

Four index Einstein field equations with quantum like effects from pure geometry

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Abstract

In this work I present extensions of Einstein field equations [1] into four index equations. This extensions give as natural result a energy tensor for vacuum thus for gravity field. It's all construct in spirit of two index field equations and in truth does not need any additional assumptions about field equations. Form it follows that it's natural completeness of two index equations not a true extension as it fully defines curvature tensor not only Ricci part of curvature as it happens in two index equations.

Quantum effects are divided into two parts, one is about wave function like object and measurement, next one is about spin as orientation of manifold. Wave function like object is constructed from normalized curvature invariant. That plays role of "probability" of finding object in given volume of spacetime at given interval of time. I did no present direct solutions to those equations or concrete examples where it differs from General Relativity [1].

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1 Einstein field equations

1.1 Einstein tensor

Einstein tensor is basis of General Relativity [1], from it arises left side of field equations that connects matter field represented by stress momentum tensor [1] with geometry part. That geometry part is just Einstein tensor that comes

from Bianchi identities [1]. It can be written as:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \quad (1)$$

It's most important property is that is divergence free [1] so that:

$$\nabla^\mu G_{\mu\nu} = 0 \quad (2)$$

It connects to left side of equations where stress momentum tensor [1] is present.

1.2 Stress momentum tensor

Matter sources in General Relativity [1] are all coming from right side of equation that is stress momentum tensor [1] $T_{\mu\nu}$. Units of stress momentum tensor are energy density so from it follows that on left side of equation I have units of curvature that is length to power minus two, thats why if field equations [1] I need to use Einstein constant that is equal to:

$$\kappa = \frac{8\pi G}{c^4} \quad (3)$$

Another crucial property of stress momentum tensor is that as on left side I have divergence free tensor , stress momentum tensor is divergence free meaning that matter fields are conserved locally. It means that divergence of stress momentum tensor is zero [1]:

$$\nabla^\mu T_{\mu\nu} = 0 \quad (4)$$

1.3 Field equation

That finally leads to field equation that I will call in this work a two index field equation, that is just ordinary Einstein field equation [1]:

$$G_{\mu\nu} = \kappa T_{\mu\nu} \quad (5)$$

Idea behind this work is simple extended this equation into four index equation, adding part that is responsible for vacuum energy or more precise gravity field energy.

1.4 Cosmological constant

Two index equations used today have additional term [1], that term is cosmological constant or energy of empty space Λ it modifies field equations by adding term with it to field equations [1]:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \quad (6)$$

In vacuum it will lead to:

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} \quad (7)$$

That solutions will be either de Sitter or anti-de Sitter [1] spacetime depending on sign of cosmological constant.

2 Four index Einstein tensor

2.1 Ricci part

Riemann tensor [1] can be decomposed into Ricci and Weyl parts:

$$R_{\alpha\mu\beta\nu} = C_{\alpha\mu\beta\nu} + \frac{1}{n-2} (R_{\alpha\beta}g_{\mu\nu} - R_{\alpha\nu}g_{\beta\mu} + R_{\mu\nu}g_{\alpha\beta} - R_{\beta\mu}g_{\alpha\nu}) - \frac{1}{(n-1)(n-2)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) R \quad (8)$$

This decomposition gives after contraction exactly Ricci tensor as part with Weyl vanishes when contracted as it's trace free for all indexes combinations. So isolating Ricci part but adding part that will after contraction reduce exactly to one half of Ricci scalar times metric tensor I will arrive at:

$$\frac{1}{n-2} (R_{\alpha\beta}g_{\mu\nu} - R_{\alpha\nu}g_{\beta\mu} + R_{\mu\nu}g_{\alpha\beta} - R_{\beta\mu}g_{\alpha\nu}) - \frac{1}{(n-1)(n-2)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) R - \frac{1}{2(n-1)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) R \quad (9)$$

I can sum terms with Ricci scalar and will arrive at final expression for first component of four index Einstein tensor:

$$\tilde{R}_{\alpha\mu\beta\nu} = \frac{1}{n-2} (R_{\alpha\beta}g_{\mu\nu} - R_{\alpha\nu}g_{\beta\mu} + R_{\mu\nu}g_{\alpha\beta} - R_{\beta\mu}g_{\alpha\nu}) - \frac{n}{2(n-1)(n-2)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) R \quad (10)$$

2.2 Divergence of Ricci part

From it follows that I can write divergence of this component with respect to first index where I will use Bianchi identities for same indexes of Ricci tensor

and divergence index so index α to make equations simpler:

$$\begin{aligned} \nabla^\alpha \tilde{R}_{\alpha\mu\beta\nu} &= \frac{1}{n-2} (\nabla^\alpha R_{\alpha\beta} g_{\mu\nu} - \nabla^\alpha R_{\alpha\nu} g_{\beta\mu} + \nabla_\beta R_{\mu\nu} - \nabla_\nu R_{\beta\mu}) \\ &\quad - \frac{n}{2(n-1)(n-2)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) R \end{aligned} \quad (11)$$

$$\begin{aligned} \nabla^\alpha \tilde{R}_{\alpha\mu\beta\nu} &= \frac{1}{n-2} (\nabla_\beta R_{\mu\nu} - \nabla_\nu R_{\beta\mu}) \\ &\quad - \frac{n}{2(n-1)(n-2)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) R + \frac{1}{2(n-2)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) R \end{aligned} \quad (12)$$

$$\nabla^\alpha \tilde{R}_{\alpha\mu\beta\nu} = \frac{1}{n-2} (\nabla_\beta R_{\mu\nu} - \nabla_\nu R_{\beta\mu}) - \frac{1}{2(n-1)(n-2)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) R \quad (13)$$

