Exact Values of the Proton Radius and the Gravitational Constant by Use of the "Hans de Vries"- and the "Julian Schwinger"-Term

Andreas Ball

Abstract:

In this report very exact formulas for the Proton Radius and for the Gravitational Constant are presented, at which the Fine Tuning Term of the "Hans de Vries"-Formula and also the "Julian Schwinger"-Term are applied. Some of the Formulas are constructed quite simple and the connections for one with another are presented. Many of the result values are very accurate referring to their tolerance ranges (i.e. exact in this context) and therefore lie astonishingly closely together.

"Hans de Vries"-Formula for the Fine Structure Constant α:

The exceptional "Hans de Vries"-Formula[1] is a Serie Equation and it is written by use of the Euler Figure e (= 2,7182818), the Circle Figure π and the Fine Structure Constant α [2.1], [2.2] itself:

$\alpha_{dV}^{-1} = \Gamma_\alpha^{-2} \cdot e^{\pi \cdot \pi/2}$ with

$e^{-\pi \cdot \pi/2} = 7{,}1918833558 \cdot 10^{-3}$ $[e^{\pi \cdot \pi/2} = 139{,}045636661]$

$\Gamma_\alpha = 1 + [\alpha_{dV}/(2\cdot\pi)^0 \cdot (1+\alpha_{dV}/(2\cdot\pi)^1 \cdot (1+\alpha_{dV}/(2\cdot\pi)^2 \cdot (1+\alpha_{dV}/(2\cdot\pi)^3 \cdot (1+\alpha_{dV}/(2\cdot\pi)^4 \cdot (1+\ldots)))))]$

The Term Γ_α (Hans de Vries named it just Γ[1]) can also be written by the following Formula:

$\Gamma_\alpha = 1 + [\alpha_{dV}^1/(2\cdot\pi)^0 + \alpha_{dV}^2/(2\cdot\pi)^1 + \alpha_{dV}^3/(2\cdot\pi)^3 + \alpha_{dV}^4/(2\cdot\pi)^6 + \alpha_{dV}^5/(2\cdot\pi)^{10} + \ldots]$

Equation for α_{dV}^{-1} is only iteratively, but sufficiently exact to solve. Therefore the quantity $\mathbf{\Gamma_{\alpha 0}}$ is used:

$$\boldsymbol{\alpha_{dV} = \Gamma_{\alpha 0}^2 \cdot e^{-\pi \cdot \pi/2} = 7{,}29735256866 \cdot 10^{-3}} \quad (\alpha\text{-dV})$$

$$\boldsymbol{\alpha_{dV}^{-1} = \Gamma_{\alpha 0}^{-2} \cdot e^{\pi \cdot \pi/2} = 137{,}035999096} \quad (\alpha\text{-dV-R})$$

$$\mathbf{\Gamma_{\alpha 0}} = 1 + [\alpha_0/(2\cdot\pi)^0 \cdot (1+ \alpha_0/(2\cdot\pi)^1 \cdot (1 + \alpha_0/(2\cdot\pi)^2 \cdot (1 + \alpha_0/(2\cdot\pi)^3 \cdot (1 + \ldots))))] = $$
$$= 1{,}007305829352 \quad (\Gamma\alpha 0)$$

An appropriate exact input value for α_0^{-1} (α_0^{-1} between 137,035999084 and 137,035999110) is required to get the above result value **137,035999096** (with 9 digits behind the decimal point) for α_{dV}^{-1}.
One can interprete the result value of Equation (α-dV) as a Constant, which is correct for example to CODATA2018 and further which is consistent by itself.
In the following the Fine Tuning Term $\mathbf{\Gamma_{\alpha 0}}$ is named for simplicity $\mathbf{\Gamma_{dV}}$. The abbreviation dV stands for de Vries. By that one can visibly relate the term to Hans de Vries.

The Term α/(2·π), which is engraved on the headstone of the deceased Physician and Nobel-Prize-Winner Julian Schwinger[3], is often used in the following, therefore the Quantity Δ_{JS} is introduced:

$$\Delta_{JS} = \alpha/(2\cdot\pi) = 1/(137{,}035999177 \cdot 2\cdot\pi) = 0{,}001\,161\,409\,732 \quad (\Delta_{JS})$$

Term $(1 + \Delta_{JS})$ is close to the value (nearly five digits behind the decimal point) of the Quantity Landé-Factor[2.3] g_e: "g_e / 2 (=2,00231930436... / 2 = 1 + 0,001159652180…)".

Calculation of the Proton Radius[4] r_p: Approximations are marked with a double cross

It has only a nectlectable influence for the calculation of the Proton Radius, if one uses the values of the Fine Structure Constant α given by CODATA2018, by CODATA2022 or by the "Hans de Vries"-Formula. The relative Tolerance Range of the Proton Radius r_p is by the factor $3{,}03\cdot10^6$ higher than the relative Tolerance Range of the Fine Structure Constant α.
This is shown by the following Relations:

Tolerance Range of the Proton Radius r_p: Tol_{rp}[4] = $\pm0{,}0039 \cdot 10^{-16}$ m

Radius of Proton r_p[4]: $8{,}4087 \cdot 10^{-16}$ m

$2 \cdot |Tol_{rp}| / r_p = 2 \cdot 0{,}0039 / 8{,}4087 = 9{,}276 \cdot 10^{-4}$

Tolerance Range of the Reciprocal α^{-1} of the Fine Structure Constant: $Tol_{\alpha Rec} = \pm 21 \cdot 10^{-9}$
Value of the Reciprocal α^{-1}: 137,035999177
$2 \cdot |Tol_{\alpha Rec}| / \alpha^{-1} = 2 \cdot 21 \cdot 10^{-9} / 137{,}035999177 = 3{,}065 \cdot 10^{-10}$

At next Formula (Trmα) the mass ratio and radius ratio of Electron and Proton in combination with the Fine Structure Constant α lead to a result close to figure 1. This is written to:

$$T_{rm\alpha} = 4 \cdot (r_e / r_p) \cdot (m_e / m_p) / \alpha = 1{,}000435 \quad [\approx 1] \qquad \text{(Trm}\alpha\text{)}$$

Formula (rp0) gives back the Proton Radius r_p in dependence of the Electron Radius r_e[2.4], the mass ratio Electron/Proton (= m_e[2.5] / m_p[5]) and the Fine Structure Constant α:

$$r_{p\#0} = 4 \cdot r_e \cdot (m_e / m_p) / \alpha = 8{,}4124 \cdot 10^{-16}\ \text{m} \quad [\approx r_p = 8{,}4087 \cdot 10^{-16}\ \text{m}] \qquad \text{(rp0)}$$

At Formula (rp1) and (rp2) the Fine Tuning Term Δ_{JS}, which is dedicated to Julian Schwinger, is used:

$$r_{p\#1} = [4 \cdot r_e \cdot (m_e / m_p) / \alpha] / (1 + \Delta_{JS})^{3/8} = 8{,}4086955 \cdot 10^{-16}\ \text{m} \qquad \text{(rp1)}$$

$$r_{p\#2} = [4 \cdot r_e \cdot (m_e / m_p) / \alpha] / (\Delta_{JS}^0 + \Delta_{JS}^1 - \Delta_{JS}^2)^{3/8} = 8{,}4086998 \cdot 10^{-16}\ \text{m} \qquad \text{(rp2)}$$

The fraction 3/8 at the exponent of Formula (rp1) can also be found at the Thomson Cross Section[2.6] σ_e [= $(8/3)\cdot\pi\cdot r_e^2$] with its fraction 8/3. Its reciprocal value 3/8 is also applied at the Formulas (rp2) to (rp5). Formulas (rp1) and (rp2) are far within the tolerance range (= $\pm 0{,}0039 \cdot 10^{-16}$ m)[4] of the Proton Radius r_p.

As mentioned before the fraction 3/8, which is part of the Thomson Cross Section[2.6] σ_e [= $(8/3)\cdot\pi\cdot r_e^2$] with its reciprocal, and additionally the Landé Factor[2.3] g_e of the Electron are used at Formula (rp3):

$$r_{p\#3} = [4 \cdot r_e \cdot (m_e / m_p) / \alpha] / (0{,}5 \cdot g_e)^{3/8} = 8{,}408701 \cdot 10^{-16}\ \text{m} \qquad \text{(rp3)}$$

The Landé Factor[2.3] g_e of the Electron possesses the value 2,002 319 304 360 92.

