Cantor Dust as Comprehensive Solution to the Dark Matter Puzzle

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Abstract

As sequel to [1 - 2], this work explores the gravitational consequences of Cantor Dust formation in the primordial Universe. We find that the multifractal structure of Cantor Dust (CD) can account for a wide range of galactic and cosmological phenomena, commonly attributed to either particle Dark Matter (DM) or modified Newtonian gravity (MOND). Asymptotically flat rotation curves are recovered without invoking modified force laws. Baryonic cooling and dissipation fix the extent of luminous structures at a universal acceleration scale, which leads naturally to the baryonic Tully–Fisher relation (BTFR). We survey weak lensing, dynamical friction, and cluster constraints, and outline testable observational signatures distinguishing this framework from standard cold DM scenarios. In summary, our results suggest that CD provides a *unified geometric* explanation of DM phenomenology across multiple scales.

Key words: Dark Matter, Cantor Dust, MOND phenomenology, Tully-Fisher relation, galactic rotation curves, weak lensing, CMB spectrum, Bullet Cluster, cosmic web.

1. Introduction and Motivation

Historically, DM was born out of a persistent mismatch between the gravitational field inferred from astrophysical observations and that predicted by the visible mass distribution under Newtonian gravity and General Relativity. This discrepancy manifests itself across a wide range of scales, from galactic rotation curves and velocity dispersion to gravitational lensing and large-scale structure formation. Despite decades of experimental effort, no conclusive evidence for DM particles or MOND predictions has been firmly established, motivating continued exploration of alternative theoretical frameworks.

Two empirical regularities have emerged as particularly constraining. The first is the ubiquity of flat galactic rotation curves, implying an enclosed gravitational mass that grows approximately linearly with radius. The second is the remarkable tightness of the ***radial acceleration relation*** (RAR), which correlates the observed gravitational acceleration with that predicted by the baryonic mass alone, with a characteristic transition near a universal acceleration scale. Closely related is the ***baryonic Tully–Fisher relation*** (BTFR), linking total baryonic mass to asymptotic rotation velocity with minimal scatter. These relations are difficult to reproduce naturally within standard cold DM models without fine-tuning of baryonic feedback and halo properties.

In this work, we explore an unconventional alternative suitable for the description of primordial cosmology. According to this view, DM is not a smooth 3-dimensional mass distribution but instead resides on a *singular geometric support* with non-integer effective dimension. Motivated by the filamentary nature of the cosmic web and by mathematical models of CD and multifractals, we consider DM distributions supported on a statistically isotropic network of one-dimensional filaments embedded in 3-dimensional space. Pursuing the framework of [1 - 2], gravity is formulated in terms of a

Poisson equation with a measure-valued source, allowing the gravitational field to be well defined even in the absence of a smooth density function.

Proceeding with that equation, its gravitational implications and using the remarkable analogy with the cosmic web, we further model DM as a statistically homogeneous and isotropic filamentary network. It is found that the enclosed mass scales linearly with radius despite a quadratic scaling of the total geometric filament content, a consequence of the Gauss theorem and isotropic cancellation of internal contributions. This mechanism yields asymptotically flat rotation curves without invoking modified force laws. Our framework retains standard gravity, introduces no new forces, and ties galactic phenomenology to the geometry of DM.

We caution from the outset that this paper is exclusively an *exploratory introduction* inspired by our previous contributions. Aiming for accessibility to a large audience, the paper is formatted in a bulleted style and kept at a "bird's eye view" level, avoiding jargon or excessive technical details.

Interested readers are encouraged to further explore, develop or refute the body of ideas presented here.

2. Derivation of the weak form of the Poisson equation

It is well known that, in ordinary Newtonian gravity, the gravitational potential $\Phi(x)$ generated by a *smooth mass density* $\rho(x)$ satisfies the Poisson equation

$$\nabla^2 \Phi(\mathbf{x}) = 4\pi G \, \rho(\mathbf{x})$$

in which,

- $\Phi(x)$ is the scalar gravitational potential,
- ∇^2 is the Laplacian operator in $\mathbb{R}^3$,
- $\rho(x)$ is mass density with respect to Lebesgue measure d^3x,
- G is Newton's constant.