Taking out common denominator I will arrive at final expression:

$$\nabla^\alpha \tilde{R}_{\alpha\mu\beta\nu} = \frac{1}{n-2} \left((\nabla_\beta R_{\mu\nu} - \nabla_\nu R_{\beta\mu}) - \frac{1}{2(n-1)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) R \right) \quad (14)$$

2.3 Weyl part

Weyl [1] part divergence is equal to:

$$\nabla^\alpha C_{\alpha\mu\beta\nu} = \frac{n-3}{n-2} \left((\nabla_\beta R_{\mu\nu} - \nabla_\nu R_{\beta\mu}) - \frac{1}{2(n-1)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) R \right) \quad (15)$$

This means that they differ only by a constant $n-3$, from it follows to create a field equation I need first to take that constant and divide Weyl tensor by it. Then whole tensor has to be divergence free, from it follows that I will take Ricci part and subtract Weyl dived by that constant:

$$\tilde{R}_{\alpha\mu\beta\nu} - \frac{1}{n-3} C_{\alpha\mu\beta\nu} \quad (16)$$

2.4 Four index Einstein tensor

Last expression is just four index Einstein tensor, it all properties of being divergence free and it's contractions lead to two index equations. This tensor can be written:

$$G_{\alpha\mu\beta\nu} = \tilde{R}_{\alpha\mu\beta\nu} - \frac{1}{n-3} C_{\alpha\mu\beta\nu} \quad (17)$$

3 Four index energy tensor

3.1 Matter part

I can use same form as in Riemann decomposition into matter part of total energy tensor. This tensor will reduce to stress momentum tensor so I can write it:

$$\begin{aligned} \tilde{T}_{\alpha\mu\beta\nu} = & \frac{1}{n-2} (T_{\alpha\beta}g_{\mu\nu} - T_{\alpha\nu}g_{\beta\mu} + T_{\mu\nu}g_{\alpha\beta} - T_{\beta\mu}g_{\alpha\nu}) \\ & - \frac{1}{(n-1)(n-2)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) T \end{aligned} \quad (18)$$

Now key property is to calculate divergence of this tensor with respect to first index as before, I can use property that stress momentum tensor is divergence free:

$$\nabla^\alpha \tilde{T}_{\alpha\mu\beta\nu} = \frac{1}{n-2} (\nabla_\beta T_{\mu\nu} - \nabla_\nu T_{\beta\mu}) - \frac{1}{(n-1)(n-2)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) T \quad (19)$$

Taking out common denominator:

$$\nabla^\alpha \tilde{T}_{\alpha\mu\beta\nu} = \frac{1}{n-2} \left((\nabla_\beta T_{\mu\nu} - \nabla_\nu T_{\beta\mu}) - \frac{1}{(n-1)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) T \right) \quad (20)$$

That is exactly propagation equation [1] from two index equations, where it does differ by same constant.

3.2 Vacuum energy part

From previous equations I can see a logic here, vacuum energy tensor divergence will be have to be equal to same as matter part times a constant defined before:

$$\nabla^\alpha \hat{T}_{\alpha\mu\beta\nu} = \frac{n-3}{n-2} \left((\nabla_\beta T_{\mu\nu} - \nabla_\nu T_{\beta\mu}) - \frac{1}{(n-1)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) T \right) \quad (21)$$

This tensor has to be trace free for all indexes just as Weyl tensor. That means if it has symmetries of Riemann tensor:

$$g^{\alpha\beta} \hat{T}_{\alpha\mu\beta\nu} = 0 \quad (22)$$

In vacuum solutions it just equal Weyl tensor, but I need to put Einstein constant [1] in it.

3.3 Four index energy tensor

Putting this all in one equation and one tensor I will use same logic from before so I arrive at total energy tensor:

$$T_{\alpha\mu\beta\nu} = \tilde{T}_{\alpha\mu\beta\nu} - \frac{1}{n-3} \hat{T}_{\alpha\mu\beta\nu} \quad (23)$$

This tensor is divergence free as part with geometry, it has a trace part (stress momentum tensor) and trace-less part (vacuum energy tensor).

4 Four index field equations

4.1 Equation

Combining all from before I arrive at four index equation:

$$G_{\alpha\mu\beta\nu} = \kappa T_{\alpha\mu\beta\nu} \quad (24)$$

Equation is divergence free:

$$\nabla^\alpha G_{\alpha\mu\beta\nu} = \kappa \nabla^\alpha T_{\alpha\mu\beta\nu} = 0 \quad (25)$$

It's contraction leads to two index equation [1]:

$$g^{\alpha\beta} G_{\alpha\mu\beta\nu} = \kappa g^{\alpha\beta} T_{\alpha\mu\beta\nu} \quad (26)$$

$$G_{\mu\nu} = \kappa T_{\mu\nu} \quad (27)$$

It has trace and trace-less parts that are equal to each other:

$$\kappa \tilde{T}_{\alpha\mu\beta\nu} = \tilde{R}_{\alpha\mu\beta\nu} \quad (28)$$

$$\kappa \hat{T}_{\alpha\mu\beta\nu} = C_{\alpha\mu\beta\nu} \quad (29)$$

4.2 Adding cosmological constant

I can add back to equations part with cosmological constant [1] it will be term with metric tensors and scalar, I can write that term:

$$\frac{1}{n-1} (g_{\mu\nu} g_{\alpha\beta} - g_{\beta\mu} g_{\alpha\nu}) \Lambda \quad (30)$$

So final field equations are:

$$\boxed{G_{\alpha\mu\beta\nu} + \frac{1}{n-1} (g_{\mu\nu} g_{\alpha\beta} - g_{\beta\mu} g_{\alpha\nu}) \Lambda = \kappa T_{\alpha\mu\beta\nu}} \quad (31)$$

If cosmological constant [1] is just a fixed number it's divergence vanishes but if not, it has to fulfill this equations:

$$(g_{\mu\nu} \partial_\beta - g_{\beta\mu} \partial_\nu) \Lambda = 0 \quad (32)$$