The next Formula (rp4) uses additionally the Fine Tuning Term Γ_{dV} of the "Hans de Vries"-Formula:

$$r_{p\#4} = [4 \cdot r_e \cdot (m_e / m_p) / \alpha] / \Gamma_{dV}^{(3/8)/(2\cdot\pi)} = 8{,}4087025 \cdot 10^{-16}\ \text{m} \qquad \text{(rp4)}$$

The term $[(3/8) / (2\cdot\pi)]$ at the exponent of Formula (rp4) takes nearly the value of the term $[(r_p / r_e) / 5]$, which includes the ratio "Proton Radius / Electron Radius" .
This is mathematically performed by Formula (rp5):

$$r_{p\#5} = [4 \cdot r_e \cdot (m_e / m_p) / \alpha] / \Gamma_{dV}^{(r_p/r_e)/5} = 8{,}4087027 \cdot 10^{-16}\ \text{m} \qquad \text{(rp5)}$$

It is an unusual formula similar to the "Hans de Vries"-Formula (α-dV), where the result quantity is part of its formula.

Isn't the exactness of the result values of Formulas (rp1) to (rp5) compared to the set value extraordinary? And isn't it astonishing, that the Physical Constant g_e - the Landé Factor for the Electron - and also a part - the fraction 8/3 - of another Physical Constant, namely the Thomson Cross Section σ_e, can be found at the formula (rp3) of the Proton Radius besides the Physical Constants α, r_e, m_e and m_p?

The "Hans de Vries"-Fine Tuning Term Γ_{dV}, which is used at the Formulas (rp4) and (rp5), in connection with the exponent term "$1/(2\cdot\pi)$" delivers:

$$\Gamma_{dV}^{1/(2\cdot\pi)} = 1{,}001\,159\,203\,3 \quad [\approx 0{,}5 \cdot g_e]$$

and fits to the term "$0{,}5 \cdot g_e = 1{,}001\,159\,652\,180\,46$" up to 6 digits behind the decimal point.

The reason of the close values is the following relation:

$$[1 + \alpha / (2\cdot\pi)] \approx (1 + \alpha)^{1/(2\cdot\pi)}$$

or written in a general form:

$$(1 + x / y) \approx (1 + x)^{1/y} \quad \text{with } x \ll 1 \text{ and } y > 1$$

Equation for the Age of the Universe derived by the Large Number Hypothesis[6] of Dirac/Weyl[7]:

The value of the Age of the Universe, which is already presented in the author's report[8], is applied at the formulas for the Gravitational Constant G. In this chapter the formula for the Age of the Universe is described once again.

The Age of the Universe is given to: $Age_{Univ} = 13{,}787 \pm 0{,}02 \cdot 10^9$ years[9] (Age)

The Age of the Universe Age_{Univ} in SI-Unit s is: $13{,}787 \cdot 10^9 \cdot 3600 \cdot 24 \cdot 356{,}256\ s = 4{,}35092 \cdot 10^{17}\ s$.

The maximal allowable relative tolerance range is: $(13{,}787 \pm 0{,}02) / 13{,}787$

that means a relative tolerance range from 0,99855 to 1,00145

The Large Number Equation LN_T, which is known since nearly 100 years, is written by use of the Age of the Universe, the Light Velocity c[2.7] and the Electron Radius r_e[2.4] to:

$$LN_T = Age_{Univ} \cdot c / r_e = 4{,}35092 \cdot 10^{17}\ s \cdot c / r_e = 4{,}62881 \cdot 10^{40} \quad \text{(LN-T1)}$$

A pretty simple, but harmonic Approximation of the Auxiliary Quantity LN_T can be given by the following Equation LN_{TAppr}, at which the Fine Structure Constant α[2.1] and the Circle Figure π are used:

$$LN_{TAppr} = (4\,\pi / \alpha)^{4\pi} = 4{,}62738 \cdot 10^{40} \quad \text{(LN-T2)}$$

The values of the used quantities Light Velocity c, Electron Radius r_e and Fine Structure Constant α can be taken from the section Used Data of Physical Constants at page 10.

The Equation for the Approximation $Age_{Univ\#}$ of the Universe Age is given by Equating of Equations (LN-T1) and (LN-T2). By its conversion it can be written to:

$$Age_{Univ\#} = LN_{TAppr} \cdot r_e / c = (4\,\pi / \alpha)^{4\,\pi} \cdot r_e / c = 4{,}34957 \cdot 10^{17}\ s = 13{,}7827 \cdot 10^9\ a \quad \text{(Age-Appr)}$$

The ratio of the calculated value $Age_{Univ\#}$ to the set value Age_{Univ} is:

13,7827 / 13,787 = 0,99969

The calculated value $Age_{Univ\#}$ is far within the tolerance range of the set value[9] for the Universe Age. Remarkable: with values "1,000017 · (4 π)" or "0,999988 · (4 π)" applied at the basis as well as at the exponent of Equation (Age-Appr) instead of the term "4 π", the result of Equation (Age-Appr) is outside the tolerance range of the set value. It means, that by a relative small change (for example "1 - 0,999988 = 0,00012") of the Term "4 π" it follows a relative big change of the result (1 - 0,99855 = 0,00145).

Isn't it fascinating? The exponent "4 π" is also part of the basis. The simplicity of Equation (Age-Appr)!

Calculation of the Gravitational Constant[2.8]: Approximations are marked with a double cross #

In the report[10] of George Bailey a Formula [Formula (4) of report[10]] is presented, which result (=3,3534) is close to radius ratio "r_e/r_p (=3,35122)" Electron to Proton. By use of the present values of the Physical Constants the result of George Bailey's Formula (GB0) takes the value 3,35923.
Formula (GB0) is written by use of other Constants – namely the Planck's Constant[2.9] h, the Elementary Charge[2.10] e and the Electric Field Constant[2.11] ε_0 – and the age of the Universe to:

$$h \cdot e^2 / [(4 \cdot \pi)^2 \cdot Age_{Univ\#} \cdot G \cdot c^2 \cdot m_e^2 \cdot m_p \cdot \varepsilon_0] = 3{,}35923 \quad [\approx r_e / r_p = 3{,}35122] \quad \text{(GB0)}$$

Formula (GB0) can be written by use of the Electric Field Constant[2.11] ε_0 $[= e^2 / (4 \cdot \pi \cdot c^2 \cdot m_e \cdot r_e)]$ to:

$$h \cdot (4 \cdot \pi \cdot r_e) / [(4 \cdot \pi)^2 \cdot Age_{Univ\#} \cdot G \cdot m_e \cdot m_p] = 3{,}35923 \quad \text{(GB1)}$$

Formula (GB1) can be written with the Formula "$Age_{Univ\#} = (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot r_e / c$":

$$h \cdot c / [4 \cdot \pi \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot G \cdot m_e \cdot m_p] = 3{,}35923 \quad \text{(GB2)}$$

Now Formula (GB2) is multiplied by the mass ratio r_p / r_e (= 1/3,35122), which delivers Formula (GB3):

$$h \cdot c \cdot (r_p / r_e) / [4 \cdot \pi \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot G \cdot m_e \cdot m_p] = 1{,}0023894 \quad [\approx 1] \quad \text{(GB3)}$$

In the upper formulas the Age of the Universe and the Gravitational Constant G are located in the denominator. The Age of the Universe is already determined by the Formula (Age-Appr) at page 3.
The term (4·π) appears 3 times at Formulas (GB2) and (GB3). Isn't it astonishing?

