Four index Einstein field equations with quantum like effects from pure geometry and CPT symmetry addition to metric tensor

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Abstract

In this work I present extensions of Einstein field equations [1] into four index equations. This extensions give as natural result a energy tensor for vacuum thus for gravity field. It's all construct in spirit of two index field equations and in truth does not need any additional assumptions about field equations. Form it follows that it's natural completeness of two index equations not a true extension as it fully defines curvature tensor not only Ricci part of curvature as it happens in two index equations.

In next parts of work I add CPT symmetry [2] into equations and make arguments about it's only possible extensions of metric tensor [1], additionally I add interpretation about gluing manifolds in certain way, from them follows how to avoid CTC [5] and those do not care about singularities in solutions. Finally at last part of work I add quantum like effects from pure geometry without invoking any quantization of field.

Those effects are divided into two parts, one is about wave function like object and measurement, next one is about spin as orientation of manifold. Wave function like object is constructed from normalized curvature invariant. That plays role of "probability" of finding object in given volume of spacetime at given interval of time. I did no present direct solutions to those equations or concrete examples where it differs from General Relativity [1].

Contents

Part I

Four index equations

1 Einstein field equations

1.1 Einstein tensor

Einstein tensor is basis of General Relativity [1], from it arises left side of field equations that connects matter field represented by stress momentum tensor [1] with geometry part. That geometry part is just Einstein tensor that comes from Bianachi identities [1]. It can be written as:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \tag{1}$$

It's most important property is that is divergence free [1] so that:

$$\nabla^{\mu} G_{\mu\nu} = 0 \tag{2}$$

It connects to left side of equations where stress momentum tensor [1] is present.

1.2 Stress momentum tensor

Matter sources in General Relativity [1] are all coming from right side of equation that is stress momentum tensor [1] $T_{\mu\nu}$. Units of stress momentum tensor are energy density so from it follows that on left side of equation I have units of curvature that is length to power minus two, thats why if field equations [1] I need to use Einstein constant that is equal to:

$$\kappa = \frac{8\pi G}{c^4} \tag{3}$$

Another crucial property of stress momentum tensor is that as on left side I have divergence free tensor , stress momentum tensor is divergence free meaning that matter fields are conserved locally. It means that divergence of stress momentum tensor is zero [1]:

$$\nabla^{\mu} T_{\mu\nu} = 0 \tag{4}$$

1.3 Field equation

That finally leads to field equation that I will call in this work a two index field equation, that is just ordinary Einstein field equation [1]:

$$G_{\mu\nu} = \kappa T_{\mu\nu} \tag{5}$$

Idea behind this work is simple extended this equation into four index equation, adding part that is responsible for vacuum energy or more precise gravity field energy.

1.4 Cosmological constant

Two index equations used today have additional term [1], that term is cosmological constant or energy of empty space Λ it modifies field equations by adding term with it to field equations [1]:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \tag{6}$$

In vacuum it will lead to:

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} \tag{7}$$

That solutions will be either de Sitter or anti-de Sitter [1] spacetime depending on sign of cosmological constant.

2 Four index Einstein tensor

2.1 Ricci part

Riemann tensor [1] can be decomposed into Ricci and Weyl parts:

$$\begin{aligned} R_{\alpha\mu\beta\nu} = C_{\alpha\mu\beta\nu} + \frac{1}{n-2}\left(R_{\alpha\beta}g_{\mu\nu} - R_{\alpha\nu}g_{\beta\mu} + R_{\mu\nu}g_{\alpha\beta} - R_{\beta\mu}g_{\alpha\nu}\right) \\ -\frac{1}{(n-1)(n-2)}\left(g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}\right)R \end{aligned} \tag{8}$$

This decomposition gives after contraction exactly Ricci tensor as part with Weyl vanishes when contracted as it's trace free for all indexes combinations. So isolating Ricci part but adding part that will after contraction reduce exactly to one half of Ricci scalar times metric tensor I will arrive at:

$$\begin{aligned} \frac{1}{n-2}\left(R_{\alpha\beta}g_{\mu\nu} - R_{\alpha\nu}g_{\beta\mu} + R_{\mu\nu}g_{\alpha\beta} - R_{\beta\mu}g_{\alpha\nu}\right) \\ -\frac{1}{(n-1)(n-2)}\left(g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}\right)R - \frac{1}{2(n-1)}\left(g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}\right)R \end{aligned} \tag{9}$$

I can sum terms with Ricci scalar and will arrive at final expression for first component of four index Einstein tensor:

$$\begin{aligned} \tilde{R}_{\alpha\mu\beta\nu} = \frac{1}{n-2}\left(R_{\alpha\beta}g_{\mu\nu} - R_{\alpha\nu}g_{\beta\mu} + R_{\mu\nu}g_{\alpha\beta} - R_{\beta\mu}g_{\alpha\nu}\right) \\ -\frac{n}{2(n-1)(n-2)}\left(g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}\right)R \end{aligned} \tag{10}$$

2.2 Divergence of Ricci part

From it follows that I can write divergence of this component with respect to first index where I will use Bianachi identities for same indexes of Ricci tensor and divergence index so index α to make equations simpler:

$$\nabla^{\alpha}\tilde{R}_{\alpha\mu\beta\nu} = \frac{1}{n-2}\left(\nabla^{\alpha}R_{\alpha\beta}g_{\mu\nu} - \nabla^{\alpha}R_{\alpha\nu}g_{\beta\mu} + \nabla_{\beta}R_{\mu\nu} - \nabla_{\nu}R_{\beta\mu}\right) \\ -\frac{n}{2\left(n-1\right)\left(n-2\right)}\left(g_{\mu\nu}\nabla_{\beta} - g_{\beta\mu}\nabla_{\nu}\right)R \tag{11}$$

$$\nabla^{\alpha}\tilde{R}_{\alpha\mu\beta\nu} = \frac{1}{n-2}\left(\nabla_{\beta}R_{\mu\nu} - \nabla_{\nu}R_{\beta\mu}\right) \\ -\frac{n}{2\left(n-1\right)\left(n-2\right)}\left(g_{\mu\nu}\nabla_{\beta} - g_{\beta\mu}\nabla_{\nu}\right)R + \frac{1}{2\left(n-2\right)}\left(g_{\mu\nu}\nabla_{\beta} - g_{\beta\mu}\nabla_{\nu}\right)R \tag{12}$$

$$\nabla^{\alpha}\tilde{R}_{\alpha\mu\beta\nu} = \frac{1}{n-2}\left(\nabla_{\beta}R_{\mu\nu} - \nabla_{\nu}R_{\beta\mu}\right) - \frac{1}{2\left(n-1\right)\left(n-2\right)}\left(g_{\mu\nu}\nabla_{\beta} - g_{\beta\mu}\nabla_{\nu}\right)R \tag{13}$$

Taking out common denominator I will arrive at final expression:

$$\nabla^{\alpha}\tilde{R}_{\alpha\mu\beta\nu} = \frac{1}{n-2}\left(\left(\nabla_{\beta}R_{\mu\nu} - \nabla_{\nu}R_{\beta\mu}\right) - \frac{1}{2\left(n-1\right)}\left(g_{\mu\nu}\nabla_{\beta} - g_{\beta\mu}\nabla_{\nu}\right)R\right) \tag{14}$$

2.3 Weyl part

Weyl [1] part divergence is equal to:

$$\nabla^{\alpha}C_{\alpha\mu\beta\nu} = \frac{n-3}{n-2}\left(\left(\nabla_{\beta}R_{\mu\nu} - \nabla_{\nu}R_{\beta\mu}\right) - \frac{1}{2\left(n-1\right)}\left(g_{\mu\nu}\nabla_{\beta} - g_{\beta\mu}\nabla_{\nu}\right)R\right) \tag{15}$$

This means that they differ only by a constant $n-3$, from it follows to create a field equation I need first to take that constant and divide Weyl tensor by it. Then whole tensor has to be divergence free, from it follows that I will take Ricci part and subtract Weyl dived by that constant:

$$\tilde{R}_{\alpha\mu\beta\nu} - \frac{1}{n-3}C_{\alpha\mu\beta\nu} \tag{16}$$

2.4 Four index Einstein tensor

Last expression is just four index Einstein tensor, it all properties of being divergence free and it's contractions lead to two index equations. This tensor can be written:

$$G_{\alpha\mu\beta\nu} = \tilde{R}_{\alpha\mu\beta\nu} - \frac{1}{n-3}C_{\alpha\mu\beta\nu} \tag{17}$$