And from two index equations it needs to vanish when contracted so it leads to:

$$\partial_\nu \Lambda = 0 \quad (33)$$

It shows that it has to be a constant number that does not change with any spatial or a temporal direction. Otherwise equations will be not divergence free in both four index and two index [1]. After contraction it will just lead to standard form two index equation [1]:

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa T_{\mu\nu} \quad (34)$$

4.3 Action formulation of equations

I can formulate field equations in as action [7] , derivation was already done in this paper [7], where there is same structure of field equations used. After adding tensors I will arrive at starting with gravity field:

$$S_G = \frac{1}{2\kappa c} \int \sqrt{-g} g^{\alpha\beta} g^{\mu\nu} (G_{\alpha\mu\beta\nu} + \lambda_{\alpha\mu\beta\nu}) d^D \mathbf{x} \quad (35)$$

Where those tensors are defined as before so:

$$S_G = \frac{1}{2\kappa c} \int \sqrt{-g} g^{\alpha\beta} g^{\mu\nu} \left(\tilde{R}_{\alpha\mu\beta\nu} - \frac{1}{n-3} C_{\alpha\mu\beta\nu} + \frac{1}{n-1} (g_{\mu\nu} g_{\alpha\beta} - g_{\beta\mu} g_{\alpha\nu}) \Lambda \right) d^D \mathbf{x} \quad (36)$$

Now matter field part behaves same way but without term with Einstein constant [7]:

$$S_M = -\frac{1}{c} \int \sqrt{-g} g^{\alpha\beta} g^{\mu\nu} T_{\alpha\mu\beta\nu} d^D \mathbf{x} \quad (37)$$

That can be expanded:

$$S_M = -\frac{1}{c} \int \sqrt{-g} g^{\alpha\beta} g^{\mu\nu} \left(\tilde{T}_{\alpha\mu\beta\nu} - \frac{1}{n-3} \hat{T}_{\alpha\mu\beta\nu} \right) d^D \mathbf{x} \quad (38)$$

Adding them both into one integral [7]:

$$S = \frac{1}{2\kappa c} \int \sqrt{-g} g^{\alpha\beta} g^{\mu\nu} (G_{\alpha\mu\beta\nu} + \lambda_{\alpha\mu\beta\nu}) d^D \mathbf{x} - \frac{1}{c} \int \sqrt{-g} g^{\alpha\beta} g^{\mu\nu} T_{\alpha\mu\beta\nu} d^D \mathbf{x} \quad (39)$$

$$S = \frac{1}{c} \int \sqrt{-g} g^{\alpha\beta} g^{\mu\nu} \left[\frac{1}{2\kappa} (G_{\alpha\mu\beta\nu} + \lambda_{\alpha\mu\beta\nu}) - T_{\alpha\mu\beta\nu} \right] d^D \mathbf{x} \quad (40)$$

That can be written in whole expanded form as:

$$S = \frac{1}{c} \int \sqrt{-g} g^{\alpha\beta} g^{\mu\nu} \left[\frac{1}{2\kappa} \left(\tilde{R}_{\alpha\mu\beta\nu} - \frac{1}{n-3} C_{\alpha\mu\beta\nu} + \frac{1}{n-1} (g_{\mu\nu} g_{\alpha\beta} - g_{\beta\mu} g_{\alpha\nu}) \Lambda \right) - \left(\tilde{T}_{\alpha\mu\beta\nu} - \frac{1}{n-3} \hat{T}_{\alpha\mu\beta\nu} \right) \right] d^D \mathbf{x} \quad (41)$$

4.4 Meaning of four index field equation

Four index field equation is natural extension of two index equation, what is new in this field equation that there is new tensor that is responsible for vacuum energy. This tensor is energy of matter field in vacuum. It means that it's an extension of matter field into vacuum and it consists of physical matter. This matter is not exactly like normal matter field, it is trace-less part of total energy, but still it's matter field.

This matter field is carrying energy of matter field into the vacuum, it's simplest candidate for gravity particle. From it follows most important part, in this model gravity as particle carrying energy is not need to quantize field to arrive at it- it's done in pure classical terms. And crucial is that energy of that particle being equal to Weyl tensor is an equation to be solved not identity, that leads to two equations but what is important are not two separate equations but come from a tensor that is created when they are combined:

$$G_{\alpha\mu\beta\nu} + \frac{1}{n-1} (g_{\mu\nu}g_{\alpha\beta} - g_{\beta\mu}g_{\alpha\nu}) \Lambda = \kappa T_{\alpha\mu\beta\nu} \quad (42)$$

$$\tilde{R}_{\alpha\mu\beta\nu} - \frac{1}{n-3} C_{\alpha\mu\beta\nu} + \frac{1}{n-1} (g_{\mu\nu}g_{\alpha\beta} - g_{\beta\mu}g_{\alpha\nu}) \Lambda = \kappa \left(\tilde{T}_{\alpha\mu\beta\nu} - \frac{1}{n-3} \hat{T}_{\alpha\mu\beta\nu} \right) \quad (43)$$

As this tensor is a divergence free object. It leads to simple question, why there is a minus sign in the parts with vacuum energy and Weyl tensor? Answer to that question is that otherwise two sides would not be divergence free. This gives a physical meaning to it, total tensor takes all inputs from vacuum as negative in sign. Still this is not a final story as it lacks any kind of quantum effects.