Now Formula (GB3) is reconstructed in the way, that the Gravitational Constant (=6,67430 $kg^{-1} \cdot m^3 \cdot s^{-2}$) stands alone on the left side:

$$G_{\#GB} = h \cdot c \cdot (r_p / r_e) / [4 \cdot \pi \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot m_e \cdot m_p] = 6{,}690248 \cdot 10^{-11} \; kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G-GB)}$$

At the next Formula (G1) the "Hans de Vries"-term Γ_{dV} and the fraction 1/3,05 are applied:

$$G_{\#1} = h \cdot c \cdot (r_p / r_e) / [4 \cdot \pi \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot m_e \cdot m_p] / \Gamma_{dV}^{1/3{,}05} = 6{,}6742996 \cdot 10^{-11} \; kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G1)}$$

Connection: 305 = 5 · 61; see use of figure 61 at the exponent of the next Equation!
Remarkable: N=5; N_{m1}=N-1=4; N_{p1}=N+1=6; $N^2 + N_{p1}^2 = 5^2 + 6^2 = 61$; $N^3 - N_{m1}^3 = 5^3 - 4^3 = 61$;
The Equation "$N^2 + N_{p1}^2 = N^3 - N_{m1}^3$" is only valid for N=5 in the proofed range N=1..23

At the next Formula (G2) the term $T_{rm\alpha}$ [$= 4 \cdot (r_e/r_p) \cdot (m_e/m_p)/\alpha = 1{,}000435$] of page 2 is used:

$$G_{\#2} = h \cdot c \cdot (r_p / r_e) / [4 \cdot \pi \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot m_e \cdot m_p] / T_{rm\alpha}^{5{,}49} = 6{,}6742991 \cdot 10^{-11} \; kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G2)}$$

Formula (G2) can also be written in dependence of the Square of the Light Velocity c:

$$G_{\#2a} = c^2 \cdot r_p / [2 \cdot \alpha \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot m_p] / T_{rm\alpha}^{5{,}49} = 6{,}6742991 \cdot 10^{-11} \; kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G2a)}$$

Connections: 549 = 9 · 61; see use of Figure 61 at Equation (G1)

To Formula (G-GB) the Fine Tuning Terms "$(1+\alpha)^{1{,}1 \cdot r_p/r_e}$" at Equation (G3) and
"$[1 + \alpha \cdot (r_p / r_e) / (8 \cdot \pi)]^{6 \cdot 1{,}37 \cdot r_e/r_p}$" at Equation (G4) are inserted:

$$G_{\#3} = h \cdot c \cdot (r_p / r_e) / [4 \cdot \pi \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot m_e \cdot m_p] / (1 + \alpha)^{1{,}1 \cdot r_p/r_e} = 6{,}6743001 \cdot 10^{-11} \; kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G3)}$$

$$G_{\#4} = h \cdot c \cdot (r_p / r_e) / [4 \cdot \pi \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot m_e \cdot m_p] / [1 + \alpha \cdot (r_p / r_e) / (8 \cdot \pi)]^{6 \cdot 1{,}37 \cdot r_e/r_p} = 6{,}6743000 \cdot 10^{-11} \; kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G4)}$$

$$G_{\#4a} = h \cdot c \cdot (r_p / r_e) / [4 \cdot \pi \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot m_e \cdot m_p] / [1 + \alpha \cdot (r_p / r_e) / (8 \cdot \pi)]^{(0{,}06/\alpha) \cdot r_e/r_p} = 6{,}674296 \cdot 10^{-11} \; kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G4a)}$$

Remarkable: Radius Ratio "r_p/r_e" is used as multiplicator and as a part of an exponent, both at Formulas (G3), (G4) and (G4a)! Figure 1,37 ($\approx 0{,}01/\alpha$) is used at the exponent of Equation (G4)!
Connections of the figures: $\ln(6 \cdot 137) / \ln(r_e/r_p) = 5{,}549992$ [≈ 5,55]; 6 = 555 - 549; $6 \cdot 137 - 555 - 11 = 2^8$;
$2^8 = 256$; $555 - 2^8 - 137 = 162 = 2 \cdot 3^{8/2}$; see Figures 549 and 11 at Equations (G2) and (G3)

$$G_{\#5} = h \cdot c \cdot (r_p / r_e) / [4 \cdot \pi \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot m_e \cdot m_p] / \{\Gamma_{dV} \cdot [1 + (r_p / r_e)/(8 \cdot \pi)]\}^{1/8} = 6{,}6743087 \cdot 10^{-11} \; kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G5)}$$

At Formula (G5) the figure 8 is used within the term "8·π" at the basis and as reciprocal at the exponent!

$$G_{\#6} = h \cdot c \cdot (r_p / r_e) / [4 \cdot \pi \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot m_e \cdot m_p] / [1 + 1/(\alpha \cdot m_p/m_e)]^{1/(9{,}6 \cdot \pi)} = 6{,}6742999 \cdot 10^{-11} \; kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G6)}$$

Remarkable: Mass Ratio "m_p / m_e" is used at the basis of the Fine Tuning Term.
Connections: $\alpha \cdot m_p / m_e = 13{,}3991$ [≈ 13,4]; 13,4 = 2 · 6,7; $6{,}7 \cdot 9{,}6 \cdot \sqrt{\pi} = 114{,}004$ [≈ 114];
134 + 96 - 114 = 116; see Figure 116 at Equation (G7 below)

$$G_{\#7} = h \cdot c \cdot (r_p / r_e) / [4 \cdot \pi \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot m_e \cdot m_p] / [1 + \Delta_{JS}]^{1{,}16 \cdot \sqrt{\pi}} = 6{,}6743004 \cdot 10^{-11} \; kg^{-1} \cdot m^3 \cdot s^{-2}$$

Connections: 116 = 4 · 29; see use of Figure 29 at Equation (G11) (G7)

$G_{\#8} = h \cdot c \cdot (r_p / r_e) / [4\cdot\pi \cdot (4\cdot\pi / \alpha)^{4\cdot\pi} \cdot m_e \cdot m_p] / [0{,}5 \cdot g_e]^{1{,}1618034\cdot\sqrt{\pi}} =$
$= 6{,}6742998 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G8)

Figure 1,1618034: $1 + 0{,}1 \cdot \Phi = 1{,}1618034$ [Golden Ratio $\Phi = (5 + 5\sqrt{5}) / 10 = 1{,}618034$]
$(1 + 0{,}1 \cdot \Phi) \cdot \sqrt{\pi} = 2{,}05924$ [$\approx \Phi \cdot \sqrt{\Phi} = 2{,}05817$];

$G_{\#9} = h \cdot c / [\pi \cdot \alpha \cdot (4\cdot\pi / \alpha)^{4\cdot\pi} \cdot (m_e + m_p)^2] / [1 + \Delta_{JS}]^{1/0{,}67} = 6{,}6742997 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G9)

Sum of the masses of Electron and Proton $(m_e + m_p)^2$ is applied; at the Equations before: $m_e \cdot m_p$;
Connections: 128 - 67 = 61 = 549 / 9 = 305 / 6; Figure 67 is derived by Equations (G6) and (G17)
549 - 305 - 116 = 128; Figures 549, 305, 116 are used at Equations (G2), (G1) and (G7);
see use of figure 128 at Equation (z-GB), (α-GB) and (α1)

$G_{\#10} = h \cdot c / [\pi \cdot \alpha \cdot (4\cdot\pi / \alpha)^{4\cdot\pi} \cdot (m_e + m_p)^2] / [1 + \Delta_{JS} - 4 \cdot \Delta_{JS}^2]^{1{,}5} =$
$= 6{,}674296 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G10)

Connections: 15 + 43 + 67 = 125; Figure 67 at Equations (G6) and (G17); Figure 43 at Equation (G14)
0,125 = 1/8 see Exponent 0,125 at Equation (G5)

$G_{\#11} = h \cdot c / [\pi \cdot \alpha \cdot (4\cdot\pi / \alpha)^{4\cdot\pi} \cdot m_p^2] / (1 + 2^{-1} \cdot \Delta_{JS})^{1{,}45 \cdot r_e / r_p} = 6{,}674302 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G11)

Connection: 145 = 5 · 29 Figure 29 is derived at Equation (G7)

George Bailey presents in his report[11] the following remarkable Formula (α-GB) for the Fine Structure Constant α in dependence of the Figure 128. See Equations (6) and (8) in his report[11]:

$\alpha_{GB}^{-1} = (z_{GB} / \pi) \cdot [1 + (1 + 1/ z_{GB}) / (4\cdot\pi^2 \cdot z_{GB} + 1 + 1 / z_{GB})] = 137{,}035999119$ (α-GB)

with $z_{GB} = 128 \cdot \ln(2) \cdot \ln(128) = 430{,}48590047$ (z-GB)

The result value of Equation (α-GB) is between the values of CODATA2018 and CODATA2022.
Equation (α-GB) can also be written by a serie Formula[11]!

