

# Symmetry Breaking and Exclusive Duality: Foundations for a Unified Theory of Structures

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## Abstract

This paper investigates the logical-mathematical foundations of physical reality, proposing a model based on the persistence of symmetry breaking from the real to the complex domain. We postulate the existence of two fundamental structures: the Internal Structure  $S(O)$ , defined in Hilbert Space, and the External Structure  $S(O^{-1})$ , defined in the complex field. The theoretical core of the work lies in identifying two mutually exclusive regimes of access to reality: the state of **Observation** (Potential Infinity) and the state of **Understanding** (Actual Infinity). We demonstrate that phenomenal reality and logical reality are not static, but the result of a continuous high-frequency exchange between cardinality increment and complex rotation. Furthermore, we hypothesize that such rotation is governed by a metric compatible with the Riemann Hypothesis, linking the distribution of quantum weights to the nature of prime numbers.

## 1 Introduction: The Problem of Coexistence

Modern physics struggles with an unresolved dualism between the local, deterministic description (General Relativity) and the non-local, probabilistic one (Quantum Mechanics). Historically, from Galileo to Bell, attempts have been made to find a unifying language capable of "stitching" these two visions together. Our hypothesis is that such unification cannot occur on the plane of simultaneous observation. We propose that existence itself (phenomenal) and its intelligibility (noumenal) are the result of a **fundamental symmetry breaking** manifesting in two exclusive modes. We cannot "see" the world and "understand" it at the same instant, because the symmetry conditions required for observation violate those required for structural understanding.

## 2 Mathematical Formalism: Hilbert Space and Structures

To rigorously describe the interaction between structures, we adopt the formalism of Quantum Mechanics integrated with Peano's axioms [2].

## 2.1 Internal Structure $S(O)$ in Hilbert Space

We define the Internal Structure  $S(O)$  as the set of "vertical steps" generating the system's dimensionality. In a Hilbert Space  $\mathcal{H}$ , we associate each deductive step with an orthonormal basis vector:

$$\text{Step } n \rightarrow |n\rangle \in \mathcal{H} \quad (1)$$

The operator  $O$  acts on these states defining discrete transitions:  $O|n\rangle = \lambda_n|n+1\rangle$ . This structure represents the logical and discrete backbone of reality (Operator Invariant), whose atomic constituents are prime numbers.

## 2.2 External Structure $S(O^{-1})$ and Complex Coefficients

The External Structure represents the continuous context in which vectors are immersed. Mathematically, it introduces "lateral steps," i.e., the complex coefficients allowing linear superposition of states:

$$|\psi\rangle = \sum_n c_n |n\rangle, \quad c_n \in \mathbb{C} \quad (2)$$

The crucial element of our model is identifying the origin of these coefficients. They derive directly from the action of the connection operator  $A = \varphi + i\omega$ . It is the complex metric  $A$  that provides the "weights" ( $c_n$ ) for the summation.

**Rotation vs. Generation:** The basis vectors  $|n\rangle$  are fixed. The operator  $A$  does not create new "rungs" of the ladder, but *rotates* the state vector  $|\psi\rangle$  within the pre-existing space. This rotation transforms a potentially divergent sequence into a convergent and closed geometry.

**Riemannian Generation Mechanism.** We propose the following operational conjecture: the complex metric  $A = \varphi + i\omega$  acts as a spectral operator whose eigenvalues coincide with the non-trivial zeros of the Riemann Zeta function ( $\rho = \frac{1}{2} + i\gamma_n$ ). In this framework, the complex coefficients  $c_n$  of the superposition are not random but determined by Riemann's explicit formula, which links the distribution of prime numbers (constituents of the Internal Structure) to the frequencies of the zeros (oscillations of the External Structure). Specifically, the modulus of the coefficient  $|c_n|$  is a function of the resonance between the rotation frequency  $\omega$  and the imaginary part  $\gamma_n$  of the zeros.

$$c_n \propto \sum_{\rho} \frac{x^{\rho}}{\rho} \quad (\text{From Spectrum to Structure}) \quad (3)$$

This implies that "understanding" (the unified state) is nothing more than the perception of the harmonic order (Zeros) underlying the apparent disorder of the numerical sequence (Primes/Observation).

## 3 The Core of the Theory: The Two Exclusive States

The interaction between structures generates two distinct states of being, which cannot overlap temporally. One excludes the other.

### 3.1 State A: Observation (Closed Space - Potential Infinity)

Observed reality emerges from the dynamic symmetry breaking between the **Operator**  $O$  and its **Inverse**  $O^{-1}$ .

$$O \cdot O^{-1} \neq I \implies \text{Residue } \varphi \text{ (Real Time)} \quad (4)$$

In this state, Hilbert Space **closes**. The basis vectors lose their complex connection, and the state vector collapses. **Reactivation of Cardinality**: The arrest of rotation (collapse) forces the system to reactivate the sequential increment of cardinality. Having lost totality, the internal structure must resume counting step by step ( $n \rightarrow n + 1$ ) in real time. This condition defines **Potential Infinity**: a visible and dynamic reality, but intrinsically incomplete.