Poisson equation implicitly assumes that mass is continuous with respect to volume, $\rho(x)$ exists pointwise and the source is a function, not a distribution.

As discussed in [1 - 2], all these assumptions fail for Cantor Dust (CD), which is not a smooth distribution of matter. Instead,

- mass is supported on a fractal set of Hausdorff dimension $D < 3$,
- there is no density $\rho(x)$ such that

$$\rho(\mathbf{x})\, d^3x = dM$$

As detailed in [3], the conjecture that particle masses and couplings emerge from the continuous and evolving spacetime dimensionality near the Planck scale is supported by several independent arguments. Pursuing this line of reasoning, we assume below that, on a multifractal Cantor Dust background, mass is described by a *Radon measure* $\mu(\mathbf{x})$. and the mass contained in a ball of radius r scales as

$$\mu(B_r) \sim r^D$$

The correct generalization of the Poisson equation is

$$\nabla^2 \Phi(\mathbf{x}) = 4\pi G\, \mu(\mathbf{x})$$

with the following interpretation,

- The Laplacian of Φ is no longer an ordinary function, and $\nabla^2\Phi(\mathbf{x})$ is taken to represent a distribution,
- The right-hand side is a measure, not a density,
- This whole equation is understood in the weak (distribution-like) sense.

The generalization of the Poisson equation in the formal sense of distributions is given by the *weak form* [1 - 2],

$$\boxed{\int \nabla\Phi \cdot \nabla\psi \, d^3x = 4\pi G \int \psi \, d\mu}$$

and can be derived as follows:

2.1: Multiply by a test function

Take a smooth test function $\psi(x)$ with compact support and multiply the Poisson equation,

$$\psi\,\nabla^2\Phi = 4\pi G\,\psi\,\mu$$

2.2: Integrate over space

$$\int \psi\,\nabla^2\Phi\,d^3x = 4\pi G \int \psi\,d\mu$$

Note that the right-hand side is well-defined because ψ is continuous and μ is a Radon measure.

Step 2.3: Integrate by parts (Green's identity)

Using

$$\int \psi\,\nabla^2\Phi\,d^3x = -\int \nabla\Phi\cdot\nabla\psi\,d^3x$$

and, since ψ has compact support, boundary terms vanish, we obtain:

$$\int \nabla\Phi\cdot\nabla\psi\,d^3x = 4\pi G \int \psi\,d\mu$$

This weak formulation of Poisson equation is standard practice in:

- Electrostatics with singular charge distributions,
- Newtonian gravity with point masses,

- Mathematical theory of elliptic PDEs

In a nutshell, the weak form of Poisson equation,

$$\int \nabla\Phi \cdot \nabla\psi \, d^3x = 4\pi G \int \psi \, d\mu$$

means that the gravitational field energy produced by Φ, tested against any probe function ψ, equals the total mass of Cantor Dust weighted by ψ.

If the measure becomes continuous,

$$d\mu = \rho(\mathbf{x}) \, d^3x$$

then

$$\int \psi \, d\mu = \int \psi \, \rho \, d^3x$$

and the weak equation reduces to the standard Poisson equation.

Applying the weak equation to a spherically averaged test function yields:

$$g(r) = \frac{GM(r)}{r^2} \sim r^{D-2}$$

which is the fractal formulation of the Gauss (divergence) theorem. This is the key result that drives:

- Scale-dependent gravitational acceleration,
- Hierarchical baryonic collapse,
- Generalized Jeans instability.

3. Deriving rotation curves from the weak equation

The goal here is to derive the galactic rotation curves directly from the weak Poisson equation.