$z_{GB}/\pi = 137{,}0279179826$

$\alpha_{\#1}^{-1} = (z_{GB} / \pi) / [1 - (2\cdot\pi) / (1000 \cdot z_{GB})]^{4/0{,}99} = 137{,}035999088$ (α1)

Connections of the Figures: 0,99 / 4 = 0,2475; 2475 = 5 · 5 · 99; 2475 - 2 · 1000 - 128/2 = 3 · 137

The next Formula (G12), which is dependent on the Light Velocity c, the Proton Radius r_p and the Proton mass m_p, applies the term 2^{129} ($=6{,}80564734 \cdot 10^{38}$) and is written as follows:

$G_{\#12} = [c^2 \cdot r_p / (2^{129} \cdot m_p)] \cdot \Gamma_{dV}{}^{1/1{,}37} / (1 + \Delta_{JS})^{1/100} =$
$= [6{,}6390056 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}] \cdot 1{,}0053152 = 6{,}674297 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G12)

Connections of the Figures: 1,37 · 100 = 137 [$\approx \alpha^{-1}$]; 129 = 3 · 43 see exponent 4,3 at Equation (G14)

$G_{\#13} = [c^2 \cdot r_p / (2^{129} \cdot m_p)] \cdot \{1 + 1 / [129 \cdot (2\cdot\pi)^2]\}^{27} =$
$= [6{,}6390056 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}] \cdot 1{,}0053152 = 6{,}674294 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G13)

Connections of the Figures: $\ln(2^{129}) / \ln(27) = 27{,}12998$ [≈ 27 + 13/100]; 129 + 27 + 13 = 169 = 13^2;
129 + 129 -137 = 121 = 11^2; 129 + 129 - 27 + 11 = 242 = 2 · 11^2

$G_{\#14} = [c^2 \cdot r_p / (2^{129} \cdot m_p)] \cdot \{1 + 1 / [(129 \cdot (2\cdot\pi)]\}^{4{,}3} = 6{,}674298 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G14)

Connections of the Figures: 129 = 3 · 43; see use of figure 4,3 at the next Equation (G10a)

$G_{\#10a} = h \cdot c \cdot (r_p / r_e) / [\pi \cdot (4{\cdot}\pi / \alpha)^{4\cdot\pi} \cdot \alpha \cdot (m_e + m_p)^2] / [1 + \Delta_{JS} - 4{,}3 \cdot \Delta_{JS}^2]^{1,5} =$
$= 6{,}6742998 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G10a)

Connection to Equation (G10): multiplicator 4,3 instead of multiplicator 4 at Equation (G10)

At page 14 of his other report[10] George Bailey gives the Formula (GB1) in dependence of the Age of the Universe, which leads to the high exponent 128 of basis 2. It is written as follows:

$3 \cdot e^2 \cdot Age_{Univ\#} / (2 \cdot 10 \cdot h \cdot \varepsilon_0 \cdot r_p)$ or
$3 \cdot e^2 \cdot Age_{Univ\#} / (2^2 \cdot 5 \cdot h \cdot \varepsilon_0 \cdot r_p) = 3{,}3948803 \cdot 10^{38}\quad [\approx 2^{128} = 3{,}4028237 \cdot 10^{38}]$ (GB1)

The Electric Field Constant ε_0 and the Age of Universe $Age_{Univ\#}$ (see page 3) are defined to:

$\varepsilon_0 = e^2 / (4{\cdot}\pi \cdot c^2 \cdot r_e \cdot m_e)$ and $Age_{Univ\#} = (4\pi / \alpha)^{4\pi} \cdot r_e / c = 4{,}34957 \cdot 10^{17}\ s$

The Electric Field Constant ε_0 and the Age of Universe $Age_{Univ\#}$ (page 3) set in Equation (GB1) delivers:

$3 \cdot e^2 \cdot [(4\pi / \alpha)^{4\pi} \cdot r_e / c] / \{2^2 \cdot 5 \cdot h \cdot [e^2 / (4{\cdot}\pi \cdot c^2 \cdot r_e \cdot m_e)] \cdot r_p\} =$
$= 3 \cdot (4{\cdot}\pi \cdot c^2 \cdot r_e \cdot m_e) \cdot [(4\pi / \alpha)^{4\pi} \cdot r_e / c] / (2^2 \cdot 5 \cdot h \cdot r_p) =$
$= 3 \cdot \pi \cdot c \cdot r_e \cdot m_e \cdot (4\pi / \alpha)^{4\pi} \cdot (r_e / r_p) / (5 \cdot h) = 3{,}3948803 \cdot 10^{38} = 0{,}99766565 \cdot 2^{128}$ (GB-1a)

The Radius r_e of the Electron is defined to: $r_e = \alpha \cdot [h / (2{\cdot}\pi)] / (c \cdot m_e)$

The Radius r_e of the Electron applied in Equation (GB-1a) delivers Equation (GB-1b), which contains the radius ratio r_e/r_p and leads closely to a Large Number caused by the high exponent figure 129:

$(3/5) \cdot \alpha \cdot (4\pi/\alpha)^{4\pi} \cdot (r_e / r_p) = 0{,}99766565 \cdot \mathbf{2^{129}} = 6{,}7897606 \cdot 10^{38}$ (GB-1b)

$G_{\#15} = 0{,}99^{1,31} \cdot [3 \cdot c^2 \cdot r_e / (2^{129} \cdot \pi^2 \cdot m_p)] \cdot [1 - 1 / (12{,}9 \cdot 13{,}1)^{1/(1,31 \cdot 0,336699)}] =$
$= 6{,}67429999 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G15)

Connections of the used Figures: $33 + 66 + 99 = 2 \cdot 99$; $3 \cdot 131 - 2 \cdot 129 = 3 \cdot 99 - 2 \cdot 9 \cdot 9$

$G_{\#16} = [3 \cdot \alpha \cdot c \cdot h / (2^{131} \cdot 5{\cdot}\pi \cdot m_e \cdot m_p)] / \{1 + 1 / [(129 \cdot (2{\cdot}\pi)^2]\}^{1/4} = 6{,}674303 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$

Connection of the Figures in the first term within rectangular brackets: $131 + 3 + 5 - 2 = 137$ (G16)

$G_{\#17} = [3 \cdot c^2 \cdot r_e / (2 \cdot 5 \cdot 144^{18} \cdot m_p)] \cdot \Gamma_{dV}^{5,57799} = 6{,}6743007 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G17)

Connections of the used Figures: $144 / 18 = 8 = 2^3$; $55 + 77 + 99 = 3 \cdot 7 \cdot 11$; Connection to Figure 336699 at Equation (G15) and Figure 67 at Equation (G9): $557799 - 336699 = 221100 = 2^2 \cdot 3 \cdot 5^2 \cdot 11 \cdot 67$

$G_{\#18} = [3 \cdot c^2 \cdot r_e / (2 \cdot 5 \cdot 144^{18} \cdot m_p)] / [0{,}96 \cdot \Gamma_{dV}^{3/100}] = 6{,}6743002 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G18)

Connections of the used Figures: $96 / 3 = 32 = 2^5$; $96 + 3 = 99$; $144 + 18 = 2 \cdot 9 \cdot 9$; $3 \cdot 96 - 144 = 144$

$G_{\#19} = \{h \cdot c / [\pi \cdot \alpha \cdot (4{\cdot}\pi / \alpha)^{4\cdot\pi} \cdot (m_e + m_p)^2]\} \cdot [(1 + \Delta_{JS}) / \Gamma_{dV}]^{1/[(5,57799 - 3) \cdot 1,37]} =$
$= 6{,}67430001 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G19)

Connections: Figure 557799 of Equation (G17); Figure 3 is part of the exponent at Equation (G19);
$\ln(557799) \cdot \ln(137) = 65{,}099977$ $[\approx 651/10]$; $651 - 3 \cdot 137 + 3 = 243 = 3^5$;
$137 - 3 = 2 \cdot 67$ Figure 67 at Equation (G9);

There is another Formula (T-Fr) for the Age of Universe $T_{Universe}$ given at the report[12] of Rodolfo A. Frino (See Formula 12, page 4 of Literature [12]):

$T_{Universe} \approx h^2 / [2{\cdot}\pi^2 \cdot G \cdot c \cdot m_e \cdot m_p^2] = 4{,}361856 \cdot 10^{17}\ s$ (T-Fr)

Formula (T-Fr) is now solved for the Graviational Constant G and delivers Formula (G-Fr):

$G_{Fr} \approx h^2 / [2\cdot\pi^2 \cdot Age_{Univ\#} \cdot c \cdot m_e \cdot m_p^2]$ with $Age_{Univ\#} = (4\cdot\pi / \alpha)^{4\,\pi} \cdot r_e / c$

$$G_{Fr} \approx h^2 / [2\cdot\pi^2 \cdot (4\cdot\pi / \alpha)^{4\cdot\pi} \cdot r_e \cdot m_e \cdot m_p^2] = 6{,}693157 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G-Fr)}$$

