Emergence of Classical Gravity from Continuous Spacetime Dimensions

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Abstract

We develop a geometric framework in which classical gravity emerges from spacetime endowed with continuous, non-integer effective dimensions. Spacetime is modeled as a fractal measure space, exemplified by Cantor-dust (CD) constructions, where the Hausdorff measure replaces ordinary volume. We point out that the notion of *fractal measure* plays the physical role of *gravitational mass*, leading to modified Poisson equations and nonlocal Green's functions without introducing additional matter components. The resulting gravitational kernel exhibits scale-dependent behavior determined solely by the effective dimension of spacetime. This approach provides a unified explanation for long-range gravitational anomalies and establishes a direct connection between Dimensional Regularization, nonlocal gravity, and emergent Dark-Matter phenomenology.

Key words: Fractal spacetime, Continuous spacetime dimensions, Dimensional Regularization, Dark Matter, nonlocal gravity, Cantor Dust.

1. Introduction

The standard formulation of gravity assumes that spacetime is a smooth four-dimensional manifold endowed with a metric structure. Both Newtonian gravity and General Relativity rely fundamentally on the existence of local neighborhoods, differentiable coordinates, and integer-dimensional volume elements. However, a growing body of evidence from the complex dynamics of nonlinear systems, Renormalization Group flows and astrophysical observations suggests that spacetime may exhibit nontrivial structure across scales.

Independently, Dimensional Regularization in Quantum Field Theory has long relied on the formal extension of spacetime dimension to non-integer values. While traditionally regarded as an exclusive calculational device, recent work has shown that dimensional continuation can be consistently interpreted as integration over fractal measure spaces [8]. In this context, the work of Tao [1] has demonstrated that Dimensional Regularization remains valid when the underlying spacetime is equipped with an appropriate fractal measure.

In this paper, we take this perspective seriously at the classical level. We propose that spacetime itself possesses a continuous effective dimension D, realized geometrically through Cantor-Dust–like constructions. Crucially, such spaces possess no ordinary volume, yet they carry a finite Hausdorff measure. We show that this measure must be identified with gravitational mass, as it is the only quantity to which gravity can be coupled consistently.

By reformulating the Poisson equation on a fractal measure space, we derive a nonlocal gravitational Green's function whose scaling depends explicitly on the effective spacetime dimension. No additional Dark Matter fields, modified inertia, or phenomenological force laws are introduced. Instead, gravitational anomalies arise as direct consequences of spacetime geometry itself.

We caution upfront that this report serves as an introductory study. Interested researchers are encouraged to review, develop or refute the body of ideas detailed here.

2. Mathematical Framework and Field Equations

2.1 Fractal Spacetime and Measure Structure

Following [1-7], we model spacetime as a metric measure space $(\mathcal{M}, d, \mu)$, where $\mathcal{M} \subset \mathbb{R}^n$ is a fractal support, $d(x, y)$ is the embedding metric, and μ is a Hausdorff-type measure. A prototypical realization is provided by Cantor dust, constructed as the Cartesian product of Cantor sets. Such spaces are totally disconnected, self-similar at all scales, and lack differentiable local neighborhoods [1-3].

The defining property of the fractal measure is its scaling behavior:

$$\mu(B_r(x)) \sim r^D, \qquad (1)$$

where $B_r(x)$is a metric ball of radius $r$centered at x, and D is the effective (Hausdorff) dimension of spacetime. For $D \neq n$, the ordinary Lebesgue volume vanishes identically,

$$\int_{\mathcal{M}} d^n x = 0, \qquad (2)$$

while the fractal measure remains finite and nontrivial. Consequently, all physical integrals must be defined with respect to $d\mu(x)$.

2.2 Identification of Fractal Measure with Gravitational Mass

In classical gravity, mass is defined operationally as the quantity sourcing the gravitational field through spatial integration. In a fractal spacetime (which, by definition, is endowed with *continuous spacetime dimensions* D), the only admissible generalization of volume integration is via the *fractal measure*. We therefore identify the enclosed gravitational mass as

$$M(r) \equiv \int_{B_r} d\mu(x) \sim r^D. \qquad (3)$$

This identification is *not optional*: since ordinary volume vanishes, the fractal measure is the only quantity capable of sourcing gravity. It follows that the measure plays the physical role traditionally assigned to mass density.

