<u>Cantor Dust and the Physics of Dark Matter (Chapter 1)</u>

Ervin Goldfain

Global Institute for Research, Education and Scholarship (GIRES).

Email: ervingoldfain@gmail.com

Abstract

We have recently conjectured that Dark Matter (DM) emerges from a statistically homogeneous and isotropic Cantor Dust (CD) mass distribution described by a *singular multifractal measure* [21 - 23]. The goal of this book is to show that leading DM paradigms—self-interacting, fuzzy, axion, and superfluid DM—emerge as *effective descriptions* of primordial CD. From this perspective, the multifractal representation of CD provides an *ultraviolet completion* of DM phenomenology, unifying galactic dynamics, lensing, and large-scale structure while remaining consistent with cluster-scale constraints and experimental observations.

Key words: Dark Matter, Cantor Dust, self-interacting Dark Matter (SIDM), fuzzy Dark Matter (FDM), axion Dark Matter, superfluid Dark Matter, MOND, cosmic web.

1. Introduction and Motivation

As of today, the nature of DM remains a major challenge of contemporary physics. On galactic scales, observations reveal striking regularities—flat rotation curves [1, 2], the baryonic Tully–Fisher relation (BTFR) [3], and the radial acceleration relation (RAR) [4]—that tightly couple baryonic matter to gravitational dynamics. In the standard cold Dark Matter (CDM) paradigm, these relations emerge only statistically, through complex baryon–halo interactions [5, 6]. Modified gravity approaches such as MOND reproduce these regularities more directly but require alterations to Newtonian dynamics or General Relativity [7–9] and are confronted by well-known challenges at cluster and cosmological scales [10].

As conjectured in [21 - 24], an alternative possibility is that the dark sector possesses a *nontrivial geometric structure*. Observations of the cosmic web reveal filamentary and sheet-like morphologies extending across many decades in scale [11–13], suggesting that DM may not be well described by a smooth three-dimensional density field. Motivated by this, we have developed in [21 - 23] a framework in which DM arises from a primordial *multifractal mass distribution*, echoing the generation of CD in mathematics and being characterized by a singular measure rather than a density function. Gravity is governed by the standard Poisson equation, interpreted weakly in the sense of distributions [14, 15].

In this approach, Gauss' law applied to a filamentary CD with a statistically isotropic fractal measure yields an enclosed mass that grows linearly with radius, producing a *background gravitational field* $g(r) \propto r^{-1}$. As shown in [21], this immediately implies flat rotation curves without invoking halos or modified dynamics. A correlation length inherent to the fractal geometry defines a universal acceleration scale a_0, which regulates baryonic collapse and disk formation. Importantly, this mechanism reproduces the BTFR and RAR while avoiding the conceptual difficulties of both particle DM tuning and modified gravity.

The main goal of this book is to show that several leading DM candidates (self-interacting dark matter (SIDM) [16], fuzzy dark matter (FDM) [17], axion dark matter [18], and superfluid dark matter [19]) arise naturally as *effective descriptions* corresponding to different coarse-graining or projection limits of the primordial CD. In this sense, CD provides an overall unifying framework rather than a competing model.

The book is divided into eight chapters, organized in the following way:

- **Chapter 1**: Introduction and Motivation.
- **Chapter 2**: Self-Interacting Dark Matter (SIDM).
- **Chapter 3**: Fuzzy Dark Matter (FDM).
- **Chapter 4**: Axion Dark Matter.
- **Chapter 5**: Superfluid Dark Matter.
- **Chapter 6**: Cantor Dust as Manifestation of Complex Dynamics.
- **Chapter 7**: Cosmological Evidence for Cantor Dust.
- **Chapter 8**: Summary and Concluding Remarks.

In line with [21 – 24] and for the sake of accessibility to a large audience, the book is kept at a "bird's eye view" level, avoiding jargon or excessive technical details. Interested readers are encouraged to further explore, develop or refute the body of ideas developed herein.

2. Emergence of CD Gravity

In the context of this work, the fundamental equation is *not modified gravity*, but a Poisson equation with a modified *source* [21]:

$$\nabla^2\Phi = 4\pi G\,\mu \qquad (1)$$

where μ is a singular multifractal measure with scaling

$$\langle \mu(B_r) \rangle \sim r^{D_{\text{eff}}},\ \ 1 < D_{\text{eff}} < 3, \qquad (2)$$

with strong anisotropic correlations inherited from filamentary and sheet-like geometry. Here, D_{eff} is the *enclosed (gravitational) dimension*, distinct from the intrinsic topological dimension of the support [21]. No smooth density field $\rho(\mathbf{x})$ is assumed to exist. Eq. (1) is interpreted as a *weak Poisson equation*, defined in the sense of distributions,

$$\int \nabla\Phi \cdot \nabla\psi\, d^3x = 4\pi G \int \psi\, d\mu \qquad (3)$$

for all test functions ψ. Eq. (3) defines the gravity carried by CD and it states that the gravitational field energy produced by Φ, tested against any probe function ψ, equals the total mass of Cantor Dust weighted by ψ.