3 Four index energy tensor

3.1 Matter part

I can use same form as in Riemann decomposition into matter part of total energy tensor. This tensor will reduce to stress momentum tensor so I can write it:

$$\tilde{T}_{\alpha\mu\beta\nu} = \frac{1}{n-2}\left(T_{\alpha\beta}g_{\mu\nu} - T_{\alpha\nu}g_{\beta\mu} + T_{\mu\nu}g_{\alpha\beta} - T_{\beta\mu}g_{\alpha\nu}\right) \\ -\frac{1}{\left(n-1\right)\left(n-2\right)}\left(g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}\right)T \tag{18}$$

Now key property is to calculate divergence of this tensor with respect to first index as before, I can use property that stress momentum tensor is divergence free:

$$\nabla^{\alpha}\tilde{T}_{\alpha\mu\beta\nu} = \frac{1}{n-2}\left(\nabla_{\beta}T_{\mu\nu} - \nabla_{\nu}T_{\beta\mu}\right) - \frac{1}{(n-1)(n-2)}\left(g_{\mu\nu}\nabla_{\beta} - g_{\beta\mu}\nabla_{\nu}\right)T \tag{19}$$

Taking out common denominator:

$$\nabla^{\alpha}\tilde{T}_{\alpha\mu\beta\nu} = \frac{1}{n-2}\left(\left(\nabla_{\beta}T_{\mu\nu} - \nabla_{\nu}T_{\beta\mu}\right) - \frac{1}{(n-1)}\left(g_{\mu\nu}\nabla_{\beta} - g_{\beta\mu}\nabla_{\nu}\right)T\right) \tag{20}$$

That is exactly propagation equation [1] from two index equations, where it does differ by same constant.

3.2 Vacuum energy part

From previous equations I can see a logic here, vacuum energy tensor divergence will be have to be equal to same as matter part times a constant defined before:

$$\nabla^{\alpha}\hat{T}_{\alpha\mu\beta\nu} = \frac{n-3}{n-2}\left(\left(\nabla_{\beta}T_{\mu\nu} - \nabla_{\nu}T_{\beta\mu}\right) - \frac{1}{(n-1)}\left(g_{\mu\nu}\nabla_{\beta} - g_{\beta\mu}\nabla_{\nu}\right)T\right) \tag{21}$$

This tensor has to be trace free for all indexes just as Weyl tensor. That means if it has symmetries of Riemann tensor:

$$g^{\alpha\beta}\hat{T}_{\alpha\mu\beta\nu} = 0 \tag{22}$$

In vacuum solutions it just equal Weyl tensor, but I need to put Einstein constant [1] in it.

3.3 Four index energy tensor

Putting this all in one equation and one tensor I will use same logic from before so I arrive at total energy tensor:

$$T_{\alpha\mu\beta\nu} = \tilde{T}_{\alpha\mu\beta\nu} - \frac{1}{n-3}\hat{T}_{\alpha\mu\beta\nu} \tag{23}$$

This tensor is divergence free as part with geometry, it has a trace part (stress momentum tensor) and trace-less part (vacuum energy tensor).

4 Four index field equations

4.1 Equation

Combining all from before I arrive at four index equation:

$$G_{\alpha\mu\beta\nu} = \kappa T_{\alpha\mu\beta\nu} \tag{24}$$

Equation is divergence free:

$$\nabla^{\alpha}G_{\alpha\mu\beta\nu} = \kappa\nabla^{\alpha}T_{\alpha\mu\beta\nu} = 0 \tag{25}$$

It's contraction leads to two index equation [1]:

$$g^{\alpha\beta}G_{\alpha\mu\beta\nu} = \kappa g^{\alpha\beta}T_{\alpha\mu\beta\nu} \tag{26}$$
$$G_{\mu\nu} = \kappa T_{\mu\nu} \tag{27}$$

It has trace and trace-less parts that are equal to each other:

$$\kappa\tilde{T}_{\alpha\mu\beta\nu} = \tilde{R}_{\alpha\mu\beta\nu} \tag{28}$$
$$\kappa\hat{T}_{\alpha\mu\beta\nu} = C_{\alpha\mu\beta\nu} \tag{29}$$

4.2 Adding cosmological constant

I can add back to equations part with cosmological constant [1] it will be term with metric tensors and scalar, I can write that term:

$$\frac{1}{n-1}\left(g_{\mu\nu}g_{\alpha\beta} - g_{\beta\mu}g_{\alpha\nu}\right)\Lambda \tag{30}$$

So final field equations are:

$$\boxed{G_{\alpha\mu\beta\nu} + \frac{1}{n-1}\left(g_{\mu\nu}g_{\alpha\beta} - g_{\beta\mu}g_{\alpha\nu}\right)\Lambda = \kappa T_{\alpha\mu\beta\nu}} \tag{31}$$

If cosmological constant [1] is just a fixed number it's divergence vanishes but if not, it has to fulfill this equations:

$$\left(g_{\mu\nu}\partial_\beta - g_{\beta\mu}\partial_\nu\right)\Lambda = 0 \tag{32}$$

And from two index equations it needs to vanish when contracted so it leads to:

$$\partial_\nu\Lambda = 0 \tag{33}$$

It shows that it has to be a constant number that does not change with any spatial or a temporal direction. Otherwise equations will be not divergence free in both four index and two index [1]. After contraction it will just lead to standard form two index equation [1]:

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \kappa T_{\mu\nu} \tag{34}$$

4.3 Action formulation of equations

I can formulate field equations in as action [7] , derivation was already done in this paper [7], where there is same structure of field equations used. After adding tensors I will arrive at starting with gravity field:

$$S_G = \frac{1}{2\kappa c}\int \sqrt{-g}g^{\alpha\beta}g^{\mu\nu}\left(G_{\alpha\mu\beta\nu} + \lambda_{\alpha\mu\beta\nu}\right)d^D\mathbf{x} \tag{35}$$

Where those tensors are defined as before so:

$$S_G = \frac{1}{2\kappa c}\int \sqrt{-g}g^{\alpha\beta}g^{\mu\nu}\left(\tilde{R}_{\alpha\mu\beta\nu} - \frac{1}{n-3}C_{\alpha\mu\beta\nu} + \frac{1}{n-1}\left(g_{\mu\nu}g_{\alpha\beta} - g_{\beta\mu}g_{\alpha\nu}\right)\Lambda\right)d^D\mathbf{x} \tag{36}$$

Now matter field part behaves same way but without term with Einstein constant [7]:

$$S_M = -\frac{1}{c}\int \sqrt{-g}g^{\alpha\beta}g^{\mu\nu}T_{\alpha\mu\beta\nu}d^D\mathbf{x} \tag{37}$$

That can be expanded:

$$S_M = -\frac{1}{c}\int \sqrt{-g}g^{\alpha\beta}g^{\mu\nu}\left(\tilde{T}_{\alpha\mu\beta\nu} - \frac{1}{n-3}\hat{T}_{\alpha\mu\beta\nu}\right)d^D\mathbf{x} \tag{38}$$

Adding them both into one integral [7]:

$$S = \frac{1}{2\kappa c}\int \sqrt{-g}g^{\alpha\beta}g^{\mu\nu}\left(G_{\alpha\mu\beta\nu} + \lambda_{\alpha\mu\beta\nu}\right)d^D\mathbf{x} - \frac{1}{c}\int \sqrt{-g}g^{\alpha\beta}g^{\mu\nu}T_{\alpha\mu\beta\nu}d^D\mathbf{x} \tag{39}$$

$$S = \frac{1}{c}\int \sqrt{-g}g^{\alpha\beta}g^{\mu\nu}\left[\frac{1}{2\kappa}\left(G_{\alpha\mu\beta\nu} + \lambda_{\alpha\mu\beta\nu}\right) - T_{\alpha\mu\beta\nu}\right]d^D\mathbf{x} \tag{40}$$

That can be written in whole expanded form as:

$$S = \frac{1}{c}\int \sqrt{-g}g^{\alpha\beta}g^{\mu\nu}\left[\frac{1}{2\kappa}\left(\tilde{R}_{\alpha\mu\beta\nu} - \frac{1}{n-3}C_{\alpha\mu\beta\nu} + \frac{1}{n-1}\left(g_{\mu\nu}g_{\alpha\beta} - g_{\beta\mu}g_{\alpha\nu}\right)\Lambda\right)\right.$$
$$\left. - \left(\tilde{T}_{\alpha\mu\beta\nu} - \frac{1}{n-3}\hat{T}_{\alpha\mu\beta\nu}\right)\right]d^D\mathbf{x} \tag{41}$$

4.4 Meaning of four index field equation

Four index field equation is natural extension of two index equation, what is new in this field equation that there is new tensor that is responsible for vacuum energy. This tensor is energy of matter field in vacuum. It means that it's an extension of matter field into vacuum and it consists of physical matter. This matter is not exactly like normal matter field, it is trace-less part of total energy, but still it's matter field.