4.5 Trace-reversed field equation

I can arrive at trace reversed field equations, first I take field equation:

$$\tilde{R}_{\alpha\mu\beta\nu} - \frac{1}{n-3} C_{\alpha\mu\beta\nu} + \frac{1}{n-1} (g_{\mu\nu}g_{\alpha\beta} - g_{\beta\mu}g_{\alpha\nu}) \Lambda = \kappa \left(\tilde{T}_{\alpha\mu\beta\nu} - \frac{1}{n-3} \hat{T}_{\alpha\mu\beta\nu} \right) \quad (44)$$

Expand left side and move cosmological constant part to right side of equations:

$$\begin{aligned} & \frac{1}{n-2} (R_{\alpha\beta}g_{\mu\nu} - R_{\alpha\nu}g_{\beta\mu} + R_{\mu\nu}g_{\alpha\beta} - R_{\beta\mu}g_{\alpha\nu}) \\ & - \frac{n}{2(n-1)(n-2)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) R - \frac{1}{n-3} C_{\alpha\mu\beta\nu} \end{aligned} \quad (45)$$

Compare it with Riemann tensor expression:

$$R_{\alpha\mu\beta\nu} = C_{\alpha\mu\beta\nu} + \frac{1}{n-2} (R_{\alpha\beta}g_{\mu\nu} - R_{\alpha\nu}g_{\beta\mu} + R_{\mu\nu}g_{\alpha\beta} - R_{\beta\mu}g_{\alpha\nu}) - \frac{1}{(n-1)(n-2)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) R \quad (46)$$

I can express now it in terms of Riemann tensor:

$$G_{\alpha\mu\beta\nu} = R_{\alpha\mu\beta\nu} - \frac{1}{2(n-1)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) R - \left(1 + \frac{1}{n-3}\right) C_{\alpha\mu\beta\nu} \quad (47)$$

Use two index equations [1] contractions and fact that Weyl tensor is equal to energy tensor of vacuum and move all that is not Riemann tensor to right side:

$$R_{\alpha\mu\beta\nu} = \kappa \left(T_{\alpha\mu\beta\nu} - \frac{1}{(n-1)(n-2)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) T + \left(1 + \frac{1}{n-3}\right) \hat{T}_{\alpha\mu\beta\nu} \right) \quad (48)$$

Expand components of total energy tensor:

$$R_{\alpha\mu\beta\nu} = \kappa \left(\tilde{T}_{\alpha\mu\beta\nu} - \frac{1}{n-3} \hat{T}_{\alpha\mu\beta\nu} - \frac{1}{(n-1)(n-2)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) T \right) + \kappa \left(1 + \frac{1}{n-3}\right) \hat{T}_{\alpha\mu\beta\nu} \quad (49)$$

Sort out components first of vacuum energy tensor:

$$R_{\alpha\mu\beta\nu} = \kappa \left(\tilde{T}_{\alpha\mu\beta\nu} + \hat{T}_{\alpha\mu\beta\nu} - \frac{1}{(n-1)(n-2)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) T \right) \quad (50)$$

Then matter energy tensor and will arrive at final expression and add cosmological constant again:

$$R_{\alpha\mu\beta\nu} = \kappa \left(\tilde{T}_{\alpha\mu\beta\nu} + \hat{T}_{\alpha\mu\beta\nu} - \frac{1}{(n-1)(n-2)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) T + \frac{2}{\kappa(n-1)(n-2)} (g_{\mu\nu}g_{\alpha\beta} - g_{\beta\mu}g_{\alpha\nu}) \Lambda \right) \quad (51)$$

4.6 Geodesic deviation and geodesic equation

From fact that I can write Riemann tensor in terms of energy tensors I can go one step forward and write whole equation as a geodesic deviation equation [8], where \mathbf{V} is vector tangent to geodesic curve [8]:

$$\nabla_{\mathbf{V}} \nabla_{\mathbf{V}} \xi^{\alpha} = R^{\alpha}_{\mu\beta\nu} V^{\mu} V^{\beta} \xi^{\nu} \quad (52)$$

Now I can switch right side of equation into energy instead of curvature parts:

$$\begin{aligned} \nabla_{\mathbf{v}} \nabla_{\mathbf{v}} \xi^\alpha = & \kappa \left(\tilde{T}_{\mu\beta\nu}^\alpha + \hat{T}_{\mu\beta\nu}^\alpha - \frac{1}{(n-1)(n-2)} (g_{\mu\nu} \delta_\beta^\alpha - g_{\beta\mu} \delta_\nu^\alpha) T \right. \\ & \left. + \frac{2}{\kappa(n-1)(n-2)} (g_{\mu\nu} \delta_\beta^\alpha - g_{\beta\mu} \delta_\nu^\alpha) \Lambda \right) V^\mu V^\beta \xi^\nu \end{aligned} \quad (53)$$

That leads to most general interpretation of field equations in geometric way as it is interpretation of Riemann tensor itself [9]. In general geodesic deviation says how "fast" geodesic [8] accelerate with respect to each other. They can either converge or diverge. Field equations will not lead to singularities only if both side of equations stay finite, from fact that Im defining Riemann tensor that encodes whole curvature I can impose that right side with energy content stays finite in whole manifold then solve for left side of equation. This gives control over singularities that is absent in two index equation [1]. If geodesic deviation acts on global spacetime, geodesic equation [1] [8] acts locally when effects of particle or systems of particles is small compared to source of gravity field. Geodesic equation [8] can be written as:

$$\frac{d^2 x^\mu}{ds^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0 \quad (54)$$

This equations in truth works only if I don't take a field as whole but a isolated part of a field into account. Field as whole is generated from curvature itself. It leads to idea that object not only follow locally geodesic equation but field in general has obey geodesic deviation equation [8] that is more general statement of gravity field as it has whole curvature as a source. So locally geodesic equation is still valid but in truth more general statement about field is geodesic deviation equation [8] [9]. In truth field should be considered as a general object, that's why geodesic equation [8] is not whole story. Energy makes geodesic accelerate towards each other or away from each other, this happens both in case of matter field or gravity field energy. Gravity field should be thought then in geometric sense [9] as focusing or un-focusing of geodesics. As long as both side of equations stay finite this leads to well defined curvature and from it follows field.

