Length Expansion: A Prerequisite for Understanding the Principle of the Constancy of the Speed of Light and Various Relativistic Paradoxes

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Abstract

The second principle of relativity, stating that the speed of light is constant regardless of the source's velocity, remains incompletely understood. Moreover, the speed of light is incompatible with length contraction. Beyond this, relativity still contains many thought experiments that are difficult to comprehend. These include Bell's spaceship paradox, the muon paradox, the Supplee submarine paradox, and the Ehrenfest paradox. The commonality among these problems is that logical contradictions arise during the application of length contraction. Since these problems stem from length contraction, approaching them with length expansion logically resolves all issues. This article examines whether length expansion resolves this series of problems.

Introduction

During the historical development of relativity, the length contraction hypothesis was proposed and became central to the theory of relativity. However, length contraction has never been experimentally proven to this day and exists only as a theoretical and abstract concept. Although it is hardly known, there is also an argument opposing length contraction. It was not raised suddenly. [1] [2] Today, there are various claims regarding length contraction. Some have proposed length expansion to resolve specific paradoxes, while others argue that length contraction is incompatible with the principle of the constancy of the speed of light, and that if length contraction were correct, GPS would not function with precision. [3] [4] [5] All of these points are correct, but length expansion is neither partially correct nor applicable only in accelerated frames. Relativistic length expansion does not distinguish between inertial and accelerated frames, nor is it partial or applicable only in specific situations. Furthermore, it is not introduced merely to awkwardly explain certain paradoxes. I believe length expansion, not length contraction, is correct throughout the entire theory of relativity. There is a logical necessity for the

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length expansion to be correct. If length expansion is incorrect, neither the principle of the constancy of the speed of light nor relativity theory itself holds. This paper will prove the validity of length expansion through several logical methods and present known experimental facts. It will also demonstrate that many paradoxes within relativity can be simply resolved by length expansion.

1 Theoretical proof of length expansion

1.1 Proof of length expansion with K-Calculus

K-calculus is a research method for special relativity proposed by *Bondi* that derives most special relativistic concepts, including as time dilation, the Lorentz transformation, and the velocity-addition formula. [6] Here, we will focus on the method he used to derive length contraction. Let us examine the fundamental concepts of k-calculus.

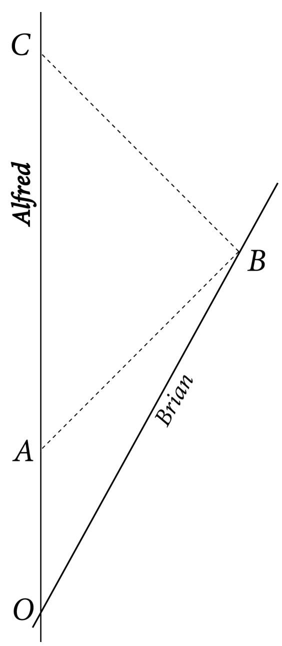


Figure 1: Alfred and Brian in motion relative to each other

Alfred and Brian are moving relative to each other in figure 1. From Alfred's point of view, Brian is moving away from him at a speed of v. When they send and receive signals, the relationship between the time measured by Alfred and the time received by Brian is given by Equation (1). Here, β represents v/c, where c is the speed of light and v is the relative velocity between the two people. Using these relationships, we can determine the value of k.

$$
\begin{aligned}
OB &= k(OA) \\
OC &= k(OB) \\
OC &= k^2(OA) \\
\therefore k &= \sqrt{\frac{1+\beta}{1-\beta}}
\end{aligned} \tag{1}
$$

In the spacetime diagram of figure 2, the horizontal axis unit distance (BC) represents one light-second (ls), and the vertical axis unit time (A_2A_3) represents one second. The light world-line always maintains a 45-degree inclination, regardless of the system's speed v. If signals are sent simultaneously from both ends of a rod separated by a unit distance, the signals will arrive at the origin at unit time intervals.

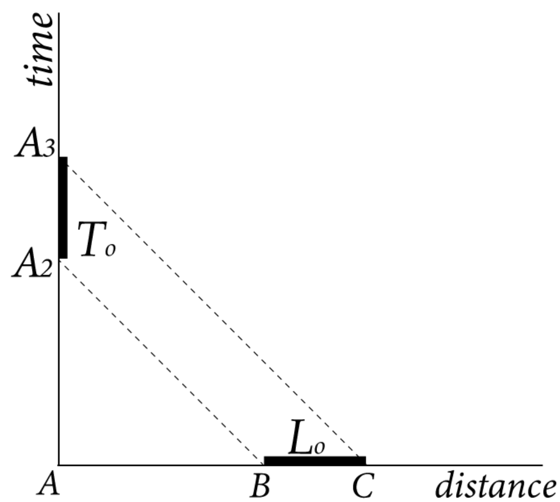


Figure 2: Unit time and unit distance

Since the speed of light is always constant, the sizes of L_o and T_o are the same. If $c = 1$, then L_o and T_o are identical. Then, how should we measure the length of a rod at rest and the length of a rod in motion? Since the rod's length is approximately one unit length, $L_o \approx 3 \times 10^8 m$, it is impossible to measure it with a ruler. Therefore, let us measure it using light. Here, we assume that no time is lost while the light reflects. In figure 3a, the dashed line represents the path of light when measuring one's own length, and in figure 3b, it shows the path of light when measuring the length of a moving opponent. In figure 4a, $t_2 - t_1$, $t_4 - t_3$, the length of an event (BC), the proper length L_o and the proper time T_o are all equal in magnitude. Since the slope of the world-line of light in the space-time diagram is 1. (Let the speed of light be 1)

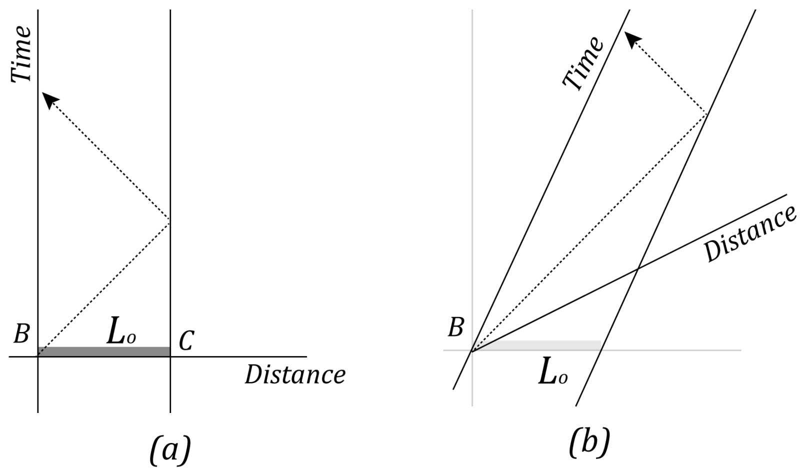


Figure 3: Light path during length measurement
a) Stationary rod, (b) Moving rod

$$\begin{aligned} & t_2 - t_1 \\ &= t_4 - t_3 \\ &= (BC) \\ &= L_o \\ &= T_o \end{aligned} \tag{2}$$

Then the length we wish to explore is not the stationary proper length, but the length of the moving counterpart. It is the spacetime interval of event (DE) in figure 4b, not 4a. This is very important. Figure 4b shows that event D is kt_1, and event E is $k^{-1}t_4$ from t_4. Therefore, the spacetime interval of event (DE) is $k^{-1}t_4 - kt_1$, which is twice the length we seek. This is also pointed out by *Hermann Bondi* in his book. [6] Thus, we can express it as shown in Equation (3).

$$\begin{aligned} k^{-1} \cdot t_4 - kt_1 &= 2L \\ t_4 - k^2 t_1 &= 2kL \\ \therefore t_4 &= 2kL + k^2 t_1 \end{aligned} \tag{3}$$

Looking at figure 4b, we can see that the value of t_3 is immediately $t_3 = k^2 t_2$. Now, we can use the values of t_4 and t_3 to obtain the desired value of L.

$$\begin{aligned} t_3 &= k^2 \cdot t_2 \\ t_2 - t_1 = t_4 - t_3 &= (BC) = L_o \end{aligned} \tag{4}$$

Since we know t_3 and t_4, we can use this to find L.

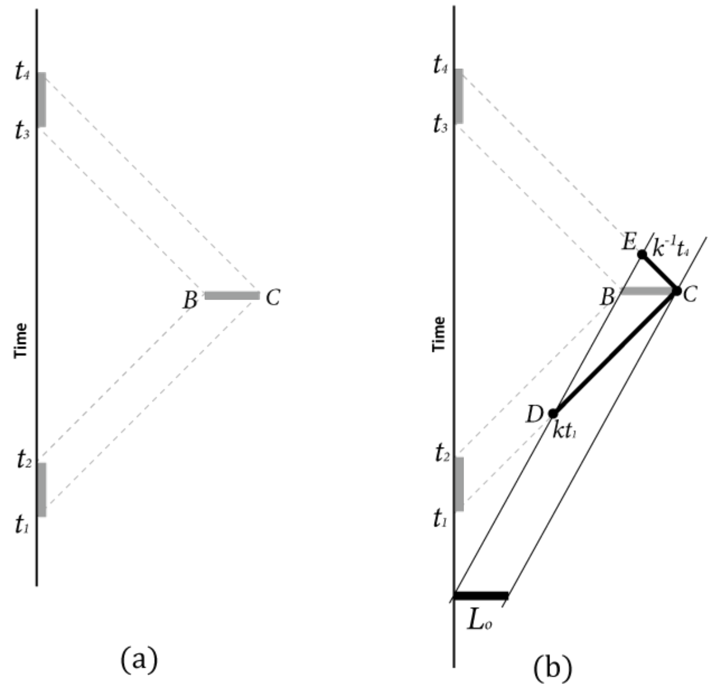


Figure 4: The spacetime interval between two events when measuring a rod
a) Stationary rod, (b) Moving rod

$$
\begin{aligned}
t_4 - t_3 &= (2kL + k^2 t_1) - (k^2 t_2) \\
t_2 - t_1 &= 2kL - k^2(t_2 - t_1) \quad \because \big(t_4 - t_3 = t_2 - t_1\big) \\
t_2 - t_1 + k^2(t_2 - t_1) &= 2kL \\
(t_2 - t_1) &= \frac{(2kL)}{1 + k^2} \\
&= \sqrt{1 - \beta^2}\, L \qquad (5)
\end{aligned}
$$

The expression up to Equation (5) was derived by *Hermann Bondi*. He expressed it as Equation (5) and considered it to be a length contraction. *Is Equation* 5 *the length-contraction formula we originally intended to derive?* The special feature of this equation is that the left side is omitted, while the right side contains the L we were seeking. The left side was clearly $t_4 - t_3 = t_2 - t_1 = (BC) = L_o$, so let us write L_o on the left side. Then we obtain the following equation.

$$L_o = \sqrt{1 - \beta^2}\, L \qquad (6)$$

Now, writing down the value of L that we aimed to find yields the following expression.

$$\therefore L = \frac{1}{\sqrt{1 - \beta^2}}\, L_o \qquad (7)$$

This is not the expression we are used to seeing. It is not a length contraction, but rather a length expansion. The expression up to Equation (5) is was clearly derived by *Bondi*. However, when we organize the formula for L that we initially sought to derive, it inevitably yields an expression in the form of length expansion rather than length contraction. Therefore, the correct length change in relativity is an expansion.

1.2 Proof of length expansion through the relativistic Doppler effect

The relativistic Doppler effect is related to the k-calculus. A stationary light source has a wavelength λ_o, as shown in figure 5a, but the wavelength of a moving light source is different. It take on an asymmetric shape, as shown in figure 5b.

