Title: The electron as a wave band: de Broglie wave phase resonance as a geometric interpretation of energy quantization in the hydrogen atom.

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Abstract

This paper presents a geometric and wave interpretation of energy quantization in the hydrogen atom, based on the de Broglie closure condition of the electron wave in a circular orbit. In this concept, energy quantization is a secondary phenomenon resulting from the fact that the electron wave in each orbit consists of exactly n full periods, and the transition to level n+1 corresponds to the addition of one full period.

Combining the wave condition with the classical equilibrium of the Coulomb and centripetal forces leads to values of the orbital radii and a discrete energy spectrum consistent with solutions of the Schrödinger equation for the hydrogen atom. It is shown that energy quantization can be interpreted as a consequence of the resonant nature of the electron's wave nature and the condition of phase uniqueness after a complete orbit around the nucleus.[1]

The model under consideration is semiclassical in nature and serves as an intuitive representation of known results from quantum mechanics. It is assumed that the allowable states correspond to configurations in which the de Broglie wave forms a standing wave containing an integer number of full periods around the orbital circumference. This condition leads directly to the quantization of angular momentum according to the Bohr model.

The waveband model provides a one-dimensional analogy of the full quantum description and can serve as a teaching tool to facilitate understanding the geometric aspects of energy quantization in the hydrogen atom. It demonstrates that energy quantization is a natural consequence of the standing wave geometry and the addition of successive full periods along the orbit as the system transitions to the next energy eigenstate after activation.

The electron is a stable eigenstate of a quantum field whose behavior in bound systems can be geometrically interpreted as the self-resonance of a de Broglie wave satisfying the condition of single-valent phase, rather than as a local particle with a classical trajectory. Self-resonance of a wave means that the condition of single-valent phase of the wave function after a complete orbit around the nucleus is satisfied, i.e., the requirement that the phase change be an integer multiple of 2π; this is equivalent to the Bohr–de Broglie condition for the closure of the de Broglie wave.

Keywords : de Broglie wave, hydrogen atom, energy quantization, standing wave, Bohr model, wave resonance.

[1]Schrödinger, E. (1926). Quantisierung als eigenwertproblem. Annalen der physik, 385(13), 437-490

Entry

The quantization of energy in the hydrogen atom is one of the fundamental phenomena of atomic physics. In the standard approach, it results from solutions to the Schrödinger equation or, in the older approach, from Bohr's postulate of quantization of angular momentum.[2]

In this paper, we assume that the electron is a de Broglie wave traveling in a circular orbit, and that stable orbits correspond only to configurations in which this wave forms a standing wave closed around the circumference. In this approach, the orbit is a geometric representation of the resonance condition, while the spherical symmetry of the state results from the orientational degeneracy of the one-dimensional wave resonance.[3]

The phase closure condition leading to the existence of standing waves is a general property of wave systems and appears in many different physical contexts.[4] [5]

The key element is the assumption that a standing wave in orbit must contain an integer number of complete sine periods. The wave closure condition leads directly to the quantization of angular momentum :

$$mvr = n\hbar \,, \quad (1.1)$$

and then – through the balance of Coulomb and centripetal forces – to the orbit radii:

$$r_n \propto n^2 \quad (1.2)$$

and to the discrete energy spectrum:

$$E_n \propto -1/n^2 \quad (1.3)$$

Energy quantization appears as a consequence of standing wave geometry, not as a separate postulate.

In this paper, we demonstrate that the quantization of energy in the hydrogen atom results solely from the condition that the de Broglie wave of the electron must form a standing wave closed in a circular orbit. This is consistent with the general principle that the allowed quantum states correspond to the resonance modes of the system. The wave closure condition leads directly to the quantization of angular momentum and then—via the Coulomb force equilibrium—to the known energy spectrum of hydrogen.

1. Model assumptions

[2]Bohr, N. (1913). XXXVII. On the constitution of atoms and molecules. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 26(153), 476-502.

[3]De Broglie, L. (1924). Recherches sur la théorie des quanta (Doctoral dissertation, Migration-université en cours d'affectation).

[4]Hecht, E. (2023). Optik. Walter de Gruyter GmbH & Co KG.

[5]Sakurai, J. J., & Napolitano, J. (2020). Modern quantum mechanics. Cambridge University Press.

The concept of orbit is used here in the Bohr sense as a representation of an equilibrium state, where energy absorption leads to a transition to another resonant state in which the phase closure condition is satisfied for a larger number of wave periods on the circumference.

Assumption 1 (matter wave) : The electron has a wave nature and is described by a de Broglie wave of wavelength $\lambda=\frac{h}{p}=\frac{h}{mv}$ (1.4)

where: h – Planck's constant, m – electron mass, v – velocity.