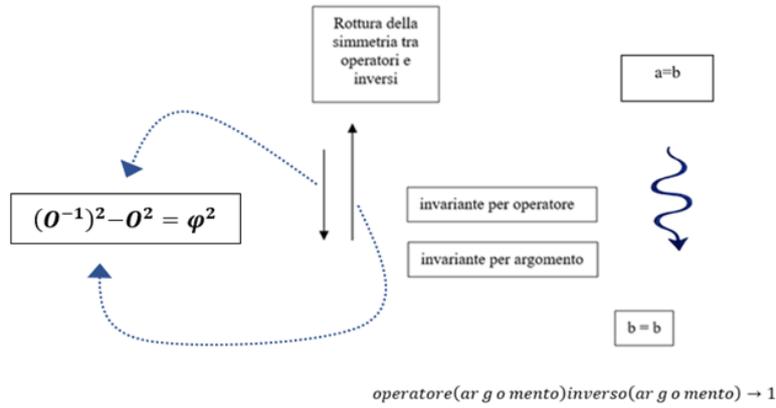


Figure 1: **Observation State**. Closed Hilbert Space. The open cycle and cardinality increment generate Potential Infinity.

### 3.2 State B: Understanding (Open Space - Actual Infinity)

Understanding emerges from a different symmetry breaking: the structural one between **Operator Invariant** ( $S(O)$ ) and **Argument Invariant** ( $S(O^{-1})$ ).

$$\|A\|^2 + \|S(O)\|^2 = \|S(O^{-1})\|^2 \implies \text{Domain } \omega \text{ (Complex Time)} \quad (5)$$

In this state, Hilbert Space **opens**. Thanks to the complex metric, states are connected in a coherent superposition. **Arrest of Cardinality**: Rotation stops the increment of cardinality. The system possesses total information within the complex phase. In accordance with the Riemannian hypothesis, in this state the complex connection reveals the hidden harmony (Zeta Zeros) underlying the distribution of discrete elements. This condition defines **Actual Infinity**: a comprehensible and complete reality, but not locally observable.

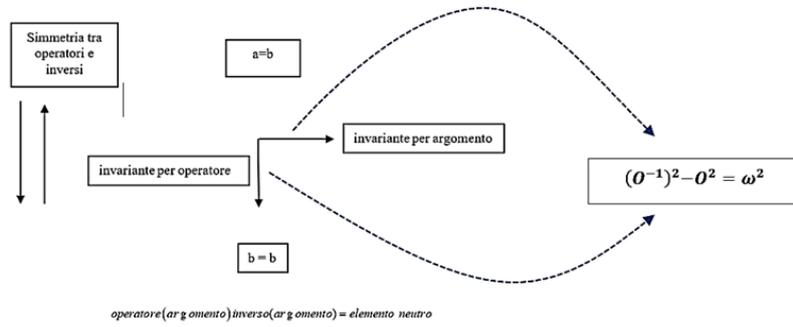


Figure 2: **Understanding State.** Open Hilbert Space. Closed geometry and resolved cardinality generate Actual Infinity.

## 4 Principle of Exclusion and Oscillation

We formulate here the Dynamic Structural Exclusion Principle:

*"It is impossible to simultaneously occupy the state of Observation and the state of Understanding. However, there exists a continuous exchange between cardinality increment and rotation."*

It is necessary to distinguish the dynamic origin of the two phenomena governing existence:

- **Oscillation (Life):** Derives from the **deviation of  $\omega$  from perfect resonance**. When the rotation frequency deviates from Riemann Zeros ( $\omega \neq \gamma_n$ ), the space closes, returning to Observation (cardinality increment). The subsequent pursuit of resonance reopens the space toward Understanding. This "beat and off-beat" generates perceived time.
- **Singularity (The End):** Derives from the **asymptotic limit**. When the components of the superposition tend to infinity ( $n \rightarrow \infty$ ), space saturation is reached. At this limit point, dynamics arrest permanently, as there is no longer room for oscillation.

## 5 Geometry of Deductive Union

Despite temporal exclusion, the two states are logically linked in the complete Deductive Procedure. The External Structure  $S(O^{-1})$  emerges as the **hypotenuse** generated by the catheti of the Complex Metric and the Internal Structure.

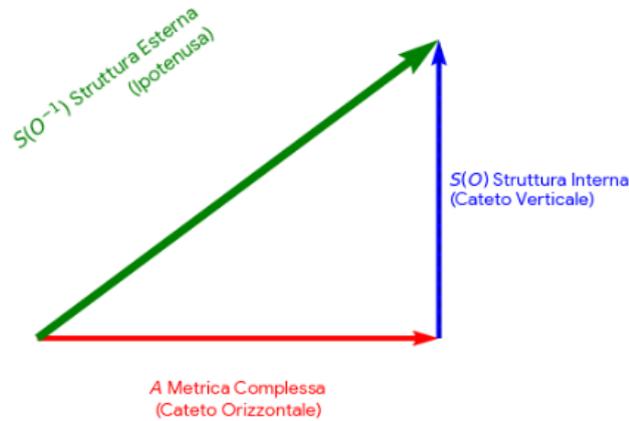


Figure 3: **Deductive Synthesis.** The right triangle unifies the two symmetry breakings into a single oscillating geometry.

## 6 Conclusions: The Pulsation of Reality

In conclusion, we have established that reality is a dialectic of alternating asymmetries. We are not statically "at real distance" or "at complex distance". We are immersed in a continuous exchange: observation (cardinality increment/Primes) feeds understanding, and understanding (rotation/Riemann) arrests observation. This fundamental pulsation is guaranteed by the imperfection of local resonance. Perfect unification occurs only at the asymptotic limit of the Singularity, where time ceases to be both real and complex, and diversity collapses into indistinct unity.

## References

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