Looking Back at Quantum Mechanics via the Quantity Momentum

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By analyzing the historical treatments of quantity momentum in different models, we revisit certain fundamental problems in quantum mechanics, e.g. how to understand quantum interference and quantum scattering. Logically, by taking over the physical picture of dressing cloud surrounding hadrons and applying it to the electromagnetic field dressed by charged particles, we find a deeper understanding of the microscopic property of electromagnetic field. It implies that the amplitude of momentum could be closely related to the oscillation of electromagnetic field dressed by charged particle. Consequently the definition of canonical momentum turns out to be interaction dependent.

I. INTRODUCTION

2025 is the International Year of Quantum Science and Technology, to memorize the birth of modern quantum mechanics [1]. At this juncture the conventional understanding of quantum mechanics is yet challenged due to a few perplexing problems, for example quantum double-slit interference, the state collapse after measurement, and the nonlocal characteristics of entangled states etc.. These conundrums were mostly born from the point of view of classical respect, namely regarding quantum entities as particles, whence they must have their localized position in space-time. In contrast to this "spatial point" viewpoint, regarding quantum entities as extended fields might give rise to consistent understanding [2]. As pointed out in the paper, the "point-particle-viewpoint" violates the Hegerfeldt's theorem [3] that tells that "particles" have no way to face quantum vacuum where there are no particles. From a historical overview across quantum physics, the Hobson's paper provides a promising direction to understand above quantum puzzles. After his paper being appeared more than one decade, the related discussions continued [4].

In present paper, we solely check the roles of the quantity momentum in different quantum models to understand certain quantum puzzles. The context shall be an examples-support to the Hobson's field attitude. The momentum is an essential quantity both in particle physics and quantum physics. Here we transfer our understanding of momentum from particle physics to quantum mechanics. Meanwhile we have compared the measurements of particle physics and quantum physics, finding that they share certain common respects but actually different. As for particle physics, it just concerns about applications of quantum mechanics in explaining scattering data. The observables of interest includes momentum, scattering angles, charge, mass and other quantum numbers of final states in scattering processes. By these observables one reasons backward and obtains the information of interactions between the incident particles. While for quantum physics nowadays, it concerns about measuring observables of moving quantum states, as well as about producing entangled states and making them stable. In this paper we try to connect the conventional particle physics with conventional quantum mechanics by reviewing the aspects of quantity momentum. But we don't involve in their nowadays research streams, that deserves independent discussions for future work.

II. DE BROGLIE RELATION

The de Broglie relation $E = h\nu$, $p = h/\lambda$, which involves quantity momentum, is an appropriate starting point for our discussion. With the relation we can estimate the wavelengths of various objects and particles to get some intuitions to micro-world. For example, a bullet has a momentum of about $3.0 N \cdot s$. A simple estimate using the above de Broglie relation shows that the wavelength of the bullet is as short as $2.0 \times 10^{-34} m$. Compared with the size of the bullet, such as one centimeter long, the wavelength of the bullet is obviously smaller than its size. After similar estimations, we recognize that any other macroscopic objects, the sun, the moon, the stars, the human beings, or any stuff we use daily etc., will convince us of the same conclusion: the wavelength scales of are much smaller than the objects. Then let's turn to the electrons moving around the nucleus, such as hydrogen atom ground state. Its momentum is around $10 eV/c$, and its wavelength happens to be the order of $1 nm$, just being the scale of its perimeter of the "orbit" around the nucleus. The electron's own size is less than $1.0 \times 10^{-21} m$, obviously its wavelength exceeds its body size so much. With the above discussions, certainly we can infer how the wavelength varies with amplitude of momentum according to the de Broglie relationship. People have some intuitions on classical waves, such as water waves and sound waves. According to the intuitions we know that sound waves can bypass obstacles of a certain scale, which is the commonality of all kinds of wave. But for classical waves, such as the sound waves that we shout, and the water waves caused by throwing a pebble, there is no clearly defined momentum, neither the similar de Broglie relation.