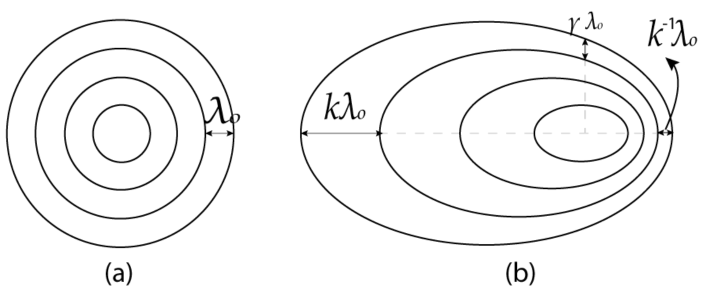


Figure 5: Transverse Doppler Effect and Wavelength Magnitude

The expression for the wavelength for an arbitrary angle is given below. (θ_o is the emission angle, and λ_o is the proper wavelength.) [7]

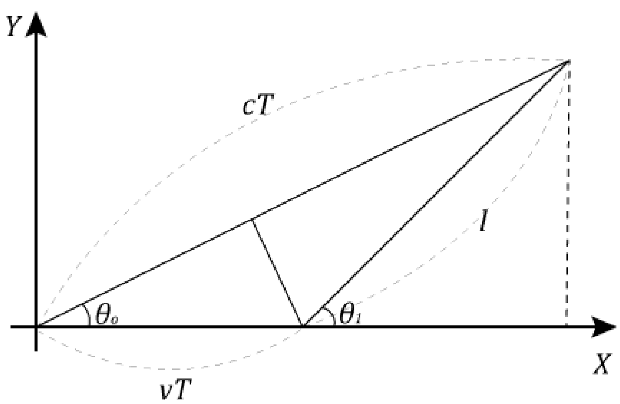


Figure 6: Relativistic Doppler Effect and Wavelength Magnitude

$$\lambda = \lambda_o \frac{1 - \beta cos\theta_o}{\sqrt{1-\beta^2}} \tag{8}$$

When $\theta_o = 0$

$$\lambda = \lambda_o \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} \tag{9}$$

When $\theta_o = \pi$

$$\lambda = \lambda_o \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} \tag{10}$$

When $\theta_o = \frac{\pi}{2}$

$$\lambda = \lambda_o \frac{1}{\sqrt{1-\beta^2}} \tag{11}$$

Since $k = \sqrt{1+\beta}/\sqrt{1-\beta}$, the Equation above can be written simply as follows.

$$\theta_o = 0 \longrightarrow \lambda = k^{-1}\lambda_o \tag{12}$$

$$\theta_o = \pi \longrightarrow \lambda = k\lambda_o \tag{13}$$

$$\theta_o = \frac{\pi}{2} \longrightarrow \lambda = \gamma\lambda_o \tag{14}$$

An easy-to-understand graphical representation of these expressions is shown in figure 7. Suppose a light-clock is aligned with the direction of travel, and light oscillates in that direction. When observing a stationary system, the wavelength magnitude remains unchanged, as shown in figure 7a. However, when a light-clock in a moving system oscillates in the direction of travel, the Doppler effect occurs, as shown in figure 7b.

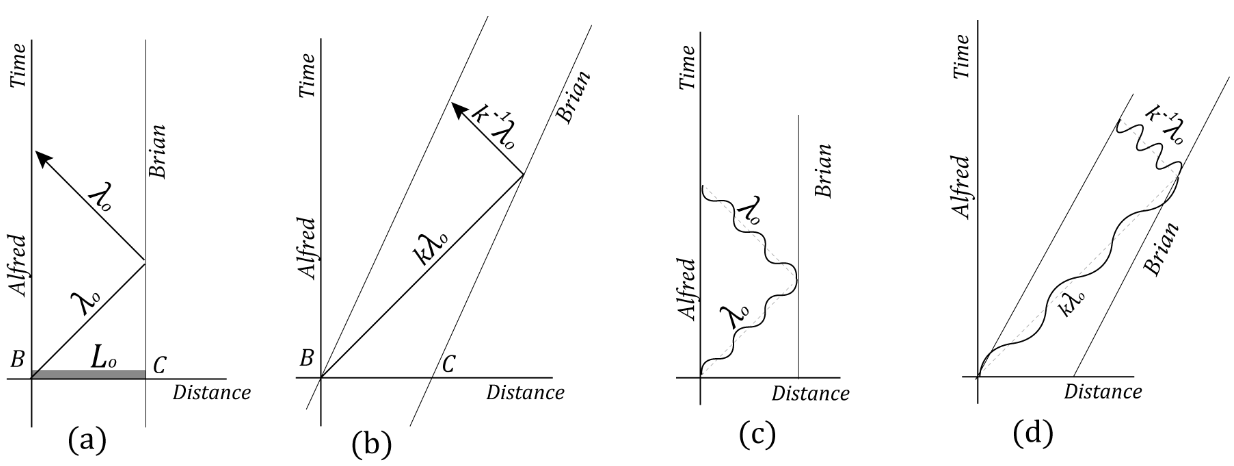


Figure 7: Relativistic Doppler effect and wavelength shown in a spacetime diagram
(a) The path of light when measuring the length of a stationary frame
(b) The path of light when measuring the length of a moving frame
(c) The shape of the light wave when measuring the length of a stationary frame
(d) The shape of the light wave when measuring the length of a moving frame

It's not just the length of the light path that changes. When you observe light traveling, the wavelength and frequency of the light change together. A representation of this as a wave is shown in figure 7c and 7d. In figure 7, both 7a and 7b refer to the legitimate lengths. Since all inertial systems are equivalent, Figure 7c and and 7d both show legitimate lengths. The only difference is that 7d represents the length when observed moving the opponent. Therefore, 7d shows the unmistakably legitimate relativistic length. The length we are looking for is this round trip $(k\lambda_o + k^{-1}\lambda_o)$ divided by 2.

$$\lambda = \frac{k\lambda_o + k^{-1}\lambda_o}{2} \tag{15}$$

$$\therefore \lambda = \gamma\lambda_o \tag{16}$$

Light, which has a wavelength λ_o in a stationary system, has a wavelength $\gamma\lambda_o$ when it moves in a reciprocating motion in the forward direction. Since wavelength is a physical quantity with a length dimension, it can be expressed in terms of length as follows:

$$\therefore L = \gamma L_o \tag{17}$$

1.3 Proof of length expansion using principle of relativity and the isotropy of light propagation

The principle of relativity is the core of relativity theory, and the isotropy of light propagation is another expression of the constancy of the speed of light. Both have been confirmed with great precision today. The method in this section is a logical proof, not a mathematical proof. If the principle of the constancy of the speed of light and the isotropy of light propagation and the principle of relativity are correct, one conclusion inevitably follows. It is not length contraction, but length expansion. While light-clocks are very helpful in studying the propagation of light, it is preferable to work with spherical mirror model rather than light-clocks to consider multiple angles. (Fig. 8) Therefore, we will focus on a spherical mirror. Suppose we have a sphere with a mirror inside and an observer at the origin. When a spherical mirror is stationary, the vibration of light aligns with our common sense. Light originating at the center reflects off the sphere's surface and returns to the center. Photons traveling along all paths reflect off the surface simultaneously, converging back at the center at the same moment. This is only natural. Now, suppose this spherical mirror is moving relative to us.

A person observing this spherical mirror from outside the sphere would see a different situation than before. (Fig. 9) Several photons have departed from the origin of the sphere at the same time; one of them will leave the sphere at point A, reflect at point B_1, and return to the origin. The sphere is moving, so the point it returns to is no longer the original starting point but rather the position it has moved to. Let's call that point C. Let A be the starting point, B_i the reflection points, and C the arrival point. Multiple photons departing from point A are each reflected at different points $B_1, B_2, B_3, ...$ and so

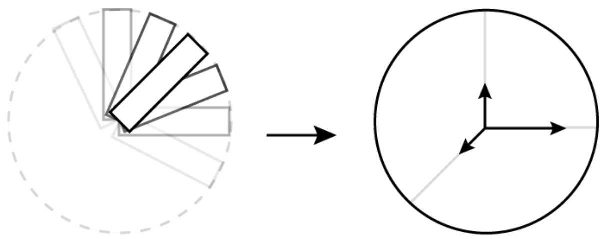


Figure 8: Cylindrical light-clock and spherical mirror clock

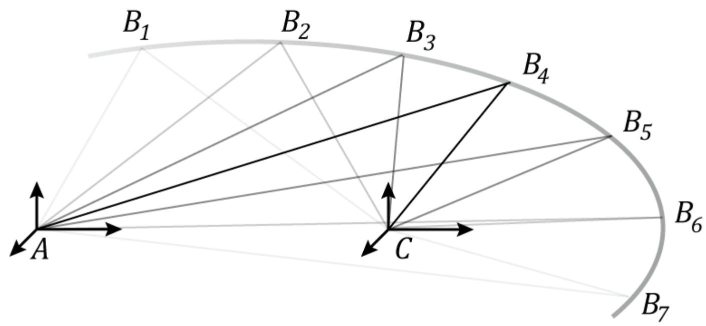


Figure 9: The point of origin, the point of reflection and the point of arrival of light

on, before arriving at point C. Connecting the points B_1, B_2, B_3, and so on, where these photons are reflected by the mirror surface, forms a single shape: an ellipse. Multiple photons leave the starting point at the same time, and they all arrive at the arrival point at the same time. This has nothing to do with the relativity of simultaneity. The relativity of simultaneity applies to two events that are spatially separated, but since the starting point is a single point, it has nothing to do with the relativity of simultaneity.

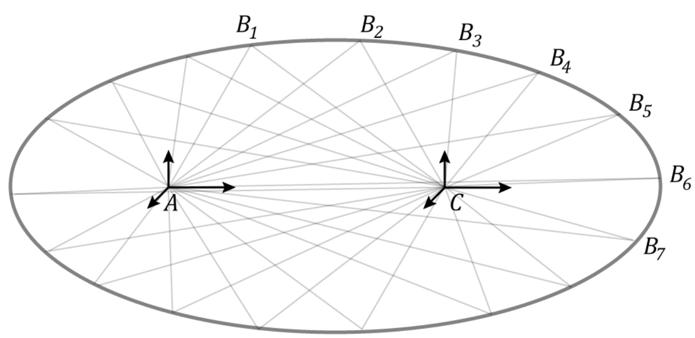


Figure 10: The set of all reflections is an ellipse

The paths of the multiple photons then have the same starting point and the same arrival point, but the reflection points are different for every photon. The lengths of the different paths must be the same; this is guaranteed by the principle of the constancy of the speed of light. As long as long as the isotropy of light propagation is ensured, the lengths of all paths for all angles are also the same. If two points are the foci and all paths have constant length, it follows that the shape formed by the sum of the reflection points is an ellipse or, in the case of three dimensions, an ellipsoid. For this reason, when a spherical mirror is in motion, it is observed to have the shape of an ellipse. The spherical and elliptical mirrors have equal status. When a person observes their own system, it appears as a spherical mirror; when observing a moving counterpart, it appears as an elliptical mirror. (Fig. 10) If the principle of relativity is correct, this is how it should be. If multiple photons originate from the same point, and that point moves and then arrives at a new location, those two positions must necessarily become two focal points. Since all paths must be of equal length, the sum of those reflection points must form an ellipse. This is what we call *the ellipse theorem*.

Ellipse theorem

When a spherical mirror moves relatively fast,
the starting and arrival points of light are the foci of an ellipse,
and the sum of the reflection points forms an ellipse.

Figure 11a shows that a sphere becomes an ellipse when it moves, indicating that it elongates in the direction of its motion. Generalizing this, all relativistic moving objects are expanded by a certain percentage in the direction of travel. There is no length contraction anywhere. Logically, if the constancy of the speed of light, the principle of relativity and the isotropy of light propagation are correct, then the length of a moving object must be expanding, not contracting.

(a)
(b)

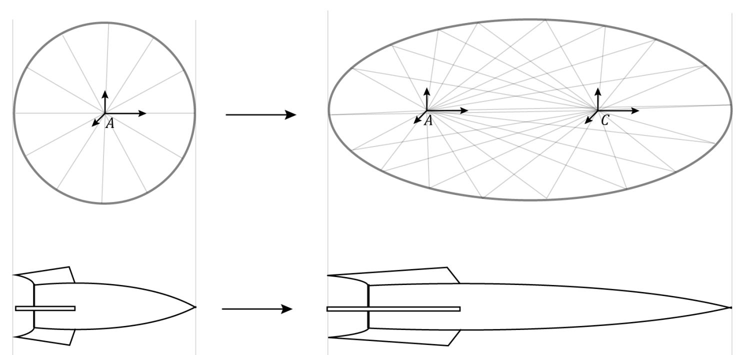


Figure 11: (a) The sphere and the sphere in motion
(b) A general object and its appearance when moving

1.4 Proof of length expansion through time dilation

If the principle of the constancy of the speed of light and time dilation are correct, we can prove that length expansion is also correct. Suppose that, in an inertial frame, a light-clock is moving at relativistic speed along the direction of travel. The speed of light in a stationary system can be expressed by Equation (18). (L_o is the distance traveled by the light in the stationary system, T_o is the time taken by the light in the stationary system, c is the speed of light, and γ is the Lorentz factor.)

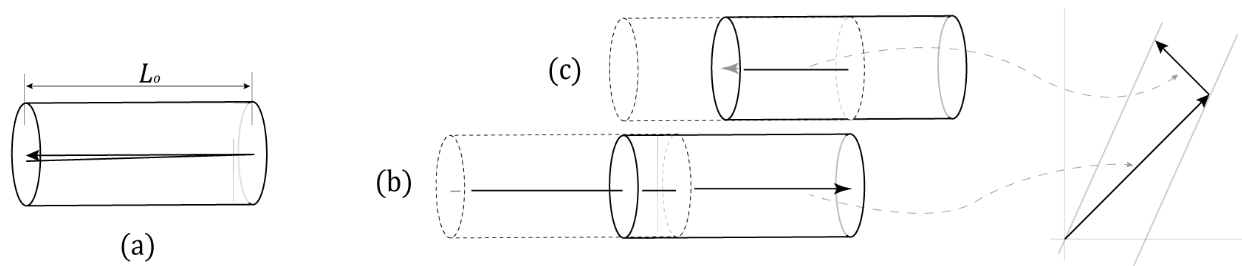


Figure 12: Light-clocks in motion when placed side by side in the forward direction
(a) Light traveling on a stationary clock
(b) Light traveling forward in a moving light-clock
(c) Light returning from a moving light-clock

$$c = \frac{L_o}{T_o} \tag{18}$$

If this light-clock is moving, the speed of light can be expressed by Equation (19). L represents the distance light has traveled in the moving frame, and T represents the time it took light to travel that distance in the moving frame. Naturally, the speed of light remains c at this time as well.

$$c = \frac{L}{T} \tag{19}$$

Since the speed of light is constant, we can combine Equations (18) and (19) to write.

$$c = \frac{L_o}{T_o} = \frac{L}{T} \tag{20}$$

Equation (20) must hold due to the principle of the constancy of the speed of light, and since time dilation implies $T = \gamma T_o$, the length must necessarily satisfy $L = \gamma L_o$. If this does not hold true, then either there is an error in the time dilation, or there is an error in the principle of the constancy of the speed of light. It is also possible to directly derive length expansion from the time dilation.

$$
\begin{aligned}
T &= \gamma T_o \\
cT &= \gamma c T_o \\
\therefore L &= \gamma L_o
\end{aligned} \tag{21}
$$

Even writing down the formula like this is actually complicated. The relationship between time, distance, and the speed of light is as follows.

$$L = cT \tag{22}$$

c is a constant, so if we set $c = 1$, we obtain the following expression.

