Phase Transition Theory and Mechanism of High-Temperature Superconductivity

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Abstract

In this paper, the author redefines the concept of phase transition in a more general sense, provides an accurate characterization method, and derives the phase transition equation and phase transition temperature formula. On this basis, the essential properties of superconducting phase transition are analyzed, and the general superconducting phase transition equation is derived. The author argues that the superconducting phase transition of high-temperature superconducting materials, including cuprates, iron-based, nickel-based superconductors, and high-pressure hydrogen-rich superconductors, still follows the conductive electron pairing mechanism. The key to the high superconducting temperature lies in the stronger space-time correlation between conductive electrons in low-dimensional structures. Based on the geometry of space-time structures, the author defines a more general space-time correlation formula between particles, reveals the logical relationship that the correlation between conductive electrons decreases geometrically with increasing dimensions from the most general conditions, provides relevant formulas, and further analyzes the intrinsic mechanism of the high superconducting phase transition temperature， and proposes the process of developing new high temperature superconductor.

1 Introduction

Nearly 40 years since the discovery of high-temperature superconducting materials by Bednorz J G and Müller K A in 1986 [9], a large number of high-temperature superconducting materials such as cuprates, iron-based, nickel-based superconductors, and high-pressure hydrogen-rich superconductors have been successively discovered (see Figure 1 for the timeline of superconducting material discoveries (Hemley, R. J., 2021, [10]). Among them, cuprates, iron-based, and nickel-based superconductors all have two-dimensional (2D) planar conductive structures. Strong correlation between conductive electrons is generally considered the main factor for the high phase transition temperature, but the exact mechanism remains controversial [15 16]. In recent years, discovered hydrogen-rich superconducting compounds [11-14] exhibit extremely high phase transition temperatures under ultra-high pressure conditions (150-200 GPa). For example, Snider, E. et al. reported that LaH_{10} has a critical superconducting temperature (Tc) of up to 250-260 K under an ultra-high pressure of 1.8 million atmospheres [12]. The phase transition mechanism of hydrogen-rich superconductors can be explained by the strong electron-phonon coupling under extreme high pressure [17]. In particular, binary hydrides tend to form metal-like hydrogen

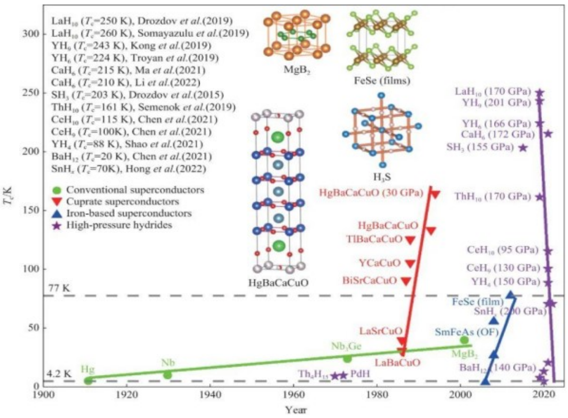


Fig . 1 Chronology of high-temperature superconductors, Hemley, R. J.[10]

structures under extreme pressure, which can be analyzed and predicted based on the BCS theory [18-21]. The discovery of numerous high-temperature hydrogen-rich superconductors at least proves that the BCS theory is still applicable to such materials. However, the extreme high-pressure condition completely limits the practical application of these superconductors.

Superconductivity, a universal phenomenon in conductors, is a macroscopic quantum effect and one of the macroscopic manifestations of the most essential material properties, indicating the existence of a universal intrinsic logic. The author has been researching phase transition and superconductivity theory since 1987 and has established and completed the relevant theories based on the self-created Geometry of Space-time Structures, which have not been formally published until now. The structural and conductive characteristics of newly discovered high-temperature superconducting materials in recent decades are basically consistent with this theory. Currently, the booming AI and related technology industries have an enormous energy demand, exacerbating the already severe global energy and environmental issues. The development and application of room-temperature superconductors are one of the key means to fundamentally solve these problems. Combining the latest research achievements in high-temperature superconductivity, this paper summarizes the author's research results for formal publication to promote and accelerate relevant experimental studies.

2 Phase Transition Theory

In a broad sense, phase transition refers to the transformation of a substance from one state of matter to another, with various forms of transformation processes. The phase transition discussed in this paper, as defined in traditional physics, refers to the transition from one uniform and stable state structure to another. The microscopic particle units composing the substance are one or a few types, and no change in microscopic particle units occurs during the phase transition; the process is reversible. Due to the diversity of phase transition processes, there are many existing phase transition theories. The phase transition theory referred to in this paper is a general theory applicable to all the above-mentioned phase transition processes.

