

On the Planck mass and length

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Novi Sad, December 2025

Keywords: Planck, proton, gravitational constant, CODATA

This paper presents a somewhat shortened version of a work previously published on the General Science Journal platform, with a simultaneous correction of the sign in equation (4).

The paper presents two complementary formulas for the Planck mass and the Planck length, which yield higher accuracy than the values reported in the CODATA reports. The formulas are proton-oriented and are analyzed on the basis of data from the CODATA reports covering the period 1986–2022. For this reason, we first present the input data used [1].

Table 1 – CODATA input data

godina	m_p ($\times 10^{-27}$ kg)	$\mu = m_p / m_e$	inverzno α	λ_p ($\times 10^{-16}$ m)
1986	1.67262310	1836.152701	137.0359895	2.10308937
1998	1.672621587	1836.1526675	137.035999760	2.103089090
2002	1.672621710	1836.1526726	137.03599911	2.1030891035
2006	1.672621637	1836.15267247	137.035999679	2.1030890862
2010	1.672621777	1836.15267245	137.035999084	2.10308910469
2014	1.672621898	1836.15267389	137.035999139	2.10308910108
2018	1.67262192369	1836.152673430	137.035999084	2.10308910336
2022	1.67262192595	1836.152673426	137.035999177	2.10308910050

The significance of this work lies in demonstrating the validity of two assumptions presented in [2], related to the proton and the mathematical constant $e^{2\pi}$, here adapted in the exponent of the formulas.

$$m_{pl} = m_p * (2\pi)^{-3/4} * 2^{cy/8 - 3/2 + 3/(4\alpha\mu + 8)} \quad (1)$$

$$l_{pl} = \lambda_p * (2\pi)^{3/4} * 2^{-(cy/8 - 3/2 + 3/(4\alpha\mu + 8))} \quad (2)$$

Here,

$$cy = e^{2\pi} = 535.491656$$

is a mathematical constant, while the known physical constants are m_p , μ , α , and λ_p , listed in the header of Table 1. **Note:** In the formulas presented, the constant α is used, whereas its inverse value, α^{-1} , is given in the table.

The aim of this paper is to analyze the effectiveness of the proposed formulas. Their derivation is omitted, as it would exceed the primary scope of the work and repeat earlier considerations

(see, for example, [2, Figs. 8 and 13]). In this sense, the paper constitutes a further application of the same conceptual framework.

The complementarity of the formulas becomes evident in the fact that the second and third terms have opposite signs; when multiplied, they cancel each other out, leaving the relation:

$$m_{pl} * l_{pl} = m_p * \hat{\lambda}_p = \hbar / c \quad (3a)$$

which represents another well-known form of the expression for \hbar/c .

Throughout the history of the CODATA reports, formulas (1) and (2) yield the results shown in Table 2, alongside the corresponding CODATA values.

Table 2 – CODATA values and results obtained using the formulas

godina	CODATA m_{pl} ($\times 10^{-8}$ kg)	Formula (1) m_{pl} ($\times 10^{-8}$ kg)	CODATA l_{pl} ($\times 10^{-35}$ m)	Formula (2) l_{pl} ($\times 10^{-35}$ m)
1986	2.176 71(14)	2.176 5116	1.616 05(10)	1.616 198 977
1998	2.176 71(16)	2.176 5097	1.616 0(12)	1.616 198 761
2002	2.176 45(16)	2.176 5098	1.616 24(12)	1.616 198 772
2006	2.176 44(11)	2.176 5097	1.6162 52(81)	1.616 198 759
2010	2.176 51(13)	2.176 5099	1.616 199(97)	1.616 198 773
2014	2.176 470(51)	2.176 5101	1.616 229(38)	1.616 198 770
2018	2.176 435(24)	2.176 5101	1.616 255(18)	1.616 198 772
2022	2.176 434(24)	2.176 5101	1.616 255(18)	1.616 198 770

The uncertainty of the results obtained using both formulas is approximately $\approx 5 \times 10^{-12}$, which is about five times smaller than the uncertainties reported in the CODATA reports.

Unified analysis (Planck mass and Planck length)

The same pattern applies to both the Planck mass and the Planck length: formulas based on proton-related constants (μ , α , m_p , $\hat{\lambda}_p$) yield numerically stable values with significantly lower uncertainty compared to historical CODATA measurements. This implies that, had this relation been known earlier, it would have enabled more precise estimates of the fundamental Planck scales than those available at the time, without introducing additional empirical parameters.

In this sense, the obtained result does not appear to be a mere numerical coincidence, but rather a manifestation of a structural relationship between the Planck quantities and the proton scale, which becomes apparent only when the constants are considered in combination rather than in isolation.

In all CODATA editions up to 2014, the experimentally determined value of the Planck length exhibits a significantly larger uncertainty than the value obtained from the formula. The formula yields a stable central value with an almost constant standard deviation that is four to five times

smaller. From 2010 onward, the CODATA central values practically converge toward the value given by the formula, albeit still with a larger experimental uncertainty.

Mirror symmetry of mass and length deviations

The CODATA values of the Planck mass and the Planck length exhibit almost mirror-symmetric deviations with respect to the values obtained from the formula: in earlier editions, the Planck mass is systematically higher and decreases over time toward the theoretical value, while the Planck length is systematically lower and increases toward it. This behavior is not accidental, but mathematically expected and follows from their structural connection through the relation:

$$m_{pl} * l_{pl} = \sqrt{\hbar c / G} * \sqrt{\hbar G / c^3} = \hbar / c \quad (3b)$$

which implies that the relative error in the gravitational constant G enters:

with a negative sign in m_{pl} ,

with a positive sign in l_{pl} .