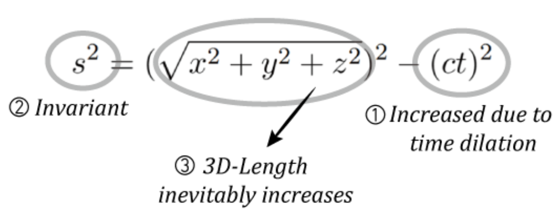
$$L = T \tag{23}$$

To understand the nature of things, when considering only variables, distance and time are the same. As shown in Equation (23), if time increases, length increases; if time decreases, length decreases. How can time dilation hold true in one inertial frame while length contraction is simultaneously possible? This is a clear error. If time dilation is correct, then length expansion must be correct. Equation (23) above says it all.

1.5 Proof of length expansion through invariants

The theory of relativity includes many invariants, among which s^2 on the left-hand side of Equation (24) is a well-known example. It is always constant regardless of the direction or speed of the system. For an inertial system $K(x, y, z, ict)$, the following expression always holds.

$$
\begin{aligned}
s^2 &= x^2 + y^2 + z^2 - c^2 t^2 \\
s^2 &= (\sqrt{x^2 + y^2 + z^2})^2 - (ct)^2
\end{aligned} \tag{24}
$$



In the second term on the right-hand side of Equation (24), c is a constant, and time t increases with increasing speed. Therefore, for s^2 on the left-hand side to remain constant, the value of $\sqrt{x^2 + y^2 + z^2}$ must increase as the speed increases. Since $\sqrt{x^2 + y^2 + z^2}$

represent the length in three-dimensional space, the nature of the expression becomes clearer if we denote it by r. Suppose there are two other inertial systems, $L(x', y', z', ict')$ and $M(x'', y'', z'', ict'')$ moving relative to the inertial system K. Then t, t', and t'' and denote the times observed by different people, and r, r', and r'' denote the three-dimensional distances observed by different people.

$$\begin{aligned} \sqrt{x^2+y^2+z^2} &= r \\ s^2 &= (r)^2 - (ct)^2 \\ &= (r')^2 - (ct')^2 \\ &= (r'')^2 - (ct'')^2 \end{aligned} \tag{25}$$

We know that the left-hand side term s^2 is always a constant invariant, and we also know that time dilates for a system moving at relativistic speeds. If length contraction is applied to r, the property that s^2 is invariant is immediately destroyed. Therefore, length contraction does not represent the correct relativistic length. Rather, length expansion must occur for s^2 to remain constant. Applying Equation (24) to light makes this even clearer. For light, since s^2 is zero, the equation can be expressed as follows.

$$\begin{aligned} & s^2 \to 0 \\ & = (ct)^2 - (r)^2 = 0 \\ & \therefore (ct)^2 = (r)^2 \\ & \therefore ct = r \\ & \therefore c = \frac{r}{t} \end{aligned} \tag{26}$$

If the equation $c = r/t$ holds in an inertial frame K, then observers in other inertial frames also measure the speed of light to be constant, so the following equation holds.

$$\begin{aligned} s^2 &= 0 \\ &= (r)^2 - (ct)^2 \\ &= (r')^2 - (ct')^2 \\ &= (r'')^2 - (ct'')^2 \end{aligned} \tag{27}$$

$$\begin{aligned} ct &= r \\ ct' &= r' \\ ct'' &= r'' \end{aligned} \tag{28}$$

$$\therefore c = \frac{r}{t} = \frac{r'}{t'} = \frac{r''}{t''} = c \tag{29}$$

So, if time is in a relationship with $t' = \gamma t$, then length must be in a relationship with $r' = \gamma r$. Then $r' = \gamma r$ must be length expansion, not length contraction.

1.6 Proof of length expansion by Kutliroff's method

This method was introduced by Kutliroff to explain the time dilation of relativity theory in a way that even high school students could understand. [8] Since the path of light is perpendicular to the direction of motion of an object, the equation is very simple, making it an excellent method for students to easily grasp an important fact of relativity theory. This is also a good way to verify that length expansion is correct.

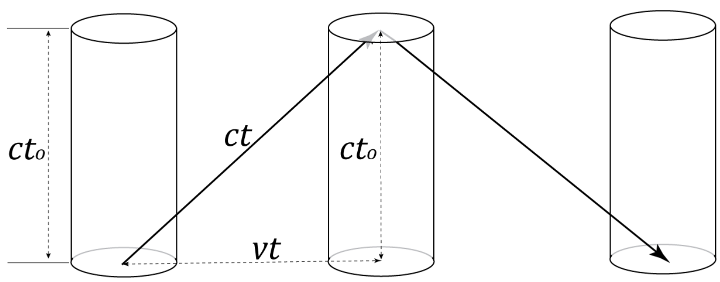


Figure 13: A light-clock moving vertically

$$
\begin{aligned}
v^2t^2 + c^2t_o^2 &= c^2t^2 \\
c^2t^2 - v^2t^2 &= c^2t_o^2 \\
c^2t^2(1 - \frac{v^2}{c^2}) &= c^2t_o^2 \qquad (30) \\
t^2(1 - \frac{v^2}{c^2}) &= t_o^2 \\
\therefore t &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}t_o \qquad (31)
\end{aligned}
$$

Now that we've derived the time dilation, let's derive the length equation. Rewriting Equation (30) yields:

$$
\begin{aligned}
c^2t^2(1 - \frac{v^2}{c^2}) &= c^2t_o^2 \\
l^2(1 - \frac{v^2}{c^2}) &= l_o^2 \\
\therefore l &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}l_o \qquad (32)
\end{aligned}
$$

The length expansion is easily proved.

1.7 Proof of length expansion using the Lorentz transformation

Roy Weinstein was the first to derive length contraction using the Lorentz transform, and he wrote in his paper that his method was recognized by everyone from Gamow to Einstein. Let's see how he derived it. [9]

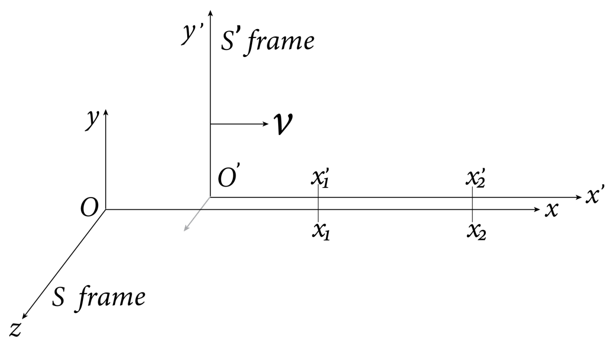


Figure 14: Two inertial systems moving past each other

Two inertial systems are moving relative to each other. One frame is S*(Alfred)* and the other is S'*(Ashley)*. From the perspective of the person in the S frame, he is at rest while the S' frame is moving. This is illustrated in figure 14. The observing entity is in the S frame, while the S' frame is moving. Therefore, applying the Lorentz transformation to a primed physical quantity must convert it into the unprimed physical quantity in the S frame. Below is the Lorentz transformation formula for moving along the x-axis. (where γ is the Lorentz factor, $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = {}^{v}/_{c}$)

$$x' = \gamma(x - vt) \tag{33}$$
$$y' = y \tag{34}$$
$$z' = z \tag{35}$$
$$t' = \gamma(t - \frac{vx}{c^2}) \tag{36}$$

Here is how Roy Weinstein derived length contraction. Suppose you measure both ends of a rod at the same time.

$$L_o = x'_2 - x'_1 \tag{37}$$
$$x'_2 = \gamma(x_2 - vt_2)$$
$$x'_1 = \gamma(x_1 - vt_1)$$
$$L_o = \gamma(x_2 - vt_2) - \gamma(x_1 - vt_1),$$
$$put)\ t_2 = t_1$$
$$L_o = \gamma(x_2 - x_1)$$
$$L_o = \gamma L$$
$$\therefore L = \frac{1}{\gamma} L_o \tag{38}$$

On the left side of Equation (37), he wrote L_o and applied the Lorentz transformation to it. To emphasize, he applied the Lorentz transformation to *the proper length*. The expression of the form of $x'_2 - x'_1$ between Alfred and Ashley is not unique. There are four different lengths between the two observers.

- Case 1: If Alfred is looking at the length inside his frame, the expression is $L_o = x_2 - x_1$. (Fig. 15a)
- Case 2: If Alfred is looking at the length of the Ashley's frame, the expression is $L = x'_2 - x'_1$. (Fig. 15b)

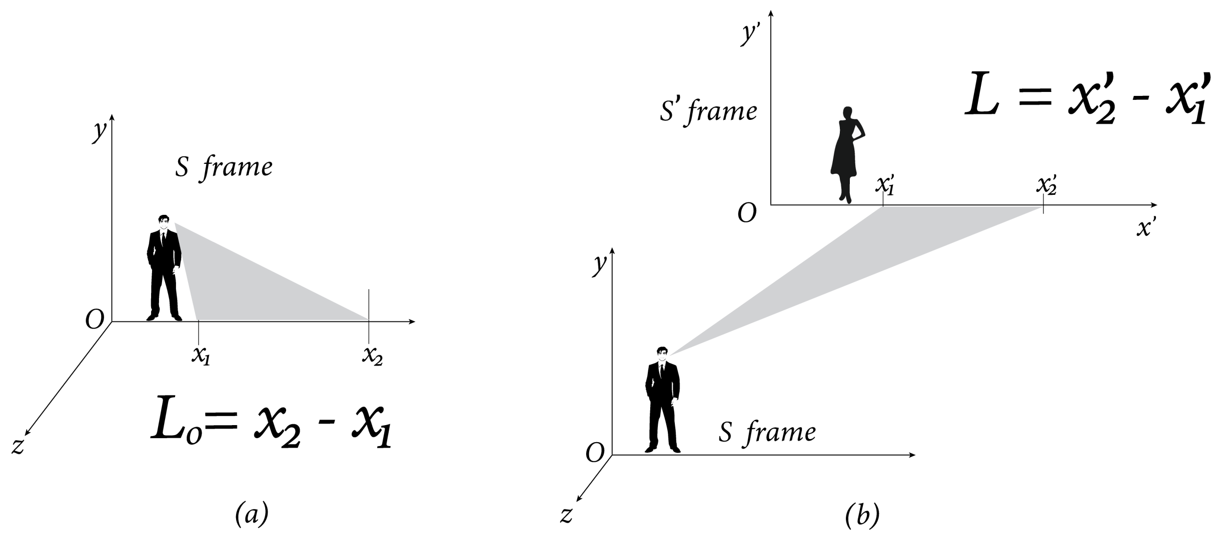


Figure 15: (a) Alfred Observing His Coordinates
(b) Alfred Observing the Other's Coordinates

- Case 3: If Ashley is looking at the length of Alfred's frame, the expression is $L = x_2 - x_1$. (Fig. 16a)
- Case 4: If Ashley is looking at the length inside her frame, then the expression is $L_o = x'_2 - x'_1$. (Fig. 16b)

This situation is well illustrated in figures 15 and 16. Here, two expressions are written in the form $x'_2 - x'_1$. They are $L = x'_2 - x'_1$ and $L_o = x'_2 - x'_1$. It is justified to substitute the Lorentz transform for the expression $L = x'_2 - x'_1$ in Case 2, but the expression $L_o = x'_2 - x'_1$ in Case 4 should not be substituted for the Lorentz transform because it is an observation from its own frame. The expression in Case 4 cannot be substituted into the Lorentz transform because it represents the proper length. The symbol L_o on the left side of $L_o = x'_2 - x'_1$ in Case 4 explicitly indicates that this equation represents the proper length. Since Roy substituted the Lorentz transform into an expression that should not be substituted, he obtained a length contraction expression.