III.   PATHS INTEGRAL

Now let's discuss the de Broglie wave of electron or proton in detail. All waves have two common properties: interference and diffraction. For example sound waves circumventing obstacles is the result of diffraction. The electron's wave is de Broglie wave, its diffractive property was verified by Thomson, Reid, Davisson and Germer immediately in 1927 [5], soon after the idea of de Broglie wave was proposed. However, the double-slit interference experiment of electron had not been confirmed by experiments until 1988 [6]. Although Jonsson claimed that he had completed the experiment in 1962 [7], since published in German it did not spread in time. That means for quite a few decades after the establishment of quantum mechanics, the double-slit interference experiment of matter waves has been a Gedanken experiment all along. This experiment is described in all textbooks of quantum mechanics, however no easy-to-understand explanation was put in the early textbooks. Nonetheless, that did not discourage certain theoretical generalizations by viewing this Gedanken experiment as fact. One famous case is the Feynman's path integral formulation in the $1940s$ [8].

The essential inference of the above Gedanken experiment is that the momentum of electron passes through both the two slits simultaneously [9], on which Feynman put a more delicate argument in his book [10]. Suppose you are observing a single electron by sending out test photons, you would know which slit each electron has passed through. However your knowledge of the slits is at the expense of that the interference fringes on the receiving screen would disappear. Because once the photon collides with the electron nearby one slit, it changes the electron's state. This experiment became a succeeding Gedanken experiment by Feynman. Feynman further assumed if a plane wave passes through two slits of a barrier simultaneously, then if opening one more slit the wave would pass through three slits simultaneously. The same shall be true if opening N slits, i.e. the wave will pass through N slits simultaneously. Extremely if the barrier is removed, the plane wave of electron will pass through each point of the imagined barrier. Imagine that the void space is full of barriers, and they were removed just now. Then if the electron propagates from point A to point B in the void space, according to above argument its paths would experience all the points in between simultaneously, and all points are equally weighted. This is Feynman's path integral method. Applying this idea to the dynamical evolution of a plane wave, after a lengthy derivation, the quantum mechanics with the form of path integral will be obtained. The path integral formalism can also lead to the Schrödinger equation under the first order perturbation approximation. The equivalence of the two methods can be proved at the quantum-mechanical level. Moreover the Path Integral Method had been proved to be a powerful tool for the later development of quantum field theory and even played an unique role in verifying renormalizability for non-Abelian field theories.

It is worth mentioning that the phase $iS/\hbar$ in the factor $e^{iS/\hbar}$ of the path integral, since the Planck constant $\hbar$ of the denominator is very small, the ratio of the usual action S over it becomes very large. In the region far from the classical paths, the exponential function in the path integral contributes null since it oscillates extremely rapid and paths in the neighborhood cancel each other. That is, although the waves of electrons are assumed to pass through all paths equally, the paths really contributing to the phase factor are those concentrated in a small "eggplant-shaped" region along the classical path. That actually produces likewise Gaussian integral. From this experience, in practical calculations one does not need to integrate over all paths, it is sufficient to integrate over merely a limited range of paths. To speak plainly, it is sufficient to consider paths' region of one or two wavelengths. By such construction and considering Heisenberg's uncertainty principle we can infer that microscopic particles are waves whose momentum is not concentrated in one point along the path, but rather extensive/diffusive approximately within one or two wavelengths perpendicular to the "path" of propagation. With this recognition we can now return to the analysis of the eigenfunctions of momentum and their Hermitian properties.

