Cosmic Structure Formation from Cantor Dust

Ervin Goldfain

Global Institute for Research, Education and Scholarship (GIRES).

Email: ervingoldfain@gmail.com

Abstract

We outline an unconventional framework for cosmic structure formation in which the underlying Dark Matter (DM) distribution is modeled as a multifractal Cantor Dust (CD). In this scenario, the fractal geometry of CD generates scale-dependent gravitational potentials that seed the collapse of baryonic matter without requiring primordial gravitational fields or particle-like DM clumps. DM resides on a multifractal support with Hausdorff dimension $D < 3$, a *gravitational scaffolding* which traps baryons into filaments, walls, and voids. Cooling baryonic gas undergoes hierarchical collapse, producing structures from the proto-galactic to stellar scale. This approach can potentially explain the emergence of Newtonian-like galaxies in the interiors of fractal potentials while resolving conceptual issues inherent in standard cosmology, including the cusp–core problem, angular momentum loss, and early formation of massive galaxies. Our results suggest that *multifractal gravitational backgrounds* provide a physically consistent and predictive alternative paradigm for the formation and evolution of cosmic structures.

Key words: Dark Matter, Cantor Dust, Cosmic Web, multifractal mass distribution, Jeans instability, baryon collapse, baryon condensation.

1. Introduction and Motivation

Understanding the formation of baryonic structures in the Universe remains a central problem in cosmology. In the standard ΛCDM paradigm, galaxies form through hierarchical merging within cold DM halos, with baryons following the gravitational potential of collisionless DM. While successful on large scales, ΛCDM faces persistent conceptual challenges, including the cusp–core problem in dwarf galaxies, the overproduction of small-scale substructure, the angular momentum catastrophe in disk formation, and the need for finely tuned feedback processes to regulate star formation.

An alternative approach considered here is the gravitational field generated not by discrete DM particles, but by a *multifractal mass distribution* produced by the continuous and evolving spacetime dimensionality near the Planck scale. Fractal or multifractal mass distributions naturally appear in analyses of large-scale structure and provide a continuous, scale-dependent gravitational field. Our approach suggests integration of fractal geometry, thermodynamic dissipation, and angular momentum transport, leading to a self-consistent picture of baryonic structure formation.

The paper is organized as follows: working assumptions are listed in the next section; section 3 elaborates on the path leading from CD to multifractal mass distributions; section 4 explains how fractal measures carry gravity, sections 5 and 6 delve into the topic of baryonic structure formation. Conclusions are detailed in the last section. An Appendix section is also included.

We caution upfront that this paper is exclusively an *exploratory work* inspired by our previous contributions. Aiming for transparency and accessibility, the paper is kept at an introductory level and formatted in a "bulleted" style, without excessive technical information.

2. Working assumptions

We assume throughout:

A1. Gravity is classical and treated as a weak field (Newtonian limit is valid on galactic scales).

A2. Mass is distributed according to a fractal measure μ, not a smooth density.

A3. The support of μ is a Cantor-type set, but physical observables are defined via coarse-graining at finite resolution ℓ.

A4. Baryons are initially considered test particles, subject to dissipation.

3. From Cantor Dust (CD) to multifractal mass distributions

We recently argued that fluctuations in the control parameter of one benchmark prototype of *complex dynamics*, the Stuart-Landau equation, generate progressive amplitude fragmentation, a process mimicking the iterative construction of Cantor sets [1, 8 - 9]. The emerging structure freezes in a cosmological phase known as *Cantor Dust*, a spontaneously broken condensate whose effective dimension flows from an integer dimension d to a non-integer dimension $< d$.

The geometric construction of Cantor Dust (CD) provides a classic example of a measure defined on a fractal set. At each iteration, the set is divided and scaled by a fixed (or random) ratio, resulting in a *sparse distribution* (Fig.1).

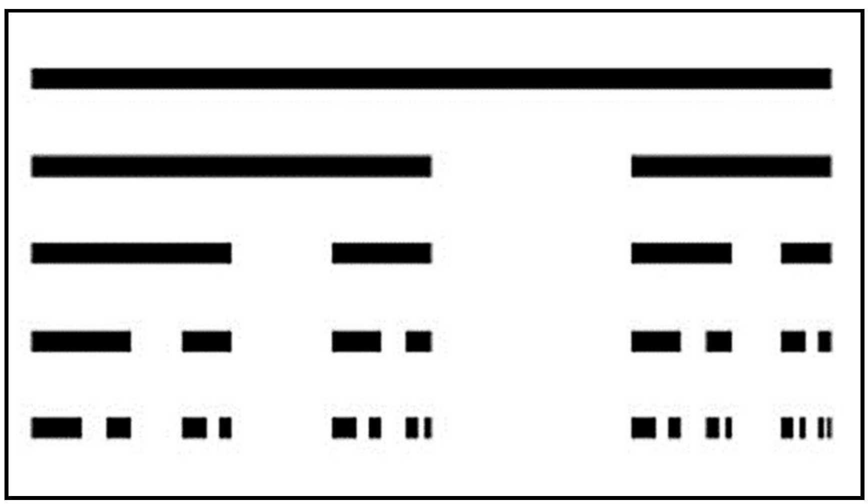

Fig. 1: Recursive construction of a two-scale Cantor set

As a result, the CD topology has three defining features,

1) *Zero Lebesgue measure*

Total mass → 0 unless the mass density ρ diverges in some erratic way.