Let's try to derive the correct expression, not the one derived by Weinstein. Alfred (S) observes, and Ashley becomes the observed object (S'). We must use $L = x'_2 - x'_1$, which

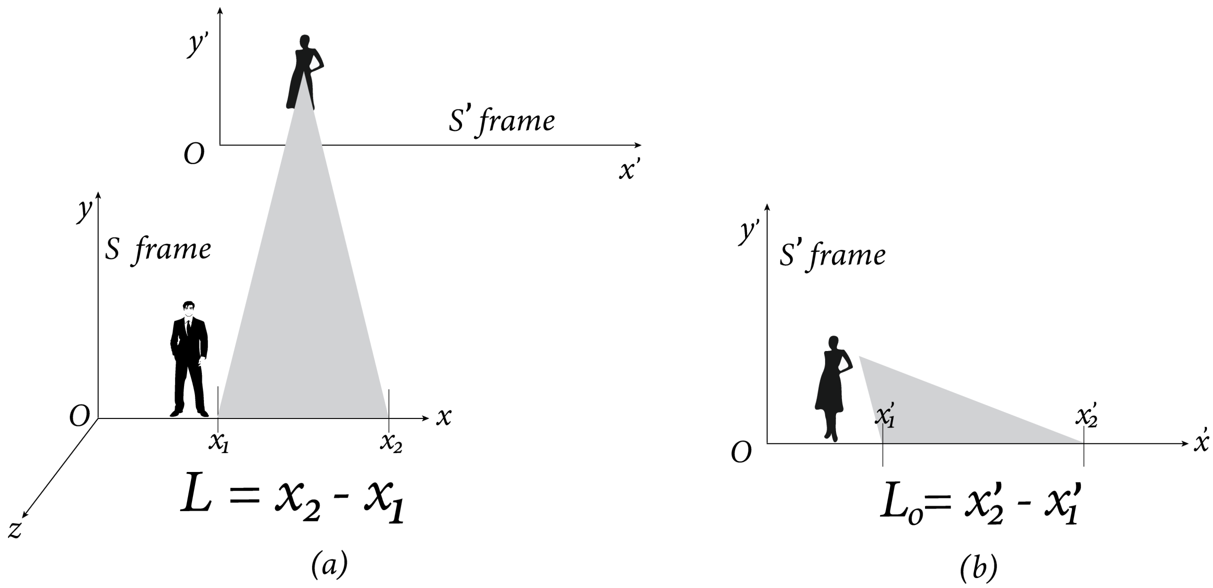


Figure 16: (a) Ashley Observing the Other's Coordinates
(b) Ashley Observes Her Coordinates

corresponds to case 2 among the four lengths. Suppose that both ends of the rod are measured simultaneously, as Roy assumes.

$$
\begin{aligned}
L &= x'_2 - x'_1 \qquad (39)\\
L &= \gamma(x_2 - vt_2) - \gamma(x_1 - vt_1),\\
&\quad put)\ t_2 = t_1\\
L &= \gamma(x_2 - x_1)\\
L &= \gamma L_o \qquad (40)
\end{aligned}
$$

Then the correct length expansion is derived exactly.

2 Experimental Proof of Length Expansion

Whether length contraction or length expansion is correct has already been proven through various experiments and observations. Looking at the experimental and observational results, it is so clear that it is easy to judge. Let us now examine several already known experiments and observational results and determine which of length contraction or length expansion is correct.

2.1 Transverse Doppler Effect

The transverse Doppler effect is widely known as proof of relativistic effects in the laboratory. The frequency shift of light emitted vertically by excited hydrogen when rotating at high speeds is not observed in classical theory but is confirmed in relativistic theory. [10]

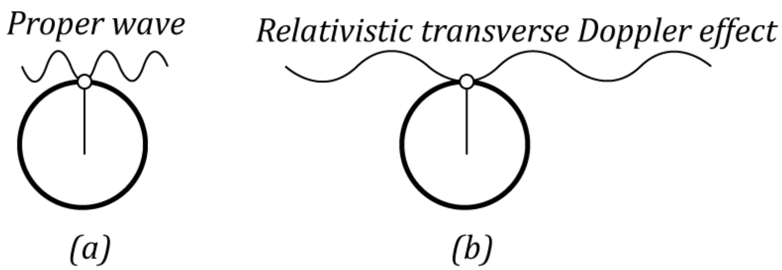


Figure 17: Relativistic Doppler Effect and Frequency

A hydrogen atom vibrating at ν_o in a stationary frame acquires a frequency ν when rotating at a relativistic velocity. This is expressed by Equation (41). We call this the *transverse Doppler effect*. Here, c is the speed of light, v is the velocity, and ν is the frequency.

$$\nu = \nu_o\sqrt{1-\frac{v^2}{c^2}} \tag{41}$$

This is identical to $t = \gamma t_o$, which represents the time dilation, differing only in expression. Since the wavelength and frequency of light are related by the following equation, this can also be expressed in terms of wavelength.

$$c = \nu_o\lambda_o \tag{42}$$
$$c = \nu\lambda \tag{43}$$

The speed of light is always constant, so the relationship $c = \nu_o\lambda_o = \nu\lambda$ holds. And since $\nu = \nu_o\sqrt{1-\beta^2}$, we have the following. Let's substitute equation (41) into the equation (43).

$$c = \left(\nu_o\sqrt{1-\frac{v^2}{c^2}}\right)\lambda \tag{44}$$
$$c = \nu_o\left(\sqrt{1-\frac{v^2}{c^2}}\right)\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)\lambda_o \tag{45}$$

For c to remain constant in Equations (42) and (43), the wavelength λ must necessarily satisfy the following relationship.

$$\lambda = \lambda_o\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{46}$$

Since wavelength is also a length, it can be expressed as follows.

$$L = L_o \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad or \quad L = \gamma L_o \tag{47}$$

This is an indisputable fact. If the transverse Doppler effect is indeed the experimental result proving the theory of relativity, then length expansion is undoubtedly correct as well.

2.2 Additional flight range of GPS satellites

GPS satellites move at high speeds within a gravitational field, so special relativistic effects must be considered. When a GPS satellite moves at approximately $4km/s$ at an altitude of about 20,000 km, it experiences a time delay $7.1\mu s$ longer each day than predicted by classical theory. (Fig. 18c) This means it would exist at a location farther away than classical theory predicts. Although it amounts to $7.1\mu s$ per day, over the course of a year it accumulates to approximately $2590\mu s$, making it a non-negligible amount of time.

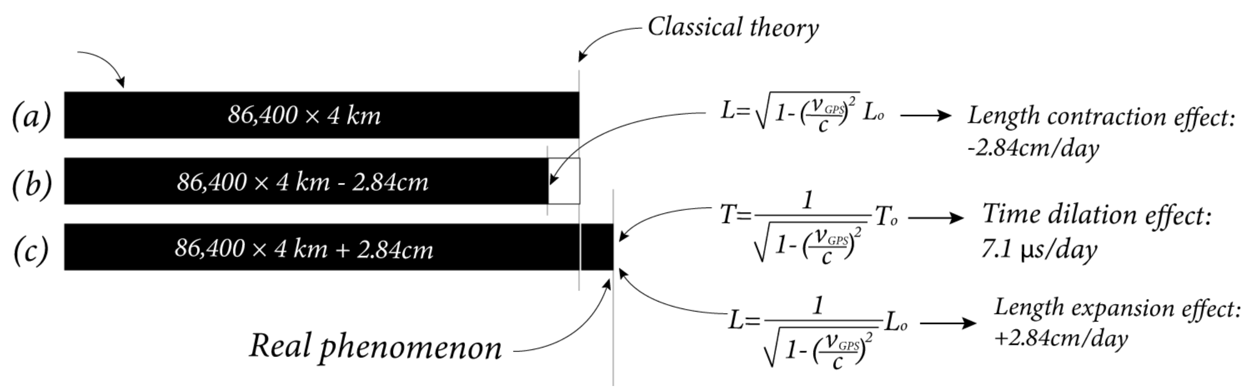


Figure 18: Interpreting the additional flight phenomenon of GPS satellites
(a) Classical model
(b) Length contraction model
(c) Length expansion and time dilation model

This can also be considered in terms of the length dimension. If the theory of length contraction is correct, GPS satellites travel a distance $2.84cm$ less each day. (Fig. 18b) Neil Ashby dismissed the theory of length contraction because it could not explain this phenomenon. [11] Sato M. argued that length contraction and the constancy of the speed of light are mutually incompatible. The correct way to interpret this seemingly contradictory phenomenon is simple. It should be interpreted not as length contraction, but as length expansion. The points predicted by the time dilation theory and the length contraction theory differ. Which theory is correct? Naturally, the time dilation theory is correct. Then why can't the values interpreted through length contraction accurately explain natural phenomena? Neil Ashby has stated the following regarding this matter: [11] *'Some things have been purposely glossed over, such as the possibility of Lorentz contraction. It actually is not important; the GPS is a timing system.'* His deliberate disregard for length contraction stems from the fact that it is not observed in experimental settings, which suggests that length contraction may not be a valid theory. The relativistic length change phenomenon is not unimportant, nor is GPS solely about the time system. Regardless of whether time change or length change is used, it must always be able to calculate accurate points and values. Only when this is possible does the reliability of relativity theory increase. Regarding the phenomenon in which predicting precise points using length contraction in GPS is impossible, Masanori Sato states the following: *'Length contraction and the principle of the constancy of the speed of light conflict with each other'*, *' If there is a Lorentz contraction, the GPS cannot work so precisely.'* [5]

That's correct. If length contraction were correct, GPS would be impossible. Furthermore, if length contraction were correct, the constancy of the speed of light would be compromised. The correct way to interpret this seemingly contradictory phenomenon is simple. It should be interpreted not as length contraction, but as length expansion. (Fig. 18c) One thing is clear: length contraction cannot explain this phenomenon, whereas length expansion explains it very naturally. The statement that the satellite traveled $7.1\mu s$ longer and traveled $2.84cm$ farther is equivalent. Therefore, both of the satellite's additional flight phenomena can be fully expressed in terms of time and distance.

2.3 Muon Reaching Sea Level

The phenomenon of muons reaching sea level is often cited as experimental evidence for length contraction. We will examine whether that is true. When a muon is created in the Earth's upper atmosphere, its speed is approximately $0.998c$ and its lifetime is about $2.2\mu s$. According to classical theory, a muon reaches the end of its lifetime after traveling approximately 650 meters. After that, the muon decays and cannot reach sea level. However, muons created at an altitude of about $10km$ are actually detected at the Earth's surface. [12] (Since the muon's speed is $0.998c$, the Lorentz factor γ is approximately 16) This is easily explained by time dilation. (Fig. 19)

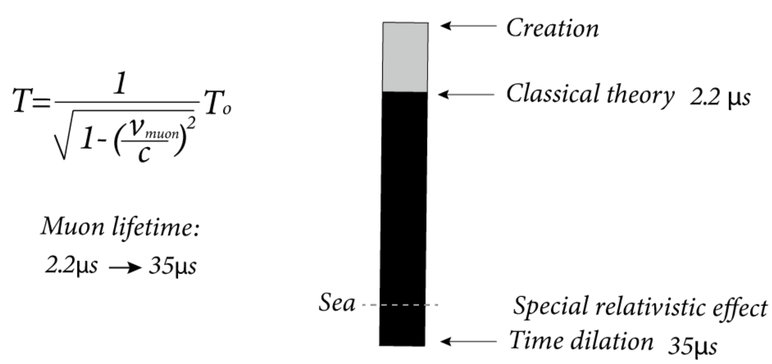


Figure 19: Interpreting muons that reach the sea surface as a result of time dilation

Explaining this phenomenon as time dilation is highly reasonable. However, attempting to explain it through length contraction creates a problem. Since classical theory predicts it should travel 650m, applying length contraction here implies the muon should only travel 40m before decaying. This fails to explain the actual phenomenon of muons reaching the sea surface. (Fig. 20a)

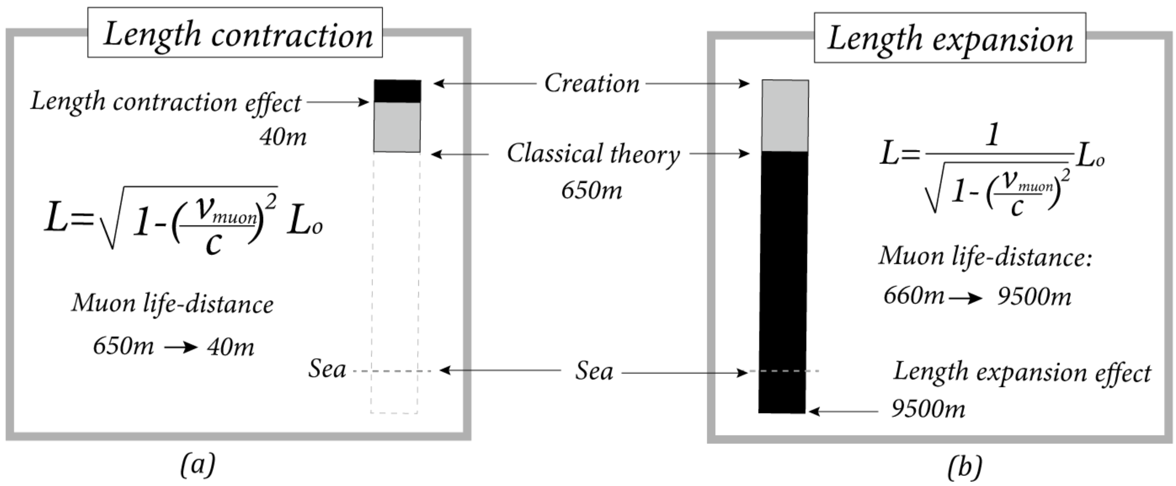


Figure 20: Differences between two interpretations of muon arrival at sea level

Time dilation explains natural phenomena well, but length contraction does not. Therefore, time dilation is a good theory, but length contraction is not a good theory. The theory of length contraction fails to explain this phenomenon, but length expansion explains it naturally. (Fig. 20b) The theory of length contraction cannot explain this phenomenon, yet some people attempt to explain it anyway. They argue that the observation should be made from the muon's perspective, not the Earth's. From the muon's standpoint, spatial distances contract, allowing it to reach the sea surface. To make such a claim, data measured from the muon's perspective must be provided. One should not make such a claim before presenting that data and evidence. David and James are the ones who observed the phenomenon of muons reaching sea level and submitted the paper. [12] They did not observe this phenomenon by riding on muons; they clearly observed muons while standing on Earth. Therefore, explaining this phenomenon from the muon's perspective remains an unrealized hypothesis. To explain this phenomenon through length contraction, one must present data or evidence measured or observed from the muon's perspective.