IV.   SCATTERING

The eigenfunction of momentum is written as $\frac{1}{\sqrt{2\pi}}e^{ipx/\hbar}$, and its normalization can only be cast into the form of a delta function $\delta(x - x')$; according to its mathematical definition, plane wave spreads throughout the entire space region rather than a limited area. Practically we rarely need such "pure" plane wave, actually that is mathematical wave. And the proof of its Hermitian property of momentum cannot be realized using partial integrals with the eigen form of plane waves [11]. The obstacle is that the delta function would be generated, due to that we have assumed a plane wave being spatially extensive to infinity. Thus according to uncertainty principle ($\triangle x \cdot \triangle p \sim \hbar$, $\triangle x \to \infty$, $\triangle p \to 0$) the momentum of the electron is naturally having a fixed value. Such defined momentum we would call it here "mathematical momentum". From this point of view, Feynman's path integral unveils partially the essence of momentum, which allows us to understand it deeper: The momentum of a micro-particle might have an amplitude, which is diffusive/extensive within a limited extent (its value and direction). This conclusion is consistent with the aforementioned experiment of double-slit interference, i.e. the momentum of an electron passes through both of the two slits of barrier simultaneously. The "plane" of a plane wave, at least theoretically, doesn't necessarily extend to

infinity, it spreads over merely a finite region, for instance over one or two wavelength.

One can understand the momentum property further by collisions in high-energy physics [12], whence the maximal value of particles' momentum has to be cut in calculating the corrections of cross-section. That is, the wavelength of microscopic particles has to be set a minimal value according to the concrete scattering process, so as to avoid divergence in calculating the high-order perturbations. Now let's turn to scattering events in which electrons are involved. As discussed above, interference and diffraction are the commonalities of all waves; a more intuitive expression of these two properties is "circumventing" while encountering obstacles, i.e. a wave can bypass an obstacle and continue to propagate when the size of the obstacle is comparable with or smaller than the wavelength. When electron collides with atoms, the effect varies with the energy. When the wavelength of electron is comparable with atomic scale, the electron may diffract via the seams between atoms, that is, certain kind of "circumventing" effect occurs, and no hard "collision" actually happens; The initial confirmations of existence of de Broglie wave (electron with about $10eV$) falls into this category [5]. As the energy increases to about $100eV$ (much greater than the binding energy of electrons in atomic molecules), the wavelength of electron becomes shorter and thus cannot circumvent, then electron would be scattered away by electrons in the orbits. We know in most cases scattering and diffraction occur simultaneously. Next, if increase the electron energy moreover to $1MeV$, then wavelength shall continue to shorten, and the incident electron can no longer "see" the electrons in orbits and can travel freely among orbits. Although now the wavelength is yet a bit larger comparing with nucleus, it can scatter and diffract with the nucleus (which might involve both protons and neutrons). Next continue to increase electron energy, and when the energy reaches around $10MeV$, the electron can finally "see" the nucleus and could go through the interstices between protons or neutrons. Furthermore, the energy continues to increase to about $100MeV$, while the protons or neutrons cannot be circumvented and would be knocked out of the nucleus to be free. Continue to increase the energy of electron to $1GeV = 1000MeV$, the electron can break the proton/neutron and produce many final state particles. The proton is composed of three quarks, and there are interstices among quarks. While the energy of the electron continues to increase greatly, the wavelength is shortened more and more. Penetrating into the interstices, it pries the quarks out of nucleons, whereby finally form new mesons, baryons, etc. To increase the energy of the electron furthermore, it would collide/scatter with the quarks in nucleon. Through the measurement of the final-state particles in specific experiments, we can infer reversely what reactions happen for certain energy scales. In summary, electrons with different energies (momentum) see different scenes.

In above discussions, we just employ the properties of de Broglie wave to explain the effects of "circumventing" or "prying out" the structure of the target particle, without considering the interaction details between the incident particles and the target particles, such as electromagnetic interaction between electron and proton. The electromagnetic potential between electron and proton is roughly in the well-known form proportional to $\frac{1}{r}$. When an electron carries very large momentum and collides heading on the proton, according to the form of the potential energy, the electromagnetic attraction seems tending to infinity. The fact is that we have never seen an incident electron colliding with a proton in experiments then releasing infinite kinetic energy. Though quantum field theory provides a mature method to calculate scattering processes to avoid infinity with which the leading-order scattering amplitude is not divergent, the divergence appears when calculating the next-leading-order contribution. It roots certainly in the potential $\frac{1}{r}$. Now the conventional approach to dealing with divergence relies on truncation, namely cutoff, i.e. integral contributions with momentum greater than a certain value shall not be involved. To speak alternatively, when de Broglie wavelength is smaller than a certain value, the contribution becomes unphysical.