This matter field is carrying energy of matter field into the vacuum, it's simplest candidate for gravity particle. From it follows most important part, in this model gravity as particle carrying energy is not need to quantize field to arrive at it- it's done in pure classical terms. And crucial is that energy of that particle being equal to Weyl tensor is an equation to be solved not identity, that leads to two equations but what is important are not two separate equations but come from a tensor that is created when they are combined:

$$G_{\alpha\mu\beta\nu} + \frac{1}{n-1}\left(g_{\mu\nu}g_{\alpha\beta} - g_{\beta\mu}g_{\alpha\nu}\right)\Lambda = \kappa T_{\alpha\mu\beta\nu} \tag{42}$$

$$\tilde{R}_{\alpha\mu\beta\nu} - \frac{1}{n-3}C_{\alpha\mu\beta\nu} + \frac{1}{n-1}\left(g_{\mu\nu}g_{\alpha\beta} - g_{\beta\mu}g_{\alpha\nu}\right)\Lambda = \kappa\left(\tilde{T}_{\alpha\mu\beta\nu} - \frac{1}{n-3}\hat{T}_{\alpha\mu\beta\nu}\right) \tag{43}$$

As this tensor is a divergence free object. It leads to simple question, why there is a minus sign in the parts with vacuum energy and Weyl tensor? Answer to that question is that otherwise two sides would not be divergence free. This gives a physical meaning to it, total tensor takes all inputs from vacuum as negative in sign. Still this is not a final story as it lacks any kind of quantum effects.

4.5 Trace-reversed field equation

I can arrive at trace reversed field equations, first I take field equation:

$$\tilde{R}_{\alpha\mu\beta\nu} - \frac{1}{n-3} C_{\alpha\mu\beta\nu} + \frac{1}{n-1} \left(g_{\mu\nu} g_{\alpha\beta} - g_{\beta\mu} g_{\alpha\nu} \right) \Lambda = \kappa \left(\tilde{T}_{\alpha\mu\beta\nu} - \frac{1}{n-3} \hat{T}_{\alpha\mu\beta\nu} \right) \tag{44}$$

Expand left side and move cosmological constant part to right side of equations:

$$\begin{aligned} &\frac{1}{n-2} \left(R_{\alpha\beta} g_{\mu\nu} - R_{\alpha\nu} g_{\beta\mu} + R_{\mu\nu} g_{\alpha\beta} - R_{\beta\mu} g_{\alpha\nu} \right) \\ -&\frac{n}{2(n-1)(n-2)} \left(g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\beta\mu} \right) R - \frac{1}{n-3} C_{\alpha\mu\beta\nu} \end{aligned} \tag{45}$$

Compare it with Riemann tensor expression:

$$\begin{aligned} R_{\alpha\mu\beta\nu} = C_{\alpha\mu\beta\nu} &+ \frac{1}{n-2} \left(R_{\alpha\beta} g_{\mu\nu} - R_{\alpha\nu} g_{\beta\mu} + R_{\mu\nu} g_{\alpha\beta} - R_{\beta\mu} g_{\alpha\nu} \right) \\ &- \frac{1}{(n-1)(n-2)} \left(g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\beta\mu} \right) R \end{aligned} \tag{46}$$

I can express now it in terms of Riemann tensor:

$$G_{\alpha\mu\beta\nu} = R_{\alpha\mu\beta\nu} - \frac{1}{2(n-1)} \left(g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\beta\mu} \right) R - \left(1 + \frac{1}{n-3} \right) C_{\alpha\mu\beta\nu} \tag{47}$$

Use two index equations [1] contractions and fact that Weyl tensor is equal to energy tensor of vacuum and move all that is not Riemann tensor to right side:

$$R_{\alpha\mu\beta\nu} = \kappa \left(T_{\alpha\mu\beta\nu} - \frac{1}{(n-1)(n-2)} \left(g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\beta\mu} \right) T + \left(1 + \frac{1}{n-3} \right) \hat{T}_{\alpha\mu\beta\nu} \right) \tag{48}$$

Expand components of total energy tensor:

$$R_{\alpha\mu\beta\nu} = \kappa \left(\tilde{T}_{\alpha\mu\beta\nu} - \frac{1}{n-3} \hat{T}_{\alpha\mu\beta\nu} - \frac{1}{(n-1)(n-2)} \left(g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\beta\mu} \right) T + \left(1 + \frac{1}{n-3} \right) \hat{T}_{\alpha\mu\beta\nu} \right) \tag{49}$$

Sort out components first of vacuum energy tensor:

$$R_{\alpha\mu\beta\nu} = \kappa \left(\tilde{T}_{\alpha\mu\beta\nu} + \hat{T}_{\alpha\mu\beta\nu} - \frac{1}{(n-1)(n-2)} \left(g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\beta\mu} \right) T \right) \tag{50}$$

Then matter energy tensor and will arrive at final expression and add cosmological constant again:

$$\begin{aligned} R_{\alpha\mu\beta\nu} = \kappa \Bigg(&\hat{T}_{\alpha\mu\beta\nu} + \frac{1}{n-2} \left(T_{\alpha\beta} g_{\mu\nu} - T_{\alpha\nu} g_{\beta\mu} + T_{\mu\nu} g_{\alpha\beta} - T_{\beta\mu} g_{\alpha\nu} \right) \\ -&\frac{2}{(n-1)(n-2)} \left(g_{\alpha\beta} g_{\mu\nu} - g_{\alpha\nu} g_{\beta\mu} \right) T - \frac{2}{(n-1)(n-2)} \left(g_{\mu\nu} g_{\alpha\beta} - g_{\beta\mu} g_{\alpha\nu} \right) \Lambda \Bigg) \end{aligned} \tag{51}$$

4.6 Geodesic deviation and geodesic equation

From fact that I can write Riemann tensor in terms of energy tensors I can go one step forward and write whole equation as a geodesic deviation equation [8] , where **V** is vector tangent to geodesic curve [8]:

$$\nabla_{\mathbf{V}}\nabla_{\mathbf{V}}\xi^{\alpha} = R^{\alpha}_{\mu\beta\nu}V^{\mu}V^{\beta}\xi^{\nu} \tag{52}$$

Now I can switch right side of equation into energy instead of curvature parts:

$$\nabla_{\mathbf{V}}\nabla_{\mathbf{V}}\xi^{\alpha} = \kappa\left(\hat{T}^{\alpha}_{\mu\beta\nu} + \frac{1}{n-2}\left(T^{\alpha}_{\beta}g_{\mu\nu} - T^{\alpha}_{\nu}g_{\beta\mu} + T_{\mu\nu}\delta^{\alpha}_{\beta} - T_{\beta\mu}\delta^{\alpha}_{\nu}\right)\right.$$
$$\left. - \frac{2}{(n-1)(n-2)}\left(\delta^{\alpha}_{\beta}g_{\mu\nu} - \delta^{\alpha}_{\nu}g_{\beta\mu}\right)T - \frac{2}{(n-1)(n-2)}\left(\delta^{\alpha}_{\beta}g_{\mu\nu} - \delta^{\alpha}_{\nu}g_{\beta\mu}\right)\Lambda\right)V^{\mu}V^{\beta}\xi^{\nu} \tag{53}$$

That leads to most general interpretation of field equations in geometric way as it is interpretation of Riemann tensor itself [9]. In general geodesic deviation says how "fast" geodesic [8] accelerate with respect to each other. They can either converge or diverge. If field only terms that always has negative contribution is cosmological constant but energy of gravity field can have negative contributions as well. Field equations will not lead to singularities only if both side of equations stay finite, from fact that Im defining Riemann tensor that encodes whole curvature I can impose that right side with energy content stays finite in whole manifold then solve for left side of equation. This gives control over singularities that is absent in two index equation [1]. If geodesic deviation acts on global spacetime, geodesic equation [1] [8] acts locally when effects of particle or systems of particles is small compared to source of gravity field. Geodesic equation [8] can be written as:

$$\frac{d^2x^{\mu}}{ds^2} + \Gamma^{\mu}_{\alpha\beta}\frac{dx^{\alpha}}{ds}\frac{dx^{\beta}}{ds} = 0 \tag{54}$$

This equations in truth works only if I don't take a field as whole but a isolated part of a field into account. Field as whole is generated from curvature itself. It leads to idea that object not only follow locally geodesic equation but field in general has obey geodesic deviation equation [8] that is more general statement of gravity field as it has whole curvature as a source. So locally geodesic equation is still valid but in truth more general statement about field is geodesic deviation equation [8] [9]. In truth field should be considered as a general object, that's why geodesic equation [8] is not whole story. Energy makes geodesic accelerate towards each other or away from each other, this happens both in case of matter field or gravity field energy. Gravity field should be thought then in geometric sense [9] as focusing or un-focusing of geodesics. As long as both side of equations stay finite this leads to well defined curvature and from it follows field.