At Formula (G-Fr) the classic Electron Radius r_e is replaced by its Formula $[r_e = \alpha \cdot h / (2\,\pi \cdot m_e \cdot c)]$. This reduces the exponent of quantity h at Formula (G-Fr) from h^2 to h^1:

$$G_{Fr} \approx h \cdot c / [\pi \cdot \alpha \cdot (4\cdot\pi / \alpha)^{4\cdot\pi} \cdot m_p^2] = 6{,}693157 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G-Fr)}$$

The use of the Fine Tuning Term $[1 + \Delta_{JS}]^{2{,}43}$ at above Formula leads to Formula (G20):

$$G_{\#20} = \{h \cdot c / [\pi \cdot \alpha \cdot (4\cdot\pi / \alpha)^{4\cdot\pi} \cdot m_p^2]\} / [1 + \Delta_{JS}]^{2{,}43} = 6{,}674305 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G20)}$$

The exponent 2,43 can be written as: $2{,}43 = 3^5 / 10^2$

The use of the Fine Structur Constant α instead of the "Julian Schwinger"-Term Δ_{JS} at Equation (G20) leads to Equations (G21) and (G22), at which the figure 13 is visible :

$$G_{\#21} = \{h \cdot c / [\pi \cdot \alpha \cdot (4\cdot\pi / \alpha)^{4\cdot\pi} \cdot m_p^2]\} / [1 + \alpha]^{1{,}3\cdot r_p / r_e} = 6{,}674306 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G21)}$$

$$G_{\#22} = \{h \cdot c / [\pi \cdot \alpha \cdot (4\cdot\pi / \alpha)^{4\cdot\pi} \cdot m_p^2]\} / [1 + \alpha]^{5{,}2\cdot(m_e/m_p)/\alpha} = 6{,}674297 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2} \qquad \text{(G22)}$$

Connections: $52 = 4\cdot 13$; $13\cdot 52 - 137 = 539 = 7^2\cdot 11$; $137 + 13 + 4 = 154 = 2\cdot 7\cdot 11$; figure 2 is the multiplicator in front of figure 7, before it is the exponent of figure 7;

$\ln(137)\cdot\ln(13)\cdot\ln(4)\cdot\pi^{-2{,}5} = 1{,}000053$ [≈ 1]; $137 + 13 + 4 - 25 = 3\cdot 43$ figure 43 at Equation (G14);

$\ln(137) = 4{,}919981$ [≈ 4,92; $492 = 4\cdot 123$]; $4\cdot 137 - 492 + 25 = 9\cdot 9$; $137 - 123 = 2\cdot 7$

The following Equation (αSP) is an extension of Equation (αHR) [13] of R. Heyrovska, which is dependent on the Golden Ratio Φ $[= (5+5\cdot\sqrt{5})/10 = 1{,}618033989]$ and which also uses the figure 3^5 (=243). Equation (αHR) [13] of R. Heyrovska is written as follows:

$$\alpha_{HR}^{-1} = 360/\Phi^2 - 2/\Phi^3 = 137{,}035628095 \qquad (\alpha HR)$$

Stergios Pellis extended upper Equation by the term "$3^5 \cdot \Phi^5$", which possesses equal exponents[13]:

$$\alpha_{SP}^{-1} = 360/\Phi^2 - 2/\Phi^3 + 1/(3^5\cdot\Phi^5) = 137{,}035999165 \qquad (\alpha SP)$$

The result of Equation (αSP) lies within the tolerance range of CODATA 2022.

Stergios Pellis' Equation (αSP) is now extended with the term "$-1/(5^8\cdot\Phi^8)$" and leads to Equation (α2):

$$\alpha_{\#2}^{-1} = 360/(1^2\cdot\Phi^2) - 16/(2^3\cdot\Phi^3) + 1/(3^5\cdot\Phi^5) - 1/(5^8\cdot\Phi^8) = 137{,}0359991103 \qquad (\alpha 2)$$

One can observe the Fibonacci-Sequence for the exponents with starting figures 2 and 3: 2 – 3 – 5 – 8!
The basis of the multipliers in front of the Φ-Terms have starting figures 1 and 2: 1 – 2 – 3 – 5 (– 8)!
Connection of the figures: $360 - 16 = 8\cdot 43$ see use of figure 43 at Equations (G14) and (G10a), page 5

One can transcribe Equation (α2) in the way, that the figure 1 is located in the numerator for every term containing the Fibonacci-Sequence "1 – 2 – 3 – 5 – …" for the Multiplicators and "2 – 3 – 5 – 8 – …" for the Exponents! This can be seen at the second block of Equation (α2a).
The transcribed Equation (α2a) is given to:

$$\alpha_{\#2a}^{-1} = [0{,}5/\Phi^0 - 2/\Phi^1 + 360/\Phi^2 - (3/8)/\Phi^3] + [1/(1^2\cdot\Phi^2) - 1/(2^3\cdot\Phi^3) + 1/(3^5\cdot\Phi^5) - 1/(5^8\cdot\Phi^8) \ldots] = 137{,}0359991103 \qquad (\alpha 2a)$$

In the first block of Equation (α2a) new terms appear at this transcribed Formula. Astonishingly at one of the terms the fraction "3/8" is visible, which is the exponent of Equation (rp3) and which is the reciprocal of the fraction of the Thomson Cross Section[2.6] σ_e $[= (8/3)\cdot\pi\cdot r_e^2]$.
The exponents of the Golden Ratio Φ within the first rectangular brackets comprise the figures n = 0 .. 3.
Connection of the used Figures: $360\cdot 3/8 - 5 - 2 = 128$ see Figure 128 at next Equation (α13).

Result of Equation (α2a) is very close to the one of Equation (α13). Is that random?

The fraction 8/3, of which the reciprocal is used at Equation (α2a), can also observed at the term M_{T1} of Equation (α13), which is presented below.

Serie Term $\boldsymbol{\Gamma}_{MT1}$ of the following Equation (α13), which is presented in the author's report [14], uses the Multiplication Term "1 - (8/3)/44.444", the Figures 12 and 128. The start value α_0^{-1} (= 137,035999110) is chosen for the Reciprocal of the Fine Structure Constant:

$M_{T1} = 1 - \alpha_0 \cdot (8/3)/44{,}444 = 0{,}9995621545$ [8/3: Fraction of the Thomson Cross Section σ_e]

$\boldsymbol{\Gamma}_{MT1} = 1 + M_{T1} \cdot (\alpha_0^1/(2\cdot\pi)^0 + \alpha_0^2/(2\cdot\pi)^1 + \alpha_0^3/(2\cdot\pi)^3 + \alpha_0^4/(2\cdot\pi)^6 + \alpha_0^5/(2\cdot\pi)^{10} + \alpha_0^6/(2\cdot\pi)^{15} + \dots) =$
$= 1{,}007302630526$

$$\alpha_{\#13}^{-1} = 128 \cdot \boldsymbol{\Gamma}_{MT1}^{\,12/1{,}28} = 137{,}0359991095 \qquad (\alpha 13)$$

By a tiny change of the exponent "12/1,28" to "(12/1,28) $\cdot$ (1 $\pm 1{,}1\cdot 10^{-8}$)" the result values of Equation (α13) even lie outside the lower tolerance value (137.035999063) of CODATA2018 and outside the upper tolerance value (137.035999198) of CODATA2022.

There is a fascinating connection of the figures of Equations (z-GB), (α2a) and (α13). Please remember the used figures **2, 12, 128, 360, 44444 and the fraction 8/3** at these Equations!
The Equation (z-GB) is repeated once more:

$z_{GB} = 128 \cdot \ln(2) \cdot \ln(128) = 430{,}48590047$

The connections with these figures are as follows:

$C_{1a} = \ln(2) \cdot \ln(128) \cdot \ln(44444) = 35{,}992607$ [$\approx 36 = 360 / 10$]
$C_1 = \ln(2) \cdot \ln(128) \cdot \ln(44444) \cdot [0{,}5 \cdot g_e]^{1/(1{,}8\cdot\pi)} = 35{,}999985$ [≈ 36; $36 \cdot 0{,}5 / 10 = 1{,}8$]

$C_{2a} = \pi \cdot \ln(2) \cdot \ln(8/3) \cdot \ln(128) \cdot \ln(360) = 60{,}998644$ [$\approx 61 = 25 + 36 = 5^2 + 6^2$; $6/5 \cdot 10 = 12$]
$C_2 = \pi \cdot \ln(2) \cdot \ln(8/3) \cdot \ln(128) \cdot \ln(360) \cdot [0{,}5 \cdot g_e]^{(3/8)/(2\cdot\pi\cdot\pi)} = 60{,}999987$ [≈ 61]

$C_2 - C_1 = 60{,}999987 - 35{,}999985 = 25{,}000002$ [≈ 25];

Exponent $(3/8)/(2 \cdot \pi^2) = 0{,}0189977219$ [$\approx (18 + 0{,}997722) \cdot 10^{-3}$; $18 \cdot 20 = 360$; $99 - 77 = 22$;
$99 + 77 + 22 - 18 = 180 = 360 / 2$]

In the report[13] of Stergios Pellis an exact Equation is presented for the Golden Ratio Φ in dependence on the Figure 666:

$\Phi = [2 - 1/\sin(666°)] / 2 = 1{,}618033989$

The Golden Ratio is also known as the Divine Figure. And it can be derived by the Figure 666!
Because of this circumstance one has to overthink the meaning of 666 given by the Bible. The reader can get more information about this topic in the author's report [14].