Why is it unphysical? The conventional quantum field theory didn't give an explanation. In fact, the unphysical scale is determined by both of the wavelength of the incident particle and the "size" of the stationary target particle. If the target particle is assumed to be stationary, then when the wavelength of incident particle is smaller than the radius of target particle to a certain ratio, the target particle might be broken apart to fragments according to the above discussions. Then such scattering is no longer elastic scattering, and the next smaller reaction scale is involved. Therefore, for elastic scattering at certain energy scale, the wavelength shall have a minimal value. Correspondingly, the momentum has a maximal value. At this minimal wavelength scale, the effect of "circumventing" occurs, which is also responsible for avoiding certain divergence caused by $\frac{1}{r}$ potential. Textbooks teach us to calculate renormalization, but rarely indicate that the physics is just "circumventing".

In the aforementioned scattering, the target particles are assumed to be sizable, i.e. having a certain shape, such as protons. Then what if the two colliding particles in scattering process are named non-splitable "point" particles? For example, in the scattering between electron and positron. They attract each other with opposite charges, the closer the distance in between them is, the greater their attraction is. Then is it going to release infinite kinetic energy in such scattering? Experiments have confirmed that even if they annihilate each other, it will not release infinite energy. According to the current experimental results, the electron/positron has no internal structure and is a pure "point" particle with charge. An electron/positron carries several physical quantities that characterize its identity: mass, charge, and spin. Because electron/positron also participates in weak interaction, actually it also carries its weak hypercharge. Here, for the sake of simplicity the hypercharge will not be considered here. We will now mainly

analyze the charge of electrons: how do we know that a particle has charge? The answer is by measurement using another charge, namely the test charge. Then how does the test charge feel the charge of an electron? Certainly, via the electric field around the electron. To speak in classic terminology, it is through virtual photon field around the electron. In addition, according to quantum field theory and modern high-energy experiments, two bare charges never touch each other, the only way they sense each other is by virtual field. Therefore, an electron exists as an inseparable entirety together with its mass, charge, and virtual field around. One noticed that the electron spin would also give rise to tiny magnetic field, which we ignore them here. The virtual fields are spatially extended and diffusive within a certain range. So, the physical electron shall not be a point, but a "bulk", albeit very small. Will this "bulk" be felt by de Broglie wave of the positron in collision? When the wavelength of the positron is small enough to be comparable to the size of the "bulk", will the positron "circumvent"? The answers are all yes. Let's first explore the general properties of "bulk", and then answer why yes.

V.   WAVE AND PARTICLE FORMING AN ENTIRETY, NOT DUALITY

Let's first give a rapid answer to above questions by a thought experiment in this paragragh. Considering a hydrogen atom, the volume occupied by the "electron cloud" (or say, hydrogen scale) is about 10^6 times larger than that of the atomic nucleus. Naturally, the "electron cloud" is composed of the electron itself and the field lines dressed around it. A direct inference is that most matter on the earth is essentially supported by "electron clouds". Now, let us assume that if the nucleus of hydrogen is removed, the electron then becomes a free electron with its original average kinetic energy. The question here then is: would the electron collapse into a point with just its charge and mass? Clearly not—instead it would occupy the similar spatial volume as it does around the nucleus. If a particle with a wavelength scale comparable to this spatial volume———i.e. corresponding to a kinetic energy of about 10 eV———were incident upon the electron, the particle would evidently be diffracted away the region, the same as it were diffracted by the hydrogen atom. Then the situation would remain the same if the incident particle were a positron.