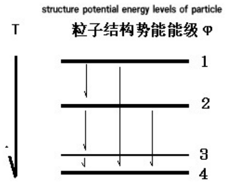
2.1 Space-time Statistical Thermodynamic Mechanism of Phase Transition

According to the space-time statistical thermodynamics theory [1], for any uniform and stable state j of a system, there exist definite total internal energy (U_j), internal energy density (u_j), and energy level ε_{ji}) of structural particle unit i, which have a one-to-one positive correlation with the number of degenerate states (Z_j and z_{ji}) of the corresponding same-energy and same-type structural form set. For simplicity, only thermodynamic systems composed of single-type particles (atoms, molecules, or ions) are considered here. Assuming the total number of particles in the system is N, the energy level ε_{ji}, thermal kinetic energy ϵ_{ji}, and ordered structural potential energy φ_{ji} of particle unit i in state j , satisfy: $U_j = \sum_{i=1}^{N} \varepsilon_{ji} = \sum_{i=1}^{N} (\epsilon_{ji} + \varphi_{ji})$. These quantities are all functions of the system temperature T. In an equilibrium state, the structure is stable, and all particles in a single-type particle system have the equal status relationship in space-time statistics. The ordered structural potential energy (φ_{ji}) of particles is the same and stable; the thermal kinetic energy levels (ϵ_{ji}) of all particles are statistically random, following a stable energy level distribution with the same average values,$\overline{\varepsilon_{ji}}$ and $\overline{\epsilon_{ji}}$, then, $U_j = \sum_{i=1}^{N} \overline{\varepsilon_{ji}} = \sum_{i=1}^{N} (\overline{\epsilon_{ji}} + \varphi_{ji})$.

In the equilibrium state, the temperature, total energy, and average energy level $\overline{\varepsilon_{ji}}$ of single particle remain constant, but particles continuously undergo thermal transitions between different energy levels, the space-time structure states of these particles all belong to the degenerate state of

$\overline{\varepsilon_{ji}}$; the overall state sets of the system are composed of the combination and superposition of subsets of the same-type space-time structural forms of all particles. The definite set (number of states)of degenerate states of the system is macroscopically characterized as a stable equilibrium state. The energy levels at all levels and the corresponding number of degenerate states in the system are monotonically increasing functions of temperature. Phase transition is the discontinuous jump of this functional relationship at a specific temperature.

Different phases of the same substance have different space-time structures, resulting in different system structural potential energies (i.e., the interaction energy between all particles in the system), internal energy densities, and corresponding numbers of system degenerate states. Phase transition is the reversible transformation between different phases of the same substance at a constant phase transition temperature (Tc). The thermal kinetic energy ($\overline{\epsilon_{ji}}$) of the same particle unit is the same at the same temperature. Therefore, the essence of material phase transition is the significant, discontinuous, and jump-like change in its structural potential energy density and internal energy density. The phase transition process is a collective jump of the unit particles of a substance between two different structural potential energies.



持续降温过程粒子在不同结构能级
之间群体跃迁引发相变机理示意图

Fig.2 Schematic diagram of particle energy level transition during continuous cooling process [1]

A single-substance system can have multiple states with different structural potential energies. During the continuous equilibrium cooling process from high temperature under isobaric conditions, the structural units(particles) of the system can undergo multiple collective jumps from relatively high ordered structural potential energy to lower ordered structural potential energy, resulting in multiple phase transition processes (as shown in the figure 2).

2.2 Energy Levels of Ordered Structural Potential Energy and Corresponding Phase Transition Characteristics

For an isobaric system with equilibrium temperature change (e.g., continuous cooling), the interaction structural potential energy level $\varphi_j(T)$ between system unit particles generally has multiple discontinuous energy levels within the temperature change range, which can be expressed as:

$$\varphi(T)=\begin{cases}\varphi_1 & T\geq T_{c_1}\\ \varphi_2 & T_{c_1}\geq T\geq T_{c_2}\\ \vdots & \vdots\\ \varphi_{\mathrm{j}} & T_{cj-1}\geq T\geq T_{cj}\\ \vdots & \vdots\\ \varphi_J & T_{c_{J-1}}\geq T\geq T_{cJ}=0\end{cases} \tag{1}$$

According to Equation (1), during the cooling process of an isobaric isovolumetric equilibrium system, the structural unit particles of the system undergo a collective ordered jump from phase j to phase j+1 at the phase transition temperature T_{cj}. The energy level change $\Delta\varepsilon_j$ of a single particle i and the released thermal internal energy ΔQ_{ij}, then, there is:

$$\Delta Q_{\mathrm{ij}}=\Delta\varepsilon_j=\Delta\varphi_j=\varphi_j-\varphi_{j+1}=h\nu_{ij} \tag{2}$$

where $h\nu_{ij}$ is the optical-thermal quantum emitted by particle i when jumping from phase j to phase j+1 at the phase transition temperature T_{cj}.