4.7 Free-fall and inertial observer

Free-fall is basis of all gravity phenomena. Field equation can be sorted into two parts, one deals with spacetime curvature one deals with energy of source of gravity field. From free-falling observer perspective there is only motion of gravity field and it's energy. To be more precise inertial observer always will "see" cause of motion a true one. Non inertial observer will not as his perspective is distorted by its motion. From free-falling observer perspective it's the source of gravity field that is in motion- a material source. What non inertial observer sees is it's opposite it will state that free-falling observer is in motion and he is at rest. It means that inertial observer will explain motion of source of gravity by it's motion and it's energy, but observer that moves with gravity source will explain it by change in geometry of spacetime. That means that inertial observer states that there is only motion of both source object and it's gravity field on other hand source moving observer will state that there is geometry of spacetime that is responsible for motion of objects. This can be summed up in two statements, first statement says that source of gravity effects is motion of objects from it follows energy of field and second is that source is spacetime curvature:

$$\text{Inertial observer} \rightarrow \kappa T_{\alpha\mu\beta\nu} \tag{55}$$

$$\text{Non inertial observer} \rightarrow G_{\alpha\mu\beta\nu} \tag{56}$$

It means that from physical interpretation perspective field equation is a statement that inertial motion and non inertial motion perspectives are same and equal to each other. This gives new meaning to just statements about mathematical model itself. Where I still assume that non inertial observer moves with source of gravity field. This statement that two possible ways of seeing gravity, one as a motion of source that gravitates and one as spacetime curvature or more precise change in shapes and volumes of spacetime is key to physical interpretation of those equations. Physical interpretation then can be written as a single statement:

$$\boxed{\text{Motion of non inertial observer=Motion of inertial observer}} \tag{57}$$

That translates to field equations. It may seem very radical to create that much mathematical complexity to state this kind of simplistic idea. But in truth it's not about mathematical model itself but about how to give physical meaning to it. Gravity is about motion of objects in gravity field thus spacetime. But motion needs to be always defined with respect to inertial observer, here it can be defined by non-inertial observer and those two perspectives are both valid. From fact that full curvature effects can be defined by energy tensors and both sides need to be equal I can assume that field equation is statement about motion, motion is always depended on energy or equally valid spacetime changes of volumes and shapes. This means that changes in shapes and volumes require energy to create them. But it can be split into two parts, one part states that there is only energy and from it follows motion and second part says that there is change in spacetime geometry and from it follows motion but in both cases result is two causes of motion. From it follows that in truth gravity field is all about motion. Those two cases of cause of motion are equal and it's not like in two index equation [1] where energy "states" how object move. It's motion that be seen as caused by energy or change in spacetime geometry. Both are equally valid to say that spacetime geometry causes motion is same as saying that it's energy that causes motion. One implies other and can be separated.

Part II
CPT metric tensor extensions

5 Why where is need to have CPT symmetry in field equations?

5.1 Imaginary spacetime extension

CPT symmetry [2] is one of basic symmetry in physics. Key question is why there is any need to extend standard metric tensor into any other form? My motivation is that extension should not have imaginary numbers but extend itself into adding CPT symmetry [2]. Let me start by why imaginary numbers are not any helpful in gravity theory in case of this work. Imaginary numbers in general will create a Hermitan metric [3] [4] and from it follows a spacetime interval:

$$ds^2 = h_{\mu\overline{\nu}} dz^{\mu} dz^{\overline{\nu}} \tag{58}$$

Now I can use property of complex numbers and split it into a block matrix, instead of using a complex dimensions I will use two times the real dimensions with special metric:

$$\begin{pmatrix} R_{\mu\nu} & -I_{\mu\nu} \\ I_{\mu\nu} & R_{\mu\nu} \end{pmatrix} \tag{59}$$

Where $R_{\mu\nu}$ is the real part of Hermitan metric [3] [4] and $I_{\mu\nu}$ is imaginary part. Let me assume that real part is somehow connected to spacetime curvature. Or more precise it decodes solutions to real two index equations or four index equations. I will re-write it then as:

$$\begin{pmatrix} g_{\mu\nu} & -I_{\mu\nu} \\ I_{\mu\nu} & g_{\mu\nu} \end{pmatrix} \tag{60}$$

And here there is simple problem that can be seen by instant, if I take this two times real space it has two copies of on manifold but two sets of coordinates, it means that it will in general not follow symmetries of real solutions to field equations. That's why I will try to use CPT symmetry as extension not a Hermitan manifold.

5.2 CPT symmetry metric tensor

I write again a CPT [2] symmetric metric tensor [1] by simple guess first start with real spacetime that is solution to four index equations, then create a copy of it that is CPT [2] reversed. I can write this type of metric tensor:

$$\begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & g_{\mu\nu}(\overline{x}) \end{pmatrix} \tag{61}$$

It meas that I can take a metric tensor and reverse all it's coordinates [2] by inversion of coordinates:

$$x^{\mu} = -\overline{x}^{\mu} \tag{62}$$

From it follows that I have two copies of metric that live in two times the normal spacetime dimensions. It means that in this case spacetime interval is equal to:

$$ds^2(x,\overline{x}) = g_{\mu\nu}(x)\,dx^\mu dx^\nu + g_{\mu\nu}(\overline{x})\,d\overline{x}^\mu d\overline{x}^\nu \tag{63}$$

It gives additional conditions for metric itself. Still it lacks interaction between two copies of spacetime, I will add cross term interaction as $A_{\mu\nu}(x,\overline{x})$ so metric becomes:

$$\begin{pmatrix} g_{\mu\nu}(x) & A_{\mu\nu}(x,\overline{x}) \\ A_{\mu\nu}(x,\overline{x}) & g_{\mu\nu}(\overline{x}) \end{pmatrix} \tag{64}$$

It has to be symmetric so $A_{\mu\nu}(x,\overline{x}) = A_{\nu\mu}(x,\overline{x})$ otherwise metric tensor will not be a metric tensor, it will not be symmetric. Form it follows that CPT [2] symmetric spacetime interval is equal to:

$$ds^2(x,\overline{x}) = g_{\mu\nu}(x)\,dx^\mu dx^\nu + g_{\mu\nu}(\overline{x})\,d\overline{x}^\mu d\overline{x}^\nu + A_{\mu\nu}(x,\overline{x})\,dx^\mu d\overline{x}^\nu + A_{\mu\nu}(x,\overline{x})\,d\overline{x}^\mu dx^\nu \tag{65}$$

Using fact that it's symmetric tensor I will arrive at:

$$ds^2(x,\overline{x}) = g_{\mu\nu}(x)\,dx^\mu dx^\nu + g_{\mu\nu}(\overline{x})\,d\overline{x}^\mu d\overline{x}^\nu + 2A_{\mu\nu}(x,\overline{x})\,dx^\mu d\overline{x}^\nu \tag{66}$$

Using fact of CPT symmetry [2] I will arrive at:

$$ds^2(x,\overline{x}) = g_{\mu\nu}(x)\,dx^\mu dx^\nu + g_{\mu\nu}(\overline{x})\,dx^\mu dx^\nu - 2A_{\mu\nu}(x,\overline{x})\,dx^\mu dx^\nu \tag{67}$$

Where i just used fact that:

$$dx^\mu = -d\overline{x}^\mu \tag{68}$$

5.3 CPT coordinates

I will use special type of coordinates that will denote both CPT [2] reversed coordinates and normal ones, those coordinates can be written as a vector $x^A = (x^\mu, \overline{x}^\mu)$. It means that I just take normal coordinates x^μ and it's CPT [2] reversed part $\overline{x}^\mu$ into one single vector that has double the dimensions of real vector only. Using those coordinates I can write metric tensor:

$$g_{AB}(x,\overline{x}) = \begin{pmatrix} g_{\mu\nu}(x) & A_{\mu\nu}(x,\overline{x}) \\ A_{\mu\nu}(x,\overline{x}) & g_{\mu\nu}(\overline{x}) \end{pmatrix} \tag{69}$$

And spacetime interval:

$$ds^2 = g_{AB}dx^A dx^B \tag{70}$$

That gives final formalism of those coordinates that is needed to add them into four index field equations.