Now let's analyse the concept of "bulk" in the previous paragraph concisely. Actually the concept is common for microscopic particles. In hadron physics, we also call such "bulk" as "cloud", i.e. "dressing cloud". So in terms of hadron physics, we name the "bulk" around the electron a dressing cloud. At low energy scale, there is only photon cloud around electron. For proton or neutron, their peripheries are also photon cloud, in addition there are π meson clouds etc. in more inner parts. Take proton as an example, the surrounding cloud types will increase with energy. Aside from photon cloud, there are π meson clouds, ρ meson clouds, gluon clouds, etc., and these cloud scales will become smaller in order successively, just as onion-shell structure (This is by no means an accurate expression, just metaphor.). Here we are concerned about only the "bulk" formed by the electromagnetic field (i.e., the photon cloud) around charges. Let's go back to the "bulk" of electrons/positrons. To understand such "bulk" in depth is to understand the microscopic properties of the electromagnetic field around electron.

For the properties of classical electromagnetic fields, they are completely described by Maxwell's equations. It is also given the equations of motion of classical charges in electromagnetic fields, that is, Newton's equations involving the Lorentz force. From the above knowledge we can infer that classical charges would radiate when accelerated. But why does the periodic motion of the electrons in the hydrogen atom not radiate? Posing this question at the time at the beginning of the last century showed that the scientific community did not understand the microscopic properties of the electromagnetic field around electric charge. To resolve this problem, scientists constructed the theory of quantum mechanics, which can explain all of the experimental results, and it is of course the reliable theory. So, we should employ quantum mechanics to understand the microscopic properties of the electromagnetic field around electric charge. Taking a hydrogen state as an example, the explanation in the quantum mechanics textbook establishes a mapping between the set of most probable spatial coordinates (namely the orbital) for electron and the corresponding state function. These spatial coordinates are regarded in priori as the positions of the charge points. With the previous discussions of measuring charge, we conclude that one observes the existence of a charge by sensing its electric field via a test charge. So we infer immediately that the most probable spatial region of a hydrogen state is the most dense region of the virtual photon fields. Then let's recall Sommerfeld-Wilson quantization condition, $\oint p \cdot dq = n\hbar$, which is equivalent to the angular momentum quantization condition. This condition implies a "default" integral path. Obviously, this path lies in the dense region of virtual photon field. Moreover, concerning that de Broglie once employed the standing matter wave of an electron along a ring to explain the above quantization condition [13], we summarize that the condition seemingly implies the oscillation of the dressed photon fields. To speak strictly, such oscillation is the only way to meet the wave hypothesis of de Broglie. An ideal quantum wave should has its oscillations ubiquitously (covering all spacetime), only the photon fields around electron charge behave in such manner could electron become matter wave. We put in such context that we regard the electron's mass, charge, spin and its dressed fields as a entirety, named electron. Furthermore, de Broglie-relation suggests that there

exist a relationship between such oscillation and momentum, and thus angular momentum as well. To summarize: the "bulk" is made of virtual field, which keeps oscillating, and the oscillation shall be certainly related with momentum.