Above Equation one can write as follows:

$\Phi = [2 - 1/\sin(666 \cdot 2\cdot\pi/360)] / 2 = 1{,}618033989$

Connections of the figures: $\ln(666) \cdot \ln(2 \cdot \text{Phi}) / \pi = 2{,}430248$ [$\approx 2{,}43 = 243 / 100 = 3^5 / 100$];
$666 - 2 \cdot 243 = 180 = 360 / 2$ see Figure 243 at Equation (G20) and (αSP) at page 7;
first four digits of Φ are 1 6 1 8: $666 + 360 + 3 \cdot 243 - 1618 =$ **137** [$\approx \alpha^{-1}$]; $3 \cdot 243 = 3^6$;
$1618 - 3^5 = 1375 = 11 \cdot 5^3$ basis and exponent exchanged: 3^5 --> 5^3

<u>Formulas with exponents containing the mass ratio (r_p/r_e) and multiples of 5:</u>

$$G_{\#23} = \{h \cdot c / [\pi \cdot \alpha \cdot (4\cdot\pi / \alpha)^{4\cdot\pi} \cdot m_p^2]\} / [1 + 2 \cdot \Delta_{JS} \cdot (r_p / r_e)]^{1{,}215\cdot r_e / r_p}$$
$$= 6{,}6743005 \cdot 10^{-11} \ \text{kg}^{-1} \cdot \text{m}^3 \cdot \text{s}^{-2} \qquad (G23)$$

Connection of Figure 1215: $1215 = 5 \cdot 3^5 = 5 \cdot 243$ Figure 243 is used at Equations (G20), (αSP) and (α2)
Connection to Figure 725 at Equation (G24): $\ln(1215) \cdot \ln(725) / \pi = 14{,}88999$ [$\approx 14{,}89$];
$1489 - 1215 = 274 = 2 \cdot 137$ [$\approx 2 \cdot \alpha^{-1}$];
$1489 - 1215 + 725 = 999$; $1215 - 999 = 216 = 6 \cdot 6 \cdot 6$

$G_{\#24} = \{h \cdot c / [\pi \cdot \alpha \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot m_p{}^2]\} / [1 + \Delta_{JS} \cdot (r_p / r_e)]^{0,725 \cdot (r_e / r_p) \cdot (r_e / r_p)}$
$= 6,6743001 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G24)

Connection: $725 = 5^2 \cdot 29$ see Figure 29 at Equations (G7) and (G11); $1215 - 725 = 490 = 2 \cdot 5 \cdot 7^2$;
$\ln(725) \cdot \pi^2 = 65,003$ [≈ 65]; $137 - 65 = 2^3 \cdot 3^2$; $\ln(725) - \ln(137) + 1 = 2,66619$ [$\approx 8/3$]
One digit Primes 2, 3, 5, 7 are used or derived, respectively at Equations (G23) and (G24)

$G_{\#25} = \{h \cdot c / [\pi \cdot \alpha \cdot (4 \cdot \pi / \alpha)^{4 \cdot \pi} \cdot m_p{}^2]\} / [1 + (r_p / r_e) / \pi]^{\pi \cdot (r_p / r_e) / 30,15}$
$= 6,6743000 \cdot 10^{-11}\ kg^{-1} \cdot m^3 \cdot s^{-2}$ (G25)

Connections of Figure 3015: $3015 = 5 \cdot 3^2 \cdot 67$; $3015 - 1215 - 725 = 1075 = 5^2 \cdot 43 = 5^2 \cdot (3^3 + 2^4)$;
Figure 67 is used at Equation (G9), Figure 43 at Equation (G14); see Figures 725 and 1215 before

Please look at the exactness of Equations (G23) to (G25), although the exponents of these Equations consist of the radius ratio r_e/r_p.
Formula (G-Fr) of Frino's report[12] is a combination of Formula (G-GB) of Bailey[10] and Formula (Trmα) presented at page 2.

Conclusion:

By use of the "Hans de Vries"- and the "Julian Schwinger"-Term one is able to perform very accurate Formulas for the Proton Radius and the Gravitational Constant. The fraction of a Formula of a Physical Constant is the exponent of several Fine Tuning-Terms for the Proton Radius, further Physical Constants themselfes are part of the exponents of several Fine Tuning-Terms at their formulas.
It may be worth for an investigation by experts of mathematics, if the "exact" [i.e. very accurate] results won by the Fine Tuning Terms are random. In this context the author indicates to his report [14], in which also many Fine Tuning Terms are presented for several Physical Constants leading to "exact" results.

In the german Magazin *Spektrum der Wissenschaft* from december 2025, the following statement of the Nobel-Prize-Winner Gerard ‘t Hooft is submitted by a reader:

“The reason, why there is nothing New, is that all equally think”.

The author takes the liberty to change this statement:

“The reason, why there is seldom something New,
is that the mayority equally thinks and
is inactive or try to make badly what they do not like”.

If there is a Divine Being, who was able to create the Universe with billions of galaxies, then one has to ask referring to Physical Constants: Would this Divine Being not be able performing incredibles formulas, which are connected with each other?
This Divine Being apparantly is full of humour and it certainly will not fulfill the wishes (which refer to the form of physical formulas) of oldfashioned thinking.
When a – even renowned – scientist says, that a "fantastic" formula is not physically plausible, is he/she right or not? The Divine Being might only smile about it, because it knows that the future will bring the truth. The famous Physicist Max Planck made an appropriate statement about this oldfashioned thinking.

In this context fits the Formula of Planck's Constant h without SI-Units, which is already presented in Literature [14]:

$h_{wU} = 6,62607015 \cdot 10^{-34} = (144 \cdot 666)^{-6.659942071}$ wU: without SI Units

What does the reader think about this Formula and its exponent close to 6,66? Can this be random?

Another proof of the significance of the figures 144 and 666 are the following formulas, which work with the sin- and cos-functions and which lead to the coefficient of the Golden Ratio using these figures.

The Golden Ratio can be written in dependence of the Figure 666:

$\Phi = [2 - 1/\sin(666°)] / 2 = 1{,}618033989$

The Golden Ratio can be written in dependence of the Figure 144:

$\Phi = [2 - 1/\cos(144°)] / 2 = 1{,}618033989$

One has to imagine: more or less exact SI-Units "kg, m and s" have been used since some 100 years. The Planck's Constant has been influenced the history of the Universe since billions of years and the mankind have been imposed up certain values to this Constant since the year 1899.
Isn’t it strange, that by the figures 144 and 666 and by an exponent, which is very close to the figure 6,66, one is able to describe the value of Planck's Constant - without considering the SI-Units - and that by the figures 144 and 666 one is able to calculate the Coefficient of the Golden Ratio by two simple Formulas, which are equally constructed. Please look at the (quite) accurate and "fantastic" Formulas at next pages!

The just presented possesses a bit of the fairy-tale called *The Emporer’s New Clothes*:
One day a child or a juvenile will read this report or an similar one and he/she will proclaim: these formulas cannot be random, there must be something higher causing this!
And this statement - maybe posted - might change everything!