3.1 Assume spherical symmetry and isotropic test functions

By analogy with the geometry of the cosmic web, let's assume statistical isotropy of the CD and choose a test function $\psi(r)$ that depends only on radius. The weak equation reduces to:

$$\int_0^\infty \frac{d\Phi}{dr}\frac{d\psi}{dr}\, 4\pi r^2 dr = 4\pi G \int_0^\infty \psi(r)\, dM(r)$$

3.2 Integration by parts

Integrate the left-hand side:

$$\int \frac{d\Phi}{dr}\frac{d\psi}{dr}\, r^2 dr = -\int \frac{d}{dr}\left(r^2 \frac{d\Phi}{dr}\right)\psi(r)\, dr$$

Thus,

$$\frac{d}{dr}\left(r^2 \frac{d\Phi}{dr}\right) = G\,\frac{dM}{dr}$$

3.3 Fractal mass profile

Using:

$$M(r) = M_0 r^D$$

we obtain:

$$\frac{d}{dr}\left(r^2 \frac{d\Phi}{dr}\right) = GDM_0 r^{D-1}$$

Integrating gives,

$$r^2 \frac{d\Phi}{dr} = \frac{GDM_0}{D} r^D = GM(r)$$

Thus:

$$g(r) \equiv \frac{d\Phi}{dr} = \frac{GM(r)}{r^2} \sim r^{D-2}$$

3.4 Rotation velocity

For circular motion one has

$$\frac{v^2(r)}{r} = g(r)$$

Hence:

$$v^2(r) = rg(r) = GM(r)r^{-1} \sim r^{D-1}$$

We are led to:

$$\boxed{v(r) \sim r^{(D-1)/2}}$$

It is seen that *flat galaxy rotation* curves correspond to $D \approx 1$ a condition associated with *filamentary Cantor Dust*. In this picture, rotation curves emerge directly from the weak Poisson equation, with no modification of Newton's law.

4. Derivation of the Tully-Fisher relationship

The baryonic Tully–Fisher relation (BTFR) states

$$M_{\mathrm{b}} \propto v_{\infty}^{4}$$

where M_{b} is baryonic mass and v_{∞} the asymptotic rotational velocity.

To derive the Tully-Fisher relationship from Cantor Dust (CD) gravity, we proceed as follows:

4.1 Asymptotic rotation velocity

From our earlier result of section 3.4, we have:

$$v^{2}(r) = \frac{GM(r)}{r}$$

For a fractal background of dimension $D = 1$, that is, for filamentary CD,

$$M(r) = M_{0}r$$

Thus,

$$v^2(r) = GM_0 \Rightarrow v(r) = v_\infty = \text{const.}$$

This identifies:

$$M_0 = \frac{v_\infty^2}{G}$$

Baryons collapse inside a radius R_b where they dominate locally:

$$M_\mathrm{b} \sim M(R_\mathrm{b}) = M_0 R_\mathrm{b}$$

Cooling and dissipation fix R_b dynamically by the condition:

$$g(R_\mathrm{b}) \sim a_0$$

where a_0 is a universal acceleration scale emerging from the CD fractal background. This reflects a balance condition between baryon cooling and baryon heating by gravitational acceleration The above condition is justified because:

- Gravity sets the dynamical time,
- Cooling shuts off below a critical acceleration,

- Fractal background supplies a universal a_0,
- Dissipation naturally drives baryons to that threshold.

Using:

$$g(r) = \frac{v_\infty^2}{r}$$

we obtain:

$$R_\mathrm{b} = \frac{v_\infty^2}{a_0}$$

4.2 Emergence of the baryonic Tully-Fisher scaling

Combining relations of the previous paragraph yields

$$\boxed{M_\mathrm{b} = \frac{v_\infty^2}{G} \cdot \frac{v_\infty^2}{a_0} = \frac{1}{G a_0} v_\infty^4}$$

which reproduces the baryonic Tully–Fisher scaling. It can be concluded that, in CD gravity, Tully–Fisher scaling is a *geometric identity*, not a phenomenological law.