2) *Disconnected support*

No continuous mass distribution ⇒ no smooth Newtonian potential.

3) *No well-defined Poisson equation, since*

$$\nabla^2 \Phi = 4\pi G \rho$$

breaks down if ρ is a singular measure.

Because CD is a sparse distribution that cannot be described by a continuous density function, we adopt below a different mass definition consistent with the concept of *measure*.

Let $\mathcal{C} \subset \mathbb{R}^3$ be a Cantor Dust structure built by iterative removal of segments with scale ratio $0 < \lambda < 1$ and iteration number N. By definition, the total mass of dust is a probability measure μ (also called *Radon measure*) on $\mathcal{C}$ such that [2 - 3]

$$\mu(\mathcal{C}) = M_{\text{tot}}$$

The Hausdorff dimension of the dust is given by

$$D = \frac{\log N}{\log(1/\lambda)}, \qquad 0 < D < 3$$

where $D = 1$ corresponds to 1D *filamentary dust,* whereas $D = 2$ and $D = 3$ denote, respectively, 2D $sheets$ and 3D $volume - filling$ dust.

Next, define the mass inside a ball of radius r:

$$M(r) \equiv \mu(B_r)$$

Then, by definition, the Hausdorff measure obeys the scaling law:

$$\boxed{M(r) \sim r^D}$$

Now**,** let's introduce the *coarse-grained density* at a resolution scale equal to the ball radius, $r = \ell$:

$$\rho_\ell(x) = \frac{\mu(B_\ell(x))}{\ell^3}$$

Using the scaling law written above, we find:

$$\rho_\ell(x) \sim \ell^{D-3}$$

which means that, since $D < 3$, the density diverges as the resolution scale drops down but remains finite for any physical $\ell > 0$.

Furthermore, allowing the local scaling exponent to vary:

$$\mu\big(B_r(x)\big) \sim r^{D(x)}$$

introduces a singularity spectrum $f(\alpha)$, where $\alpha = D(x)$. This spectrum defines a *multifractal mass distribution*, not a mono-fractal CD. Stated differently, once coarse-grained, CD necessarily generates a multifractal mass distribution characterized by local scaling exponents $D(x)$.

A key observation is that, unlike the mass observable of standard physics (inertial attribute or energy content), the mass defined in the CD framework is strictly a *topological concept.*

4. How a fractal measure carries gravity

In the weak field limit, the Newtonian potential satisfies

$$\nabla^2 \Phi = 4\pi G\, \mu$$

In the context of CD, this is understood in the *distribution sense*:

$$\int \nabla\Phi \cdot \nabla\psi \, d^3x = 4\pi G \int \psi \, d\mu$$

for all test functions ψ. Define next the spherically averaged mass for a ball radius r ,

$$M(r) = \int_{|x|<r} d\mu$$

By Gauss's theorem (valid for measures):

$$\frac{d\Phi}{dr} = -\frac{GM(r)}{r^2}$$

Insert the scaling:

$$\frac{d\Phi}{dr} \sim r^{D-2}$$

and integrate to obtain:

$$\boxed{\Phi(r) \sim \begin{cases} r^{D-1}, & D \neq 1 \\ \ln\, r, & D = 1 \end{cases}}$$

This is the *gravitational potential generated by a CD mass distribution*. The three gravitational regimes induced by this potential are discussed in the Appendix A.

Two important clarifications are now in order:

- Throughout this work, the radial variable r plays a *dual* role. It denotes both 1) the radius of the ball B_r used to define the spherically averaged enclosed mass $M(r) = \mu(B_r)$ of the CD measure and 2) the physical radial distance from the center of the resulting spherically averaged gravitational potential. These two notions coincide once statistical isotropy is assumed, since the gravitational field depends only on the *integrated mass* within B_r and all baryonic observables (rotation curves, lensing, orbital dynamics) are functions of this radial distance alone.
- There is a key distinction between the *intrinsic dimension* (D_{int}) which characterizes the geometry of the mass support itself (e.g.,

filaments, sheets), and the *enclosed dimension* D which governs the radial scaling of the mass contained within a sphere. For statistically isotropic embeddings in 3D space, spherical averaging introduces one additional dimension,

$$D = D_{\text{int}} + 1,$$

subject to the upper bound $D \leq 3$. This relation follows from geometric measure theory and reflects the integration of a lower-dimensional set over the radial direction. Consequently, intrinsic dimensions should never be inserted directly into force or potential formulas without first converting to the enclosed dimension.