6 Field equations with CPT symmetry

6.1 Field equations with no interaction

Simplest part of field equations would be taking a no cross term field equation with CTP [2] symmetry metric tensor. It's just two copies of same spacetime but one is CPT reversed [2] compared to another. It means that I can split field equation into a block with two copies of same tensors but reversed:

$$G_{ABCD} + \lambda_{ABCD}(x, \overline{x}) = \begin{pmatrix} G_{\alpha\mu\beta\nu}(x) + \lambda_{\alpha\mu\beta\nu}(x) & 0 \\ 0 & G_{\alpha\mu\beta\nu}(\overline{x}) + \lambda_{\alpha\mu\beta\nu}(\overline{x}) \end{pmatrix} \tag{71}$$

$$T_{ABCD} = \begin{pmatrix} T_{\alpha\mu\beta\nu}(x) & 0 \\ 0 & T_{\alpha\mu\beta\nu}(\overline{x}) \end{pmatrix} \tag{72}$$

Where parts with cosmological constant [1] are equal:

$$\lambda_{\alpha\mu\beta\nu}(x) = \frac{1}{n-1}\left(g_{\mu\nu}(x)\, g_{\alpha\beta}(x) - g_{\beta\mu}(x)\, g_{\alpha\nu}(x)\right)\Lambda \tag{73}$$

$$\lambda_{\alpha\mu\beta\nu}(\overline{x}) = \frac{1}{n-1}\left(g_{\mu\nu}(\overline{x})\, g_{\alpha\beta}(\overline{x}) - g_{\beta\mu}(\overline{x})\, g_{\alpha\nu}(\overline{x})\right)\Lambda \tag{74}$$

That leads to simple in terms of those components field equation:

$$G_{ABCD} + \lambda_{ABCD}(x, \overline{x}) = \kappa T_{ABCD} \tag{75}$$

$$\begin{pmatrix} G_{\alpha\mu\beta\nu}(x) + \lambda_{\alpha\mu\beta\nu}(x) & 0 \\ 0 & G_{\alpha\mu\beta\nu}(\overline{x}) + \lambda_{\alpha\mu\beta\nu}(\overline{x}) \end{pmatrix} = \kappa \begin{pmatrix} T_{\alpha\mu\beta\nu}(x) & 0 \\ 0 & T_{\alpha\mu\beta\nu}(\overline{x}) \end{pmatrix} \tag{76}$$

In general case this will be not possible to put this as so simple block when I add interaction term, interaction term will lead into nonlinear effects changing all parts of field. Those equations without interaction term add new effect to relativity, that effect is two manifolds instead of one. Those two are present when calculating total distance in spacetime.

6.2 Field equations with interaction term

When there is interaction term present I can't express it as a block tensor. But I can express metric tensor in terms of it:

$$g_{AB}(x, \overline{x}) = \begin{pmatrix} g_{\mu\nu}(x) & A_{\mu\nu}(x, \overline{x}) \\ A_{\mu\nu}(x, \overline{x}) & g_{\mu\nu}(\overline{x}) \end{pmatrix} \tag{77}$$

It means that I arrive at more general four index field equation:

$$G_{ABCD}(x, \overline{x}) + \lambda_{ABCD}(x, \overline{x}) = \kappa T_{ABCD}(x, \overline{x}) \tag{78}$$

Where left side is based on a metric and so is tensor but it has independent components that give energy of interaction and need to be solved. It means that I should express this equation in full form:

$$\tilde{R}_{ABCD} - \frac{1}{n-3} C_{ABCD} + \lambda_{ABCD}(x, \overline{x}) = \kappa\left(\tilde{T}_{ABCD} - \frac{1}{n-3}\hat{T}_{ABCD}\right) \tag{79}$$

Where both sides use this kind of coordinates and cosmological constant [1] part is equal to:

$$\lambda_{ABCD}(x, \overline{x}) = \frac{1}{n-1}\left(g_{AC}(x, \overline{x})\, g_{BD}(x, \overline{x}) - g_{AD}(x, \overline{x})\, g_{BC}(x, \overline{x})\right)\Lambda \tag{80}$$

7 Connecting manifolds

7.1 Why CPT symmetry gives practical results

CPT symmetry [2] will have some very important results in terms of connecting manifolds and possible CTC [5] in solutions or it's being not present. Idea is very simple manifolds and it's CPT reversed image [2] can be connected. For making this reasoning more grounded let's start with simple metric of universe one de Sitter universe [1] and one of universe with a bounce like behavior. I will start by writing metrics for both of them:

$$ds^2 = -c^2dt^2 + e^{2Ht}\left(dr^2 + r^2d\Omega^2\right) \tag{81}$$

$$ds^2 = -c^2dt^2 + a_0^2\cosh^2\left(t\lambda\right)\left(dr^2 + r^2d\Omega^2\right) \tag{82}$$

First universe is not possible to glue it's end to it's beginning in infinite future universe is point like, function e^{2Ht} goes to zero and in infinite future it goes to infinity. In case of second universe there is possibility to glue ending of universe to it's beginning as both of them are infinite expanded universes. I will add it's CPT reversed [2] pairs:

$$ds^2 = -c^2d\bar{t}^2 + e^{2H\bar{t}}\left(d\bar{r}^2 + \bar{r}^2d\overline{\Omega}^2\right) \tag{83}$$

$$ds^2 = -c^2d\bar{t}^2 + a_0^2\cosh^2\left(\bar{t}\lambda\right)\left(d\bar{r}^2 + \bar{r}^2d\overline{\Omega}^2\right) \tag{84}$$

Now if I want to glue CPT reversed universe [2] with it's normal part in first case it's not possible as they only connect at moment of time equal to zero and do not connect at infinite future or past. But again it's possible in second case. In first case I can use another coordinates of de Sitter spacetime [1]:

$$ds^2 = -\left(1 - \frac{\Lambda r^2}{3}\right)c^2dt^2 + \left(1 - \frac{\Lambda r^2}{3}\right)^{-1}dr^2 + r^2d\Omega^2 \tag{85}$$

Now it can be seen why first metric wasn't time symmetric, in this coordinates it does not matter do I reverse order of time and space as will arrive at same result. So in truth I don't have to glue this kind of spacetime as it already time and space reversal symmetric [2]. To show it more clearly I can use another coordinates:

$$ds^2 = -dT^2 + \frac{1}{H^2}\cosh^2\left(HT\right)d\Omega_3^2 \tag{86}$$

Where $d\Omega_3^2$ encodes metric of hyper-sphere. Now it can be clearly seen that this metric can be glued of infinite past and future. So all metric so far can be glued.

7.2 Why to glue universe with itself?

In general CPT universe [2] is not connect to normal one otherwise it shares same space-time and can have interaction term but if there is no interaction term they do not interact with each other. Still need of CPT reversed universe [2] to exist means that any solution has to have infinite past and future. From it follows that I can glue not two universes but same universe leading to CTC [5] or I need to assume that there is infinite number of in this case universes casually connected. That don't give rise to CTC [5]. In both cases gluing is good extensions of those solutions as it says clearly how to extend solutions. Last thing I can do is to glue normal universe with CPT [2] reversed one. This gives still CTC [5] but between two copies of same universe that is not causality breaking as all that happens in one universe is exact copy of what happens in second one.

7.3 Black holes in GP metric

I can use black hole metric in GP coordinates [1] for same logic with connecting manifold I will write two expressions for both universe and CPT universe [2]:

$$ds^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - 2\sqrt{\frac{r_s}{r}} dr c dt - dr^2 - r^2 d\Omega^2 \quad (87)$$

$$ds^2 = \left(1 - \frac{r_s}{\bar{r}}\right) c^2 d\bar{t}^2 - 2\sqrt{\frac{r_s}{\bar{r}}} d\bar{r} c d\bar{t} - d\bar{r}^2 - \bar{r}^2 d\overline{\Omega}^2 \quad (88)$$

And again using symmetry of those coordinates second solutions gives a white whole where in our universe there is a black hole, and a black hole where in our universe there is a white hole. Key question is can they be glued in each universe. In infinite future we have a black hole in infinite past we have a white hole. Can we connect a black hole to a white hole? Short answer is yes, object falling into a black hole and coming out of a white hole in past possible. And it's only possible if there is some kind of bounce that gets rid of singularity or more precise that bounce can happen even with singularity, as when I assume that object compresses to a point then gets out of this point on other side. So black hole metric can generate CTC [5], when glued.