4.7 Field equations as connection with two index equations

I can re-write field equations using two equations that are separated, one equation is just two index field equation [1] written in four index form and one deals with Weyl tensor [1]:

$$\tilde{R}_{\alpha\mu\beta\nu} = \kappa \tilde{T}_{\alpha\mu\beta\nu} \quad (55)$$

$$C_{\alpha\mu\beta\nu} = \kappa \hat{T}_{\alpha\mu\beta\nu} \quad (56)$$

As first equation can be reduced to two index equation [1] I can re-write field equations as:

$$G_{\mu\nu} = \kappa T_{\mu\nu} \quad (57)$$

$$C_{\alpha\mu\beta\nu} = \kappa \hat{T}_{\alpha\mu\beta\nu} \quad (58)$$

Where there is connection between two index equations and four index equations, this connection is covariant derivative. When I do take covariant derivative of vacuum energy tensor I need to arrive exactly at same expression as for energy momentum part:

$$\nabla^\alpha \hat{T}_{\alpha\mu\beta\nu} = \nabla^\alpha \tilde{T}_{\alpha\mu\beta\nu} \quad (59)$$

At first it may seem as it's just propagation equation [1] from two index equation [1] but key here is that vacuum energy tensor is equation to be solved not an identity. It means that it agrees with two index equation [1] in form of propagation equation [1]:

$$\nabla^\alpha C_{\alpha\mu\beta\nu} = \kappa \nabla^\alpha \hat{T}_{\alpha\mu\beta\nu} \quad (60)$$

$$\begin{aligned} \frac{n-3}{n-2} \left((\nabla_\beta R_{\mu\nu} - \nabla_\nu R_{\beta\mu}) - \frac{1}{2(n-1)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) R \right) = \\ \frac{n-3}{n-2} \kappa \left((\nabla_\beta T_{\mu\nu} - \nabla_\nu T_{\beta\mu}) - \frac{1}{(n-1)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) T \right) \end{aligned} \quad (61)$$

$$\begin{aligned} (\nabla_\beta R_{\mu\nu} - \nabla_\nu R_{\beta\mu}) - \frac{1}{2(n-1)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) R = \\ \kappa \left((\nabla_\beta T_{\mu\nu} - \nabla_\nu T_{\beta\mu}) - \frac{1}{(n-1)} (g_{\mu\nu} \nabla_\beta - g_{\beta\mu} \nabla_\nu) T \right) \end{aligned} \quad (62)$$

It means that change in energy tensor of vacuum is connected to change in matter field, or stating opposite change of vacuum energy tensor is connected to change of matter field. From it follows that if vacuum energy stays finite there is need for matter field to stay finite if I started from finite values. From it follows that each change in gravity field is connected to change in matter field and matter field change are connected to gravity field, if those changes remain finite whole equations will remain finite. It's imposing on two index equations [1] that energy momentum tensor [1] stays finite is equally imposing on vacuum energy to stay finite from it follows Weyl tensor so gravity field itself. This separation of field equations shows how on one hand it's close to two index equations [1] and where it does differ. As to create a whole field equations I need to use four index formalism. It follows from conservation equations that are written in four index formalism. So in truth I can't state that those two are equally valid and can be separated into two sets of one four index and one two index [1] equations. It means that I need to use whole field equations or trace reversed ones, that capture four index dynamics rather than using separated equations. It means that connections between those two equations two index [1] and four index does not mean that they are exactly same equations.

5 Quantum effects form geometry

5.1 Is there matter field or mater particles?

In physics there are two notions of matter, matter as field and matter as a particle. I will postulate that matter field is more of a field than of a distinct particles that can be independent of field itself. Let me start by in general I can think of gravity as a particle (source) and it's field generated by that particle (effect in space). It does work this way for all other forces, my assumption is that matter field and effect of that matter field (gravity field) both are same thing but affecting two kinds of curvature effects. Matter field affects Ricci curvature [1] and, gravity affects Weyl curvature [1]. But both are forms of same field and both carry energy. Key question would be can one be turn into another? First in static solutions there is clear separation between gravity field and matter field. But in general non-static solutions it's possible to have this kind of separation. If Weyl curvature [1] mixes with Ricci curvature [1], I have both matter field and gravity energy in same point of spacetime acting as one field. This is key insight into understanding how quantum like effects emerge from pure geometry of spacetime. Gravity field is in truth not localized in a point, same as wave function in quantum mechanics. It's spread across all space. I will assume that this spreading of gravity field is not only hint of how to get into quantum effects only from pure geometry but possibly a hint about another path, instead of quantizing gravity, trying to build model that uses only gravity to explain quantum effects. First I will start by a simplest construction that is normalized curvature scalar.

5.2 Normalized curvature scalar

From trace revered field equations I can directly calculate curvature scalar [1] $K = R_{\alpha\mu\beta\nu}R^{\alpha\mu\beta\nu}$ using only energy parts. I will write it as:

$$R_{\alpha\mu\beta\nu} = \kappa \left(\hat{T}_{\alpha\mu\beta\nu} + \frac{1}{n-2} (T_{\alpha\beta}g_{\mu\nu} - T_{\alpha\nu}g_{\beta\mu} + T_{\mu\nu}g_{\alpha\beta} - T_{\beta\mu}g_{\alpha\nu}) \right. \\ \left. - \frac{2}{(n-1)(n-2)} (g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}) T + \frac{2}{(n-1)(n-2)} (g_{\mu\nu}g_{\alpha\beta} - g_{\beta\mu}g_{\alpha\nu}) \Lambda \right) \quad (63)$$

$$R^{\alpha\mu\beta\nu} = \kappa \left(\hat{T}^{\alpha\mu\beta\nu} + \frac{1}{n-2} (T^{\alpha\beta}g^{\mu\nu} - T^{\alpha\nu}g^{\beta\mu} + T^{\mu\nu}g^{\alpha\beta} - T^{\beta\mu}g^{\alpha\nu}) \right. \\ \left. - \frac{2}{(n-1)(n-2)} (g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\beta\mu}) T + \frac{2}{(n-1)(n-2)} (g^{\mu\nu}g^{\alpha\beta} - g^{\beta\mu}g^{\alpha\nu}) \Lambda \right) \quad (64)$$