At low temperatures, due to the weakened random thermal motion energy of particles in an isobaric substance system, certain weak interactions between particles become macroscopically apparent. At this time, the fine structure of some ordered structural potential energy levels of particles can be observed, leading to many wonderful phases that cannot be detected at room temperature.

Near absolute zero, according to the space-time statistical theory, for any internal equilibrium multi-particle system, the thermal motion energy levels of particles tend to a definite minimum energy level, the number of degenerate states of the total space-time structure form set of the system tends to a single definite state or a few definite states, the space-time structure conformation of particles in the system tends to be completely ordered, and the system entropy S=0. For single-particle fluid substances, when the temperature approaches absolute zero, the particle system tends to be completely ordered, forming the same state or quasi-same state (e.g., Fermi quanta, which are in different states but have a definite state combination, or can be understood as the same system state), i.e., superfluidity, so electrons exhibit superconductivity, while atoms exhibit superfluidity (e.g., liquid helium). Therefore, metallic substances with free electrons inside all exhibit superconductivity at ultra-low temperatures. At this time, the free electrons in the metal exhibit collective resonant motion, and pairing is only part of the connotative properties of this collective macroscopic quantum state. Due to the isotropic space-time structure of hydrogen atoms, liquid hydrogen has no solid phase under normal pressure.

2.3 Phase Transition Equation and Calculation of Phase Transition Temperature T_c

Consider a multi-particle system composed of one type of microscopic particle, which only exchanges heat with the external environment. Let m_i be the mass of particle units, m_e the electron mass, and N the total number of system unit particles (e.g., atoms or electrons). Assume the system particles reversibly jump from the high-energy phase j to the low-energy phase j+1 by releasing heat ΔQ. The average ordered structural potential energies of particles in the high-energy phase j and low-energy phase j+1 are φ_j and φ_{j+1}, respectively, with $\Delta\varphi_j = \varphi_j - \varphi_{j+1}$. The thermal kinetic energies of particles are $\bar{\epsilon}_j$ and $\bar{\epsilon}_{j+1}$, and the energy levels are $\bar{\varepsilon}_j$ and $\bar{\varepsilon}_{j+1}$, and the total internal energy and its change are U_j， U_{j+1} and ΔU, respectively, and the total entropy change is ΔS; the temperature remains constant at the phase transition temperature throughout the process. Using Equations (1) and (2), we obtain:

Energy Level Equations:
$$\begin{cases}\bar{\varepsilon}_j = \bar{\epsilon}_j + \varphi_j = \bar{\epsilon}_j + (\sum_{q=J-1}^{j} \Delta\varphi_q + \varphi_J) \\ U_j = N\bar{\varepsilon}_j = N(\bar{\epsilon}_j + \varphi_j) = N\bar{\epsilon}_j + N(\sum_{q=J-1}^{j} \Delta\varphi_q + \varphi_J)\end{cases} \quad (3)$$

Related Quantity Equations:
$$\begin{cases}\bar{\epsilon}_j = \bar{\epsilon}_{j+1} \\ \Delta\varphi_j = \varphi_j - \varphi_{j+1} = h\nu_j \\ \Delta U = U_{j+1} - U_j =- N\Delta\varphi_j =- Nh\nu_j = \Delta Q \\ \Delta S = k_B \ln(M_{\text{Ш}j}/M_{\text{Ш}j+1}) = \Delta Q/T_{cj}\end{cases} \quad (4)$$

$$T_{cj} =- N\Delta\varphi_j/(k_B \ln(M_{\text{Ш}j+1}/M_{\text{Ш}j}) \quad (5)$$

In Equation (4), $M_{\text{Ш}j}$ and $M_{\text{Ш}j+1}$ are the numbers of space-time structure states (degenerate states) of the system completely in the high-energy phase j and low-energy phase j+1 at the phase transition temperature, respectively; k_B is the Boltzmann constant; φ_J is the ordered structural potential energy of the system in the ground state at absolute temperature T=0. The system of equations is not specific and is universally applicable. Assuming the system starts to heat up from absolute zero and undergoes multiple phase transitions to reach phase j, the ordered structural potential energy of particles increases from the lowest ground state φ_J up to φ_j, the increase in the ordered structural potential energy level of particles is Δφ， then $\Delta\varphi = \varphi_j - \varphi_J \approx \sum_{q=J-1}^{j} \Delta\varphi_q$. For particles to jump from the lowest ground state J to phase j, their corresponding thermal kinetic energy must be sufficient to overcome the bound potential well depth Δφ , $k_B T_{cj} = h\nu_{jJ} \sim \Delta\varphi$, then,

$$\mathrm{T_{cj}} \approx \Delta\varphi/\mathrm{k_B} \approx \frac{1}{\mathrm{k_B}}\sum_{\mathrm{q=J-1}}^{\mathrm{j}} \Delta\,\varphi_{\mathrm{q}} \qquad (6)$$