Concluding this section,

- CD does not gravitate as a *density*; it gravitates as a *measure*.
- Coarse-grained measures generate multifractal mass scaling.
- Multifractal mass scaling generates long-range gravitational wells if the dust dimension satisfies $2 < D < 3$.

- These wells trap baryons and seed galaxies.

5. Baryonic structure formation from CD-induced gravitation

5.1 Dynamical equations for baryons

In classical Euler hydrodynamics, a baryonic fluid element of density ρ_b obeys the Newton equation of motion,

$$\boxed{\ddot{\mathbf{x}} = -\nabla\Phi - \frac{1}{\rho_b}\nabla P + \mathbf{a}_{\mathrm{diss}}}$$

where Φ is the potential, P the pressure and $\mathbf{a}_{\mathrm{diss}}$ the acceleration produced by dissipative forces. Neglecting pressure and the dissipative acceleration and recalling that mass $M(r)$ follows the fractal scaling law

$$M(r) \sim r^D$$

leads to gravitational acceleration

$$g(r) = |\nabla\Phi| = \frac{GM(r)}{r^2} \sim r^{D-2}.$$

and the radial equation of motion becomes:

$$\boxed{\ddot{r} = -k\, r^{D-2}}$$

with $k = GM_0$. Note that the above equation is not assumed — it follows directly from the combined use of the Gauss theorem along with fractal mass scaling.

There are 3 cases of interest here,

- For $D = 3$: harmonic potential
- For $D = 2$: constant acceleration
- For $2 < D < 3$: sub-harmonic confining force

The last case shows that, for an enclosed mass dimension $2 < D < 3$, the gravitational acceleration scales as $g(r) \propto r^{D-2}$, yielding a force that increases with radius but more slowly than the harmonic case $g \propto r$, hence producing *sub-harmonic confinement*. In this situation, baryons are trapped into a smooth, long-range and confining potential, despite evolving in a background with ill-defined density ρ .

5. 2 Jeans instability in a fractal gravitational background

Jeans instability explains how vast, diffuse interstellar clouds transform into the observed stars and stellar systems, connecting thermodynamic cloud properties (density, temperature) to the rate and scale of star formation. Specifically, it describes the gravitational collapse of gas clouds in space, leading to star formation, and occurring when gravity exceeds internal gas pressure [4 - 6].

In the context of our work, we first define the *free-fall time*:

$$t_{\mathrm{ff}}(r) \sim \sqrt{\frac{r}{g(r)}} \sim r^{(3-D)/2}$$

Next, define the *sound-crossing time*:

$$t_s \sim \frac{r}{c_s}$$

Collapse occurs if:

$$t_{\mathrm{ff}} < t_s \Rightarrow r^{(3-D)/2} < \frac{r}{c_s}$$

which yields:

$$\boxed{r > r_J \sim c_s^{\frac{2}{D-1}}}$$

It follows that this generalized Jeans length exists *only if* $D > 1$.

6. Dissipation and baryonic condensation

Baryons radiate, collide, and lose angular momentum. They settle into local minima of the smooth fractal potential. The dark sector remains diffuse — baryons *do not trace* the Cantor dust microscopically. Once baryons begin to dominate locally,

- the potential becomes locally Newtonian,
- star formation proceeds as in standard cosmology,
- galaxy morphology decouples from fractal support.

In summary,

- CD multifractal gravity produces extended, smooth potential wells that naturally seed baryonic collapse and galaxy formation,
- Fractality is only gravitationally visible.

7. Conclusions

We have outlined a self-consistent framework for baryonic structure formation in which the gravitational potential is provided by a multifractal CD distribution. The key results are:

1. **Fractal gravitational potentials:** The Cantor dust generates scale-dependent gravitational fields that seed baryonic collapse without invoking particle dark matter halos.
2. **Hydrodynamic baryonic collapse:** Our approach hints that dissipative processes in the baryonic fluid, including radiative cooling and viscous angular momentum transport, drive condensation into early galaxies and stars.

3. **Generalized Jeans instability:** Collapse occurs hierarchically as the Jeans scale dynamically decreases with cooling, leading naturally to multi-scale structure formation.
4. **Emergence of Newtonian galaxies:** Baryons dominate the local potential in collapsed regions, producing galaxies with standard Newtonian dynamics while retaining the fractal gravitational envelope at larger scales.
5. **Conceptual advantages over ΛCDM:** This framework can potentially avoid small-scale problems of standard cosmology, including the cusp–core issue, angular momentum loss, and excessive reliance on feedback, offering a physically transparent mechanism for early galaxy formation.