2.4 Long-Range Flight of Muons in Circular Particle Accelerators

Additional muon flight phenomena are observed even within circular particle accelerators. The muon lifetime is $2.2\mu s$, and their speed within the accelerator is extremely high, resulting in a Lorentz factor γ of approximately 30. [13] Suppose a muon particle is generated at point A inside the accelerator, as shown in figure 21a. According to classical theory, it should be found at point B. However, it is actually detected at point

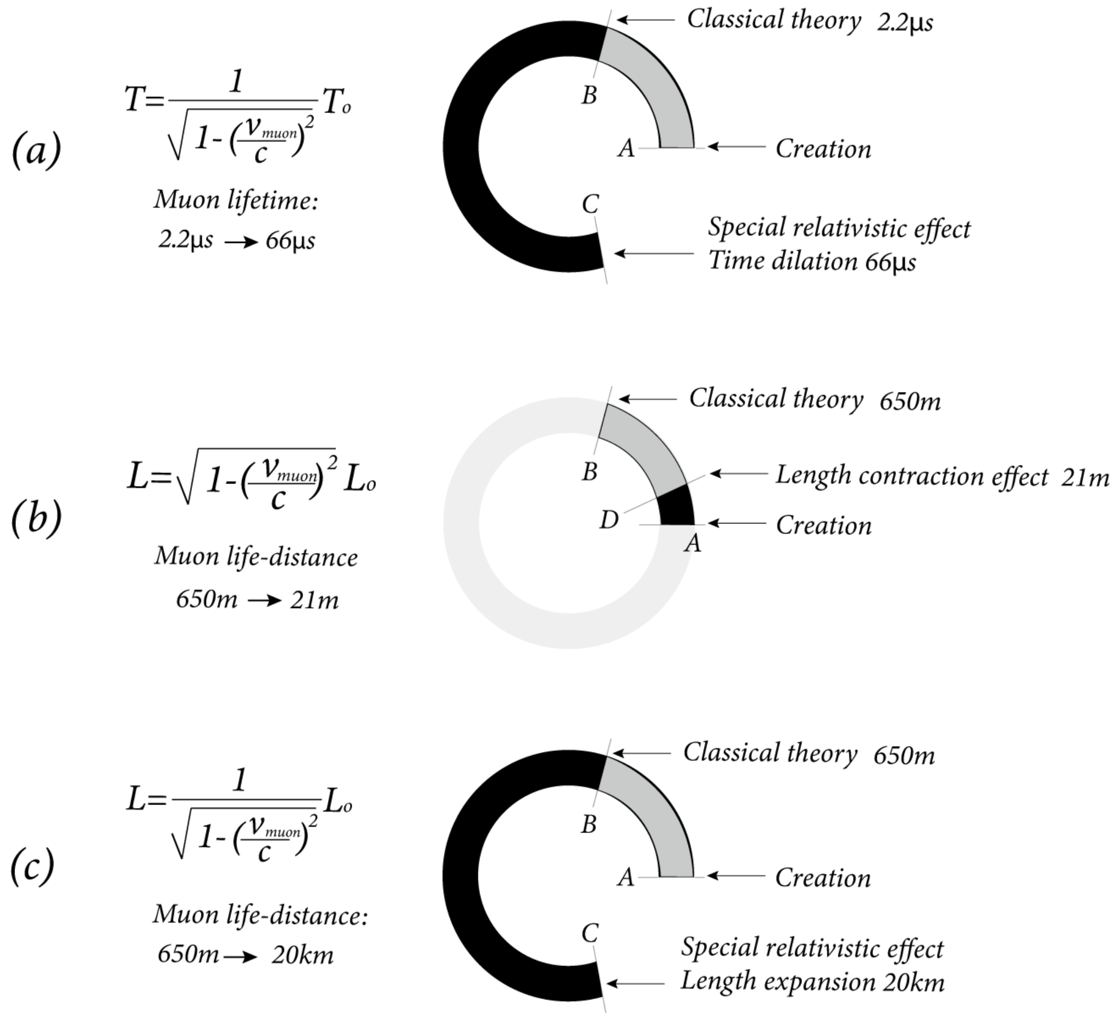


Figure 21: Interpretation of the long-range flight phenomenon of muons
(a) Time dilation (b) Length contraction (c) Length expansion

C. This is not an abstract thought experiment, but a real observational result within a particle accelerator. It is a natural phenomenon that no one can deny. If a theory cannot explain this phenomenon, it is not a good theory. It can be explained by time dilation, but not by length contraction. Let's interpret this phenomenon using the theory of length contraction. (Fig. 21b) If the muon's flight distance becomes shorter, it cannot explain the phenomenon of muons traveling long distances. Classically, a muon particle can travel about 650 meters, but if length contraction is correct, it would be 30 times shorter, flying only about 21 meters before decaying. Therefore, the phenomenon of muon particles being detected at point D cannot be explained by the length contraction theory, but it can be very simply explained by the length expansion theory. (Fig. 21c) Its position is exactly the same as that predicted by time dilation. According to classical theory, the muon should only travel about 650 meters. However, its flight distance increases by a factor of 30 to approximately 20 kilometers, which sufficiently explains the observed phenomenon. Therefore, the long-distance flight of muon particles within a circular particle accelerator provides strong evidence for length expansion.

2.5 Cross Collision

I believe the experiments and observational evidence examined thus far are sufficient to determine whether length contraction or length expansion is correct. If additional evidence is still needed, I propose the following method.

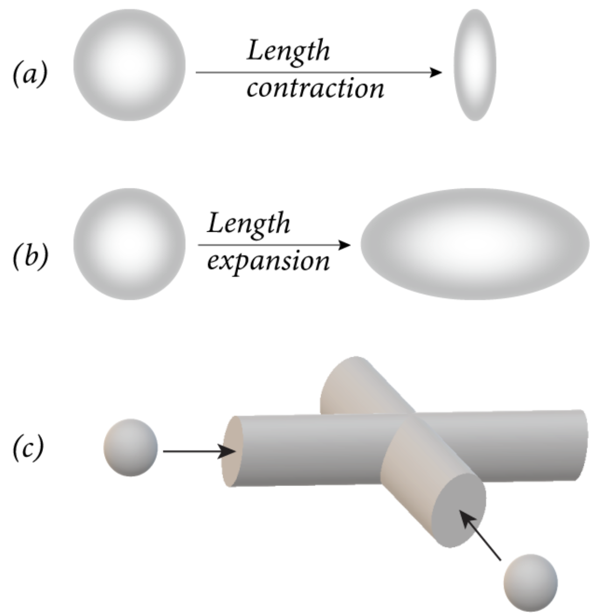


Figure 22: (a) Length contraction model (b) Length expansion model (c) Cross collision

If the length contraction theory is correct, the particle size will decrease as shown in figure 22a. If the length expansion theory is correct, the particle size will increase, as shown in 22b. If particles accelerate in opposite directions within a particle accelerator and collide, we would be unable to distinguish between the two theories. However, if they collide perpendicularly, as shown in figure 22c, it will be possible to determine which of the two theories is correct. Figure 22c depicts two particles colliding perpendicularly from a classical mechanics perspective. At this point, the particle shape remains spherical and undeformed. (A particle shape is assumed to be spherical.)

Figure 23a shows the collision assuming length contraction is correct, while figure 23b shows the collision assuming length expansion is correct. The probability changes when they collide. Let P be the probability predicted classically when particles collide perpendicularly. When multiple particles collide perpendicularly, their collision cross section probability changes due to relativistic effects. If the length contraction theory is correct, the probability of a perpendicular collision will be $(1/\gamma)^2 P$. If the length expansion is correct, it will be $\gamma^2 P$. Quantitative data are not actually necessary. When the particle's speed is gradually increased, if the collision frequency decreases progressively,

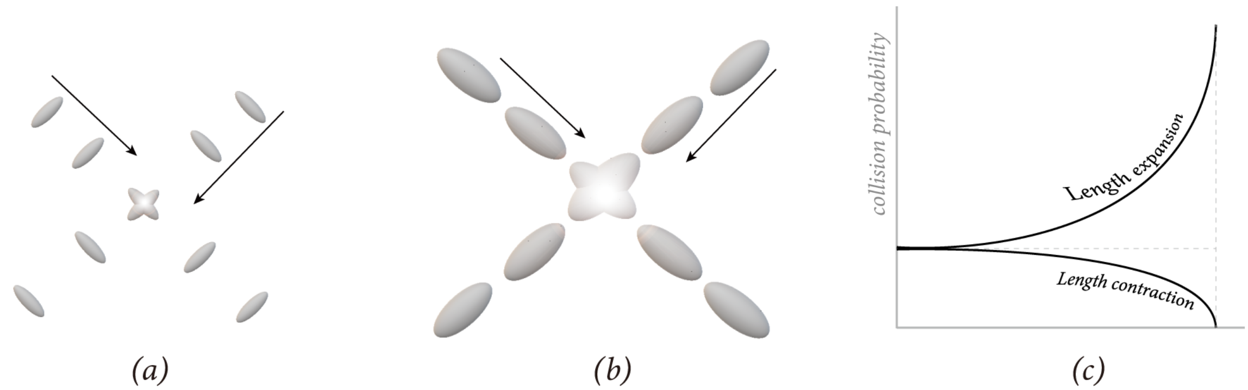


Figure 23: (a) Length contraction model (b) Length expansion model
(c) Relativistic Effects of Cross Collisions

the length contraction theory is correct; if it increases progressively, the length expansion theory is correct. (Fig. 23c) If you find it difficult to believe the proof, experimental evidence, or observational evidence for length expansion presented thus far, you can verify the truth by performing this experiment.

3 Various Paradoxes Arising from Length Contraction

Though length contraction has been central to relativity theory for over a century, it remains experimentally unproven. Rather than being proven, it has only deepened the confusion. The most prominent source of this confusion is the principle of the constancy of the speed of light. Understanding the principle of the constancy of the speed of light is not unrelated to length contraction. The fact that the principle remains incomprehensible is entirely the fault of the theory of length contraction. If the theory of length contraction were removed from relativity theory, the constancy of the speed of light could be understood intuitively. Furthermore, the theory of length contraction generates numerous additional paradoxes. When length contraction is introduced into relativity theory, paradoxes inevitably arise. There are many paradoxes, such as the muon reaching sea level, Bell's spaceship paradox, and Supplee's submarine paradox. The paradox of the constant speed of light was also one such type of paradox. We will examine why these are problematic and explore ways to resolve them.

3.1 The Paradox of the Constancy of the Speed of Light

There are two things about the speed of light that remain incomprehensible today. The first is how the speed of light remains constant regardless of the speed of an inertial frame. The second is how the phenomenon of length contraction is compatible with the constancy of the speed of light. To examine these two problems, let us look at the definitions of time and length. One second is defined as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom. Using this, let's also define the unit of distance called a ‘light-second.’ As a unit of distance, one light-second (1 ls) is the distance light travels in the time it takes a cesium atom to undergo 9,192,631,770 oscillations. This can be illustrated as follows. In figure 24, light travels from A to B. The time taken for light to travel this distance is one second, and the distance traveled is one light-second(ls). Naturally, in this situation, light travels at a constant speed equal to the speed of light.