The phase transition temperature is proportional to the interaction energy between particles. Strong correlation between electrons is the key to the high $\mathrm{T_c}$ of superconductors.

3 Wave Vector Characterization and Superconducting Phase Transition Theory

Superconductivity is a phase transition phenomenon in which the resistance of conductors such as metals drops abruptly to zero when the temperature reaches a specific critical temperature (T c) during cooling. The superconducting state is a collective ordered single macroscopic quantum state where all conductive electrons coexist collaboratively. The random thermal scattering of conductive electrons by phonons (lattice structure) transforms into a coupling induction that "assists" electrons in forming this unified macroscopic quantum state. When Tc→0, free electrons in the conductor actually condense into the same ground state (or near-ground state). Superconductivity only involves the interaction relationships between electron-electron, electron-lattice, or electron-lattice-electron, and wave vector characterization is relatively simple and direct.

3.1 Wave Vector Characterization and Correlation between Electrons [1, 2]

According to the wave-particle duality, the wave vector $\vec{\mathrm{K}}$ is defined as $\vec{\mathrm{K}}=\vec{\mathrm{P}}/\hbar=2\pi\vec{\mathrm{P}}/\mathrm{h}$, $\mathrm{k} = 2\pi/\lambda$, h is Planck's constant, $\hbar$ is the reduced Planck's constant, P is the momentum of the free particle, and λ is the wavelength. Essentially, for any microscopic equilibrium system, its space-time space, momentum-energy space, and wave vector-frequency space are mutually reflective, with transformation relationships governed by Fourier transforms.

The correlation between a large number of free electrons in a conductor can be analyzed using statistical averaging logic. For simplicity, only single-element lattice structures are considered, and the wave vectors of lattice atoms, free electrons, and optical-thermal quanta are denoted as $\vec{K}_s$, $\vec{K}_e$ and $\vec{K}_p$, respectively.

The correlation between any two microscopic particles $\vec{\mathrm{K}}_\mathrm{i}$ and $\vec{\mathrm{K}}_\mathrm{j}$ is defined as $\vec{\mathrm{K}}_\mathrm{i}\bullet\vec{\mathrm{K}}_\mathrm{j}$. Considering the corresponding interaction energy significance and adding a coefficient ξ correction, the correlation interaction $\mathrm{V_{ij}}$ between any two microscopic particles $\vec{\mathrm{K}}_\mathrm{i}$ and $\vec{\mathrm{K}}_\mathrm{j}$ is defined as:

$$\mathrm{V_{ij}}=\xi\vec{\mathrm{K}}_\mathrm{i}\bullet\vec{\mathrm{K}}_\mathrm{j} \qquad (7)$$

As interaction mediating particles, photons and phonons have no mutual interaction, $\vec{\mathrm{K}}_\mathrm{i}\bullet\vec{\mathrm{K}}_\mathrm{j}=0$.

For the correlation interaction between electrons $\vec{\mathrm{K}}_\mathrm{i}$ and $\vec{\mathrm{K}}_\mathrm{j}$, let $\mathrm{m_e}$ denote the electron mass, introducing $\xi=\hbar^2/2\sqrt{\mathrm{m_i m_j}}$, we obtain:

$$\mathrm{V_{ij}}=\frac{\vec{\mathrm{P}}_\mathrm{i}\bullet\vec{\mathrm{P}}_\mathrm{j}}{2\sqrt{\mathrm{m_i m_j}}}=\frac{\hbar^2\vec{\mathrm{K}}_\mathrm{i}\bullet\vec{\mathrm{K}}_\mathrm{j}}{2\sqrt{\mathrm{m_i m_j}}}=\frac{\hbar^2\vec{\mathrm{K}}_\mathrm{i}\bullet\vec{\mathrm{K}}_\mathrm{j}}{2\mathrm{m_e}} \qquad (8)$$