2.3 Gravitational Field Equation on Fractal Spacetime

The Poisson equation of Newtonian gravity is generalized to fractal spacetime by replacing the Laplacian with a fractional or measure-adapted operator $\mathcal{L}_D$:

$$\mathcal{L}_D \Phi(x) = 4\pi G\, \mu(x). \qquad (4)$$

Here, $\mu(x)$ denotes the local density of fractal measure, and $\mathcal{L}_D$ reduces to the standard Laplacian only when $D = 3$. For isotropic fractal geometries, $\mathcal{L}_D$ may be represented by a *fractional Laplacian*,

$$\mathcal{L}_D \equiv (-\Delta)^{D-1/2}. \qquad (5)$$

2.4 Nonlocal Gravitational Green's Function

The gravitational potential is expressed in terms of a Green function $G(x, y)$ defined by

$$\mathcal{L}_D G(x, y) = \delta_\mu(x, y), \qquad (6)$$

where $\delta_\mu(x, y)$is the Dirac delta distribution with respect to the fractal measure:

$$\int f(y)\, \delta_\mu(x, y)\, d\mu(y) = f(x). \qquad (7)$$

The solution of Eq. (6) yields

$$\Phi(x) = \int G(x, y)\, d\mu(y). \qquad (8)$$

Dimensional analysis fixes the asymptotic form of the kernel:

$$G(x, y) \sim \frac{1}{|\, x - y \,|^{D-2}}, \qquad D \neq 2. \qquad (9)$$

For $D = 3$, Newtonian gravity is fully recovered, $G \sim 1/|\, x - y\,|$. Note that, for $D < 3$, gravity is enhanced by long-range interactions, while for $D > 3$ it becomes limited-range interaction, that is, *screened*.

2.5 Emergent Force Law and Dimensional Gravity

Gravitational acceleration follows directly from the potential:

$$g(r) = -\nabla\Phi(r) \sim \frac{GM(r)}{r^2} \sim r^{D-2}. \qquad (10)$$

Thus, the force law is determined entirely by the effective spacetime dimension. Unlike MOND or alternative gravitation theories, *no modification* of the gravitational constant or field equations is required [4-7].

2.6 Action Principle on Fractal Spacetime

Only to the extent that the action principle holds as *effective approximation* in fractal spacetime, the gravitational action can be written as

$$S = \int d\mu(x) \left[\frac{1}{16\pi G} R + \mathcal{L}_{\text{matter}}\right]. \qquad (11)$$

Variation with respect to the metric or measure yields field equations in which curvature responds directly to changes in the fractal measure. Even in the absence of conventional matter fields, *spacetime itself carries gravitational mass.*

2.7 Relation to Dimensional Regularization

Tao has shown that dimensional regularization is mathematically consistent when integrals are defined over fractal measure spaces [1, 7]:

$$\int d^D x \;\leftrightarrow\; \int d\mu(x). \qquad (12)$$

In this framework, propagators take the form

$$G(x,y) = \int \frac{d\mu(k)}{|\,k\,|^D} e^{ik\cdot(x-y)}, \qquad (13)$$

demonstrating that nonlocal gravitational kernels arise naturally from the geometry of spacetime rather than from ad-hoc modifications.

2.8 Stochastic and Nonlocal Effects

Inherent fluctuations in the fractal measure,

$$\mu(x) \to \mu(x) + \delta\mu(x), \qquad (14)$$

lead to correlated gravitational force fluctuations:

$$\langle F(x)F(y) \rangle \sim \nabla_x \nabla_y G(x,y). \qquad (15)$$

These correlations generate effective self-interactions and transport phenomena without introducing new particle degrees of freedom. These observations carry key consequences in modeling Dark Matter and primordial gravity [4-7].