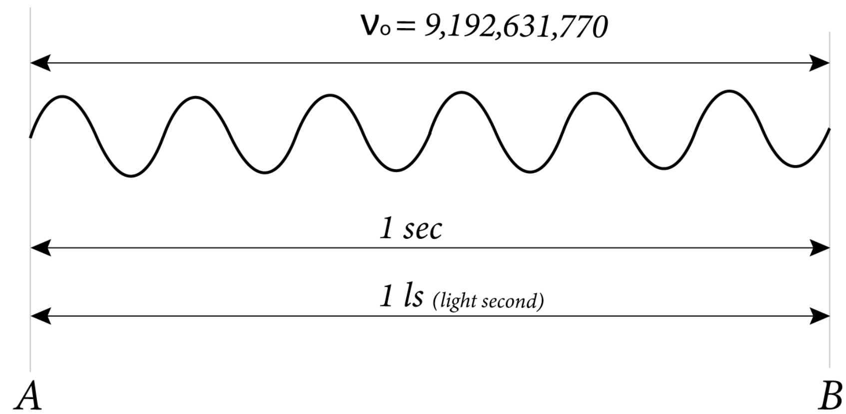


Figure 24: Definition of unit time (sec) and unit distance (ls)

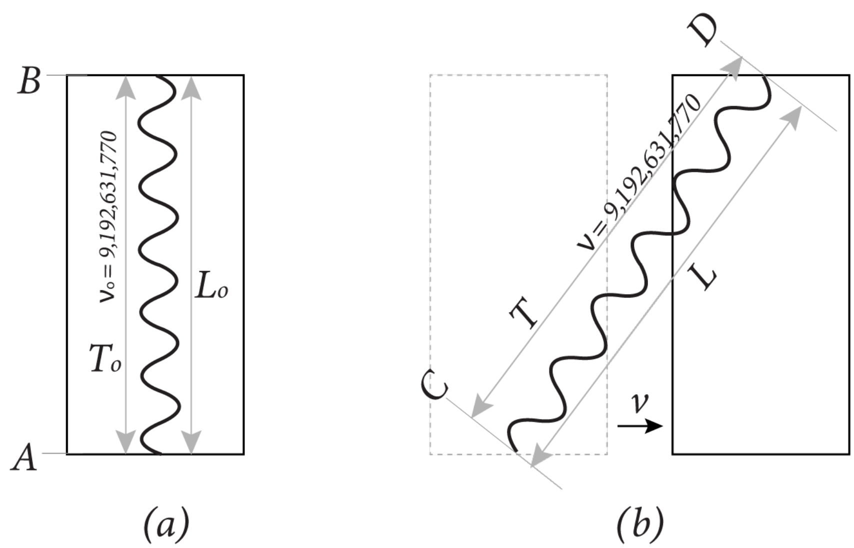


Figure 25: Unit time(sec) and unit length(ls) in two scenarios.

Figure 25a shows the proper time and proper length. When observed by another observer moving at a relative velocity of $-v$, Figure 25a would appear as shown in 25b. Even then, light oscillated 9,192,631,770 times between C and D, so naturally, 25b also represents one second of time and one light-second of distance. Let us place 25a and 25b from figure 25 side by side in figure 26 and compare their sizes.

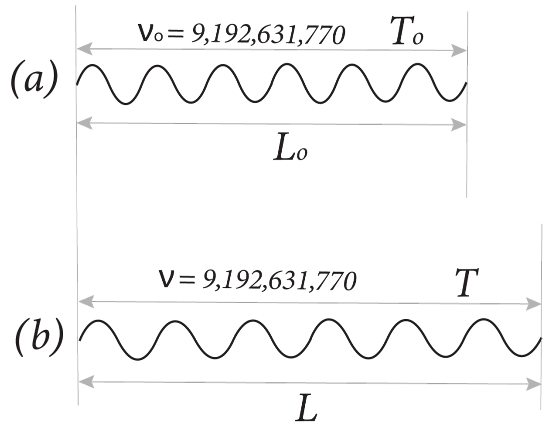


Figure 26: Comparison of two times and two lengths

It is clear from figure 26, 26b is larger than 26a. The magnitude of time is larger, and the magnitude of length is also larger. Because time is longer, we refer to this as time dilation or time expansion. And the length is also clearly larger in 26b than in 26a. Therefore, length expansion is correct. And the ratio of these magnitudes depends on the Lorentz factor γ. The core issue is that both 26a and 26b represent a legitimate second. This is because the light emitted by cesium oscillated 9,192,631,770 times in both 26a and 26b. This strictly adheres to the definitions of one second and one light-second. As we are well aware of time dilation, the following relationship holds between the time and length in figures 26a and 26b.

$$T > T_o \tag{48}$$

If Equation (48) holds between 26a and 26b, then the length naturally satisfies the following relationship.

$$L > L_o \tag{49}$$

Instead of this inequality, it can be expressed as an equality using the following equation.

$$T = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}\, T_o = \gamma T_o \tag{50}$$

$$L = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}\, L_o = \gamma L_o \tag{51}$$

The derivations of Equations (50) and (51) were already examined in Section 1.6. If there is a problem with Equation (51), it lies in the definition of length. In figure 26b, the length L is the distance traveled by light during 9,192,631,770 oscillations of a cesium atom in a specific state. Crucially, it is impossible to distinguish between 26a and 26b in any inertial frame. Moreover, they are completely equivalent in terms of validity. This is guaranteed by the principle of relativity. Figure 26a has a time unit of one second, and 26b also uses a time unit of one second. The unit length in figure 26a is one light-second(ls), and the unit length in 26b is also one ls. Although the values of one second and one ls differ between 26a and 26b, the ratio of time to distance remains constant. The value of '*unit distance per unit time*' in 26a is exactly the same as the value of '*unit distance per unit time*' in 26b. Although the shape of the wave and the size of the wavelength may vary, the value of the '*unit distance per unit time*' always remains constant. Figure 27 shows the wavelength changes in various cases. Figure 27c shows a blue shift, while 27(d) illustrates continuous changes in wavelength within a gravitational field. Just as the '*unit distance per unit time*' ratio is constant in 27a and 27b, it is also constant in 27c and 27d.

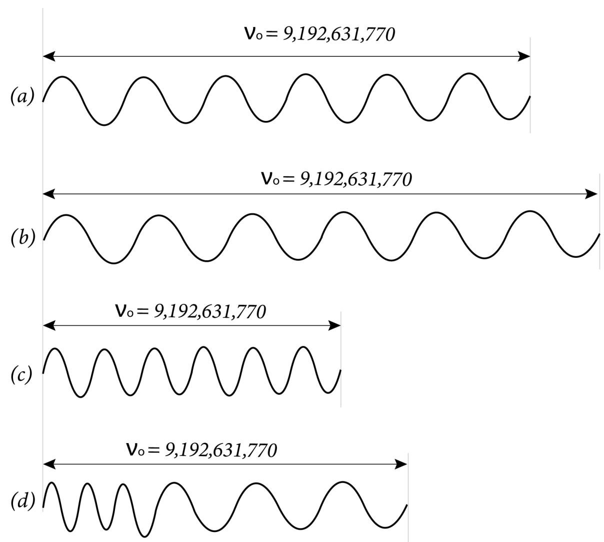


Figure 27: Various forms of unit time and unit length

Therefore, no matter how fast any frame moves, this ratio can only be constant. This is why the speed of light remains constant regardless of the frame's speed. Expressed mathematically, it is as follows:

$$c = \frac{L_{proper}}{T_{proper}} = \frac{L_o}{T_o} = \frac{L_{red}}{T_{red}} = \frac{L_{blue}}{T_{blue}} = \frac{L}{T} = \frac{\gamma L_o}{\gamma T_o} = \frac{L_{grav}}{T_{grav}} = c \tag{52}$$

The different subscripts in Equation (52) represent various reference frames. We define two physical quantities using one. That is, we define both time and length simultaneously using light. When observing the light-clock of a moving counterpart, the wavelength always increases. Therefore, if time dilates as light travels, space naturally dilates as well. Since the rate of increase is constant, the value of '*the unit distance per unit time*' remains constant at all times. This is the key point regarding the first question.

The second question is: '*How can the principle of the constancy of the speed of light be compatible with length contraction?*' In principle, the speed of light must be constant in all inertial frames. However, substituting the time dilation formula ($T = \gamma T_o$) and the length contraction formula ($L = (1/\gamma)L_o$) into the equation describing the speed of light leads to a logical contradiction. Since the speed of light is always constant, it must always hold that $c = L/T = L_o/T_o$. Let us then start with Equation (53).

$$c = \frac{L}{T} \tag{53}$$

$$c = \frac{L \leftarrow (L = 1/\gamma L_o)}{T \leftarrow (T = \gamma T_o)}$$

$$= \frac{(1/\gamma)L_o}{\gamma T_o}$$

$$= \frac{1}{\gamma^2}\frac{L_o}{T_o} \tag{54}$$

$$put)v \to c$$

$$\therefore c = 0 \tag{55}$$

Substituting the length contraction formula into the constancy of light speed yields a speed of light equal to zero. This logical contradiction is entirely attributable to length contraction. Length contraction is not a theory of relativity. It is merely a hypothesis temporarily devised to rescue the ether hypothesis. Once the absence of the ether was confirmed, the length theory of relativity should have been rebuilt from the ground up. Therefore, the solution to the second question is simple: abandon length contraction and introduce length expansion. If length expansion replaces length contraction as the legitimate theory of length in relativity, then the second problem will be resolved automatically following the first.

3.2 Bell's Spaceship Paradox

Two identical spaceships are connected by a thin string. While at rest, this string remains intact. The core question is: what happens to this string when both spaceships move at the same high speed? Many people concluded that '*the string between the two spaceships breaks*'. From a spacetime diagram perspective, their conclusion is entirely logical and reasonable. The space between the two spaceships expanded, and the spaceship shortened due to the length contraction effect, so it is only natural that the string would break. The invariant quantity $s^2 = x^2 + y^2 + z^2 - c^2t^2$ can be expressed in two dimensions as $s^2 = x^2 - c^2t^2$. Figure 28 shows only the first quadrant of this graph; as the spaceship's speed increases, the two axes approach the 45-degree line.

When they are at rest, the two spaceships are connected by a string. The two ends of the rope are points E and F, and its length is d_1. As the spaceship's speed increases, the distance between the points on the string becomes points P and Q, and the distance becomes d_2. Looking closely at the graph, it is clear that $d_2 > d_1$. However, if the spaceship's length decreases due to the length contraction effect, the string must break. Most people think the spaceship is a rigid body, and the string represents spatial distance.

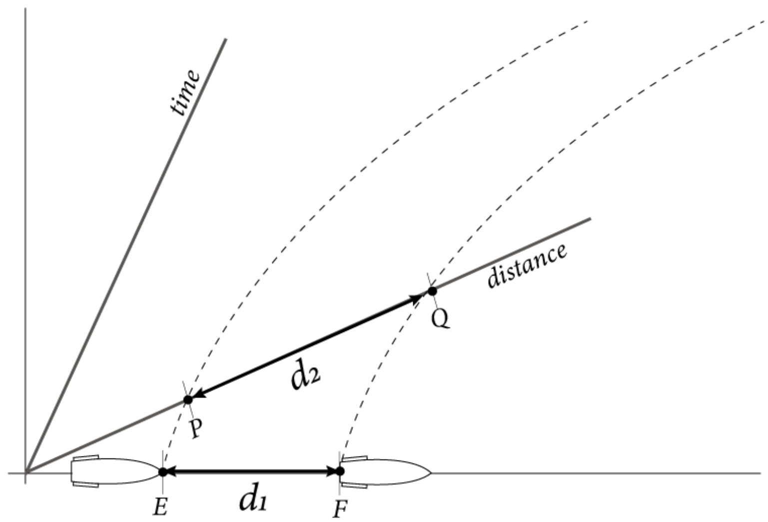


Figure 28: The gap between two spaceships

Therefore, they believe the rigid body shrinks due to the length contraction effect, while the spatial distance increases due to coordinate transformation. The formula Jerrold Franklin used to calculate the distance d_2 is as follows. [4]

$$
\begin{aligned}
x_P &= \gamma(x_E - vt) \\
x_Q &= \gamma(x_F - vt) \\
d_2 &= x_Q - x_P = \gamma d_1 \\
\therefore d_2 &= \gamma d_1
\end{aligned}
\tag{56}
$$

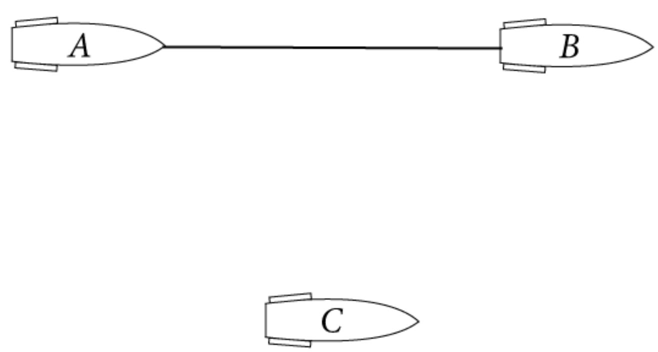


Figure 29: Two different inertial frames

d_2 is γ times longer than d_1. This equation is clearly different from the length contraction. Since this equation differs from the length contraction equation, let's provisionally call it *space expansion*. Maudlin holds a slightly different view, but the conclusion reached by most researchers studying this paradox is that the string breaks due to length contraction and space expansion. However, this view may protect the concept of unverified length contraction, but it raises several serious problems. The first problem is the destruction of the principle of relativity. Spaceships A and B, which are tethered together, and spaceship C are moving relative to each other. (Fig. 29) Let us examine this from the perspective of spaceship C. Spaceships A and B are tethered together and traveling at a constant speed, while spaceship C remains stationary. Therefore, according to his claim, the string could break.