7.4 Universe with singularity at beginning

Now I will assume a universe matter only dominated without cosmological constant, that starts with singularity, metric for this universe will be very simple as its only representative case:

$$ds^2 = -c^2 dt^2 + a_0^2 t^2 \left(dr^2 + r^2 d\Omega^2\right) \quad (89)$$

$$ds^2 = -c^2 d\bar{t}^2 + a_0^2 \bar{t}^2 \left(d\bar{r}^2 + \bar{r}^2 d\overline{\Omega}^2\right) \quad (90)$$

In case of this universe singularity occurs at moment of time equal to zero. So it may seem that I glue them at that moment but from CPT symmetry [2] I need to assume that time goes before time equals zero. So in truth I need to glue them in infinite future to infinite past that could lead to CTC [5]. This case is another example of time symmetric solutions when applying CPT [2] symmetry.

7.5 Tower like model and CTC with two universes

There is simple alternative to CTC [5] that is tower like model of gluing universes or more precise manifolds. In case of universe each universe follows a new one that is casually connected to another one and this one is to another one and so on for infinity. It means that before that manifold there is always another manifold. Same applies to a black hole, in each universe there is a new white hole forming out of a black hole in our universe. It gets rid of CTC [5] but at cost of creating infinite distinct universes. Question is CTC [5] solution true or tower like model is a simple one as CTC [5] are causality breaking so there is only tower like model possibility left. On the other hand in spirit of CTP symmetry [2] I can create universe with it's CPT reversed copy connected forming a CTC but this time a not causality breaking loop as both universes are same but with reversed direction of time and space. Infinite number of universes may seem like a logical step , but it does not take into account second universe. If I connect both of them only way to connected them is by gluing infinite future of one universe to infinite past of another, and then gluing infinite future of second one to infinite past of first one, forming a loop that is not causality breaking. Formally I can write that:

I consider two CPT reversed spacetimes [2] $(\mathcal{M}_x, g_x)$ and $(\mathcal{M}_{\overline{x}}, g_{\overline{x}})$, together with submanifolds $A \subset \mathcal{M}_x$ and $B \subset \mathcal{M}_{\overline{x}}$ identified by a diffeomorphism $\phi : A \to B$. The glued spacetime is defined as the quotient

$$\mathcal{M} = (\mathcal{M}_x \sqcup \mathcal{M}_{\overline{x}})/\sim,$$

where the equivalence relation identifies each $p \in A$ with $\phi(p) \in B$. This construction produces a single manifold in which $\mathcal{M}_x$ and $\mathcal{M}_{\overline{x}}$ share the common boundary determined by ϕ.

7.6 Multiverse of CTC connected universes

I can extend this construction by adding infinite number of parallel universes to make a true tower like multiverse , where each universe and anti-universe connect by always opposite ends. One universe infinite past connects another universe infinite past leading to two possible paths when falling in loop, one leads back to the loop another one leads to parallel universe. Formally I can write that:

$$\mathcal{M}_\infty = \left(\bigsqcup_{n\in\mathbb{Z}} \left(\mathcal{M}_x^{(n)} \sqcup \mathcal{M}_{\overline{x}}^{(n)}\right) \right) /\sim, \tag{91}$$

That can be represented by a simple graph:

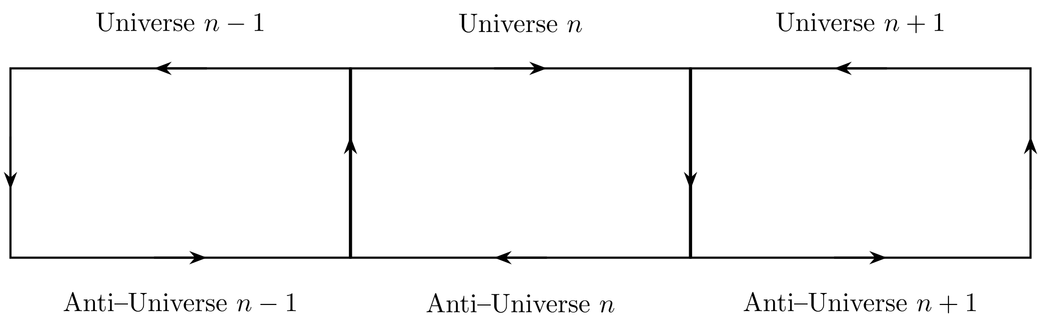

Figure 1: Diagram of glued universes with it's anti-universe pairs.

Where n denotes each of universe in this tower of parallel universes. Same happens with anti-universes they are connected by infinite pasts. As shown in 7.6. It shows that anti-universe can go to universe $n-1$ and n universe can go to anti-universe $n+1$. It means that there is ordering in both space and time. Gluing is done as show in diagram, where vertical line shows gluing. Infinite future of universe n is glued with infinite past of anti-universe n and $n+1$, infinite future of anti-universe n is glued with infinite past of universe $n-1$ and universe n infinite past. It forms both each universe/anti-universe loop and connects all possible universes.

Part III
Quantum effects

8 Quantum effects form geometry

8.1 Is there matter field or mater particles?

In physics there are two notions of matter, matter as field and matter as a particle. I will postulate that matter field is more of a field than of a distinct particles that can be independent of field itself. Let me start by in general I can think of gravity as a particle (source) and it's field generated by that particle (effect in space). It does work this way for all other forces, my assumption is that matter field and effect of that matter field (gravity field) both are same thing but affecting two kinds of curvature effects. Matter field affects Ricci curvature [1] and, gravity affects Weyl curvature [1]. But both are forms of same field and both carry energy. Key question would be can one be turn into another?

First in static solutions there is clear separation between gravity field and matter field. But in general non-static solutions it's possible to have this kind of separation. If Weyl curvature [1] mixes with Ricci curvature [1], I have both matter field and gravity energy in same point of spacetime acting as one field. This is key insight into understanding how quantum like effects emerge from pure geometry of spacetime. Gravity field is in truth not localized in a point, same as wave function in quantum mechanics. It's spread across all space. I will assume that this spreading of gravity field is not only hint of how to get into quantum effects only from pure geometry but possibly a hint about another path, instead of quantizing gravity, trying to build model that uses only gravity to explain quantum effects. First I will start by a simplest construction that is normalized curvature scalar.

8.2 Normalized curvature scalar

From trace revered field equations I can directly calculate curvature scalar [1] $K = R_{\alpha\mu\beta\nu}R^{\alpha\mu\beta\nu}$ using only energy parts. I will write it as:

$$R_{\alpha\mu\beta\nu} = \kappa\left(\hat{T}_{\alpha\mu\beta\nu} + \frac{1}{n-2}\left(T_{\alpha\beta}g_{\mu\nu} - T_{\alpha\nu}g_{\beta\mu} + T_{\mu\nu}g_{\alpha\beta} - T_{\beta\mu}g_{\alpha\nu}\right)\right.$$
$$\left. - \frac{2}{(n-1)(n-2)}\left(g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu}\right)T - \frac{2}{(n-1)(n-2)}\left(g_{\mu\nu}g_{\alpha\beta} - g_{\beta\mu}g_{\alpha\nu}\right)\Lambda\right) \quad (92)$$

$$R^{\alpha\mu\beta\nu} = \kappa\left(\hat{T}^{\alpha\mu\beta\nu} + \frac{1}{n-2}\left(T^{\alpha\beta}g^{\mu\nu} - T^{\alpha\nu}g^{\beta\mu} + T^{\mu\nu}g^{\alpha\beta} - T^{\beta\mu}g^{\alpha\nu}\right)\right.$$
$$\left. - \frac{2}{(n-1)(n-2)}\left(g^{\alpha\beta}g^{\mu\nu} - g^{\alpha\nu}g^{\beta\mu}\right)T - \frac{2}{(n-1)(n-2)}\left(g^{\mu\nu}g^{\alpha\beta} - g^{\beta\mu}g^{\alpha\nu}\right)\Lambda\right) \quad (93)$$