Assumption 2 (circular orbit): The electron occupies a circular orbit of radius r_n around the nucleus.

Assumption 3 (wave band, standing wave): Only such orbits are allowed in which the de Broglie wave forms a standing wave closed on the circumference:

$$2\pi r_n=n\lambda_n \quad n=1,2,3,... \quad (1.5)$$

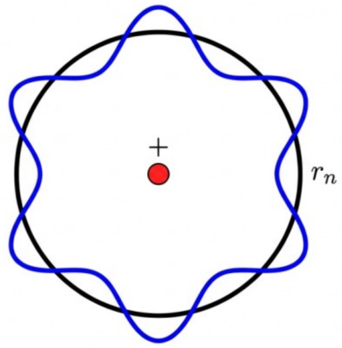

Figure 1. The de Broglie wave creates a standing wave closed around the circumference, a graphic example

2. Wave closure condition → quantization of angular momentum

From the closure condition $2\pi r_n=n\lambda_n$ and the de Broglie relation:

$$\lambda_n=\frac{h}{mv_n} \quad (1.6)$$

we get:

$$mv_n*2\pi r_n= nh \quad (1.7)$$

We obtain the quantization condition for angular momentum:

$$mv_n*r_n=\frac{nh}{2\pi}= n\hbar \quad (1.8)$$

Which is identical to Bohr's postulate, but here it follows from the wave condition . The angular momentum is quantized not because it is assumed so, but because the wave must close as a standing wave with n complete periods along the circumference.

The transition from level n to n +1 n+1 corresponds to a state in which the phase closure condition is satisfied for one more wave period along the circumference, which translates into a change in the value of the orbital angular momentum from $n * \hbar$ to (n +1) * $\hbar$.

3. Balance of Forces: Motion in a Coulomb Field

The electron is attracted to the nucleus by the Coulomb force.

$$F_C = \frac{k e^2}{r_n^2} \quad (1.8)$$

which balances the centripetal force for circular motion:

$$Fd = \frac{m v_n^2}{r_n} \quad (1.9)$$

Equilibrium condition: $Fd = F_C$

$$m v_n^2 = \frac{k e^2}{r_n} \quad (1.10)$$

Orbit radii After eliminating v_n we get:

$$r_n = n^2 * \frac{\hbar^2}{mk e^2} \quad (1.11)$$

Character $r_n \propto n$ 2 was not assumed; it follows solely from: wave closure (standing wave with n periods), classical equilibrium of forces.

4. Energy of an electron in orbit n

Total electron energy:

$$E_n = K_n + V_n \quad (1.12)$$

Where kinetic energy:

$$K_n = \frac{1}{2} * \frac{m v_n}{2} \quad (1.13)$$

potential energy in the Coulomb field:

$$V_n = -\frac{k e^2}{r_n} \quad (1.14)$$

From the power balance equation:

$$E_n = \frac{m e^4}{2\,(4\,\pi\,\varepsilon_0)^2\,\hbar^2} * \frac{1}{n^2} (1.15)$$

Total energy:

$$E_n = -\frac{m k^2 * e^4}{2\hbar^2} \cdot * \frac{1}{n^2} \quad (1.16)$$

which corresponds to the Rydberg formula.

So: **:** $E_n \propto -\frac{1}{n^2}$ **(1.17)**

We have obtained the standard form of the energy spectrum of hydrogen. The form of the energy spectrum of the hydrogen atom, described by this relationship, was originally determined empirically based on the analysis of line spectra.[6]

Energy quantization is not assumed, but is a consequence of the wave closure condition, specifically the requirement that the band contain an integer number of periods, with adjacent levels differing by exactly one full period.

5. Wave interpretation

The self-resonance of a wave means that the condition of the wave function's phase being single-valued after a complete revolution around the nucleus is met, i.e., the requirement that the phase change be an integer multiple of 2 π . This is equivalent to the Bohr–de Broglie condition for the closure of a matter wave.

The electron is a stable eigenstate of a quantum field whose behavior in bound systems can be geometrically interpreted as the self-resonance of a de Broglie wave, rather than as the motion of a particle along a classical trajectory.

[6]Rydberg, J. R. (1890). XXXIV. On the structure of the line-spectra of the chemical elements. The London, Edinburgh, and Dublin philosophical magazine and journal of science, 29(179), 331-337.Rydberg, J. (1890). On the Structure of the Line-Spectra of the Chemical Elements. Philosophical Magazine, vol. 29, pp. 331–337.

Figure2. Axis trainer model as an illustration of the geometric superposition of resonant de Broglie wave states corresponding to different energy levels and leading to a spherically symmetric probability distribution.