$$K(x) = R_{\alpha\mu\beta\nu}R^{\alpha\mu\beta\nu} \quad (65)$$

This will lead to last final expression for normalized curvature scalar:

$$\alpha = \int_{ct_A}^{ct_B} \int_{\Sigma_t} \sqrt{-g} K(x) d^{n-1} x dx^0 \quad (66)$$

$$N(x) = \frac{1}{\alpha} \int_{ct_A}^{ct_B} \int_{V^{n-1}} \sqrt{-g} K(x) d^{n-1} x dx^0 \quad (67)$$

Where key here is that this scalar represents total curvature and comes from trace reversed field equations. It plays role like wave function in quantum mechanics. With change that instead of calculating probability of finding particle in given volume there is curvature density in given volume compared to total curvature. That curvature field has simple interpretation it's density of field compared with some region of space and time. Formally it says how much curvature affects given region of spacetime compared to total curvature. Key here is that it has to be finite in order to make this integral possible so all singular solutions are discarded.

5.3 Non-normalized curvature scalar

From it I can create just normal integral that is not normalized, this integral will have direct meaning as if there is enough curvature in given volume of space and given time interval I can detect particle. First I will define a particle focus region where most of curvature of particle or system of particles is focused then will need for object to be measured only if flow with some region of space and given interval of time is equal to that value of curvature:

$$N_0 = \int_{ct_a}^{ct_b} \int_{V_0^{n-1}} \sqrt{-g} K(x) d^{n-1} x dx^0 \quad (68)$$

Where there is not only focused volume V_0^{n-1} but time focused time interval $\Delta t = t_b - t_a$. Now to detect particle there has to be at least that amount of curvature, it means when I do measurement in given volume and time interval particle can be only detected if this curvature in this region is greater or equal to that value:

$$N_0 \leq \int_{ct_A}^{ct_B} \int_{V^{n-1}} \sqrt{-g} K(x) d^{n-1} x dx^0 = \alpha N(x) \quad (69)$$

To detect particle or system of particles I need at least N_0 value of curvature. Detecting particle acts as focusing of curvature into some volume of space in given time interval. Particle will be detected only if there is enough curvature in focusing region on the detector. Focusing comes from interaction of curvature field with other objects or even with detector itself.

5.4 How well I can measure objects

More precise I do measure objects position less less volume goes into integral, from it follows less total curvature I do take into integral. Smaller the time interval of measurement it too makes curvature less focused. It means that If i want to measure position precise I need to take into account big interval of time, and opposite fast measurements need big amount of space im measuring. This can be expressed that If I take small volume of space need to take big interval of time to detect a particle:

$$N_0 \leq \int_{ct_1}^{ct_2} \int_{\epsilon^{n-1}} \sqrt{-g} K(x) d^{n-1} x dx^0 \quad (70)$$

$$\int_{ct_1}^{ct_2} \sqrt{-g} K(x) dx^0 \gg \int_{\epsilon^{n-1}} \sqrt{-g} K(x) d^{n-1} x \quad (71)$$

And opposite if I take small time interval I need big volume of space:

$$N_0 \leq \int_{ct_{\epsilon_1}}^{ct_{\epsilon_2}} \int_{V^{n-1}} \sqrt{-g} K(x) d^{n-1} x dx^0 \quad (72)$$

$$\int_{V^{n-1}} \sqrt{-g} K(x) d^{n-1} x \gg \int_{\epsilon_1}^{\epsilon_2} \sqrt{-g} K(x) dx^0 \quad (73)$$

If I do take a small volume of space and small time interval particle becomes hardest to detect.

5.5 Energy of system invariant

I can use Einstein constant to arrive at same invariant but now instead of calculating curvature I do calculate total energy density squared. This can be expressed just as:

$$\int_{ct_A}^{ct_B} \int_{V^{n-1}} \sqrt{-g} \Phi(x) d^{n-1} x dx^0 = \frac{1}{\alpha \kappa^2} \int_{ct_A}^{ct_B} \int_{V^{n-1}} \sqrt{-g} K(x) d^{n-1} x dx^0 \quad (74)$$

From it follows that I can write field equations in scalar form as:

$$K(x) = R_{\alpha\mu\beta\nu} R^{\alpha\mu\beta\nu} \quad (75)$$

$$\begin{aligned} \Phi(x) = & \left(\hat{T}^{\alpha\mu\beta\nu} + \frac{1}{n-2} (T^{\alpha\beta} g^{\mu\nu} - T^{\alpha\nu} g^{\beta\mu} + T^{\mu\nu} g^{\alpha\beta} - T^{\beta\mu} g^{\alpha\nu}) \right. \\ & - \frac{2}{(n-1)(n-2)} (g^{\alpha\beta} g^{\mu\nu} - g^{\alpha\nu} g^{\beta\mu}) T + \frac{2}{(n-1)(n-2)} (g^{\mu\nu} g^{\alpha\beta} - g^{\beta\mu} g^{\alpha\nu}) \Lambda \Big) \\ & \left(\hat{T}^{\alpha\mu\beta\nu} + \frac{1}{n-2} (T^{\alpha\beta} g^{\mu\nu} - T^{\alpha\nu} g^{\beta\mu} + T^{\mu\nu} g^{\alpha\beta} - T^{\beta\mu} g^{\alpha\nu}) \right. \\ & - \frac{2}{(n-1)(n-2)} (g^{\alpha\beta} g^{\mu\nu} - g^{\alpha\nu} g^{\beta\mu}) T + \frac{2}{(n-1)(n-2)} (g^{\mu\nu} g^{\alpha\beta} - g^{\beta\mu} g^{\alpha\nu}) \Lambda \Big) \end{aligned} \quad (76)$$

$$\Phi(x) = \frac{1}{\kappa^2} K(x) \quad (77)$$

That scalar form of field equations is invariant form of equations , if it's finite it meaning that both fields stay finite, spacetime has no singularities.