I need to change indexes to metric with CPT symmetry [2]:

$$R_{ABCD} = \kappa \Bigg(\hat{T}_{ABCD} + \frac{1}{n-2} \left(T_{AC} g_{BD} - T_{AD} g_{BC} + T_{BD} g_{AC} - T_{BC} g_{AD} \right)$$
$$- \frac{2}{(n-1)(n-2)} \left(g_{AC} g_{BD} - g_{AD} g_{BC} \right) T - \frac{2}{(n-1)(n-2)} \left(g_{AC} g_{BD} - g_{AD} g_{BC} \right) \Lambda \Bigg) \tag{94}$$

$$R^{ABCD} = \kappa \Bigg(\hat{T}^{ABCD} + \frac{1}{n-2} \left(T^{AC} g^{BD} - T^{AD} g^{BC} + T^{BD} g^{AC} - T^{BC} g^{AD} \right)$$
$$- \frac{2}{(n-1)(n-2)} \left(g^{AC} g^{BD} - g^{AD} g^{BC} \right) T - \frac{2}{(n-1)(n-2)} \left(g^{AC} g^{BD} - g^{AD} g^{BC} \right) \Lambda \Bigg) \tag{95}$$

$$K(x, \overline{x}) = R_{ABCD} R^{ABCD} \tag{96}$$

This will lead to last final expression for normalized curvature scalar:

$$\alpha = \int_{ct_A}^{ct_B} \int_{c\bar{t}_A}^{c\bar{t}_B} \int_{\Sigma_t} \int_{\Sigma_{\bar{t}}} \sqrt{-g} K(x, \overline{x}) \, d^{n-1}x d^{n-1}\overline{x} dx^0 d\overline{x}^0 \tag{97}$$

$$N(x, \overline{x}) = \frac{1}{\alpha} \int_{ct_A}^{ct_B} \int_{c\bar{t}_A}^{c\bar{t}_B} \int_{V^{n-1}} \int_{\overline{V}^{n-1}} \sqrt{-g} K(x, \overline{x}) \, d^{n-1}x d^{n-1}\overline{x} dx^0 d\overline{x}^0 \tag{98}$$

Where key here is that this scalar represents total curvature and comes from trace reversed field equations. It plays role like wave function in quantum mechanics. With change that instead of calculating probability of finding particle in given volume there is curvature density in given volume compared to total curvature. That curvature field has simple interpretation it's density of field compared with some region of space and time. Formally it says how much curvature affects given region of spacetime compared to total curvature. Key here is that it has to be finite in order to make this integral possible so all singular solutions are discarded.

8.3 Non-normalized curvature scalar

From it I can create just normal integral that is not normalized, this integral will have direct meaning as if there is enough curvature in given volume of space and given time interval I can detect particle. First I will define a particle focus region where most of curvature of particle or system of particles is focused then will need for object to be measured only if flow with some region of space and given interval of time is equal to that value of curvature:

$$N_0 = \int_{ct_a}^{ct_b} \int_{c\bar{t}_a}^{c\bar{t}_b} \int_{V_0^{n-1}} \int_{\overline{V_0}^{n-1}} \sqrt{-g} K(x, \overline{x}) \, d^{n-1}x d^{n-1}\overline{x} dx^0 d\overline{x}^0 \tag{99}$$

Where there is not only focused volume V_0^{n-1} but time focused time interval $\Delta t = t_b - t_a$. Now to detect particle there has to be at least that amount of curvature, it means when I do measurement in given volume and time interval particle can be only detected if this curvature in this region is greater or equal to that value:

$$N_0 \leq \int_{ct_A}^{ct_B} \int_{c\bar{t}_A}^{c\bar{t}_B} \int_{V^{n-1}} \int_{\overline{V}^{n-1}} \sqrt{-g} K(x, \overline{x}) \, d^{n-1}x d^{n-1}\overline{x} dx^0 d\overline{x}^0 = \alpha N(x, \overline{x}) \tag{100}$$

To detect particle or system of particles I need at least N_0 value of curvature. Detecting particle acts as focusing of curvature into some volume of space in given time interval. Particle will be detected only if there is enough curvature in focusing region on the detector. Focusing comes from interaction of curvature field with other objects or even with detector itself. Still position of particle or system of particles is still well defined but it's motion not as whole field interacts with other objects. And from that non-linear interaction comes wave like movement of well defined particle in a field that interacts with other fields. By well define I mean that particle focusing volume where there is most energy of particle is well defined. More precise I will take measurement in space of particle or system of particles less volume goes into space part of integral. If less volume goes into spacetime integral I get overall less total curvature current. Same pattern follows for time interval, smaller time interval less curvature flows in that time interval.

9 Spin

9.1 Spin as a manifold orientation

Last example is idea that spin is orientation of manifold in space. So for example for spin one half particles I need to use hyper-sphere. What I do mean by orientation of manifold in space? If i take a vector that goes form center of hyper-sphere and points to it's surface I will call it orientation vector. This vector says how this manifold is pointing in space. When manifold rotates it will rotate around this vector. In general there are infinite number of possible orientations on any manifold. When I do measurement I project orientation of manifold onto my measurement axis.

From it follows that I get two possible outcomes, orientation changes to being up or down with measurement axis. As there are only two possible solutions that align with measurement axis that are vectors one is pointing up one is point down with measurement axis. Orientation vector can't be split into smaller vectors as manifold can have only one orientation it explains why there are only two possible states of spin. I will start with simple math let me write black hole solutions [1] from it it can be easy seen that it's a solution of a hyper-sphere:

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2 d\Omega^2 \tag{101}$$

Isolate part with black hole radius so skip part with time components:

$$ds^2 = \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2 d\Omega^2 \tag{102}$$

Write hyper-sphere metric with not a constant radius:

$$ds^2 = \frac{dr^2}{\left(1 - \frac{r^2}{R^2(r)}\right)} + r^2 d\Omega^2 \tag{103}$$

Where radius of hyper-sphere is defined as $R(r) = \sqrt{\frac{r^3}{r_s}}$ same can be done with inside metric:

$$ds^2 = -\frac{1}{4}\left(3\sqrt{1-\frac{r_s}{r_g}} - \sqrt{1-\frac{r^2 r_s}{r_g^3}}\right)^2 c^2dt^2 + \frac{dr^2}{1-\frac{r^2 r_s}{r_g^3}} + r^2d\Omega^2 \quad (104)$$

Where again I take only spatial part without time:

$$\frac{dr^2}{1-\frac{r^2 r_s}{r_g^3}} + r^2d\Omega^2 \quad (105)$$

$$ds^2 = \frac{dr^2}{1-\frac{r^2}{R^2}} + r^2d\Omega^2 \quad (106)$$

Where in this case radius is equal to $R = \sqrt{\frac{r_g^3}{r_s}}$ so it shows clear that those solutions give exactly hyper-sphere so come from matter field with spin one half. Same can be done with de Sitter universe [1] in coordinates that I did use before:

$$ds^2 = -dT^2 + \frac{1}{H^2}\cosh^2(HT)\, d\Omega_3^2 \quad (107)$$

Where here whole hyper-sphere does change with time not only radial part. But again I can use another coordinates that will show that is true hyper-sphere , ones used before:

$$ds^2 = -\left(1-\frac{\Lambda r^2}{3}\right)c^2dt^2 + \left(1-\frac{\Lambda r^2}{3}\right)^{-1}dr^2 + r^2d\Omega^2 \quad (108)$$

Here radius is constant of hyper-sphere and is equal to:

$$R = \sqrt{\frac{3}{\Lambda}} \quad (109)$$

So again metric can be written as in spatial parts:

$$ds^2 = \frac{dr^2}{1-\frac{r^2}{R^2}} + r^2d\Omega^2 \quad (110)$$

That shows that two base solutions used in cosmology and black holes are in truth a hypes-sphere solutions so their use matter field with spin one half.

9.2 Simple projection of orientation vector model

I will start with only space components of metric as I care only about them. First denote spatial part of metric as h_{ab} and its orientation vector as s^a, that vector length squared is equal to:

$$L^2 = h_{ab}s^as^b \quad (111)$$

Now I need to add measurement axis vector that will be projected onto orientation axis. This will be denoted as v^a and it's length squared is defined same way:

$$L^2 = h_{ab} v^a v^b \tag{112}$$

Both have to lie on same sphere or hyper-sphere and have same length equal to its radius. So I can set that:

$$h_{ab} v^a v^b = h_{ab} s^a s^b = L^2 \tag{113}$$

Now i create two vectors that are projections of measurement axis vector on spin state, they represent spin up and down, so two orientations projection of orientation vector can take. One state has to be rotated π radians compared to another for sphere so spin one and one has to have state 2π for spin one half. That represents spin up and down states. Total length of final vector is sum of orientation vector with its measurement axis vector normalized by total length first for sphere so spin one:

$$\psi_+ (\theta) = \frac{L + L\cos(\theta)}{2L} \tag{114}$$

$$\psi_- (\theta) = \frac{L + L\cos(\pi - \theta)}{2L} \tag{115}$$

Then for hyper-sphere so spin one half:

$$\psi_+ \left(\frac{\theta}{2}\right) = \frac{L + L\cos\left(\frac{\theta}{2}\right)}{2L} \tag{116}$$

$$\psi_- \left(\frac{\theta}{2}\right) = \frac{L + L\cos\left(\pi - \frac{\theta}{2}\right)}{2L} \tag{117}$$

This can be re-written using formula for angle between vectors, and lengths using only metric tensors:

$$\cos(\theta) = \frac{h_{ab} s^a v^b}{\sqrt{h_{ab} s^a s^b}\sqrt{h_{ab} v^a v^b}} = \frac{h_{ab} s^a v^b}{L^2} \tag{118}$$

Idea is that before measurement orientation vector is random in direction as it's unknown and there is infinite number of possible orientation so there is exactly zero probability of guessing right one, after measurement we project that vector into our measurement axis and get known state of orientation, when we measure another axis there is new projection and so on. So measurement does change orientation state from s^a to $s^{a'}$ with probability equal to $\psi_+ \left(\frac{\theta}{2}\right)$ or $\psi_- \left(\frac{\theta}{2}\right)$ for up or down state and spin one half. Where state up represents orientation vector align with measurement axis and down opposite to it.

