Numbers represented by sums of three

(1^{st},2^{nd},3^{rd} &4^{th} powers)

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Abstract

This paper has solution for equation with degree's (1,2,3 & 4). The author has also shown solution for equation for degree (1 & 7). He have also provided an identity where a fourth power is represented by (first, second & third powers).

Part one:

In part two we have considered simultaneous equation of degree one & seven.

But we first consider the below set of simultaneous equation for degree's one to four:

Consider the below set, consisting of equation's for degree (1,2,3 & 4):

$$a+b+c=p$$

$$a^2+b^2+c^2=q$$

$$a^3+b^3+c^3=r$$

$$a^4+b^4+c^4=s$$

Where, (p,q,r,s) are of given form below as:

$$(p,q,r,s)=[(6n,2(6n^2+1),12n(2n^2+1),2(24n^4+24n^2+1)]$$

We have the identity:

$$(a+b+c)^4=(p)^4=4pr+3q^2-6s+12abc(p)$$

Hence, $bc=\left[\frac{w}{12pa}\right]$

& $(b+c)=(p-a)$

where, $w=(p^4-4pr-3q^2+6s)$

we have: $(ab+bc+ca)=(\frac{p^2-q}{2})$

hence: $a(b+c)+bc=a(p-a)+\left(\frac{w}{12pa}\right)$

hence:

$12p(a^3)-12p^2(a^2)+(6p^3-6pq)a-w=0$ ------ (1)

For n=2, we have, $(p,q,r,s,w)=(12{,}50{,}216{,}962{,}8640)$

Hence equation (1) becomes:

$$a^3-12a^2+47a-60=0, \ \ or$$

$$(a-3)(a-4)(a-5)=0$$

Hence, $(a,b,c)=(3{,}4{,}5)$

Summary:

For n=2, we have, $(p,q,r,s)=)=(12{,}50{,}216{,}962)$ &

we get, $(a,b,c)=(3{,}4{,}5)$

Thus numerical solution to the above system of simultaneous equation is:

$$(a,b,c)=(3{,}4{,}5)$$

we also get another solution if we take:

For n=3, we have, $(p,q,r,s)=(18{,}110{,}684{,}4322)$ &

we get, $(a,b,c)=(5,6,7)$

Thus numerical solution to the above system of simultaneous equation is:

$$(a,b,c)=(3,4,5)$$

“continued below”

Part two:

Next we consider another set of simultaneous equation:

The below set is for degree (2 & 7)

$$a^2+b^2+c^2=p$$

$$a^7+b^7+c^7=q$$

Where (p,q) are given as:

$$(p,q)=[2(1+3n^2),14n(1-n^2)(1+3n^2)^2]$$

We apply the condition: $(a+b+c)=0$

We have the below identity’s :

$$(ab+bc+ca)=-[\ \frac{p}{2}\ ]$$

$$(a^7+b^7+c^7)=-7abc(ab+bc+ca)^2=q$$

Hence, $bc=\frac{4q}{7ap^2}$

Also, $(b+c)=(-a)$

We have, $-[\frac{p}{2}]=(ab+bc+ca)=(-a^2+bc)=\left(-a^2+\frac{4q}{7ap^2}\right)$

Hence, $14a^3p^2-7ap^3-8q=0\ \ ----(1)$

For, $n = 3$, we have: $(p, q) = (56, -263424)$

Hence equation (1) becomes:

$$(a^3 - 28a + 48) = 0$$

or

$$(a - 4)(a - 2)(a + 6) = 0$$

Thus: $(a, b, c) = (4,2, -6)$ is a solution.

we also get another solution:

If using the above method, we take, (n=4) then we have,

$$(p, q) = (98, -2016840)$$

& we get the solution, $(a, b, c) = (5,3, -8)$

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