The conclusions of the above paragraph can be reached exclusively via historically retrospective analysis, since even the probabilistic interpretation of the wave function was not considered important at the beginning years of quantum theory. It was impossible at the time for people to reach such micro-properties of electromagnetic field. On the other hand, although quantum mechanics proposes de Broglie waves, it does not clearly or accurately define what the physics of de Broglie wave is. The term "probability amplitude" for wave function proved to be abstract for all beginners. In contrast the original terminology of "matter wave" shall be closer to physics, i.e. wave features displayed by microscopic massive particles. The essential quantity in the Schrödinger equation representing the de Broglie wave/(matter wave) is the wave function Ψ. We know that the interference and diffraction of matter wave can be derived from the wave function Ψ. According to the nature of wave, matter wave Ψ can "sense" the size of an obstacle and "circumvent" it, e.g. a low-energy electron bypassing a proton (or another electron). This "circumventing" can even occur in the scattering of two "point" charges, such as between an electron and a positron. Because their scattering result does not produce infinite energy. This suggests that the "bulk" made of photon fields around the "point" charge works effectively and can be sensed by positron wave Ψ. It will not lose generality to assume that the two entities, the wave represented by the wave function Ψ and the "bulk" (another microscopic charge together with its cloud) of the obstacle, are on the same foot, whose roles of being wave or being "bulk" could be exchange mutually. The positions of "being circumvented" and "circumventing" can be interchanged, similar to the change of two coordinates for two objects. With such recognition we conclude that the electron's de Broglie wave is the same as "bulk", being made of the photon virtual fields (photon cloud), which keep oscillating around electron. So for point charges "photon virtual fields" exist to cause interaction. But for "bulks", the "photon virtual fields" exist to cause wave diffraction effects. The above conclusion is implied but never explicitly stressed in quantum mechanics. As byproduct, we have answered the question again how electron and positron circumvent each other in their scattering.

VI. THE ESSENCE OF MOMENTUM

The above understanding is a new recognition rather than a new explanation, and the mathematical language shall be completely the same as conventional quantum mechanics. In summary, we have given more essence to the virtual photon fields (photon cloud). The photon cloud provides the oscillation required for the wave on the one hand, as well as the interaction on the other hand. To speak alternatively we endow the wave Ψ with physics: the wave is caused by the oscillation of the virtual photon fields around charge, which is vulnerable under measurement. For microscopic particles, de Broglie wavelength is closely related to momentum by $E = h\nu$, $p = h/\lambda$. Combining the arguments in the previous paragraphs, we can infer further: Momentum shall be the characterization of the oscillating amplitude of the virtual photon fields, which is an observable; and the wave function Ψ is the characterization of the wave effect generated by the oscillation, which can be measured by interference and diffraction. So far, we have almost understood the microscopic properties of electromagnetic fields dressed by electric charges. Momentum and de Broglie waves are associated with this physical picture. The expression for this association is the de Broglie relation. When we consider the Schrödinger equation for an electron to move in an external magnetic field, we notice the indistinguishability of electromagnetic momentum $e\vec{A}$ and particle's classical momentum. They give rise to the canonical momentum $\vec{P} = m\vec{v} + e\vec{A}$, which turns out to be an observable, whereas classical momentum $m\vec{v}$ is no longer an observable. The physics of canonical momentum is consistent with the aforementioned picture that virtual-field-oscillation is related to quantum momentum, and its significance shall not be limited to the topic here. Interested readers can study in depth by applying the canonical momentum to other micro-interactions.

Now let's turn to Feynman's Gedanken experiment to see what happens when a photon "watching" the slits in the electron double-slit experiment: originally Feynman explained the process in terms of interaction, now we can also explain it in terms of wave. If explained by interaction, this process is equivalent to adding an external electromagnetic field to an electron while it's passing through the slits, and this external field interacts with the charge of the electron, disturbing the original "paths"(states) of the charge. If explained by wave, then because the external photon field disturbs the original oscillation modes of the photon cloud (virtual fields surrounding electrons), consequently varies the direction of group velocity of the electron wave passing through the two slits, finally the interference fringes disappear. Certainly, it also varies the direction of the momentum effectively. It should be emphasized here that an electron momentum passes through both slits simultaneously, concomitant with its "cloud".