In a thermodynamic system (conductor), the space-time correlation states of two electrons with wave vectors $\vec{\mathrm{K}}_\mathrm{i}$ and $\vec{\mathrm{K}}_\mathrm{j}$ may have multiple degenerate states, denoted by $\mathrm{d_{ij}}$ as the number of degenerate states and M as the total number of system states. The contribution of the state where the two electrons have wave vectors $\vec{\mathrm{K}}_\mathrm{i}$ and $\vec{\mathrm{K}}_\mathrm{j}$ to the correlation interaction V is,

$$\mathrm{V_{ij}}=\mathrm{d_{ij}}\hbar^2\vec{\mathrm{K}}_\mathrm{i}\bullet\vec{\mathrm{K}}_\mathrm{j}\ /2m_e\mathrm{M}\ ,\ \mathrm{i}\neq\mathrm{j} \qquad (9)$$

Summing over all possible wave vector states of the two electrons gives the correlation interaction between any two electrons in the multi-particle system. Considering that the probability of any state in a thermodynamic system is negatively correlated with its energy level, i.e., proportional to the Boltzmann distribution $\mathrm{e}^{-\varepsilon/\mathrm{k_B T}}$, this relationship is used for correction. Summing over all possible wave vector states $\vec{\mathrm{K}}_\mathrm{i}$ and $\vec{\mathrm{K}}_\mathrm{j}$ of electrons i and j and normalizing, we obtain:

$$V=\frac{\Sigma_i^{N-1}\Sigma_j^{N}\xi d_{ij}\,\vec{K}_i\bullet\vec{K}_j e^{-\xi\vec{K}_i\vec{K}_j/\beta}}{M}=\frac{\Sigma_{\vec{K}_i}\Sigma_{\vec{K}_j}\xi d_{ij}\,\vec{K}_i\bullet\vec{K}_j\,e^{-\xi\vec{K}_i\vec{K}_j/\beta}}{M} \tag{10}$$

where $M=\Sigma_i^{N-1}\Sigma_j^{N}d_{ij}\,e^{-\xi\vec{K}_i\vec{K}_j/\beta}=\Sigma_{\vec{K}_i}\Sigma_{\vec{K}_j}d_{ij}\,e^{-\xi\vec{K}_i\vec{K}_j/\beta}$ is the normalization constant, N is the total number of electrons, $\xi=\hbar^2/2m_e$，$\beta=k_BT$. Equation (10) represents the general correlation interaction between electrons (particles) in a conductor (system).

For the same electron (i=j), Equation (10) becomes the kinetic energy of the electron (particle) in the classical sense. In other words, summing over all wave vector values of electrons $\vec{K}_i$ in Equation (10) includes both the structural potential energy and kinetic energy of electrons, which is essentially the electron energy level.

Assume the unit cell size of the conductor lattice is a×b×c, the wave vector quantization condition is:

$$\begin{cases}K_x a=2\pi n_x\\ K_y b=2\pi n_y\\ K_z c\ =2\pi n_z\end{cases}\Rightarrow\begin{cases}K_x=\kappa_x n_x\\ K_y=\kappa_y n_y\\ K_z=\kappa_z n_z\end{cases} \tag{11}$$

where n_x，n_y，n_z are positive integers，$\kappa_x=2\pi/a$，$\kappa_y=2\pi/b$，$\kappa_z=2\pi/c$ are constants; for single-element lattices with isotropy, $\kappa_x=\kappa_y=\kappa_z$. Equation (11) determines the summation range of Equation (10), i.e., the lattice structure determines V.

The structural potential energy of solids is almost stable at room temperature or low temperature, especially for calculating the superconducting phase transition temperature Tc of superconductors. As long as the lattice structure of the material is known, the structural potential energy of electrons (correlation interaction V) can be calculated just like calculating energy bands.

A simplified estimation of V in Equation (10) can be performed[2]. For metallic conductors with a single-atom structure, and isotropy, at temperature T=0, the electron wave vector $\vec{K}_i$ has a maximum value $\vec{K}_H$ (Fermi surface, corresponding to the maximum value of n_i as n_0). For fermions, $d_{ij}=1$, and the sum over all $\vec{K}_j$ for any $\vec{K}_i$ in Equation (10) is equal. Thus, the double summation over two variables in Equation (10) simplifies to a single-variable summation. Further simplifying by setting $\vec{K}_i=\vec{K}_0=(n_0,0,0)$, we finally obtain，$V\sim n_0^2\sim K_H^2$.

3.2 Superconducting Phase Transition and Calculation of Phase Transition Temperature T_C

Using wave vectors $\vec{K}$ to characterize the space-time structure of particles, lattice atoms, electrons, and optical-thermal quanta are denoted as $\vec{K}_s$, $\vec{K}_e$ and $\vec{K}_p$, respectively.