Now let's look at it from the perspective of spaceships A and B instead. Spaceships A and B are stationary, while spaceship C is moving rapidly in the opposite direction. From the perspective of A and B, since they are stationary, they would judge that the string is not breaking, but the string suddenly snaps. The string snapped without cause simply because the C spaceship passed by, while they remained stationary. The snapped string and the intact string are undoubtedly distinct physical phenomena governed by different physical laws. According to the principle of relativity, physical laws must apply identically in all inertial reference frames. In this case, both observers are moving at constant velocity, yet the physical laws appear to apply differently depending on whose perspective is being considered. For observers A and B, the physical law states that the string does not break, while for observer C, it states that the string does break. So, whose claim is correct, and whose observation is true? Does the other person see an illusion? Both observers can be wrong, but both cannot be right. This directly contradicts the principle of relativity. Concluding that the string breaks means abandoning the principle of relativity in order to preserve the length contraction.

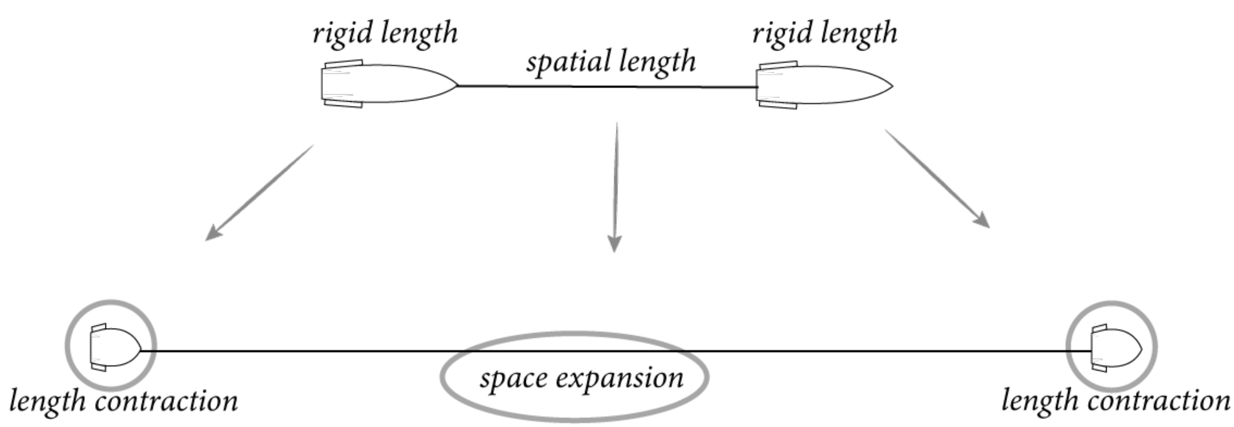


Figure 30: The uncomfortable combination of length contraction and space expansion

Those who claim the string breaks believe that while the space between spaceships expands, the rigid spaceships themselves contract. According to their argument, the phenomena of '*space expansion*' and '*length contraction*' occur simultaneously. (Fig. 30) Are spaceships rigid bodies, but the string is not? Considering that the atoms composing a spaceship are mostly empty space, the spaceship itself is also largely empty space and not an ideal rigid body. When viewing the spaceship and the string as a single object, two different physical laws apply to that one object. This is a regression in science. Maudlin was aware of all these logical flaws, so he made an entirely different argument.

Such a serious problem is actually easily resolved. If interpreted as length expansion rather than length contraction, the problem never arises in the first place.

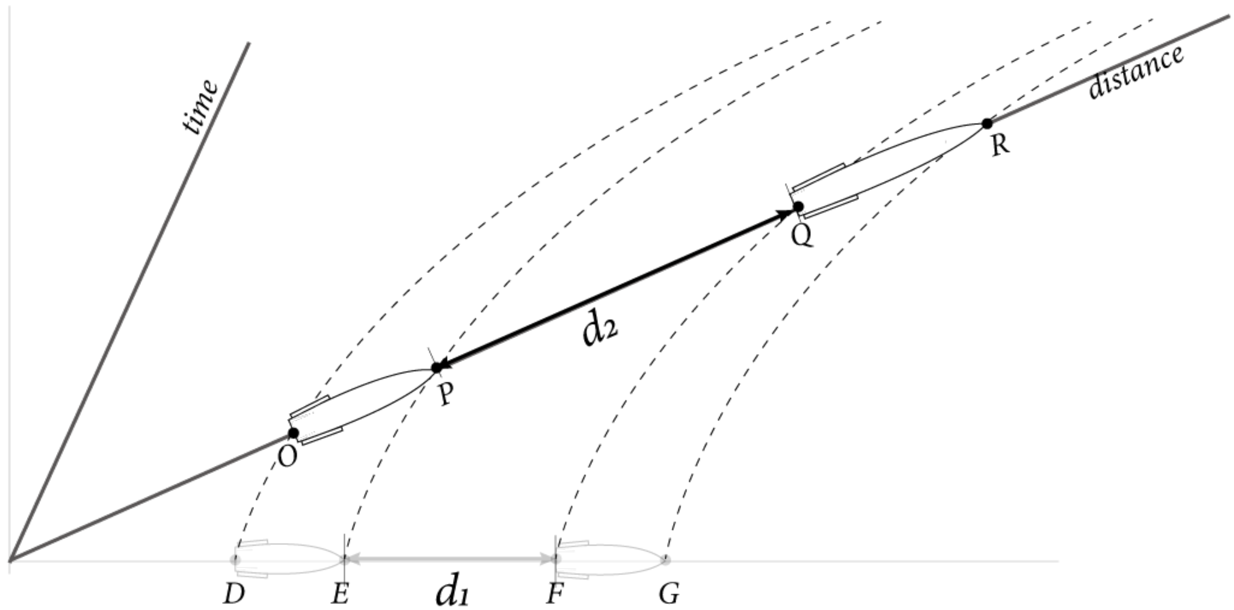


Figure 31: Bell's paradox interpreted through length expansion

See figure 31. Let the two ends of the left spaceship be D and E, and the two ends of the right spaceship be F and G. Then, the distance between the spaceships is (EF). As the speeds of these four points increase, moving their world-lines directly results in points O, P, Q, and R. The left spaceship's segment (DE) increases to become (OP), and the relationship $(OP) = \gamma(DE)$ holds between these two points. This is stretched due to the coordinate transformation, so no tension occurs. The right spaceship also extends from (FG) to (QR), and the relationship $(QR) = \gamma(FG)$ holds between them as well. Since these are stretched due to coordinate transformation, the string between the spaceships experiences no tension at all. By the exact same principle, the line (EF) becomes (PQ), and the relationship $(PQ) = \gamma(EF)$ also holds between them. Naturally, the total length (DG) also equals (OR), and the relationship $(OR) = \gamma(DG)$ holds between them. The overall relationship can be written as follows.

- Left spaceship: $(OP) = \gamma(DE)$
- Right spaceship: $(QR) = \gamma(FG)$
- String between the two spaceship: $(PQ) = \gamma(EF)$
- Sum of the two spaceship and the string: $(OR) = \gamma(DG)$

The interpretation of length expansion is straightforward. Everything simply expands by the Lorentz factor through a coordinate transformation. When length contraction was first proposed, it was based on the assumption of a hypothetical substance called the ether, so factors like stress on the ether were also considered. However, the ether does not exist and should not be considered. Since the spaceship and the string expand together, the string does not break, and the result is consistent regardless of the observer. If everything expands together, observers on spaceships A and B would judge that the string remains unbroken, and observers on spaceship C would agree. Then it would not violate causality, nor would two physical phenomena appear on a single object. Interpreting it as length expansion naturally resolves everything.

3.3 Supplee's submarine paradox

This is a paradox in relativity published by Supplee. [14] There is a submarine with neutral buoyancy, floating at a constant depth in the ocean. What would happen to this submarine if it were to travel through the ocean at relativistic speeds? To those aboard the submarine, the seawater will appear to contract, thereby increasing in density. Therefore, they would conclude that the submarine they are aboard is surfacing. Conversely, people observing the submarine from outside would conclude that it is contracting, increasing its density, and will therefore sink to the seabed. (Fig. 32) Whose judgment is correct? We call this *the submarine paradox*. Let the density of the submarine when stationary be d_1, and the density when moving be d_2.

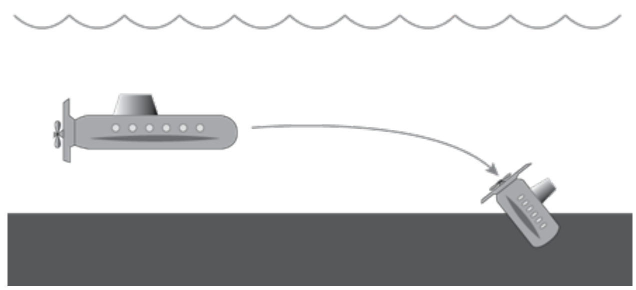


Figure 32: Submarine sinking due to length contraction and density increase

$$d_1 = \frac{M_o}{V_o} \tag{57}$$

$$d_2 = \frac{M}{V} = \frac{\gamma M_o}{(1/\gamma)V_o} = \gamma^2 d_1 \tag{58}$$

$$\therefore d_1 \neq d_2 (d_1 < d_2) \tag{59}$$

This is not a simple paradox. Like Bell's spaceship paradox, it raises significant issues for the validity of the principle of relativity. The principle of relativity states that the laws of physics are the same for all observers in inertial frames, yet this situation implies different physical laws apply depending on the observer. Of course, the ocean where the submarine floats is not an inertial frame, but to simplify the problem, let us assume that in terms of the direction of motion, it behaves like an inertial frame. People inside the submarine will claim it is surfacing, while an observer outside the submarine will claim it is sinking. Length contraction theory offers no good solution. However, length expansion solves this problem very easily. The observational results from inside and outside are the same; their conclusion are identical. Now let's consider this problem using length expansion. (Fig. 33)

$$d_1 = \frac{M_o}{V_o} = \frac{\gamma M_o}{\gamma V_o} = \frac{M}{V} = d_2 \tag{60}$$

$$\therefore d_1 = d_2 \tag{61}$$

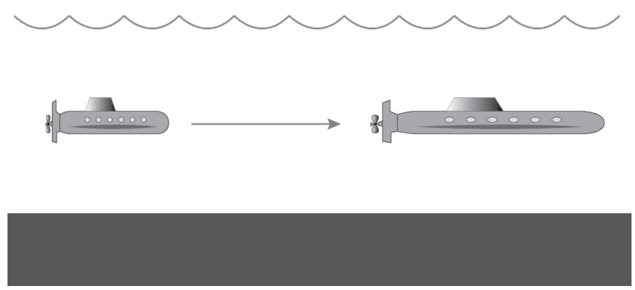
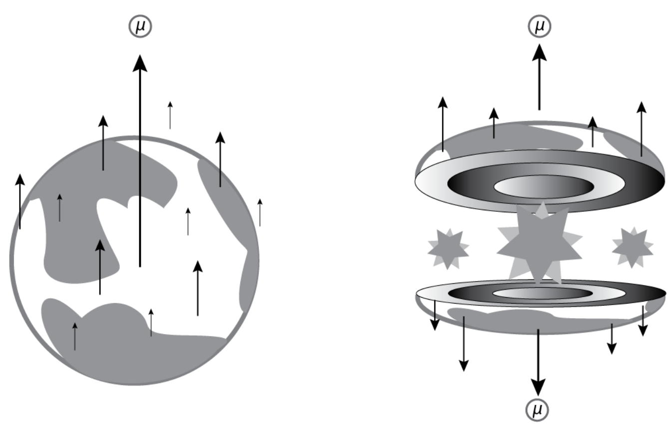


Figure 33: Submarine maintaining neutral buoyancy
due to length expansion and mass increase

Density is mass per unit volume. Just as the speed of light remains constant regardless of an object's speed, the ratio of mass to volume remains constant because both are factors in the numerator and denominator. Therefore, as shown in Equation (60), a submarine with neutral buoyancy when stationary maintains that same neutral buoyancy while in motion. If a submarine moves quickly, its length increases, its volume expands, and its mass increases proportionally by the same amount. Therefore, as shown in Equation (60), its density remains unchanged. Consequently, whether stationary or moving, regardless of speed or direction, the submarine always floats with neutral buoyancy. Consequently, observations made by observers inside the submarine and those outside yield consistent results. Interpreting this as length contraction presents a highly perplexing problem, but interpreting it as length expansion makes everything make sense.