The above three equivalent concepts, de Broglie wave, the microstructure of electromagnetic field around electron, and dressing cloud, are all described by $\Psi(x)$ in the Schrödinger equation. They are not quantum observables. However, the observable effects of the three quantities can be reflected by the properties of momentum (such as quantization conditions, renormalization scale, etc.) or wave properties such as interference and diffraction (which

can also be attributed to the direction of the momentum). Momentum can be viewed as representative observable of these three concepts. According to the experimental results of high energy physics, taking the proton as an example, the surrounding clouds will increase with the energy. Except for the photon cloud, the π meson cloud, the ρ meson cloud, and the gluon cloud will appear successively, and various cloud scales are different, smaller and smaller. And these π meson clouds, ρ meson clouds, gluon clouds, etc. represent other interaction fields, and also represent de Broglie waves at different microscopic scales. Although we express the de Broglie waves at different scales using the same formalism of momentum, the physics of these momentums has changed. Because the virtual fields represented by the momentum have changed. Therefore, when the interaction of a new scale occurs, the dynamic momentum and the interaction potential are coupled into the canonical momentum of this scale, then the canonical momentum and the potential field are written as the covariant differential operator $D_\mu = \partial_\mu - gA_\mu$. So far, we conclude that the concept of quantum momentum is closely related to interactions, and the connotation of quantum momentum itself is not independent of interactions.

VII.   SUMMARY AND DISCUSSIONS

This paper starts from the de Broglie relation, which relates two quantities momentum and wavelength. We look back at quantum mechanics from the perspective of hadron physics. Summarizing a few different applications of momentum operators in models, it indicates that the application of ideal momentum (mathematical momentum) is practically limited. Mostly it is constrained by certain conditions, i.e. it never ever runs over whole momentum space. These constraints on momentum allow us to further analyze the deeper meaning of wave-particle duality: "point" particle (charge) together with its surrounding waves form an inseparable entirety, which appears as a cloudy "bulk" instead of a point. And the momentum amplitude relates to the oscillation of this cloud bulk, suggested by it's de Broglie relation. More generally, there shall be no zero-dimensional "point" in the true physical world [14, 15], the "point" is born from math and makes sense in math. After understanding quantum mechanics to this level, many relevant puzzles would become clear.

In addition, although quantum mechanics was summarized solely from phenomena relating to electromagnetic interaction (field), principles of quantum physics are originally thought to be independent of interactions, i.e. they are general. However after reviewing the particle physics and looking backward, logically and strictly, if the quantum principles are supposed to be universal for all interactions (different fields), then certain experimental tests shall be called for. Therefore in order to be tested, designing experiments from other microscopic interactions become interesting and meaningful. Such investigation might also be helpful for our deeper understanding of quantum puzzles.

VIII.   ACKNOWLEDGEMENTS

[1] https://quantum2025.org

[2] A. Hobson, Am. J. Phys. 81, 211 (2013).

[3] Gerhard C. Hegerfeldt, Annalen der Physik 7, 716-725 (1998); "Remark on causality and particle localization," PRD 10 (1974), 3320-3321.

[4] Charles T. Sebens, Synthese (2022)200:380

[5] C. Davisson and L. H. Germer Phys. Rev. 30, 705(1927); G. P. Thomson, A. Reid,(1927). Nature. 119, 890(1927).

[6] A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, and H. Ezawa, Am. J. Phys. 57, 117 (1989);

[7] C. Jonsson, Am. J. Phys. 42, 4 (1974).

[8] R. P. Feynman, The space-Time approach to non-relativistic quantum mechanics, Rev. Mod. Phys.,367, 1948.

[9] P. A. M. Dirac, The Principles of Quantum Mechanics, Oxford, New York, 4th edition(1958).

[10] R. P. Feynman, Quantum mechanics and Path Integrals, 1965 by McGraw-Hill, Inc.

[11] Leonard I. Schiff, Quantum Mechanics, third edition, McGraw-Hill Inc. (1968).

[12] Roger G. Newton, Scattering theory of waves and particles, second edition, @1966, 1982 by Springer-Verlag New York, Inc.

[13] Jammer, Max (1989). The conceptual development of quantum mechanics. The history of modern physics (2nd ed.). Los Angeles (Calif.): Thomas publishers. ISBN 978-0-88318-617-6. De Broglie standing wave.

[14] Hai-Jun Wang, J. Math. Phys. 49, 033513 (2008)
[15] Hai-Jun Wang, J. Math. Phys. 52, 033510 (2011)