on programming the simulations of the growing open networks and carried out a significant part of it. All authors have read and agreed to the published version of the manuscript.

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