A key observation is that, in this formulation, there is no particle number density and there is no notion of microscopic scattering

2.1 CD Gravity as Background Field

Integrating the Poisson equation (1) over a ball B_r and applying the divergence theorem yields

$$\oint_{\partial B_r} \nabla\Phi \cdot d\mathbf{S} = 4\pi G\, \mu(B_r). \qquad (4)$$

For a statistically homogeneous and isotropic CD network, filament entry–exit cancellations reduce the effective monopole contribution by one power of r, giving [21]

$$\mu(B_r) \sim \lambda r, \qquad (5)$$

where λ is the linear mass density of the filament network. The associated background gravitational field is

$$g_{\mathrm{bg}}(r) = \frac{G\mu(B_r)}{r^2} \sim \frac{G\lambda}{r}, \qquad (6)$$

implying asymptotically flat rotation curves.

2.2 Emergence of a Universal Acceleration Scale

A reasonable assumption is that the fractal description is valid down to a correlation length ℓ, below which the statistical isotropy assumption breaks down. Evaluating eq. (6) at this scale defines a universal acceleration [21]

$$a_0 \equiv g_{\mathrm{background}}(\ell) = \frac{G\lambda}{\ell} \qquad (7)$$

Since this scale depends only on microscopic geometric parameters of CD and is therefore universal.

Next chapters aim to show that the CD framework enables multiple effective descriptions, depending on how the measure μ is coarse-grained or projected. The couple of diagrams below illustrate the main points underlying the rationale of this book.