6 Spin

6.1 Spin as a manifold orientation

Last example is idea that spin is orientation of manifold in space. So for example for spin one half particles I need to use hyper-sphere. What I do mean by orientation of manifold in space? If i take a vector that goes from center of hyper-sphere and points to it's surface I will call it orientation vector. This vector says how this manifold is pointing in space. When manifold rotates it will rotate around this vector. In general there are infinite number of possible orientations on any manifold. When I do measurement I project orientation of manifold onto my measurement axis.

From it follows that I get two possible outcomes, orientation changes to being up or down with measurement axis. As there are only two possible solutions that align with measurement axis that are vectors one is pointing up one is point down with measurement axis. Orientation vector can't be split into smaller vectors as manifold can have only one orientation it explains why there are only two possible states of spin. I will start with simple math let me write black hole solutions [1] from it it can be easy seen that it's a solution of a hyper-sphere:

$$ds^2 = - \left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2 d\Omega^2 \quad (78)$$

Isolate part with black hole radius so skip part with time components:

$$ds^2 = \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2 d\Omega^2 \quad (79)$$

Write hyper-sphere metric with not a constant radius:

$$ds^2 = \frac{dr^2}{\left(1 - \frac{r^2}{R^2(r)}\right)} + r^2 d\Omega^2 \quad (80)$$

Where radius of hyper-sphere is defined as $R(r) = \sqrt{\frac{r^3}{r_s}}$ same can be done with inside metric:

$$ds^2 = -\frac{1}{4} \left(3\sqrt{1 - \frac{r_s}{r_g}} - \sqrt{1 - \frac{r^2 r_s}{r_g^3}} \right)^2 c^2 dt^2 + \frac{dr^2}{1 - \frac{r^2 r_s}{r_g^3}} + r^2 d\Omega^2 \quad (81)$$

Where again I take only spatial part without time:

$$\frac{dr^2}{1 - \frac{r^2 r_s}{r_g^3}} + r^2 d\Omega^2 \quad (82)$$

$$ds^2 = \frac{dr^2}{1 - \frac{r^2}{R^2}} + r^2 d\Omega^2 \quad (83)$$

Where in this case radius is equal to $R = \sqrt{\frac{r_g^3}{r_s}}$ so it shows clear that those solutions give exactly hyper-sphere so come from matter field with spin one half. Same can be done with de Sitter universe [1] in coordinates that I did use before:

$$ds^2 = -dT^2 + \frac{1}{H^2} \cosh^2(HT) d\Omega_3^2 \quad (84)$$

Where here whole hyper-sphere does change with time not only radial part. But again I can use another coordinates that will show that is true hyper-sphere, ones used before:

$$ds^2 = -\left(1 - \frac{\Lambda r^2}{3}\right) c^2 dt^2 + \left(1 - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (85)$$

Here radius is constant of hyper-sphere and is equal to:

$$R = \sqrt{\frac{3}{\Lambda}} \quad (86)$$

So again metric can be written as in spatial parts:

$$ds^2 = \frac{dr^2}{1 - \frac{r^2}{R^2}} + r^2 d\Omega^2 \quad (87)$$

That shows that two base solutions used in cosmology and black holes are in truth a hyper-sphere solutions so their use matter field with spin one half.

6.2 Simple projection of orientation vector model

I will start with only space components of metric as I care only about them. First denote spatial part of metric as h_{ab} and its orientation vector as s^a , that vector length squared is equal to:

$$L^2 = h_{ab} s^a s^b \quad (88)$$

Now I need to add measurement axis vector that will be projected onto orientation axis. This will be denoted as v^a and its length squared is defined same way:

$$L^2 = h_{ab} v^a v^b \quad (89)$$

Both have to lie on same sphere or hyper-sphere and have same length equal to its radius. So I can set that:

$$h_{ab}v^av^b = h_{ab}s^as^b = L^2 \quad (90)$$

Now i create two vectors that are projections of measurement axis vector on spin state, they represent spin up and down, so two orientations projection of orientation vector can take. One state has to be rotated π radians compared to another for sphere so spin one and one has to have state 2π for spin one half. That represents spin up and down states. Total length of final vector is sum of orientation vector with its measurement axis vector normalized by total length first for sphere so spin one:

$$\psi_+(\theta) = \frac{L + L \cos(\theta)}{2L} \quad (91)$$

$$\psi_-(\theta) = \frac{L + L \cos(\pi - \theta)}{2L} \quad (92)$$

Then for hyper-sphere so spin one half:

$$\psi_+\left(\frac{\theta}{2}\right) = \frac{L + L \cos\left(\frac{\theta}{2}\right)}{2L} \quad (93)$$

$$\psi_-\left(\frac{\theta}{2}\right) = \frac{L + L \cos\left(\pi - \frac{\theta}{2}\right)}{2L} \quad (94)$$

This can be re-written using formula for angle between vectors, and lengths using only metric tensors:

$$\cos(\theta) = \frac{h_{ab}s^av^b}{\sqrt{h_{ab}s^as^b}\sqrt{h_{ab}v^av^b}} = \frac{h_{ab}s^av^b}{L^2} \quad (95)$$

Idea is that before measurement orientation vector is random in direction as it's unknown and there is infinite number of possible orientation so there is exactly zero probability of guessing right one, after measurement we project that vector into our measurement axis and get known state of orientation, when we measure another axis there is new projection and so on. So measurement does change orientation state from s^a to $s^{a'}$ with probability equal to $\psi_+\left(\frac{\theta}{2}\right)$ or $\psi_-\left(\frac{\theta}{2}\right)$ for up or down state and spin one half. Where state up represents orientation vector align with measurement axis and down opposite to it.