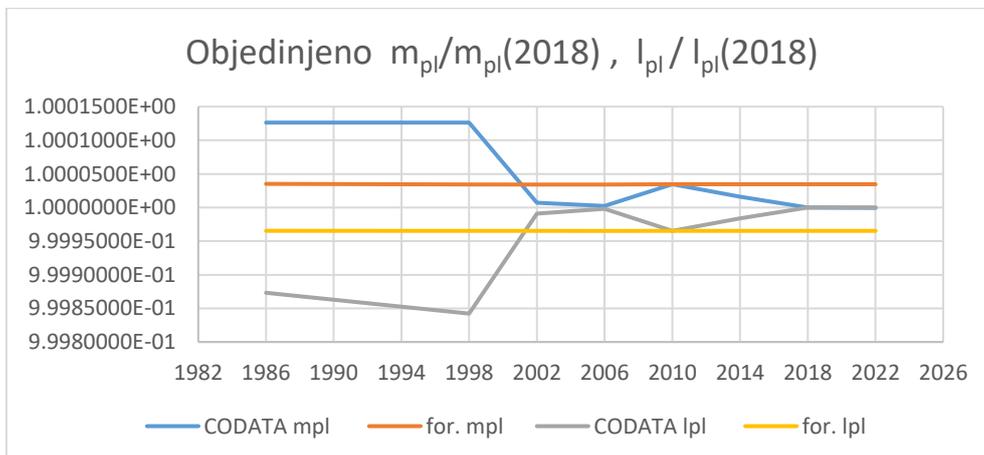
Mathematically expressed:

$$-\delta m_{pl} / m_{pl} = \delta l_{pl} / l_{pl} = \delta G / 2G \quad (4)$$

Consequently, any change in the estimate of the gravitational constant produces mirror-symmetric deviations of the Planck mass and the Planck length. The proposed formulas do not use G as an input quantity; instead, they rely exclusively on microphysical constants (μ , α , m_p , λ_p) and yield mutually consistent values. The complementary deviations of the CODATA values are a direct reflection of the shared uncertainty in G , whereas the values obtained from the formulas remain stable. This effect is illustrated in the figure below, with reference to CODATA 2018,

$$m_{pl} / m_{pl}(2018) \quad \text{and} \quad l_{pl} / l_{pl}(2018)$$

which enables a comparison of the behavior of the mass and length parameters throughout the CODATA history.



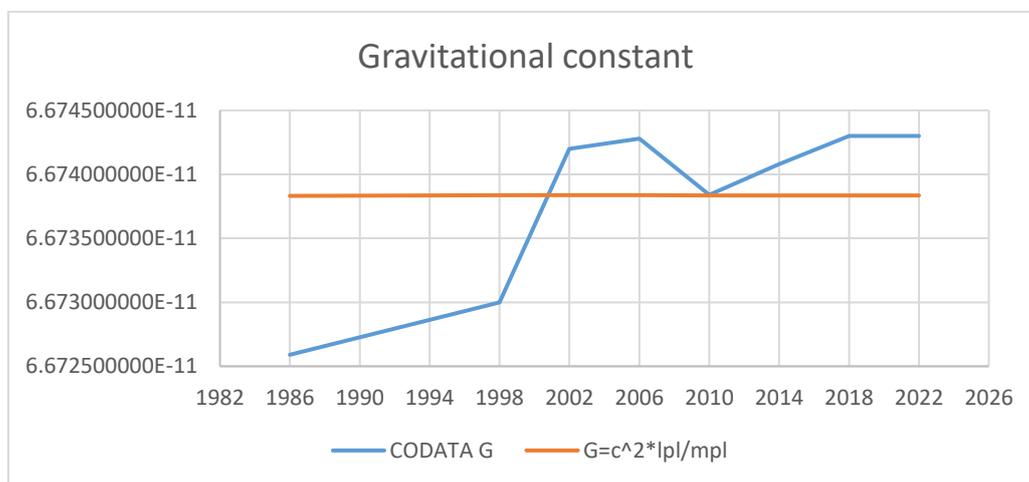
If, instead of taking the speed of light and Planck's constant as exact, the mass and wavelength of the proton were taken as exact, the theoretical uncertainty of the formula would be further reduced and limited solely by the precision of the dimensionless constants, which would be more favorable than the SI definition of the Planck quantities based on the gravitational constant. In that case, the uncertainties would be around $\sigma \approx 5 \times 10^{-14}$.

Gravitational constant

If the gravitational constant is expressed through the relation involving the Planck mass and Planck length:

$$G = c^2 * l_{pl} / m_{pl} \quad (5)$$

and using the values obtained from the corresponding formulas, a comparative chart can be produced against the experimental data throughout the history of CODATA recommendations.



The chart shows that theoretical knowledge of the relations for the Planck parameters would have provided more accurate and consistent values of the gravitational constant in the earlier period, further confirming the validity of the proposed formulas.

Conclusion

The proposed formulas, linked to the proton, highlight the significance of Planck mass and length and confirm the utility of c and proton-oriented constants.

The causes of the relatively large uncertainty of the Planck values in the CODATA reports have been shown.

REFERENCES

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