9.3 Spin state when there is correlation between states

When there is a correlation between states [6] its best candidate to test will this formalism break. I will use correlation between states where angle between them is always anti-correlated [6] it means that if one particle is spin up one is spin down. Here I have two body system [6], but I can use same logic. Now If i take axis between one system and another system as angle θ and there is no correlation [6] will arrive at same equation as before but if I assume anti-correlation I will arrive at states:

$$\psi_+(\theta) = \frac{L + L\cos(\vartheta + \pi)}{2L} \tag{119}$$

$$\psi_-(\theta) = \frac{L + L\cos(\vartheta)}{2L} \tag{120}$$

Then for hyper-sphere so spin one half:

$$\psi_+\left(\frac{\theta}{2}\right) = \frac{L + L\cos\left(\pi + \frac{\vartheta}{2}\right)}{2L} \tag{121}$$

$$\psi_-\left(\frac{\theta}{2}\right) = \frac{L + L\cos\left(\frac{\vartheta}{2}\right)}{2L} \tag{122}$$

As first angle is always rotated by π compared to second so I have change in angle equal to π:

$$\phi_+ = \pi + \vartheta \quad \phi_- = \vartheta \tag{123}$$

$$\phi_+ - \phi_- = \pi \tag{124}$$

I can use inverse correlation [6] but then I need to be aware that those states are now giving opposite relation:

$$\phi_+ = \vartheta \quad \phi_- = \vartheta + \pi \tag{125}$$

$$\phi_+ - \phi_- = -\pi \tag{126}$$

It means that states are reversed. Up state is now down state and down state is now up state, as I need to flip orientation (from minus sign). In case where there is no correlation angles between are random. So if one vector has angle ϑ another one has $\pi - \vartheta$, as total angle between spin state up and down is equal to π:

$$\vartheta + \pi - \vartheta = \pi \tag{127}$$

In case of correlated states of two systems I ask what is correlation [6] of them if I chose random axis to measure them, angle is angle between measurement axis just like in Bell theorem [6]. Now I can again use same logic and get correct answer for correlation between both anti-correlated states and not correlated states:

$$E_{\text{Correlated}}(\vartheta) = \psi_+(\vartheta) - \psi_-(\vartheta) \tag{128}$$

$$E_{\text{Correlated}}(\vartheta) = \frac{L + L\cos(\vartheta + \pi)}{2L} - \frac{L + L\cos(\vartheta)}{2L} = \sin^2\left(\frac{\vartheta}{2}\right) - \cos^2\left(\frac{\vartheta}{2}\right) = -\cos(\vartheta) \tag{129}$$

$$E_{\text{Random}}(\vartheta) = \psi_+(\vartheta) - \psi_-(\vartheta) \tag{130}$$

$$E_{\text{Random}}(\vartheta) = \frac{L + L\cos(\vartheta)}{2L} - \frac{L + L\cos(\pi - \vartheta)}{2L} = \cos^2\left(\frac{\vartheta}{2}\right) - \sin^2\left(\frac{\vartheta}{2}\right) = \cos(\vartheta) \tag{131}$$

That is exactly result one would expect from quantum theory [6] but here it's done only by geometric relation between orientation vectors.

Part IV
Summary

In this section I did present geometric explanation of quantum effects without invoking any kind of quantum postulates. It was done in pure geometric way. From it follows that there is a way possible way of getting quantum effects just from pure geometry and instead of quantazing spacetime and relativity find effects in it that are quantum in nature. Connecting all those things into one framework. Key question is can those effects explain observed quantum experiments?

And answering that question is beyond this work as it only does state foundation of mathematical formalism for those. Solving those equations for spacial cases is just solving normal two index equations [1] but there could be possible new solutions that come from this four index equations. Adding CPT [2] symmetry into metric tensor can lead to model of particle physics generated by interaction between two fields. In general interaction term $A_{\mu\nu}(x, \overline{x})$ makes possible to create interactions between matter field and antimatter field. Even without it there is both matter and antimatter field in CPT reversed universe [2]. That leads to metric changes, as there is simple interaction between metrics.

This means that CPT revered universe [2] can have real effects on our universe. That would manifest in form of metric changes. On the other hand I did present gluing theorem for universes and black holes that can rid of any CTC [5] in those by using tower like model. It works both in case of singularities and with bounce models. This gluing is not arbitrary as it has a strong need that two manifolds are casually connected, from it follows that each manifold form CPT [2] symmetry has to have infinite past and future and if I can connect both sides there is a possibility of creating CTC [5] that can be removed by adding tower like infinite manifolds. On the other hand I can assume more in spirit of CPT [2] symmetry a universe that is glued with its reversed copy, and its solutions I will choose.

Field equation in general a four index one is not a extension of two index equation [1] but it's full form. I can say that it's same equation but written in more general way , that way is simplest way. It does not add new objects to two index equation as in truth it still deals with curvature of spacetime. What is new is interpretation of energy of vacuum that is just extension of matter field energy into space.

That extension is crucial as it's where it differs in interpretation form two index equation [1]. Still this is natural order of things following from fact that gravity field is not localized just line in two index equations [1]. From this un-localized energy filed of gravity comes idea about how to get quantum effects from just pure curvature, where in simple terms if energy of field is not localized I can assume that object in field itself is either not localized when measured as field and object are not two separate things but one thing that is combined in four index field equation.

This generates new view on how quantum effects emerge from spacetime, without invoking any additional conditions other than field equations and scalar that is invariant of them. Lastly spin is understood as orientation of manifold. Where there is infinite number of orientations for given shape and I did show that two most used solutions for black holes and de Sitter spacetime are in truth a hyper-spheres so they explain how matter field with spin one half propagates. I last subsection about spin I did put forward simplest possible mode of how orientation vector projection onto spin axis works. Where still only pure geometric phenomena was used. It's not full model but a great approximation how whole model would look.

I can summarize this work as two statements, four index field equations are natural extension of two index field equations [1] and there is possibility that quantum phenomena can be explained by geometric means. Additionally adding CPT symmetry [2] is possible resolution of both particle interaction geometric model and independent of singularity solution to a black holes, as from it follows model of two connected universes where they form a CTC [5] without breaking of causality. It would suggest that our universe is connected to CPT inverse universe forming a loop.

Are those equations just same equations as two index equations [1]? That question remains open and is beyond scope of this paper. They could be just same equations but in another formalism or they could lead to new solutions that are non existent in two index equation [1]. Still what is new are quantum effects from pure geometry and CPT [2] symmetric metric tensor. Even if those are only new expressions of same equations this four index expression gives new formalism for Relativity.

Where is biggest change is that vacuum from it follows Weyl tensor [1] is now controlled by a unknown functions to solve that is energy tensor of vacuum. It leads in some case to possibility of new solutions as there is a way to control how vacuum behaves. Still there is need to check are two scenario possible in physical real world. Another hint is that normalized curvature scalars work only when they have finite total value, it means that in whole model solutions with infinite curvature value are non normalize-able. From it follows that there is possibility of selection only those solutions that are normalize-able. As it should be in reality.

Presented model of spin as manifold orientation is simplest possible one. Still it arrives at correct conclusions showing that this approach seems promising in general. Still its not a final model as things are more complex for many systems.

So summarizing this work, I did present possible extensions of General Relativity [1] with four index equations that are self consistent. Next I added CPT [2] symmetry to it and finally quantum like effects in last part of this work. Still did not present any solutions to those equations or examples that differ from two index equations [1] as it should be done in future work.

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