6.3 Spin state when there is correlation between states

When there is a correlation between states [6] its best candidate to test will this formalism break. I will use correlation between states where angle between them is always anti-correlated [6] it means that if one particle is spin up one is spin down. Here I have two body system [6], but I can use same logic. Now

If i take axis between one system and another system as angle θ and there is no correlation [6] will arrive at same equation as before but if I assume anti-correlation I will arrive at states:

$$\psi_+(\theta) = \frac{L + L \cos(\vartheta + \pi)}{2L} \quad (96)$$

$$\psi_-(\theta) = \frac{L + L \cos(\vartheta)}{2L} \quad (97)$$

Then for hyper-sphere so spin one half:

$$\psi_+\left(\frac{\theta}{2}\right) = \frac{L + L \cos\left(\pi + \frac{\vartheta}{2}\right)}{2L} \quad (98)$$

$$\psi_-\left(\frac{\theta}{2}\right) = \frac{L + L \cos\left(\frac{\vartheta}{2}\right)}{2L} \quad (99)$$

As first angle is always rotated by π compared to second so I have change in angle equal to π :

$$\phi_+ = \pi + \vartheta \quad \phi_- = \vartheta \quad (100)$$

$$\phi_+ - \phi_- = \pi \quad (101)$$

I can use inverse correlation [6] but then I need to be aware that those states are now giving opposite relation:

$$\phi_+ = \vartheta \quad \phi_- = \vartheta + \pi \quad (102)$$

$$\phi_+ - \phi_- = -\pi \quad (103)$$

It means that states are reversed. Up state is now down state and down state is now up state, as I need to flip orientation (from minus sign). In case where there is no correlation angles between are random. So if one vector has angle ϑ another one has $\pi - \vartheta$, as total angle between spin state up and down is equal to π :

$$\vartheta + \pi - \vartheta = \pi \quad (104)$$

In case of correlated states of two systems I ask what is correlation [6] of them if I chose random axis to measure them, angle is angle between measurement axis just like in Bell theorem [6]. Now I can again use same logic and get correct answer for correlation between both anti-correlated states and not correlated states:

$$E_{\text{Correlated}}(\vartheta) = \psi_+(\vartheta) - \psi_-(\vartheta) \quad (105)$$

$$\begin{aligned} E_{\text{Correlated}}(\vartheta) &= \frac{L + L \cos(\vartheta + \pi)}{2L} - \frac{L + L \cos(\vartheta)}{2L} = \sin^2\left(\frac{\vartheta}{2}\right) - \cos^2\left(\frac{\vartheta}{2}\right) \\ &= -\cos(\vartheta) \end{aligned} \quad (106)$$

$$E_{\text{Random}}(\vartheta) = \psi_+(\vartheta) - \psi_-(\vartheta) \quad (107)$$

$$\begin{aligned} E_{\text{Random}}(\vartheta) &= \frac{L + L \cos(\vartheta)}{2L} - \frac{L + L \cos(\pi - \vartheta)}{2L} = \cos^2\left(\frac{\vartheta}{2}\right) - \sin^2\left(\frac{\vartheta}{2}\right) \\ &= \cos(\vartheta) \end{aligned} \quad (108)$$

That is exactly result one would expect from quantum theory [6] but here it's done only by geometric relation between orientation vectors.

7 CPT symmetry metric tensor extension

7.1 CPT metric

I will add CPT symmetric metric tensor [2] instead of extending spacetime to a complex Hermitan spacetime [4] as this choice allows me to use real spacetime solutions. In contrast Hermitan [4] complex extensions give only new complex solutions without any way to fit a real spacetime solutions. I will start by defining a new metric tensor that will be denoted as g_{AB} and it's coordinates that will be denoted as $x^A = (x^\mu, \bar{x}^\mu)$ where bar represents CPT reversed coordinates [2] that fulfill $x^\mu = -\bar{x}^\mu$. For now I will skip charge change but in general I need to assume too that each charge is flipped so that $q = -\bar{q}$. Metric tensor is just:

$$g_{AB} = \begin{pmatrix} g_{\mu\nu}(x) & A_{\mu\nu}(x, \bar{x}) \\ A_{\mu\nu}(x, \bar{x}) & \bar{g}_{\mu\nu}(\bar{x}) \end{pmatrix} \quad (109)$$

Where $A_{\mu\nu}$ is interaction term between two metrics that are just CPT reversed copy of each other [2].

7.2 Field equations for CPT spacetime

Using metric from before I can define a field equations:

$$G_{ABCD} + \frac{1}{n-1} (g_{AC}g_{BD} - g_{AD}g_{BC}) \Lambda = \kappa T_{ABCD} \quad (110)$$

Where those index run from one copy of spacetime to it's CPT [2] copy so its $2n$ dimensional spacetime. Interaction term states how two copies of spacetime will affect each other. I can define a spacetime interval for CPT [2] metric tensor:

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu + \bar{g}_{\mu\nu}(\bar{x}) d\bar{x}^\mu d\bar{x}^\nu + A_{\mu\nu}(x, \bar{x}) dx^\mu d\bar{x}^\nu + A_{\mu\nu}(x, \bar{x}) d\bar{x}^\mu dx^\nu \quad (111)$$

Where I can use fact that $A_{\mu\nu}$ is symmetric tensor so I arrive at:

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu + \bar{g}_{\mu\nu}(\bar{x}) d\bar{x}^\mu d\bar{x}^\nu + 2A_{\mu\nu}(x, \bar{x}) dx^\mu d\bar{x}^\nu \quad (112)$$

Interaction term does define interaction between two CPT [2] reversed copies of universes that live in their respected dimensions.

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