The collective resonant order of the electron fluid, forming a quantum state $\vec{K}$ (constant state, including $\vec{K}=0$), that is the superconducting phase.

In the ground state of the system, the particles, multi-particle sets, total wave vector $\vec{K}$, and temperature are all approximately zero, which is naturally a completely ordered state (pure superfluid or superconducting) state. Of interest is the critical state where both the normal state and superconducting state coexist with Tc much higher than zero, where $\vec{K}_s$, $\vec{K}_e$and $\vec{K}_p$may be small but non-zero.

Within the same unit cell, two electrons cannot be in the same state (note: the Pauli exclusion principle has applicable conditions, such as "interval" limitations). To achieve the overall coordinated order of the wave vectors of all adjacent electrons within the same or multiple unit cell regions, it is necessary to rely on $\vec{K}_s$ and $\vec{K}_p$, i.e., through the coupling of Σ（$\vec{K}_s+\vec{K}_e+\vec{K}_p$）. Due to the periodicity of the lattice structure, if the wave vector coupling combination of all electrons within a single or multiple adjacent unit cells (⊙) is zero, the static superconducting state of the system can be achieved (the total wave vector of the system and the sum of wave vectors within any small envelope interval are all zero), i.e.:

$$\Sigma_{\odot}\left(\vec{K}_s+\vec{K}_e+\vec{K}_p\right)=\mathbf{0} \tag{12}$$

This already includes the meaning of electron pairing. This is the wave vector equation for realizing scale-invariant superconducting phase transition.

The collective ordered motion state of the electron fluid, i.e., the superconducting state, means that the sum of electron wave vectors at all scales in the conductor (including unit cells) takes a fixed value $\vec{K}_{e0}$. At ultra-low temperatures, the optical-thermal quanta inside the solid are almost completely lattice vibration wave vectors, so $\vec{K}_p$ in Equation (12) can be neglected, leading to:

$$\sum_{\odot}（\vec{K}_s+\vec{K}_e+\vec{K}_p） = \sum_{\odot}（\vec{K}_s+\vec{K}_e） = \vec{K}_{e0} \qquad （13）$$

Equation (13) is the characterization equation for conductors to exhibit the superconducting quantum state, representing the strong adjacent coupling interaction.

Equation (13) can be used to calculate the ordered potential energy level difference Δφ corresponding to the low-temperature superconducting phase transition of conductors. $\sum_{\odot}$ is the sum over multiple adjacent atoms and electrons in the lattice, where $\sum_{\odot}（\vec{K}_e\cdot\vec{K}_s）$ is exactly the wave vector characterization form of the system's ordered structural potential energy constraint Δφ experienced by free electrons in the conductor. In the non-relativistic case, the thermal internal energy of a particle is $\epsilon = P^2/2m$, with $\vec{P}=\hbar\vec{K}$ and $|\vec{K}|=\sqrt{2m\epsilon}/\hbar$. Let:

$$\Delta\varphi = \frac{1}{q}\sum_{\odot}\frac{\hbar^2}{2\sqrt{m_e m_s}}\vec{K}_e\cdot\vec{K}_s \qquad （14）$$

where q is the number of electrons in the unit cell. Δφ is related to the mass of lattice atoms, confirming the existence of the isotope effect in superconductivity.

At temperature Tc, the total number of degenerate states of the system in the fully normal state and fully superconducting state are $M_{\text{Шj}}$ and $M_{\text{Шj+1}}$, respectively. For superconducting phase transition, the low-energy phase j+1 is taken as the system ground state, and the number of states $M_{\text{Шj+1}}$ can be a small integer (e.g., 1). For free electrons inside metals, the Fermi-Dirac statistical distribution approximation is adopted. For a free electron quantum gas with volume V and number N of free electrons, the number of degenerate states of a single electron $m_{\text{Шi}}$ is approximately $4\pi V(2m_e)^{3/2}\sqrt{\varepsilon}/(2\pi\hbar)^3$ (Wang Zhengxing, Modern Physics, Peking University Press, 2008, P358). Taking the electron energy level ε as the average value $\bar{\varepsilon}$ at Tc, the total number of degenerate states of the electron quantum gas system $M_{\text{Шj}}$ is, $M_{\text{Шj}} = \prod_i^N m_{\text{Шi}} = (4\pi V(2m_e)^{3/2}\sqrt{\bar{\varepsilon}}/(2\pi\hbar)^3)^N$. Using Equations (5) and (14), we obtain:

$$T_c=\frac{\hbar^2\sum_{\odot}(\vec{K}_e\cdot\vec{K}_s/\sqrt{m_e m_s})}{2qk_B\ln[4\pi V(2m_e)^{3/2}\sqrt{\bar{\varepsilon}}/(2\pi\hbar)^3]} \qquad （15）$$