<u>Used Data of Physical Constants:</u>

Electric Field Constant ε_0[2.11]:	$8{,}854\,187\,8188(14) \cdot 10^{-12}$ C V^{-1} m^{-1}
Electron Charge e[2.10]:	$1{,}602\,176\,634 \cdot 10^{-19}$ C
Fine Structure Constant α[2.1]:	$7{,}297\,352\,5643(11) \cdot 10^{-3}$
Reciproke of Fine Structure Constant $1/\alpha$[2.2]:	137,035 999 177(21)
Gravitational Constant G[2.8]:	$6{,}67430(15) \cdot 10^{-11}$ m^3 kg^{-1} s^{-2}
Landé Factor for the Electron g_e[2.3]:	2,002 319 304 360 92 (36)
Light velocity c[2.7]:	299 792 458 m/s
Mass of Electron m_e[2.5]:	$9{,}109\,383\,7139(28) \cdot 10^{-31}$ kg
Mass of Proton m_p[5]:	$1{,}672\,621\,925\,95(52) \cdot 10^{-27}$ kg
Planck’s Constant h[2.9]:	$6{,}626\,070\,15 \cdot 10^{-34}$ J s
Radius of Electron r_e[2.4]:	$2{,}817\,940\,3205(13) \cdot 10^{-15}$ m
Radius of Proton r_p[4]:	$0{,}84087(39) \cdot 10^{-15}$ m

The figures in the brackets behind the data descibe the uncertainty referring to the last digits of the given value[2].

Calculations were performed with the LibreOffice Calc-Software.

Addendum: "Fantastic" Formulas by use of the figures 144 and 666

Approximations are marked with a double cross #

Planck's Constant h **w**ithout SI **U**nits: $h_{wU} = 6{,}62607015 \cdot 10^{-34}$ [wU: without SI Units]

The very exact and extraordinary Equation (h1) for the Planck's Constant is dependent on the Landé-Factor g_e and the "Hans de Vries"-Term Γ_{dV}. The exponent figure 3357 is explained further below.

$$h_{wU\#1} = (144 \cdot 666)^{-6.66} \cdot 0{,}999^{-6{,}66/10} \,/\, [\Gamma_{dV} / (0{,}5 \cdot g_e)]^{1/3357} = 6{,}62607014999 \cdot 10^{-34} \qquad \text{(h1)}$$

Connections of the figures: $3357 = 10 \cdot 333 + 3 \cdot 3 \cdot 3$; $666 + 333 = 999$; $666 - 333 = 333$;
$3357 = (3+3+3) \cdot 333 + 360$ Figure 360 is used at Equations (αHR) and (α2)
$360 = 10 \cdot 36 = 2 \cdot 5 \cdot 2^2 \cdot 3^2 = (2+3) \cdot 2^3 \cdot 3^2$

The next Equation (h2) is also remarkable, but is not as accurate as Equation (h1).

$$h_{wU\#2} = (144 \cdot 666)^{-6.66} \cdot 0{,}999^{-6{,}66/10} \,/\, \Gamma_{dV}^{1/3993} = 6{,}6260701509 \cdot 10^{-34} \qquad \text{(h2)}$$

Connections of the figures: $3 \cdot 11^3 = 3993$; $3993 = 3996 - 3 = 12 \cdot 333 - 3 = 6 \cdot 666 - 3$;
Connection to figure 3357 above: $3993 - 3357 = 636$; $2 \cdot 0{,}636 = 1{,}272$ ($\approx \sqrt{\Phi} = 1{,}27202$);
the Golden Ratio Φ can be derived by the trigonometrical functions Sinus and Cosine, which is presented at page 10.

The next Equation (h3) is a bit more accurate than Equation (h2).

$$h_{wU\#3} = (144 \cdot 666)^{-6.66} \cdot (0{,}666 \cdot 3/2)^{-2/3} \cdot [(1+\Delta_{JS}) / \Gamma_{dV})]^{1/2457} = 6{,}6260701493 \cdot 10^{-34} \qquad \text{(h3)}$$

Connections of the figures: $3993 - 2457 = 1536 = 3 \cdot 512 = 3 \cdot 2^{3 \cdot 3}$; $0{,}666 \cdot 3/2 = 0{,}999$

Light Velocity c **w**ithout SI **U**nits: $c_{wU} = 299792458$ [wU: without SI Units]

In the author's report [14] the very good Approximation (c1) and the exact Equation (c2) are presented:

$$\begin{aligned} c_{\#1} = (144^3 + 666^3) &+ 3 \cdot (144^2 + 666^2) + \\ &+ 6 \cdot (144^1 + 666^1) + \\ &+ 9 \cdot (144^{0,5} + 666^{0,5}) + \\ &+ 12 \cdot (144^{0,25} + 666^{0,25}) = 299792458{,}79 \end{aligned} \qquad \text{(c1)}$$

Deviation: $c_{\#1} - c_{wU} = 0{,}79$; See the terms with multiplicators starting from 3 and increasing by 3!
Exponents starting from 2 and decreasing by factor 0,5!

One has to consider, that each of the three terms "$666^{0,5}$, $666^{0,25}$ and $144^{0,25}$" possesses a crooked value and all together deliver a value, which fits with the other terms very well to the set value c_{wU}.
Remarkable: with the second link of above Equation (c1) this good approximation for the sun diameter in unit km is yielded: **$3 \cdot (144^2 + 666^2)$ km = 1392876 km** [$\approx$ Ø$_{Sun}$]

Exact Formula for the light velocity c_{wU}:

$$c_{\#2} = 144^3 + 666^3 + 3 \cdot (144^2 + 666^2) + (40/9) \cdot 144 + 7 \cdot 666 = 299792458 \qquad \text{(c2)}$$

This is the exact value, deviation Zero !!!
The figures of the last two termes are: 40, 9, 144, 7, 666
Connections: $7^2 = 40 + 9$; $40 \cdot 144 - 7 \cdot 666 - 9 = 1089 = 33^2 = (3 \cdot 11)^2$; $666 - 9 \cdot 40 - 144 + 7 = 13^2$;
$666 - 9 \cdot 40 - 144 = 2 \cdot 9^2$; $7 \cdot 40 - 144 - 11 = 5^3$; $7 \cdot 40 + 9^1 = 17^2$; $7 \cdot 40 + 9^2 = 19^2$;
$666 - 144 - 9^2 = 9 \cdot 7^2$; Primes from 2 to 19 are given or derived; $144 - 7 = 137$ [$\approx \alpha^{-1}$]

The first Fine Tuning Term $\Gamma_{dV}^{2/0{,}99}$ of Equation (c3) is dependent on the "Hans de Vries"-Term. The second Fine Tuning Term is dependent on the "Hans de Vries"- and "Julian Schwinger"-Term.

$$c_{\#3} = 666^3 \cdot \Gamma_{dV}^{2/0{,}99} \cdot [\Gamma_{dV} \cdot (1+\Delta_{JS})]^{1/320} = 299792457{,}4572 \qquad \text{(c3)}$$

Connections of the figures: $320 - 2 \cdot 99 = 122 = 2 \cdot 61$ see figure 61 at Equation (G2);

$320 - 2 \cdot 99 + 3 = 125 = 5^3$ see figure 0,125 (=1/8) at Equation (G5);
$666 - 320 - 2 \cdot 99 - 3 = 145 = 5 \cdot 29$ see figure 29 at Equation (G7);
$320 = 5 \cdot 2^6$; $320 - 137 = 183 = 3 \cdot 61$; $61 - 29 = 32 = 2^5$

The second Fine Tuning Term of Equation (c4) is dependent on the "Julian Schwinger"-Term and the Figure 0,99, which can be observed two times. The exponent figure 337 is explained below.

$$c_{\#4} = 666^3 \cdot \Gamma_{dV}{}^{2/0,99} / [0,99 \cdot (1 + \Delta_{JS})]^{1/337} = 299792458,5405 \qquad \text{(c4)}$$

Connections of the figures: there are the figures without decimal points: 666, 3, 2, 99, 99, 337;
$337 - 99 - 99 + 2 + 3 = 144$ figure 144 is connected to 666;
$666 - 337 - 3 - 2 = 324 = 2 \cdot 2 \cdot 9 \cdot 9$; $324 - 99 - 99 - 3 - 2 = 121 = 11^2$;
$666 + 337 + 99 + 99 + 3 + 2 = 1206 = 2 \cdot 9 \cdot 67$;
$337 - 99 - 99 - 3 - 2 = 134 = 2 \cdot 67$; Figure 67 is used at Equation (G9)

It is extremely astonishing, that the half of the sum of Equations (c3) and (c4) delivers a very accurate result of the Light Velocity c_{wU}. This can be observed at Equation (c5):

$$c_{\#5} = 0,5 \cdot (c_{\#3} + c_{\#4}) = 0,5 \cdot (299792457,4572 + 299792458,5405) = 299792457,9988 \qquad \text{(c5)}$$