3.4 Two Muon Paradoxes

The core of the muon paradox is that muon particles, which should not have been able to reach the sea surface, did reach it. Viewed through the lens of classical theory, this is impossible. [12] Yet it is a natural phenomenon that muon particles, created in the sky, travel long distances and eventually reach the sea surface. A good theory must be able to explain this natural phenomenon logically and accurately. There are two ways to explain this phenomenon. One uses time dilation, and the other uses length contraction. The method using time dilation to explain this phenomenon is very natural and logical, but the method using length contraction to explain it is not. When explaining this phenomenon through length contraction, everything is described from the muon's perspective. It is claimed that the Earth rapidly approaches the muon. Therefore, the distance from the muon's point of origin to the sea surface contracts, allowing the muon to reach that distant location. (Fig. 34a) However, this is not science. Humans have never observed this phenomenon from the muon's perspective, nor can they ever do so. Did Frisch and Smith observe this phenomenon from the muon's perspective? That's not it. They observed this phenomenon from Earth, from the perspective of Earthlings. Therefore, interpreting the phenomenon of muons reaching sea level from the muon's perspective is an unverified fact. Only interpretation from the perspective of Earthlings is possible. Truly, to claim that observation or interpretation from the muon's perspective is possible, one would need to present evidence of observing Earth while riding a muon. If observation from the muon's perspective is possible, a series of very serious problems arise. From the muon's perspective, it is at rest, and the Earth must be moving rapidly toward it at a relativistic speed. (Fig. 34a) If there's only one muon, that might be possible, but what if muons fall simultaneously at both the North and South Poles? Should Earth race toward the northern muons, or should it race toward the southern muons? If neither, then must Earth split apart? (Fig. 34b)



(a) (b)

Figure 34: Earth split by two subatomic particles

A series of hard-to-explain logical flaws occurs. Explaining it through length expansion makes everything logical. If we say the time and distance the muon can travel increase, allowing it to reach sea level, the issue is easily resolved. There's no need to consider it from the muon's perspective; we can interpret this phenomenon from the Earthling's viewpoint. This does not lead to bizarre logic such as the Earth rushing toward muons or the Earth splitting apart.

This phenomenon occurs not only in the Earth's atmosphere but also inside circular accelerators. The phenomenon of muon lifetime extension is not unique to the natural world. Within particle accelerators, muons travel at extremely high speeds (Fig. 35a), sometimes reaching a Lorentz factor of 30. Classically interpreted, this particle should only travel 650 meters and should not be detected beyond that distance. However, this particle is being detected at distances far greater than this. From the perspective of classical theory, this is a paradox. Furthermore, the theory of length contraction only deepens this paradox. When interpreting this phenomenon, if one insists on explaining it using the theory of length contraction, there is one solution. This interprets the problem from the muon's perspective, similar to the previous "muon reaching sea level" phenomenon. From the muon's perspective, it is not the muon rotating inside the circular accelerator, but rather the Earth rapidly rotating relative to the muon. From the muon's perspective, this is equivalent to the Earth rushing toward the muon, allowing the muon to reach sea level. This creates an even more serious problem than before. Because the particle accelerator is attached to the Earth's surface, for this to be possible, the Earth would have to rotate 10,000 times per second. (Fig. 35b) Then neither the Earth nor anything on it would remain. No one would actually think this could happen. Going further in this situation, let us consider the case of two muons rotating in opposite directions. To collide two particles within a particle accelerator, they must be made to rotate in opposite directions. Then, in which direction should the Earth rotate for these particles? Thus, this method of interpretation using the theory of length contraction only generates additional paradoxes.

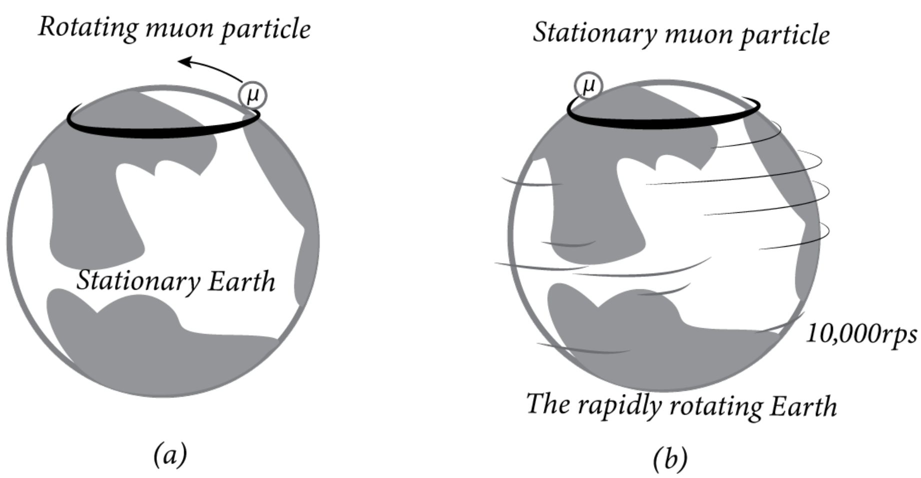


Figure 35: Earth rotating 10,000 times per second

The solution to this contradictory situation is surprisingly simple. Simply abandon the length contraction hypothesis and adopt the length expansion theory. Interpreting it as length expansion is equivalent to observing from the Earth's perspective. When observed from the Earth's perspective, the muon is rotating while the Earth remains stationary. Everything is normal.

3.5 Ehrenfest Paradox

It is known that objects moving at relativistic speeds undergo length contraction. Then what happens to objects rotating at high speeds? The Ehrenfest Paradox arose during the exploration of this question. [15] [16] [17] Understanding this through the theory of length contraction is very difficult. Many people have researched this problem to interpret it, but no satisfactory results have yet emerged. Objects rotating at nearly relativistic speeds are not found on Earth, but they are actually observed around black holes. Therefore, analysis of high-speed rotating objects is absolutely necessary in this situation. Let us approach this problem assuming that length contraction is correct. If rotating matter possesses sigma bonds, these substances will separate as orbital overlap decreases when rotating rapidly. (Fig. 36) Consequently, not only molecules, atoms, and protons, but any state whatsoever will be unable to form bonds.

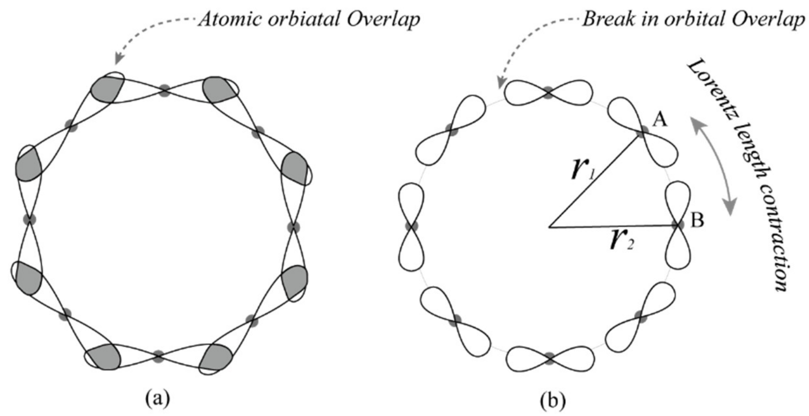


Figure 36: Rotating objects interpreted by the theory of length contraction
(a) Object rotating at low speed
(b) Object rotating at high speed

Their radii r_i remain constant, and the distance between points A and B does not change. At high speeds, the two orbitals eventually separate due to Lorentz contraction. Therefore, most objects rotating relativistically find it difficult to maintain a bonded state. Interpreted this way, the existence of rotating matter around a black hole cannot be explained.

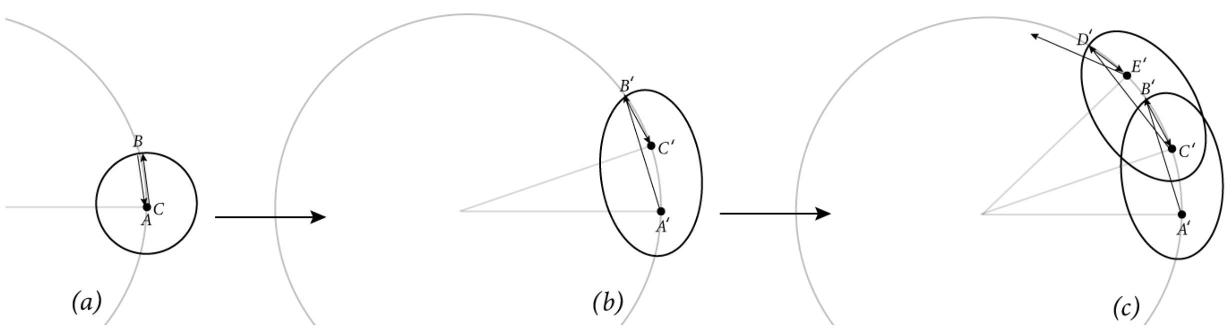


Figure 37: Spherical mirror in circular motion, light paths within it, and vibration mechanism

However, explaining it through length expansion makes this problem relatively easy to understand. Figure 37 illustrates *the ellipse theorem* in rotational motion. (Section 1-3, Ellipse Theorem: When a spherical mirror moves relatively fast, the starting and arrival points of light are the foci of an ellipse, and the sum of the reflection points forms an ellipse.) Figure 37a shows the path of light within a stationary spherical mirror, while figure 37b shows the path of light within a rotating spherical mirror. Figure 37c shows the continuous mechanism of 37b. Just as with linear motion, when considering rotational motion in small increments, it must satisfy the ellipse theorem. Line segment $A'B'C'$ is longer than line segment ABC. (Fig. 37b) This indicates that lengths do not shorten but lengthen when traveling at relativistic speeds, demonstrating that length expansion, not contraction, is correct. This follows the same logic as the validity of Equation (49) in Chapter 3.1. Inside a spherical mirror moving in a circular path, the following equation holds.

$$L_{A'B'C'} > L_{ABC} \tag{62}$$

Applying the ellipse theorem, as shown in figure 37c, the previous focus of the ellipse becomes the starting point for the next cycle, allowing light to propagate continuously. Thus, the circumference of an object moving in a circular path at high speed increases by a factor of γ. Therefore, the circumference of a rapidly rotating circle satisfies Equation (63).

$$2\pi\gamma r > 2\pi r \tag{63}$$

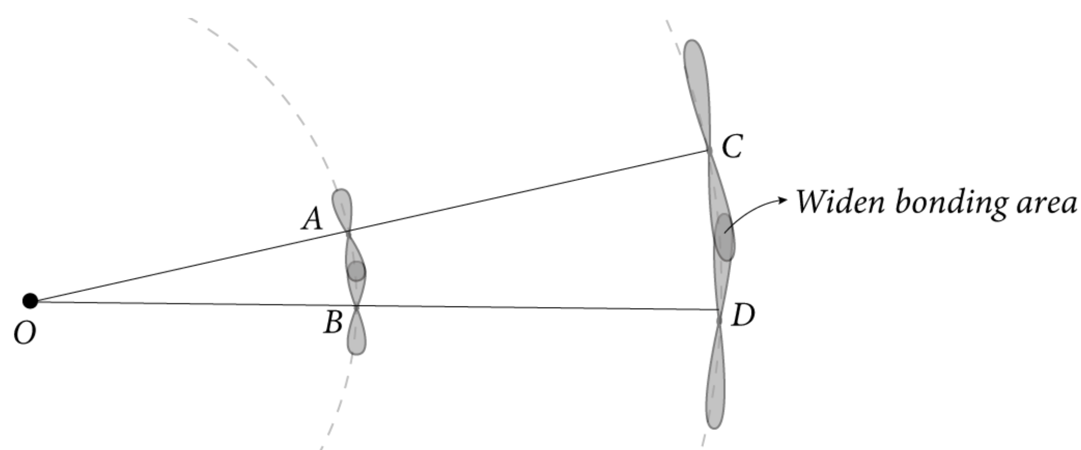


Figure 38: The overlap region of orbitals increases due to length expansion, making it possible to explain the existence of matter

As the circumference of a rapidly rotating circle increases, the overlap of the object's molecular orbitals also increases. (Fig. 38) As the orbitals extend in the direction of motion, their overlap increases, thereby preventing the substance from decomposing and allowing it to maintain its bonded state continuously. At this point, the radius remains constant regardless of speed. It may seem strange that the radius remains unchanged while the circumference increases by a factor of γ. However, this issue is resolved by considering that the length is measured by the light's round-trip motion within the light-clock or spherical mirror. The path length of the circular motion of the spherical mirror($2\pi r$) and the path length of the light oscillating within it ($2\pi\gamma r$) can be considered separately. Because the light in the light-clock undergoes round-trip motion while the spherical mirror moves in a circle, the radius remains constant even as the circumference increases. Thus, it becomes possible to explain the existence of objects rotating at relativistic speeds.

4 Conclusion

The theory of relativity became a major theory of modern science, recognized for its logical completion of electromagnetism and its predictive power over natural phenomena. Behind this success, there are several additional points that warrant examination. One question is why the speed of light remains constant regardless of the speed of the light source or the observer. When combined with the phenomenon of length contraction, it creates a critical logical flaw. Another issue is that it exhibits too many contradictions when interpreting various observational phenomena or thought experiments. The theory of relativity, which emerged to correct the flaws of classical mechanics, itself exhibits deficiencies in several areas. Many of these deficiencies are related to length contraction. To logically understand the principle of the constancy of the speed of light, various experimental values, and the paradoxes without contradiction, we must set aside length contraction and introduce length expansion. Introducing length expansion resolves many issues. It strengthens the logic of relativity itself and further enhances its ability to predict new natural phenomena. Therefore, the legitimate theory of length in relativity should be length expansion, not length contraction.

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