Contrary to the intuitive notion of random blurring, the spherical symmetry of the state arises systematically from the orientational degeneracy of the one-dimensional wave resonance. This work does not imply the existence of a deterministic electron trajectory or the possibility of predicting the results of individual measurements. Its goal is to demonstrate the geometric and structural origins of the probability distribution.

Discussion

The presented waveband model, in which the electron state is visualized as a one-dimensional wave resonance satisfying the phase closure condition with exactly n full periods on the circumference, is in agreement with both the semiclassical Bohr model and the basic principles of quantum mechanics.[7]

The orbital wave closure condition is a special case of the general principle that quantum systems admit only states that satisfy the resonance conditions that lead to the existence of standing waves. [8]This phenomenon is universal and occurs in many different physical systems.[9]

The proposed model applies exclusively to states with zero orbital angular momentum and describes only the principal quantum number n . It does not take into account relativistic corrections or quantum electrodynamic effects such as the Lamb shift, whose relative magnitude in the hydrogen atom is of the order of 10e−4.

The behavior of an electron in a stationary state is neither chaotic nor random in a structural sense. However, regularity and stability pertain to the structure of the quantum state, and the results of individual position measurements remain probabilistic, consistent with the general principles of quantum mechanics.

The waveband model is not a new physical theory, but a visual reinterpretation of the uniqueness condition for the wave function phase. Its goal is to clarify the relationship between the semiclassical visualization and the formal quantum description, without introducing new dynamical assumptions or additional parameters.

Although the model is based on the resonance condition for a circular orbit, the author is aware that a complete description of the hydrogen atom's spectrum requires the use of the Dirac equation and the inclusion of corrections from quantum electrodynamics. In this sense, the proposed

[7]Griffiths, D. J., & Schroeter, D. F. (2018). Introduction to quantum mechanics. Cambridge university press.
[8]Tipler, P. A., & Llewellyn, R. (2003). Modern physics. Macmillan.
[9]Born, M., & Wolf, E. (2013). Principles of optics: electromagnetic theory of propagation, interference and diffraction of light. Elsevier.

approach should be considered a basic interpretive framework, with respect to which subtle energy deviations can be analyzed.

The spherical symmetry of the probability distribution is interpreted here as a consequence of the orientational degeneracy of the one-dimensional resonance, rather than as an effect of the dynamical rotation of the object. The stationary state corresponds to a superposition of equivalent geometric orientations, which is a visual analogy of s- type orbitals in quantum mechanics.

The novelty of the work is not the resonance condition itself, known from semi-classical models of the atom, but its consistent geometric interpretation in the form of a wave band and the explicit connection of the $n \rightarrow n$ +1 transition with the addition of one full wave period satisfying the phase closure condition.

CONCLUSIONS :

The presented derivation is formally equivalent to the semiclassical Bohr–de Broglie model. The novelty of this work lies in the inversion of the interpretative logic: the wave resonance condition is treated as a primary assumption, while the quantization of angular momentum and energy appears as its consequence.

This paper shows that the discreteness of the energy of bound states in the hydrogen atom can be understood as a result of the self-consistent phase condition (self-resonance) of the electron wave, rather than as an independent, primary assumption about energy quantization. In this approach, energy quantization results from the existence of allowed resonance states that satisfy the phase closure condition.

The proposed model provides an intuitive, geometric interpretation of discrete energy levels, consistent with both the Bohr model and the quantum mechanical formalism described by the Schrödinger equation. The presented derivation does not introduce new dynamical postulates and remains consistent with classical wave resonance principles.

A one-dimensional wave resonance can be semiclassically visualized as a mode on a circle. However, a single orientation of such a circle does not constitute a physical state of the atom but merely an illustration of the local phase configuration. The stationary state corresponds to the superposition of all equivalent resonance orientations, leading to a spherical symmetry of the probability density.

The described spherical symmetry is not a result of the dynamical rotation or motion of the object, but a property of the quantum state with zero orbital angular momentum.

Bibliography:

[1] Schrödinger, E. (1926). Quantisierung als eigenwertproblem. Annalen der physik, 385(13), 437-490

[2] Bohr, N. (1913). XXXVII. On the constitution of atoms and molecules. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 26(153), 476-502.

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[6] Rydberg, J. R. (1890). XXXIV. On the structure of the line-spectra of the chemical elements. The London, Edinburgh, and Dublin philosophical magazine and journal of science, 29(179), 331-337.Rydberg, J. (1890). On the Structure of the Line-Spectra of the Chemical Elements. Philosophical Magazine, vol. 29, pp. 331–337.

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