Overall, modeling DM as a multifractal CD provides a predictive alternative to particle-based DM models. It unifies the formation of baryonic structures with a fundamental geometric property of the underlying mass distribution and suggests new directions for testing multifractal cosmology via rotation curves, weak lensing, and high-redshift galaxy surveys.

Bottom line is that, while classical gravity of General Relativity remains fundamental in the early and late stages of Universe evolution, it *emerges* from CD and multifractal mass distributions in the primordial Universe.

Our plan is to expand these findings in an upcoming contribution [7]. It is found that the gravitational behavior of CD unifies galactic rotation curves, Tully–Fisher scaling, MOND phenomenology, and weak lensing as direct consequences of Newtonian gravity acting on a fractal mass measure. Once gravity is sourced by a *fractal measure*, in addition to baryon structure formation, phenomena like weak lensing and merger phenomenology become geometrically constrained and acquire natural explanations - as many puzzles of ΛCDM cosmology appear to simultaneously go away.

APPENDIX A:

Gravitational regimes from Cantor-Dust mass distributions

In this Appendix section we classify the gravitational potential and force generated by a spherically averaged Cantor-Dust mass distribution. Throughout this section, the symbol D denotes the *enclosed (radial) mass dimension* defined by the scaling of the mass measure within a ball of radius r,

$$M(r) \equiv \mu(B_r) \sim r^D.$$

This definition must be clearly distinguished from the *intrinsic* (Hausdorff or correlation) dimension of the underlying fractal support. The results below depend only on the enclosed scaling $M(r)$, irrespective of the microscopic geometry.

For a spherically averaged mass measure, Gauss' law remains valid in the weak sense,

$$\oint_{S_r} \nabla\Phi \cdot d\mathbf{S} = 4\pi G M(r),$$

which yields the radial gravitational acceleration

$$g(r) = -\frac{d\Phi}{dr} = \frac{GM(r)}{r^2} \sim r^{D-2}.$$

Integrating once gives the asymptotic potential,

$$\Phi(r) \sim \begin{cases} r^{D-1}, & D \neq 1, \\ \ln\, r, & D = 1. \end{cases}$$

These relations provide a complete classification of gravitational regimes. The qualitative behavior of the force and potential depends on the value of D:

- $0 < D < 1$: The force decays faster than Newtonian and the potential converges at large radius. Such configurations are weakly binding and non-confining.
- $D = 1$: The force scales as $g(r) \sim r^{-1}$and the potential grows logarithmically. This corresponds to marginal binding, analogous to an isothermal configuration.

- $1 < D < 2$: The force still decays with radius but more slowly than Newtonian. Rotation curves decline, and the potential grows sublinearly. These configurations describe extended but non-confining halos.
- $D = 2$: The force is constant, $g(r) = \text{const}$, and the potential grows linearly. The circular velocity is independent of radius, yielding flat rotation curves.
- $2 < D < 3$: The force increases with radius as $g(r) \sim r^{D-2}$while remaining weaker than the harmonic case. The potential grows faster than linear but slower than quadratic, producing sub-harmonic confinement.
- $D = 3$: The force is linear in radius and the potential is quadratic, corresponding to harmonic confinement as generated by a uniform density distribution.

This classification exhausts all possible large-scale gravitational behaviors arising from Cantor-like mass measures in 3 dimensions.

References

1. Goldfain, E., (2025), "Dark Matter as Cantor Dust: Numerical Evidence", preprint https://doi.org/10.13140/RG.2.2.29108.92807/1

2. Falconer, K. J., (1985), *The Geometry of Fractal Sets*, Cambridge Univ. Press.

3. Evertsz, C. J. G. and Mandelbrot, B. B., (1992), "Multifractal Measures", in *Chaos and Fractals, New Frontiers of Science,* Springer-Verlag.

4. Peebles, P. J. E., (1980), *The Large-Scale Structure of the Universe,* Princeton Univ. Press.

5. Peebles, P. J. E., (1993), *Principles of Physical Cosmology*, Princeton Univ. Press.

6. Binney J. and Tremaine S., (2008), *Galactic Dynamics (2nd Edition)*, Princeton Univ. Press.

7. Goldfain, E., (2025), "Cantor Dust as Comprehensive Solution of the Dark Matter Puzzle", work in progress.

8. Goldfain, E., (2024), "Cosmological Structure Formation and Fractal Spacetime", preprint https://doi.org/10.32388/WUQZE8

9. Goldfain, E., (2023), "Dark Matter as Dimensional Condensate", preprint https://doi.org/10.32388/8A8579