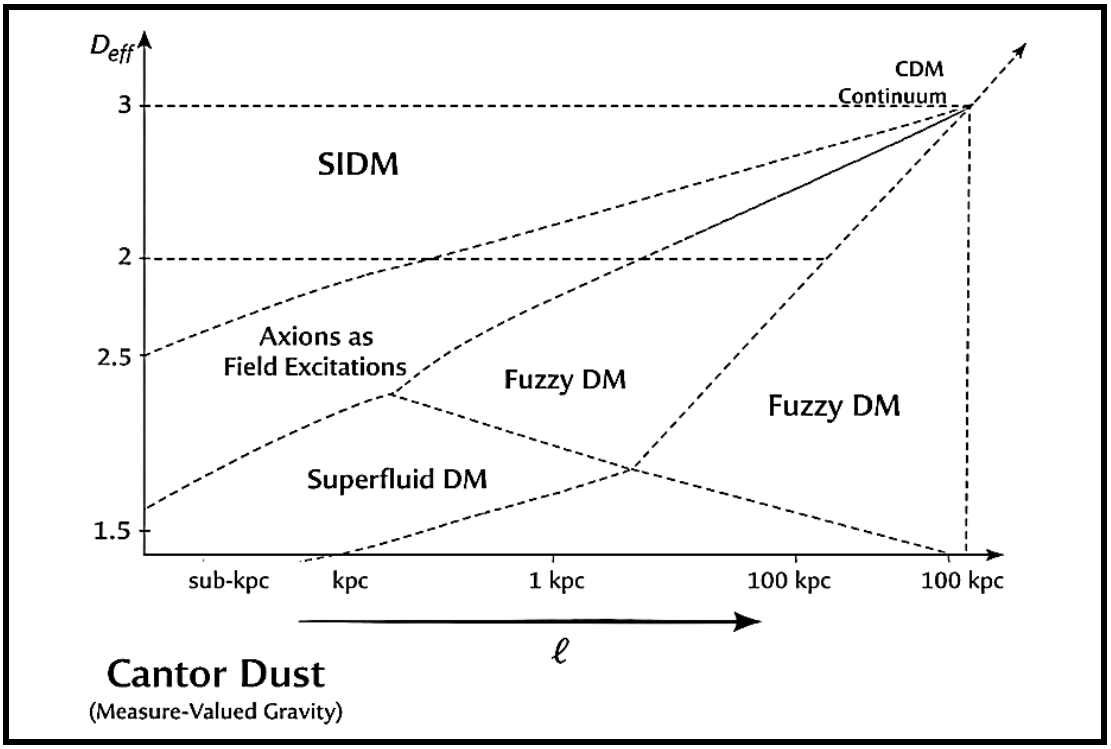

Fig. 1: Cantor Dust Phase Diagram of Dark Matter

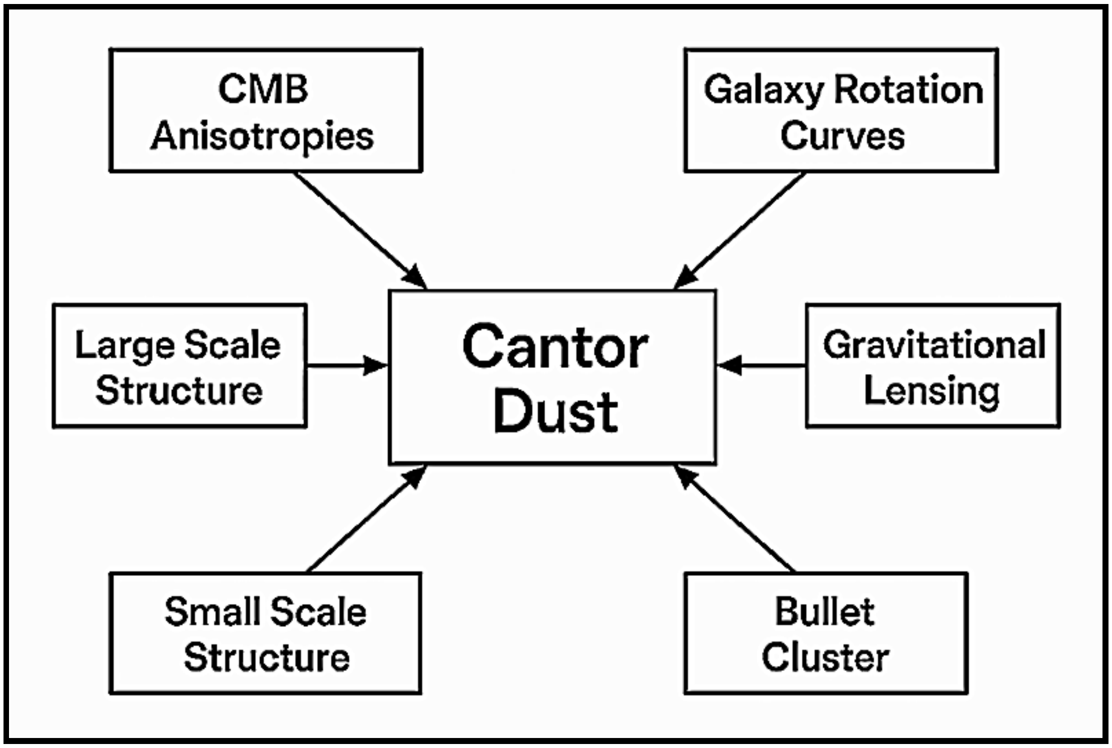

Fig. 2: Cantor Dust as source of Dark Matter phenomenology

References

1. V. C. Rubin, N. Thonnard, and W. K. Ford, Astrophys. J. **238**, 471 (1980).
2. S. McGaugh, Astrophys. J. **632**, 859 (2005).
3. S. McGaugh et al., Phys. Rev. Lett. **117**, 201101 (2016).

4. S. McGaugh, F. Lelli, and J. Schombert, Phys. Rev. Lett. **117**, 201101 (2016).
5. J. Navarro, C. Frenk, and S. White, Astrophys. J. **462**, 563 (1996).
6. A. Pontzen and F. Governato, Nature **506**, 171 (2014).
7. M. Milgrom, Astrophys. J. **270**, 365 (1983).
8. J. Bekenstein, Phys. Rev. D **70**, 083509 (2004).
9. R. H. Sanders and S. McGaugh, Annu. Rev. Astron. Astrophys. **40**, 263 (2002).
10. D. Clowe et al., Astrophys. J. Lett. **648**, L109 (2006).
11. J. Bond, L. Kofman, and D. Pogosyan, Nature **380**, 603 (1996).
12. V. Springel et al., Nature **435**, 629 (2005).
13. M. Cautun et al., Mon. Not. R. Astron. Soc. **441**, 2923 (2014).
14. L. Schwartz, *Théorie des distributions* (Hermann, Paris, 1950).
15. R. Evans and R. Gariepy, *Measure Theory and Fine Properties of Functions* (CRC, 1992).
16. D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. **84**, 3760 (2000).

17. W. Hu, R. Barkana, and A. Gruzinov, Phys. Rev. Lett. **85**, 1158 (2000).

18. P. Sikivie, Rev. Mod. Phys. **93**, 015004 (2021).

19. J. Khoury, Phys. Rev. D **91**, 024022 (2015).

20. M. Markevitch et al., Astrophys. J. **606**, 819 (2004).

21. E. Goldfain, preprint https://doi.org/10.13140/RG.2.2.12791.28321

22. E. Goldfain, preprint https://doi.org/10.13140/RG.2.2.36487.46242/1

23. E. Goldfain, preprint https://doi.org/10.13140/RG.2.2.29108.92807/1

24. E. Goldfain, preprint https://doi.org/10.13140/RG.2.2.28465.90723/4