3. Phenomenological Implications

3.1 Galactic Rotation Curves

In the present framework, the gravitational acceleration produced by a fractal spacetime of effective dimension D scales as [4-7]

$$g(r) \sim r^{D-2}. \qquad (16)$$

For $D < 3$, this leads to a slower radial decay of the gravitational field than in Newtonian gravity. The circular velocity of a test particle orbiting at radius r,

$$v_c^2(r) = r\, g(r), \qquad (17)$$

scales as

$$v_c(r) \sim r^{(D-1)/2}. \qquad (18)$$

For a background having $D \approx 1$ (filamentary Cantor Dust), the velocity becomes approximately constant, reproducing the observed flat rotation curves of spiral galaxies without invoking Dark Matter halos. Importantly, this behavior arises from the geometric scaling of the fractal measure rather than from modifications of inertia or phenomenological interpolation functions. The effective dimension may vary weakly with scale, allowing smooth interpolation between Newtonian dynamics in the inner regions and flat rotation curves at large radii. Similar findings extend to the *baryonic Tully-Fisher relation* [4-7].

3.2 Gravitational Lensing

Gravitational lensing depends on the integrated gravitational potential along the line of sight. In fractal spacetime, the potential is given by

$$\Phi(x) = \int G(x,y)\, d\mu(y), \qquad (19)$$

with the nonlocal kernel $G(x,y) \sim\mid x-y \mid^{D-2}$.

Because fractal measure acts as gravitational mass, lensing probes the same geometric quantity responsible for dynamical effects. The deflection angle α scales as

$$\alpha \sim \int \nabla_{\perp}\Phi\, dl \sim \int d\mu, \qquad (20)$$

implying enhanced lensing relative to visible matter alone. This may naturally explain the observed coincidence between dynamical mass inferred from rotation curves and lensing mass, without introducing collisionless Dark Matter particles. Since lensing depends only on the measure distribution, the framework may predict consistent lensing signals even in systems with low baryonic content.

3.3 Emergent Self-Interacting Dark Matter Phenomenology

Fluctuations of fractal measure induce correlated force fluctuations:

$$\langle F(x)F(y)\rangle \sim \nabla_x \nabla_y G(x,y). \qquad (21)$$

These correlations lead to effective momentum exchange between trajectories moving through spacetime, giving rise to transport phenomena analogous to *self-interacting dark matter* (SIDM) [4].

The effective scattering rate scales with the variance of measure fluctuations,

$$\Gamma_{\text{eff}} \sim \int \langle \delta\mu(x)\, \delta\mu(y)\rangle\, G^2(x,y)\, d\mu(y), \qquad (22)$$

yielding cross sections that are:

- velocity dependent,

- enhanced in low-velocity environments,
- suppressed in high-velocity systems.

These properties align with SIDM phenomenology inferred from dwarf galaxies, galaxy clusters, and halo cores, while avoiding the introduction of new particle species. In this picture, SIDM behavior is not fundamental but emergent, arising from fractal geometry rather than microscopic interactions.

3.4 Summary of Potential Observational Signatures

When fully developed, our framework can likely predict:

1. Flat or slowly rising rotation curves without Dark Matter halos,
2. Gravitational lensing consistent with dynamical mass estimates,
3. Velocity-dependent self-interaction effects emerging from geometry,
4. Smooth transitions between regimes controlled by effective dimension $D(r)$.

These signatures may open clear observational avenues for testing gravity as an emergent consequence of continuous spacetime dimensions.

A full report on these observations and similar findings is planned to be provided in [4].

4. Conclusions

We have indicated that classical gravity can be derived from spacetime endowed with continuous, non-integer effective dimensions. In such geometries, exemplified by Cantor Dust (CD) constructions, ordinary volume vanishes and is replaced by a fractal measure. This measure is *not a mathematical artifact* but a physical quantity that must be identified with gravitational mass.

The resulting gravitational field equations are intrinsically nonlocal and stochastic, with Green's functions determined by the fractal dimension of spacetime. Deviations from Newtonian gravity emerge naturally when the effective dimension differs from three, providing a geometric explanation for phenomena traditionally attributed to Dark Matter. Importantly, this framework does not modify the fundamental principles of gravity but generalizes the notion of spacetime on which they act.

Our results establish a direct link between Dimensional Regularization, fractal geometry, and classical gravity. They suggest that gravitational dynamics may ultimately be governed not by additional matter components but by the dimensional structure of spacetime itself. This perspective opens a new avenue for understanding gravitational anomalies as emergent phenomena rooted in geometry rather than particle physics.

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