3.3 Mechanism of High-Temperature Superconductivity

The discussions in the previous two sections of this chapter are of general significance. As long as there is an attractive coupling interaction between adjacent electrons in the lattice, electrons in the conductor can form holographic even homomorphic states at any scale, resulting in superconducting phase transition. Even pairing is natural, and the key is the magnitude of the interaction structural potential energy between electrons. At extremely low temperatures close to absolute zero, this interaction is universally present; therefore, high-temperature superconductors must have a larger ordered structural energy of attractive interaction between electrons.

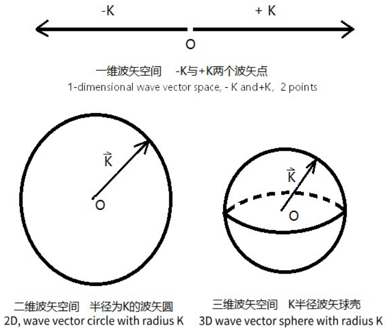
The author has found [1, 2] that the degree of freedom (dimension) of conductive electrons is a universal and key factor determining the magnitude of the interaction energy between electrons. The author proves that the interaction between microscopic particles in low-dimensional spaces is stronger than that in similar high-dimensional space structures, and particles exhibit stronger correlation. For the electron fluid in a conductor, if it is in a 2D planar space, the interaction strength between electrons may be an order of magnitude larger than that in 3D space. Therefore, all discovered cuprate, iron-based, and nickel-based superconductors so far have 2D planar conductive structures. The strong electron correlation in high-pressure hydrogen-rich superconductors is caused by the change of the H-H atomic spacing (compression) induced by external extreme pressure (not inherent to their own structure) [14]. This "temporary" high-pressure compressed state with small volume can be approximately explained by the strong 0-1 dimensional correlation between electrons.

In Section 3.1 of this chapter, the author has generally proven that the strong correlation interaction V (or $\Delta\varphi$) of conductive electrons is at the same energy level as the Fermi surface at temperature T=0, proportional to the square of the corresponding wave vector $\vec{\mathrm{K}}_{\mathrm{H}}$. In Reference [2], the author derived the relationship between the wave vector moduli $|\vec{\mathrm{K}}_{\mathrm{H1}}|$, $|\vec{\mathrm{K}}_{\mathrm{H2}}|$ and $|\vec{\mathrm{K}}_{\mathrm{H3}}|$of conductive electrons in 1D, 2D, and 3D structures, i.e.,

$$|\vec{\mathrm{K}}_{\mathrm{H1}}|:\ |\vec{\mathrm{K}}_{\mathrm{H2}}|:\ |\vec{\mathrm{K}}_{\mathrm{H3}}|=\sqrt{2\pi}|\vec{\mathrm{K}}_{\mathrm{H3}}|^{2}:\ \sqrt[3]{2|\vec{\mathrm{K}}_{\mathrm{H3}}|^{4}}\ :1 \qquad (16)$$

The higher the dimension, the geometrically smaller the$|\vec{\mathrm{K}}_{\mathrm{H}}|$; but $\sum_{\odot}$ **is** just the sum over multiple adjacent atoms and electrons. The essence of this relationship is further demonstrated below.