5. Equivalence with MOND in the deep-field limit

In MOND, the deep-field regime satisfies:

$$g^2 = a_0 g_{\mathrm{N}}$$

where $g_{\mathrm{N}} = GM_{\mathrm{b}}/r^2$. For $D = 1$:

$$g(r) = \frac{GM(r)}{r^2} = \frac{GM_0}{r}$$

Next, define:

$$a_0 \equiv GM_0$$

Thus:

$$g(r) = \sqrt{a_0 \frac{GM_{\mathrm{b}}}{r^2}}$$

or equivalently:

$$g^2 = a_0 g_{\mathrm{N}}$$

The results of this section show that,

- MOND's interpolation function is *not fundamental*.

- The MOND law emerges as *the asymptotic scaling of a fractal mass measure.*
- a_0 is a *geometric* and not a phenomenological parameter.

It is seen that filamentary CD gravity with $D = 1$ reproduces MOND's deep-field limit.

6. Concluding remarks

We have presented a geometric framework in which a CD dark sector accounts for the principal phenomenology traditionally attributed to DM. By modeling the dark component as a statistically isotropic filamentary network and formulating gravity through a Poisson equation with a measure source, we have shown that the gravitationally relevant enclosed mass scales linearly with radius despite a higher-order scaling of geometric filament content. This distinction resolves apparent contradictions between fractal geometry and observed flat rotation curves and follows directly from Gauss' law.

Within this framework, baryonic collapse is regulated by cooling and dissipation in a slowly varying gravitational field, leading to a dynamically selected radius at which the local acceleration reaches a universal scale determined by the fractal mass measure. This mechanism provides a natural explanation for the radial acceleration relation (RAR) and the baryonic Tully–Fisher relation (BFTR) without modifying Newtonian dynamics or introducing free interpolation functions. The same geometric principles govern lensing, suppress dynamical friction in filament-dominated halos, and offer testable predictions for halo–galaxy correlations and cluster collisions.

Our approach reframes the DM problem as a question of spatial support and measure rather than new particle species. While further work is required to embed this framework fully within relativistic cosmology and to confront precision cosmological data, the results presented here imply that a singular, fractal dark sector can reproduce a broad range of observations within a unified and mathematically controlled setting. This, in turn, suggests that the geometry of the dark sector may play a more fundamental role in cosmic structure and dynamics than previously assumed.

The reader is directed to [4 - 12] for background information and in-depth technical details.

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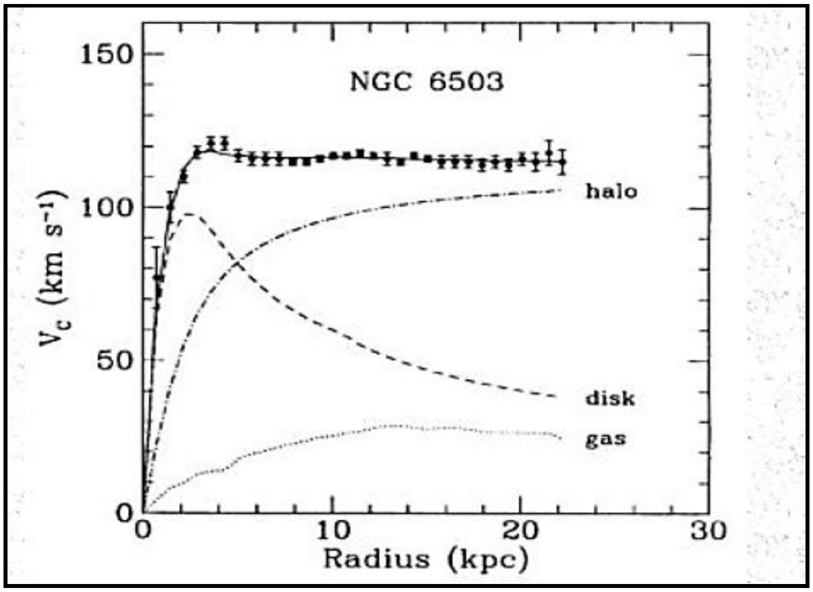

Fig. 1. Galactic rotation curve for NGC 6503 showing disk and gas contribution plus DM halo contribution needed to match the data [12].