Connections of the exponents at Equations (c3) and (c4): $337 - 320 = 17$; $9^2 - 8^2 = 9 + 8 = 17$;
$666 - 337 - 320 = 9$; $144 \cdot (8/2) - 337 - 320 = -9^2$

<u>Atomic Mass of Helium[15] without Unit u dependent on the Light Velocity c_{wU}:</u>

$m_{uHe} = 4,002602$ u (Tolerance: $\pm 2 \cdot 10^{-6}$ u)
$m_{uHe\#} = 4,002602$

$$m_{uHe\#1} = (12 \cdot 10^8 / c_{wU}) \cdot [1 - 1/(144 \cdot 666)]^4 = 4,00260220 \qquad \text{(muHe1)}$$

Deviation: $m_{uHe\#1} - m_{uHe\#} = 1,96 \cdot 10^{-7}$

$$m_{uHe\#2} = (12 \cdot 10^8 / c_{wU}) \cdot [1 - 1/(144 \cdot 666)]^{4,002602} = 4,00260209 \qquad \text{(muHe2)}$$

Deviation: $m_{uHe\#2} - m_{uHe\#} = 0,874 \cdot 10^{-7}$

Remarkable: Formula ($m_{uHe2\#}$) with the exponent $m_{uHe\#}$ (=4,002602) is more accurate as Formula ($m_{uHe1\#}$) with the exponent 4. Equations are already presented at the author's report [14], page 9.

Please look also at the "fantastic" Formulas for the Norm Temperature T_N, for the Norm Pressure p_N and for the Term "$p_N \cdot T_N$" at the author's report [14], page 11.
By use of the Fine Structure Constant α, which is valid in the whole Universe, very accurate and visibly well structured Approximations are won for the choice Norm Data T_N and p_N!

Literature and wikipedia.de-Entries:

The data of the physical Constants are taken from the entries of Wikipedia Germany. The Physical Constants given in the corresponding entries refer mostly to CODATA 2022.

[1] www.chip-architect.com/news/2004_10_04_The_Electro_Magnetic_coupling_constant.html;
An exact formula for the Electro Magnetic coupling constant. (fine structure constant) by Hans de Vries

[2] Wikipedia.de-Entry "Physikalische Konstante"; Status January 2026

[2.1] Fine Structure Constant α:
26. CODATA Recommended Values (2022). (https://physics.nist.gov/cgi-bin/cuu/Value?alph) NIST, abgerufen am 11. Juni 2024 (englisch, Wert für die Feinstrukturkonstante)

[2.2] Reciprocal of Fine Structure Constant α^{-1}:
27. CODATA Recommended Values (2022). (https://physics.nist.gov/cgi-bin/cuu/Value?alphinv) NIST, abgerufen am 11. Juni 2024 (englisch, Kehrwert der Feinstrukturkonstante)

[2.3] Landé-Faktor g_e:
55. CODATA Recommended Values (2022). (https://physics.nist.gov/cgi-bin/cuu/Value?gem) NIST, abgerufen am 11. Juni 2024 (englisch, Wert für den Landé-Faktor des freien Elektrons)

[2.4] Electron Radius r_e:
49. CODATA Recommended Values (2022). (https://physics.nist.gov/cgi-bin/cuu/Value?re) NIST, abgerufen am 11. Juni 2024 (englisch, Wert für den klassischen Elektronenradius).

[2.5] Electron Mass m_e:
44. CODATA Recommended Values (2022). (https://physics.nist.gov/cgi-bin/cuu/Value?me) NIST, abgerufen am 11. Juni 2024 (englisch, Wert für die Elektronenmasse in Kilogramm).

[2.6] Thomson-Wirkungsquerschnitt σ_e:
50. CODATA Recommended Values (2022). (https://physics.nist.gov/cgi-bin/cuu/Value?sigmae) NIST, abgerufen am 11. Juni 2024 (englisch, Wert für den Thomson-Wirkungsquerschnitt)

[2.7] Light Velocity c:
13. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?c) NIST, abgerufen am 3. Juni 2019 (englisch, Wert für die Lichtgeschwindigkeit)

[2.8] Gravitational Constant G:
31. CODATA Recommended Values (2022). (https://physics.nist.gov/cgi-bin/cuu/Value?bg) NIST, abgerufen am 11. Juni 2024 (englisch, Wert für die Gravitationskonstante)

[2.9] Plancks Constant h:
18. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?h) NIST, abgerufen am 3. Juni 2019 (englisch, Wert für die Planck-Konstante in der Einheit Js)

[2.10] Electron Charge e:
14. CODATA Recommended Values. (https://physics.nist.gov/cgi-bin/cuu/Value?e) NIST, abgerufen am 3. Juni 2019 (englisch, Wert für die Elementarladung)

[2.11] Electric Field Constant ε_0:
16. CODATA Recommended Values (2022). (https://physics.nist.gov/cgi-bin/cuu/Value?ep0) NIST, abgerufen am 11. Juni 2024 (englisch, Wert für die elektrische Feldkonstante).

[3] Wikipedia.en-Entry "Julian Schwinger"; Status August 2025

[4] "Measuring the size of the proton"; Dr. Randolf Pohl (Max-Planck-Institut f. Quantenoptik)
https://indico.ph.tum.de/event/3056; Status August 2025

[5] Wikipedia.de-Entry "Proton"; Status January 2026
1. Die Angaben über die Teilcheneigenschaften (Infobox) sind, wenn nicht anders angegeben, entnommen aus: CODATA Recommended Values (2022). (https://physics.nist.gov/cgi-bin/cuu/Category?view=html&Atomic+and+nuclear.x=114&Atomic+and+nuclear.y=16) National Institute of Standards and Technology, abgerufen am 11. Juni 2024. Die eingeklammerten Ziffern bezeichnen die Unsicherheit in den letzten Stellen des Wertes, diese Unsicherheit ist als geschätzte Standardabweichung des angegebenen Zahlenwertes vom tatsächlichen Wert angegeben.

[6] Wikipedia.de-Entry “Large Number Hypothesis“; Status August 2025

[7] Freistetters Formelwelt: “Die mysteriöse Zahl, die überall im Universum auftaucht“; Michael Freistetter; monthly german Magazin *Spektrum der Wissenschaft, 11.2024*

[8] Exact Formulas of the Age of the Universe and of the Gravitational Constant dependent on Physical Constants; Andreas Ball;
Published at viXra: viXra submission: 2502.0074v3; 2025-04-17

[9] Wikipedia.de-Entry “Universum“; Status August 2025
2. Planck 2018 results. VI. Cosmological parameters. In: Astronomy & Astrophysics. Band 641. Planck Collaboration, 2020, S. A6, PDF Seiten 15, Tabelle 2: "Age/Gyr", letzte Spalte, doi:10.1051/0004-6361/201833910 (https://doi.org/10.1051/0004-6361%2F201833910), arxiv:1807.06209 (https://arxiv.org/abs/1807.06209), bibcode:2020A&A...641A...6P (https://ui.adsabs.harvard.edu/abs/2020A&A...641A...6P) (englisch)

[10] An universe age of 13.807 billion years and a proton radius of 0.8403 fm would fit perfectly Dirac/Eddington's Large Number Hypothesis; George Bailey
Published at viXra: viXra submission 2108.0081v2; 2022-01-23

[11] The link between the de-Vries-formula for the fine structure constant and the power-of-two value of 128; George Bailey
Published at viXra: viXra submission 2310.0136; 2023-10-29

[12] The Age of the Universe and other Cosmological Issues; Rodolfo A. Frino
Published at viXra: viXra submission 1407.0071v2; 2014-09-22

[13] Exact formula for the fine-structure constant α in terms of the golden ratio φ; Stergios Pellis
Published at viXra: viXra submission 2110.0053v3; 2021-10-13

[14] Exact Approximations of Physical Constants using the Figures Φ, π, 144 and 666;
Andreas Ball; Published at viXra: viXra submission 2406.0018v3; 2025-12-01

[15] Wikipedia.de Entry “Helium“; Status January 2026
3. Michael E. Wieser, Tyler B. Coplen: Atomic weights of the elements (IUPAC Technical Report) In: Pure and Applied Chemistry Vol. 83, No. 2, 2011, S. 359–396.
4. IUPAC, Standard Atomic Weights Revised 2013 (http://www.ciaaw.org/pubs/TSAW2013_xls.xls).