Under equilibrium and stable conditions, the motion states of all conductive electrons in a good conductor are equivalent, and their space-time structure configurations become similar space-time structure configurations, which can be accurately described by wave vectors $\vec{K}$. The equivalence of free electron motion states means that electrons have the same energy level and similar wave vectors $\vec{K}$; more precisely, they are statistically equal. The energy level distribution of free electrons is a statistical distribution with the average energy level $\bar{\varepsilon}$ as the maximum probability (i.e., the central value). The separated energy levels of electrons converge into an approximately continuous band (conduction band), and the quantized wave vectors $\vec{K}$ of electrons in the wave vector space also form an approximately continuous wave vector band. The center of the wave vector band is the wave vector $\vec{\mathrm{K}}_{\mathrm{H}}$ corresponding to the electron energy level $\bar{\varepsilon}$ (with the maximum number of degenerate states and a modulus of $|\vec{\mathrm{K}}_{\mathrm{H}}|$). The corresponding distribution form is approximately two points ($-|\vec{\mathrm{K}}_{\mathrm{H}}|$ and $|\vec{\mathrm{K}}_{\mathrm{H}}|$) in 1D wave vector space, a circle with radius $|\vec{\mathrm{K}}_{\mathrm{H}}|$ in 2D wave vector space, and a spherical shell with radius $|\vec{\mathrm{K}}_{\mathrm{H}}|$in 3D wave vector space (see Figure 3 for a schematic diagram). The wave vector $\vec{K}$ of electrons does not take continuous and uniform values from 0 to$|\vec{\mathrm{K}}_{\mathrm{H}}|$as described in traditional theories (traditional related theories are problematic, and the actual meanings of the Fermi sphere, Fermi surface, and Fermi energy level are also incorrect [4, 5, 7]); instead, it follows a statistical distribution centered on the wave vector with modulus $|\vec{\mathrm{K}}_{\mathrm{H}}|$ (maximum state). Of course, the wave vector states of conductive electrons comply with the Pauli exclusion principle. As a theoretical approximation, since the motion states of all conductive electrons in a good conductor are equivalent, it can be simply assumed that all conductive electrons have the same energy level $\bar{\varepsilon}$, and their wave vectors $\vec{\mathrm{K}}$ all have a modulus of $|\vec{\mathrm{K}}_{\mathrm{H}}|$, are isotropic, belong to the same energy level and similar states (degenerate states), and are equivalent. $\bar{\varepsilon}$ has a one-to-one proportional relationship with $|\vec{\mathrm{K}}_{\mathrm{H}}|$. Referring to the classical



一维波矢空间　-K与+K两个波矢点
1-dimensional wave vector space, - K and+K, 2 points

二维波矢空间　半径为K的波矢圆
2D, wave vector circle with radius K

三维波矢空间　K半径波矢球壳
3D wave vector sphere with radius K

Fig.3　1维、2维、3维空间波矢取值分布示意图.　Reference[2]
The distribution of wave vector values in 1D, 2D, and 3D space

kinetic energy relationship of electrons, the correlation interaction relationship, and the energy level lowering property, there must be an approximate relationship: $\bar{\varepsilon} \propto -{K_H}^2$. Obviously, the higher the spatial dimension, the broader and more dispersed the energy distribution of a single electron with the same $\bar{\varepsilon}$, and the lower the energy level of the maximum motion state and average energy level of this electron; from the perspective of wave vector space, the number of possible energy level states of electrons increases geometrically with increasing dimension, and the average energy level and average correlation interaction value decrease geometrically with increasing dimension. This is essentially a problem of space-time structure order: free particles in low-dimensional systems have fewer degrees of freedom and relatively fewer spatial arrangement and combination forms compared to similar particles in high-dimensional systems, appearing more ordered; free particles in high-dimensional spaces undergo random motion but are confined to relatively low-dimensional subspaces, and there must be a stronger constraint potential energy to maintain this special low-dimensional substructure in high-dimensional space, which is the ordered potential energy $\Delta\varphi$. All normal-pressure high-temperature superconductors have this structure; the phase transition temperature T_c can be estimated using $\Delta\varphi/k_B$.

Both 2D and 1D conductor materials may have high superconducting phase transition temperatures. The discovered high-temperature superconducting materials are basically ceramic materials, etc. Only such insulating materials are more conducive to forming independent, embedded "conductive planar structures" within large unit cells.

4 Conclusions and Discussions

In this paper, the author clarifies the intrinsic logic of general phase transitions and superconducting phase transitions in a general sense, derives the corresponding phase transition equations and phase transition temperature formulas, and demonstrates that this superconducting phase transition theory is fully applicable to high-temperature superconductors. It is pointed out that the structural characteristic of high-temperature superconducting materials is the existence of a larger ordered structural energy of attractive interaction between electrons, i.e., conductive electrons have stronger correlation in low-dimensional spaces.

The author believes that more practical high-temperature superconducting materials are 2D structural materials (which can be multi-layer stacked) with green inorganic dense insulating materials (such as highly insulating ceramic materials) as interlayers and conventional green metals (copper, silver, etc.) as intermediate layers. Recently, Zhao, J., Li, L. et al. [22] successfully prepared atomically thick 2D metals (Bi, Sn, and other metals, among which single-layer Bi has a room-temperature conductivity of 9.0×10^6 S/m, nearly 10 times higher than that of bulk Bi) using a simple van der Waals (vdW) compression method. This process can be used as one of the basic processes for preparing green 2D metallic high-temperature superconducting materials.

The potential largest application of high-temperature superconductors is in electrical wires. The development of green and economical 1D conductive structure high-temperature superconducting materials (e.g., atomic nanowires of copper, silver, etc.) seems more difficult but is the most ideal goal. Carbon nanotubes filled with atomic arrays (such as silver, copper, etc.) [23, 24] may be potential 1D high-temperature superconductors.

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